論 文 の 内 容 の 要 旨

論文題目　Combinatorial Optimization on Group-Labeled Graphs
（群ラベル付きグラフにおける組合せ最適化）

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The maximum matching and disjoint paths problem have been studied as central topics in combinatorial optimization since its beginning at the middle of the 20th century. Through the research in these topics, various useful concepts and techniques have been developed such as good characterizations by combinatorial duality and efficient algorithms using augmenting paths, which are utilized even today.

One of the highlights on these two topics is Mader's theorem (1978) for openly disjoint A-paths, where an A-path is a simple path between two vertices in a prescribed vertex subset A. He gave a good characterization by a min-max duality for the maximum number of openly disjoint A-paths in an undirected graph, which commonly generalizes the Tutte–Berge formula for maximum matchings in non-bipartite graphs and Menger's theorem for the maximum number of disjoint s–t paths.

To the problem of finding a maximum number of openly disjoint A-paths, Lovász (1980) first provided a solution by reducing it to the matroid matching problem. The matroid matching problem unifies two tractable generalizations of the maximum matching problem in bipartite graphs (the maximum matching problem in non-bipartite graphs and the matroid intersection problem), but itself is intractable in general. It was however impressive that Mader's problem, which also unifies two generalizations of bipartite matching (non-bipartite matching and disjoint s–t paths), can be solved via matroid matching.

As a further extension of Mader's problem, Chudnovsky, Geelen, Gerards, Goddyn, Lohman, and Seymour (2006) introduced the problem of finding disjoint "non-zero" A-paths in group-labeled graphs, which also includes some interesting problems in topological graph theory. They showed a min-max duality extending Mader's theorem, and later Chudnovsky, Cunningham, and Geelen (2008) proposed a polynomial-time combinatorial algorithm for this problem. Pap (2006–2008) introduced a slightly more general model, and also suggested a border between disjoint A-paths problems that enjoy nice structure (e.g., good characterizations and efficient algorithms) and those who do not.

One of the main topics of this thesis is to analyze these extended problems via matroid matching, and to clarify the border between tractable and intractable problems. We show that the most general setting among those suspected as tractable reduces to the matroid matching problem, and clarify when the reduced problem has nice structure such as tractability, good characterizations, and linear representations of reduced matroids.

Other than openly disjoint paths and topological conditions, various settings on graphs can be simply formulated using group-labeled graphs. For instance, a variety of NP-hard problems such as the Hamiltonian path problem and the k-disjoint paths problem can be formulated as the problem of finding a path of a designated label in a group-labeled graph. This fact implies that combinatorial optimization on group-labeled graphs is pretty challenging even if we focus on just one path, e.g., to determine whether a given group-labeled graph contains an s–t path whose label is in a prescribed subset of the underlying group.

As the first nontrivial step in this direction, since the situation forbidding only one label is quite easy, we investigate the problem of finding an s–t path with two labels forbidden. This problem in fact includes the 2-disjoint paths problem in undirected graphs, which was well-studied in 1980s. Inspired by and with the aid of Seymour's characterization (1980) for 2-disjoint paths, we give the first nontrivial characterization of group-labeled graphs in terms of the possible labels of s–t paths. Moreover, based on our characterization, we provide an efficient algorithm for finding an s–t path with two labels forbidden.