中性K中間子の二個のミュオンへの崩壊の分岐比の測定

小松原　健

Measurement of the Branching Ratio of the Decay of the Long-Lived Neutral Kaon into Two Muons

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Abstract

We have measured the branching ratio of the decay $K_L^0 \to \mu^+\mu^-$, a rare decay mode caused by the flavor-changing neutral current, at the neutral beamline (K0) of the KEK 12-GeV Proton Synchrotron. The experimental setup of KEK E137 experiment comprising a neutral beam followed by a vacuum decay chamber and a double-arm double-stage spectrometer with particle-identification capability was used. The recorded events were reconstructed by the tracking programs and were checked by the particle identification routine and selection criteria. 179 $K_L^0 \to \mu^+\mu^-$ events were observed in the fiducial region. The $K_L^0 \to \pi^+\pi^-$ decay events were accumulated simultaneously for normalization, and various distributions of these events (for example, the invariant-mass distribution with a resolution of 1.28 MeV/c^2) were well reproduced by a Monte Carlo calculation. The sensitivity was determined by the number of $K_L^0 \to \pi^+\pi^-$ events, the acceptance ratio from a Monte Carlo calculation, the particle identification efficiencies from the data sample, and other corrections, and it was 4.45 $\times$ 10^{-11}. The number of $K_L^0 \to \mu^+\mu^-$ events was 177.8 after the background subtraction of 1.2 events, and the result of this experiment is

$$B.R.(K_L^0 \to \mu^+\mu^-) \;=\; (7.9 \pm 0.6 \pm 0.3) \times 10^{-9} \;,$$

where the first uncertainty (7.5 %) is the *statistical* error and the second one (4 %) is the *systematic* error. This value is above the theoretical lower bound (the unitarity limit), consistent with the previous result of our experiment, and also consistent with the recent result of the BNL E791 experiment. In the Standard Model the $K_L^0 \to \mu^+\mu^-$ branching ratio gives constraints on the top-quark mass and the Kobayashi-Maskawa matrix parameter ρ. Using a theoretical model, the above result indicates that the top-quark mass is less than 310 GeV/c^2 and the heavier the mass of the top quark is the more important lower bounds are given on ρ.

Contents

Chapter 1

Introduction

1.1 Survey of the Past Experiments and Present Status

The decay of the long-lived neutral kaon into two muons, $K_L^0 \rightarrow \mu^+\mu^-$, is a flavor-changing neutral current (FCNC) process, which is allowed but highly suppressed in the Standard Model. This decay occurs via the second-order electroweak interaction, and the branching ratio, $B.R.(K_L^0 \rightarrow \mu^+\mu^-)$, is of the order of 10^{-9}. It is one of the so-called *rare* decay modes, the study of which provides useful ground for the test of the Standard Model and several theoretical models which attempt to extent the Standard Model.

Historically speaking, the $K_L^0 \rightarrow \mu^+\mu^-$ decay played an important role in the development of the Standard Model, especially in the determination of the quark structure of the electroweak interaction, in the late 1960s and early 1970s. By 1970 several experiments [1,2,3,4] had been done but had not been able to observe a positive signal for this decay at the sensitivity of the order of 10^{-8}. To explain the smallness of the branching ratio for the $K_L^0 \rightarrow \mu^+\mu^-$ decay relative to that for the flavor-changing *charged* current process $K^+ \rightarrow \mu^+\nu$, a suppression mechanism for the FCNC was introduced by Glashow, Iliopoulos, and Maiani in 1970 (GIM mechanism [5]), and the necessity for the existence of a fourth-quark was postulated before the discovery of the charm quark in 1974 [6].

$B.R.(K_L^0 \to \mu^+\mu^-)$ has a theoretical *lower* bound, often referred to as the *unitarity limit*, of $(6.83 \pm 0.32) \times 10^{-9}$ (see the next section). In 1971 an experiment reported at 90% confidence level an *upper* limit of 1.82×10^{-9} [7], a value significantly below this lower bound. However, another experiment found 9 $K_L^0 \to \mu^+\mu^-$ events corresponding to a branching ratio of $12^{+8}_{-4} \times 10^{-9}$ in 1973 [8], and this result was confirmed by two other experiments (3 events corresponding to $8.8^{+10.7}_{-5.5} \times 10^{-9}$ in 1976 [9], and 16 events corresponding to $8.1^{+2.8}_{-1.7} \times 10^{-9}$ in 1979 [10]). These three experiments are all that had been done for the measurement of $B.R.(K_L^0 \to \mu^+\mu^-)$ until the mid 1980s, and the averaged branching ratio was $9.5^{+2.4}_{-1.5} \times 10^{-9}$ [11]. The experimental error was dominated by the *statistical* error because the number of the observed events was small. It was difficult to extract physics information from this value.

Stimulated by the recent developments of experimental techniques, several new experiments were proposed in the mid 1980s to search for rare kaon decay modes with higher sensitivities than previously achieved. The decay $K_L^0 \to \mu e$, which is a lepton-flavor changing process and forbidden in the Standard Model, is a typical example of rare decay modes. A positive signal for this decay implies the existence of a new physics beyond the Standard Model. Three experiments to search for the $K_L^0 \to \mu e$ decay were performed : E137 at the National Laboratory for High Energy Physics (KEK), E780 and E791 at Brookhaven National Laboratory (BNL). The sensitivity of E780 was 10^{-9}, but E137 and E791 aimed to reach the sensitivity of 10^{-11}. The $K_L^0 \to \mu^+\mu^-$ decay and the $K_L^0 \to ee$ decay, which is also a FCNC process but much more suppressed than the $K_L^0 \to \mu^+\mu^-$ decay (see the next section), are kinematically similar to the $K_L^0 \to \mu e$ decay. Therefore, these three modes are collected simultaneously using the same detector for the $K_L^0 \to \mu e$ experiment. A few hundred of $K_L^0 \to \mu^+\mu^-$ events can be collected with the sensitivity of 10^{-11} for the $K_L^0 \to \mu e$ decay, and it is possible to make a precise measurement of $B.R.(K_L^0 \to \mu^+\mu^-)$.

E780 [12] reported their final result in 1989 : an upper limit of 1.9×10^{-9} for the $K_L^0 \to \mu e$ decay and 8 $K_L^0 \to \mu^+\mu^-$ events.[1] E137 reported their first result in

[1] They did not calculate the branching ratio.

1989 [13] based on about one third of their data sample. The upper limit for the $K_L^0 \to \mu e$ decay was 4.3×10^{-10} , and $B.R.(K_L^0 \to \mu^+\mu^-)$ was $(8.4 \pm 1.1) \times 10^{-9}$ based on 54 $K_L^0 \to \mu^+\mu^-$ events. The error was statistical only and the value was consistent with the previous values. E791 reported their result in 1989 [14], and the upper limit for the $K_L^0 \to \mu e$ decay was 2.2×10^{-10} . They observed 87 $K_L^0 \to \mu^+\mu^-$ events and obtained the branching ratio $B.R.(K_L^0 \to \mu^+\mu^-) = (5.8 \pm 0.6(stat.) \pm 0.4(syst.)) \times 10^{-9}$, which was not consistent with the first result of E137 and was lower than the unitarity limit. Recently they reported the result based on their new data [15] of 281 $K_L^0 \to \mu^+\mu^-$ events and $B.R.(K_L^0 \to \mu^+\mu^-) = (7.6 \pm 0.5(stat.) \pm 0.4(syst.)) \times 10^{-9}$. When combined with the previous 87 events this result yields $(7.0 \pm 0.5) \times 10^{-9}$. This average value is consistent with the unitarity limit.

Table A.1 lists the results of all these experiments.

In this thesis I will describe the method and result of the measurement of $B.R.(K_L^0 \to \mu^+\mu^-)$ based on the entire data sample of KEK E137 experiment, which was completed in May 1990. The motivation of this experiment was to measure the value of $B.R.(K_L^0 \to \mu^+\mu^-)$ as precisely as possible so as to be able to shed light on the mechanism of the $K_L^0 \to \mu^+\mu^-$ decay.

1.2 Theoretical Background

The leading diagrams for the $K_L^0 \to \mu^+\mu^-$ decay are shown in Fig. 1.1. These are the diagram involving the exchange of two photons (2γ), the Z^0 exchange diagrams, and the W box diagram.

The 2γ exchange diagram contributes to both the real part (when either of the two photons is not on mass shell) and the imaginary part (when both photons are on mass shell) of the decay amplitude. On the other hand, the Z^0 exchange diagrams and the W box diagram contribute only to the real part, because the gauge bosons in these diagrams are not on mass shell. Hence the imaginary part of the amplitude can be estimated [16] by the product of the decay amplitude of the decay $K_L^0 \to \gamma\gamma$ and the $\gamma\gamma \to \mu^+\mu^-$ amplitude. The former is known experimentally and the latter is calculated reliably from QED. This estimation of the imaginary part provides the theoretical lower bound for $B.R.(K_L^0 \to \mu^+\mu^-)$ [17], called the ***unitarity limit*** :

$$\frac{B.R.(K_L^0 \to \mu^+\mu^-)}{B.R.(K_L^0 \to \gamma\gamma)} \geq \frac{1}{2}\alpha^2 \left(\frac{m_\mu}{M_K}\right)^2 \frac{1}{\beta}\left[\log\left(\frac{1+\beta}{1-\beta}\right)\right]^2$$

where m_μ is the mass of the muon, M_K is the mass of the kaon, α is the fine structure constant, and β is the velocity of the muon in the rest frame of the kaon :

$$\beta = \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} = 0.905 \ .$$

Using the branching ratio for the $K_L^0 \to \gamma\gamma$ decay $B.R.(K_L^0 \to \gamma\gamma) = (5.70 \pm 0.27) \times 10^{-4}$ [18], the unitarity limit for the $K_L^0 \to \mu^+\mu^-$ decay is $(6.83 \pm 0.32) \times 10^{-9}$. The other processes that may interfere with the imaginary part change this value very little [19], and it is difficult to explain the branching ratio lower than this bound without new particles or interactions [20]. The fact that the experimental results of $B.R.(K_L^0 \to \mu^+\mu^-)$ are of the same order as that of the unitarity limit suggests that the main contribution to the $K_L^0 \to \mu^+\mu^-$ decay comes from the 2γ exchange diagram.

As for the $K_L^0 \to ee$ decay, due to the smallness of the electron mass the unitarity limit is $(3.0 \pm 0.1) \times 10^{-12}$, much less than that for the $K_L^0 \to \mu^+\mu^-$ decay. This value does not contradict the experimental upper bound for the $K_L^0 \to ee$ decay branching ratio previously achieved [18].

The Z^0 exchange diagrams and W box diagram are loop-induced processes, and they are calculable in the Standard Model. There are two unknown parameters in these diagrams, the mass of the top quark and the mixing between the top and down quarks in the charged weak currents. The precise measurement of $B.R.(K_L^0 \to \mu^+\mu^-)$ provides a constraint on these Standard Model parameters. To do so it is necessary to extract the contributions of these diagrams from the real part contribution, which is equal to the excess of the branching ratio over the unitarity limit. Details of theoretical calculations will be described and discussed in Chapter 6.

1.3 Requirements for the Experiment

For a precise measurement of the branching ratio of the order of 10^{-9} there are several points that have to be considered.

To reduce the statistical error it is essential to collect as many $K_L^0 \to \mu^+\mu^-$ events as possible in a limited time. A neutral beam line which is made of a high intensity primary proton beam with an efficient duty cycle and a large solid angle for extracting the K_L^0 beam from the target is needed. With a long decay region and a large acceptance detection apparatus, a high intensity beam leads to a high counting-rate environment for the detectors. It is necessary to have:

- Finely segmented and fast response detectors,
- Electronics modules with excellent timing resolution, and
- A fast and effective trigger scheme.

The number of triggers and the data size are expected to be large, and it is desired to have:

- A filtering system to reject useless events, and
- A fast online data-taking system.

The major source of the background for the $K_L^0 \to \mu^+\mu^-$ decay is the $K_L^0 \to \pi\mu\nu$ ($K_{\mu3}$) decay with the pion decaying in flight to or being misidentified as a muon. Neglecting the mass resolution with well-measured π, μ tracks, the reconstructed $\mu^+\mu^-$ invariant mass comes up to $\sim 9\,MeV/c^2$ below the kaon mass. When the pion decays in the spectrometer or the track is falsely reconstructed, the invariant mass may be at or above the kaon mass.[2] To reject these background events, the experiment has to be equipped with :

- An excellent mass-resolution spectrometer, and
- Two magnets for the redundant measurement of momentum.

[2] Background for the $K_L^0 \to \mu^+\mu^-$ decay will be discussed in Section 5.7.

Another background comes from the $K_L^0 \to \pi e \nu$ (K_{e3}) decay with both the pion and electron being misidentified as muons. In this case a wrong-assignment of the electron mass to the muon mass makes the $\mu^+\mu^-$ invariant mass larger than the kaon mass. To reduce this background it is important to have

- Good particle identification detectors

not only for the muon but also for the electron.

Background events caused by an accidental overlap of two $K_{\mu 3}$ decays can be reduced by the timing requirements and offline selection criteria such as the minimum distance of two tracks. But neutrons and photons, an enormous amount of which remains in the neutral beam and interact with the material in the path of the beam, may produce events that mimic kaon decays, but simulation of this effect is difficult. In addition, these interactions produce many secondary particles that increase the counting rate of the detectors and make the reconstruction of real K_L^0 decay events difficult. To reduce these difficulties,

- A reduction in the number of neutrons and photons,
- A careful collimation of the neutral beam, and
- Evacuation of the beam line and decay chamber

are indispensable.

The sensitivity for the $K_L^0 \to \mu^+\mu^-$ decay is obtained from accumulating the $K_L^0 \to \pi^+\pi^-$ decay simultaneously. The $K_L^0 \to \pi^+\pi^-$ decay is the famous CP-violating process and kinematically similar to the $K_L^0 \to \mu^+\mu^-$ decay. The branching ratio for the $K_L^0 \to \pi^+\pi^-$ decay ($\sim 0.2\%$) is large enough, and this mode can be used not only for the normalization of the K_L^0 flux but also for the monitoring of the spectrometer performance. To reduce the systematic error it is important to estimate this sensitivity factor without any bias. The differences between the $K_L^0 \to \mu^+\mu^-$ events and the $K_L^0 \to \pi^+\pi^-$ events must be estimated carefully.

Taking these requirements into account the experimental setup of KEK E137 was designed and constructed. It is described in the next chapter.

Chapter 2

Experimental Arrangements

2.1 Neutral Beam Line

A neutral beam line called the K0 channel was newly constructed for this experiment in the East Counter Hall (E-Hall) of the KEK 12-GeV Proton Synchrotron (KEK-PS) [21] in 1986. A plan view of the E-Hall is shown in Fig. 2.1.

A primary $12\,GeV$ proton beam was extracted from the main ring over a period of 0.5 *second* with each beam pulse occurring every 2.65 *seconds*. The intensity of the beam was $1.0 \sim 1.5 \times 10^{12}$ protons per pulse (ppp) on the target. Protons hit a copper target which was $10\,mm$ in diameter and $120\,mm$ long. The target was followed by four bending magnets, which swept charged particles out in the vertical direction. Primary protons were bent downward and led to a beam dump made of tungsten. A vertical view of the beam line is shown in Fig. 2.2.

A neutral beam was taken at $0\,degree$ with respect to the primary beam in order to collect as many kaons as possible, and was collimated by two brass collimators. The first collimator was fixed in the third bending magnet and shaped the beam into a cone of a $7\,mrad$ half-cone angle, that is, a solid angle of $154\,\mu sr$. The second collimator was placed between the third and the last bending magnets and subtended a half-cone angle of $9\,mrad$. It consisted of two parts, and each part was horizontally movable.

The beam line in the last bending magnet, which was $2.6\,m$ long, was evacuated and immediately followed by a $10\,m$ long vacuum decay chamber. The vacuum was kept to be better than 0.4 *Torr* during the experiment. The entrance window was made of $50\,\mu m$ stainless foil, and the upstream end of the decay chamber was $10.5\,m$ from the target. About 8 % of K_L^0's decay in the chamber, and the number of K_S^0 or Λ^0 particles produced at the target is completely negligible in the chamber. The downstream end was connected, at the center, to a tapered vacuum beam pipe for transmitting neutral particles. The beam pipe was made of aluminum with the effective thickness of $5\,mm$, and subtended a half-cone angle of $10\,mrad$ from the target. Neutral particles were finally absorbed by a beam dump, which was $3.5\,m$ downstream of the last part of the detector system. The downstream end of the decay chamber also had two exit windows for passage of secondary particles from the K_L^0 decay. The window consisted of $50.8\,\mu m$ polyester film supported by $60.8\,mg/cm^2$ carbon fiber cloth.

The experiment was performed under the condition mentioned above for the first three-quarters of the running period. In the autumn of 1989, during a KEK-PS shutdown, the following modifications were made to the K0 channel, and the experiment for the last quarter period was performed under following new conditions.

- Two new bending magnets were installed upstream of the target. They varied the K_L^0 production angle from 0 *degree* to 2 *degrees* by bending the incident proton beam before it hit the target.
- The second collimator was replaced by a new stationary collimator with a half-corn angle of $7\,mrad$.
- The beginning of the vacuum region of the beam line was extended by $2\,m$ in the upstream direction between the third bending magnet and the second (now fixed) collimator.

The purpose for these changes was to produce a *cleaner* neutral beam and decrease the counting rates of the detectors. Since neutrons are produced predominantly in the direction of primary protons, changing of the production angle decreases the neutron-to-kaon flux ratio without losing many kaons. To extend the vacuum region moves the background source in the beam line further upstream. These modifications enabled us to increase the primary beam intensity up to 2.5×10^{12} ppp.

The changes in the beam line condition as well as the detector running condition will be summarized in Section 3.3.

2.2 Outline of the Detector System

Fig. 2.3 is a schematic view of the KEK E137 detector system downstream of the decay chamber. It was a double-arm spectrometer with particle-identification capability, sandwiching the beam pipe mentioned in Section 2.1. The coordinates were defined so that +Z (downstream) was in the direction of travel of the particles in the neutral beam, +X was to the left as seen by the beam, and +Y was upward. The origin of the coordinates was the point on the beam line where the decay chamber ended. The arm on the +X (−X) side was called the left (right) arm or the L (R) arm. Each arm consisted of five drift chambers W1–W5, two bending magnets, two hodoscopes H1 and H2, a gas Čerenkov counter CH, an electromagnetic shower counter EM, and a muon identifier MU. The detectors on the L (R) arm were called W1L–W5L (W1R–W5R), H1L and H2L (H1R and H2R), CHL (CHR), EML (EMR), and MUL (MUR).

The spectrometer had two stages, and it was possible to measure a track momentum twice. W1 and W2 and the upstream magnet were placed at an angle of $50\,mrad$ with respect to the beam line so as to be nearly perpendicular to incoming particles. Helium bags made of $65\,\mu m$ thick aluminized polyethylene film were placed between drift chambers to reduce the amount of material in the spectrometer. The radiation length, nuclear interaction length, and nuclear collision length up to W5 are shown in Table B.1.

The hodoscopes were used for making the basic trigger. The timing of the hodoscope trigger served as the time origin of the drift chambers and the other counters. The gas Čerenkov counter and the electromagnetic shower counter were used mainly for electron identification. The muon identifier was the last part of the detector system. It was the only detector that tagged and identified the muon from $K_L^0 \to \mu^+\mu^-$ decay in this experiment.

Table B.2 shows the Z position and X–Y coordinates covered by each detector element.

2.3 Drift Chamber

The drift chambers used for this experiment had four sense-wire planes, X, X', Y and Y'. The wire spacing was $9\,mm$. The two upstream chambers, W1 and W2, had 128 X (X') sense wires and 96 Y (Y') sense wires and had the dimensions of $115.2 \times 86.4\,cm^2$, while the three downstream chambers, W3, W4 and W5, had 128 X (X') sense wires and 128 Y (Y') sense wires and had the dimensions of $115.2 \times 115.2\,cm^2$. The total number of sense wires was 4864 for ten chambers, and typically 10 wires were dead during the experiment. Cathode field wires were placed around each anode sense wire to form a hexagonal cell structure as shown in Fig. 2.4. Two planes, X and X' or Y and Y', were staggered by half a cell to resolve the left-right ambiguities of tracks for each wire. There were other wire planes called the guard-wire planes at the outermost sides of each chamber to shape the electric field and to prevent electrons from entering the cell from the outside.

The sense wires were gold-plated tungsten wire with a diameter of $20\,\mu m$ and strung with a tension of $40\,g$, while the field and guard wires were gold-plated aluminum wire with a diameter of $100\,\mu m$ and strung with a tension of $80\,g$. A mixture of 50 % argon and 50 % ethane at atmospheric pressure was used for the chamber gas. The front and back of each chamber was covered by $25\,\mu m$ Mylar sheet except for the back of W5, which was covered by $50\,\mu m$ Mylar sheet. The amount of material of one drift chamber was 1.08×10^{-3} radiation length for W1 – W4 and 1.17×10^{-3} radiation length for W5.

The field wires of the downstream chambers were maintained at a voltage of $-2.00\,KV$ and the guard wires were maintained at $-1.85\,KV$, while the sense wires were at the ground. The upstream chambers were operated at a potential $0.05\,KV$ lower because of their high counting rates. The drift velocity of the electrons in the gas was about $50\,\mu m/nsec$. One end of each sense wire was attached to a charge-sensitive preamplifier (FUJITSU MB43468) that was mounted directly on the chamber. The gain of the preamplifier was $12.5\,mV/\mu A$. Each preamplifier output was transmitted

by a $4m$ twisted-pair cable to a 32-channel amplifier-discriminator (PRESSEY SL560C and LeCroy MVL407) board located in a crate near the chamber. The discriminator output, which was at the differential ECL level, was sent to a time-to-digital converter (TDC) board in the electronics hut by a $30\,m$ twisted-pair flat cable.

A 32-channel TKO-standard [22] TDC board, which was developed by the KEK Electronics Group, was used for the drift chamber TDC of this experiment. The received signal was translated to a TTL level signal and delayed by a $300\,nsec$ delay, and was coincidenced with a $175\,nsec$ width gate signal, which was produced by the master trigger. The time difference between the signal and the common stop signal from the delayed master trigger was converted to charge by a time-to-amplitude converter of each channel. Only from the channels with a signal with proper timing the charges were transferred through multiplexers and digitized by an analog-to-digital converter. A time resolution of this TDC was $0.7\,nsec$, and a conversion time per channel was $12\,\mu sec$. The hit information was also available as a TTL level signal from the front panel of the board, and this signal was sent to *hardware filter* modules. The modules checked the hit pattern of each chamber, and it was required that at least one wire in the X or X' plane and one wire in the Y or Y' plane were hit. If this was not satisfied, a veto signal was sent to the data-taking system (see Section 3.2) and used as the second level trigger called the hardware-filter.

The parameters to convert the TDC channel number into a drift time were calculated from the response of the test pulse applied to an amplifier-discriminator board prior to the experiment, and were modified from a TDC distribution of the data. The space-drift time relation was represented by a cubic spline function, whose parameters were calculated by an iterative method using real events. These parameters were determined for each wire. The spatial resolutions were measured with the sum of the drift lengths of the staggered-hit wires. The resolutions were about $330\,\mu m$ for the upstream chambers and about $280\,\mu m$ for the downstream chambers, which are shown in Fig. 2.5. The chamber efficiencies were measured to be better than 98 % for the upstream chambers and better than 99 % for the downstream chambers from a test using a ^{90}Sr beta ray source.

More details of the drift chambers are given in [23].

2.4 Spectrometer Magnet

Two C-type bending magnets were identical and had the same magnetic field. The aperture of the magnet was $130\,cm \times 100\,cm$. The field was oriented in the downward ($-$Y) direction. The negative (positive) particles in the left (right) arm were bent inward and accepted by the hodoscopes. The field strength at the center of the magnet was about $2.8\,KGauss$. The total integrated field strength was $0.79\,Tm$ corresponding to a transverse momentum kick of $238\,MeV/c$. The maximum transverse momentum for the $K_L^0 \to \mu^+\mu^-$ decay is $225\,MeV/c$, and the decay particles from a nearly transverse $K_L^0 \to \mu^+\mu^-$ decay emerged from the downstream magnet nearly parallel to the Z axis. This is the basis of the triggering by the hodoscopes. The track momentum resolution of the spectrometer, $\Delta p/p$, was estimated to be

$$\Delta p/p \simeq \sqrt{0.019 \cdot (\,p\,(GeV/c)\,)^2 + 0.11} \;\;\%$$

by a Monte Carlo calculation[1].

The magnetic field in the spectrometer was measured in the experimental area with a Hall probe when the drift chambers were removed. The probe was temperature-controlled, and the accuracy of the measurement was less than $1\,Gauss$. Mesh points of the measurement were placed at $5.0\,cm$ or $2.5\,cm$ intervals in the X and Y coordinates and at $2.5\,cm$ intervals in the Z coordinate. A magnetic field map was used for the track reconstruction. We *did not* have to scale the magnetic field to obtain proper physical quantities such as the K_L^0 mass. During the experiment the field strength was monitored by an NMR flux-meter located on the face of the lower pole piece of each magnet, and was kept constant to within $0.03\,\%$.

[1]The Monte Carlo simulation system of this experiment is described in Appendix A

2.5 Hodoscopes

A schematic view of the hodoscopes is shown in Fig. 2.6. The upstream hodoscope, H1, and the downstream hodoscope, H2, were placed behind W5 and separated by $2\,m$. Each consisted of sixteen vertically arranged plastic scintillators (BICRON BC-412) whose dimensions were $7 \times 116 \times 1\,cm^3$ for H1 and $12 \times 140 \times 1\,cm^3$ for H2. Each of the scintillation counters was named H1-1, ... , H1-16 and H2-1, ... , H2-16, and was viewed by two 2-inch photomultipliers (HAMAMATSU H1161) at the top and bottom ends. Since the single counting rates were high, booster voltages were applied to the last two or three dynode stages of every 2-inch photomultiplier. The X position of the center of each of the H1 counters and that of the corresponding H2 counter were set to be equal. H1-n (n=1 to 16) counter and H2-n counter formed a pair and covered a horizontal angular divergence of $\pm 12.5\,mrad$, as shown in Fig. 2.7. The H2 counters were staggered so that adjacent counters overlapped by $4.8\,cm$.

The muons from a nearly transverse $K_L^0 \to \mu^+\mu^-$ decay exited the spectrometer nearly parallel to the beam line. On the other hand, charged particles from the dominant K_L^0 decay modes such as $K_L^0 \to \pi\mu\nu$, $K_L^0 \to \pi e\nu$ and $K_L^0 \to \pi^+\pi^-\pi^0$, had smaller transverse momenta and exited the spectrometer in the inward direction. By requiring that a charged particle traversed a hodoscope pair H1-n and H2-n in each arm, two-body decays were enhanced and the three-body decays were suppressed at the trigger level. This requirement was called the *parallel* coincidence. In the case of the $K_L^0 \to \pi^+\pi^-$ decay, whose maximum transverse momentum is $206\,MeV/c$, the pions from this decay were bent slightly inward by the spectrometer magnets. Hence, for the trigger for the $K_L^0 \to \pi^+\pi^-$ decay, the H1-H2 parallel coincidence requirement was relaxed so that the *semi-parallel* coincidence, that is, a particle which traversed either the parallel pair H1-n and H2-n or the pair H1-n and H2-(n$-$1), the adjacent counter on the beam pipe side, was required. It is illustrated in Fig. 2.8. In this trigger, not only the $K_L^0 \to \pi^+\pi^-$ decay but also the three-body semileptonic decays $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ were accepted. These semileptonic events were useful for the calibration of the response of the lepton-identification detectors.

A block diagram of the signal processing for the hodoscopes is shown in Fig. 2.9. The signal from each photomultiplier was transmitted to the electronics hut by a 5D2V coaxial cable and was discriminated. One set of the discriminator outputs, which were at the NIM level, was sent to meantimers. The other set was sent to CAMAC TDC modules, whose time resolution was 0.025 $nsec$, by 50 m $5D2V$ coaxial delay cables. The signals from photomultipliers at the top and bottom end of each counter, H1-n-U and H1-n-D [2] or H2-n-U and H2-n-D, were then mean-timed in order to process a signal, H1-n-MT [3] or H2-n-MT, whose timing was independent of the Y position of the track. One output of the meantimer was sent to a TDC module, and the other two outputs were sent to coincidence modules.

A time resolution of the meantime is shown in Fig. 2.10. For H1 the average of the up and down TDC's was used, while for the H2 the meantimer TDC was used. The timing of the signals from all the scintillation counters was adjusted by Lemo-cables so that the timing of the outputs of meantimers agreed with each other to within 0.5 $nsec$. This adjustment was made prior to the experiment using a signal from a test horizontal counter located at the vertical mid position of the hodoscope as the reference of the timing. Details of the hodoscope tuning are given in [24].

The meantimer outputs of corresponding H1 and H2 counters were fed to a coincidence module to produce a parallel coincidence signal. Sixteen coincidence outputs were sent to a fan-in module, and the output OR signal was used as a parallel signal for one arm. A coincidence signal of H1-m-MT and H2-(m−1)-MT (m=2 to 16) was also produced, and these fifteen signals were sent to another fan-in module. The output signal of this fan-in module and the parallel signal mentioned above were further OR'ed and produced a semi-parallel signal. The parallel signal and semi-parallel signal were used for the trigger signal from the hodoscopes.

[2] U stands for "up", and D stands for "down".

[3] MT stands for "meantimer".

2.6 Čerenkov Counter

A schematic view of the gas Čerenkov counter, CH, is shown in Fig. 2.11. The counter box was made of aluminum sheet, and the thickness of the sheets at the entrance and exit was 0.2 cm. Four spherical Lucite mirrors, which had a radius of 190 cm and faced upstream, were placed near the downstream end and segmented the counter into four cells, CH-1 (inside up), CH-2 (inside down), CH-3 (outside up), and CH-4 (outside down). Each mirror was 1.0 cm thick, 65 cm wide and 75 cm high, and was supported by 1.5 cm thick aluminum plates as shown in Fig. 2.11. The mirror focused light on a 5-inch photomultiplier (HAMAMATSU R1584), which was located 90 cm away from it. To compensate for the field of the spectrometer magnets, which was about 2 $Gauss$ at the surface of each photomultiplier, a compensation coil (0.5A, 40 turn) was mounted just in front of each photocathode.

The counter box was filled with air at atmospheric pressure as the radiator, the effective length of which was about 150 cm. The index of refraction is 1.000273, and the threshold momentum is 4.5 GeV/c for muons and 6.0 GeV/c for pions. The half-angle of the Čerenkov cone for electrons is 23.4 $mrad$, and in the wavelength range from 250 nm to 480 nm the number of photons emitted per cm of the path length is 0.477.

The Čerenkov counter was placed between H1 and H2. The amount of material of H1 was 2.36×10^{-2} radiation length, 1.26×10^{-2} nuclear interaction length, and 1.77×10^{-2} nuclear collision length, and the amount of material of the Čerenkov counter was 1.90×10^{-1} radiation length, 5.16×10^{-2} nuclear interaction length, and 7.62×10^{-2} nuclear collision length. The radiation length, nuclear interaction length, and nuclear collision length up to W5 (Table B.1) were smaller than these values. This fact is important in the consideration of the *nuclear interaction loss*, which will be discussed in Section 5.5.

The signal from each photomultiplier was transmitted to the electronics hut by a coaxial cable and was divided into three by a passive divider. One of these signals was sent by a 50 m $5D2V$ coaxial delay cable to a CAMAC analog-to-digital converter (ADC) module (LeCroy 2249A), the gate width of which was 60 $nsec$. A second signal was discriminated and sent to a TDC module. The remaining signal was fed into an active mixer (LRS 428A), whose output was discriminated to produce a signal used for the electron trigger. The threshold level of the discriminator was set to be equivalent to $\sim$ 1 photoelectron. A signal block diagram is shown in Fig. 2.12.

2.7 Shower Counter

The electromagnetic shower counter, EM, was placed behind H2. It consisted of sixteen vertically arranged "modules" whose dimensions were $8 \times 152\,cm^2$. Each module was made up of ten alternate layers. The first layer was a $12\,mm$ thick iron plate followed by a $6\,mm$ thick scintillator, and each of the other nine layers was an $8\,mm$ thick lead plate followed by a $6\,mm$ thick scintillator. For the shower counter KYOWA SCSN-81 plastic scintillator was used. To observe the shower development each module was divided into two sections. The forward section, EMF, consisted of the first four layers, and the backward section, EMB, consisted of the last six layers. Each section was viewed by two 2-inch photomultipliers at the top and bottom ends, and was named EMF-1, ... , EMF-16 and EMB-1, ... , EMB-16. A schematic view of the shower counter is shown in Fig. 2.13.

The forward section had 5.0 radiation lengths and the backward section had 8.7 radiation lengths. The shower maximum position for most of the electrons in this experiment was in the forward section of the counter. The gain of the photomultipliers was adjusted so as to give the same pulse hight for a minimum ionizing particle. The energy resolution was $\Delta E/E \simeq 20\%/\sqrt{E(GeV)}$.

A block diagram of the shower counter signal processing is shown in Fig. 2.14. The signal from each of the forward sections was transmitted to the electronics hut by a coaxial cable and was divided into two. One of them was discriminated and sent to a TDC module, and the other was sent to a passive mixer to mix the up and down photomultiplier signals of each section. One set of the mixer outputs was sent to ADC modules, and the other set was sent to an active mixer (ORTEC AN308/NL) in order to mix the sixteen signals from all the forward sections. In the case of the backward

sections, the up and down signals of each section were mixed. One of the mixer outputs was sent to an ADC module, and the other was then divided into two, one of which was discriminated and sent to a TDC module and the other was sent to another active mixer. The outputs of the forward mixer and backward mixer were finally added with the ratio of 4 : 6 using an attenuator. This ratio is determined as that between the numbers of scintillators in the forward and backward sections. The analog output signal was amplified and was discriminated to produce a shower counter signal for the electron trigger. The discriminator threshold level was set to correspond to a signal from a $0.5\,GeV$ electron.

2.8 Muon Identifier

The muon identifier, MU, was located behind EM. It consisted of four iron blocks, whose dimensions were $152 \times 190\,cm^2$ with thicknesses of 10, 50, 30, and $30\,cm$ from the upstream end. Each block was followed by plastic scintillators (BICRON BC-412). Six vertically arranged scintillators whose dimensions were $22 \times 162 \times 1\,cm^3$ followed the first two iron blocks. Each scintillator was viewed by two 2-inch photomultipliers at the top and bottom ends, and was named MU1-1, ... , MU1-6 and MU2-1, ... , MU2-6. The last two blocks were followed by eight horizontally arranged scintillators whose dimensions were $140 \times 21 \times 1\,cm^3$. Each scintillator was viewed by a 2-inch photomultiplier at the end opposite to the beam line, and was named MU3-1, ... , MU3-8 and MU4-1, ... , MU4-8. A schematic view is shown in Fig. 2.15. The minimum momenta of muons penetrating the iron blocks were 0.4, 1.0, 1.4, and $1.8\,GeV/c$ for MU1, MU2, MU3, and MU4.

The signal from each photomultiplier was transmitted to the electronics hut by a coaxial cable and was divided into two. One was sent to an ADC module, and another was discriminated. The discriminator outputs of MU1 and MU2 were sent to meantimers, and the up and down photomultiplier signals of each counter were meantimed as done for the hodoscope signals. The meantimer outputs were sent to a matrix coincidence module, which produced the muon trigger signal. The matrix was set so that a coincidence between a MU1 counter, MU1-n (n=1 to 6), and the corresponding MU2 counter, MU2-n, or the adjacent MU2 counters, MU2-(n−1) and MU2-(n+1), was accepted. This corresponded to the requirement that the muon momentum should be greater than $1.0\,GeV/c$, since such a muon would penetrate the first two iron blocks, and reduced the background contamination by pion punchthrough. The meantimer outputs of MU1 and the discriminator outputs of MU2, MU3, and MU4 were sent to TDC modules. A block diagram of the signal processing for the muon identifier is shown in Fig. 2.16.

Chapter 3

Data Acquisition

3.1 Trigger

A coincidence output of the parallel signals from the left and right arms was called the " Para trigger " and used as the basic trigger for the $K_L^0 \to \mu^+\mu^-$, μe , and ee decays, while that of the semi-parallel signals was called the " Spara trigger " and used for the $K_L^0 \to \pi^+\pi^-$ decay. The muon trigger was a discriminated output of the matrix coincidence (Section 2.8). For the electron trigger a coincidence output of the electron signal from the Čerenkov counter (Section 2.6) and that from the shower counter (Section 2.7) was used. For the last quarter of the running period only the Čerenkov counter signal was required, because the electron trigger rate was reduced. These signals were sent to coincidence modules to make the trigger for each decay mode, and one set of signals was sent to a coincidence register module. The pulse width of the hodoscope signal was $10\,nsec$ and that of the muon and electron signals was $20\,nsec$, so that the timing edge of the trigger signal (coincidence output) was determined by the hodoscope signal.

A trigger logic diagram of this experiment is shown in Fig. 3.1. The $K_L^0 \to \mu^+\mu^-$ trigger was produced by requiring the Para trigger and the muon triggers in both arms. For the $K_L^0 \to ee$ trigger the electron triggers were required. The $K_L^0 \to \mu e$ trigger was made from the Para trigger and the electron trigger in one arm and the muon trigger in the other arm. The $K_L^0 \to \pi^+\pi^-$ trigger was the Spara trigger only, and it was prescaled by a preset scaler. The prescaling factor was 500 or 300 or 250 depending on the trigger rate. No anticoincidence veto signal from the other counters was imposed on these triggers. One set of the trigger signals was sent to a coincidence register module, and the other was mixed by a fan-in module (LRS 429) to produce the master trigger signal. The four two-body decay modes were simultaneously accumulated. The master trigger signal was also used to make the common start signal for all the CAMAC TDC modules, the gate signal for all the CAMAC ADC modules, and the gate and common stop signals for all the TKO TDC boards.

3.2 Online Data-taking

The online data-taking system is shown in Fig. 3.2. Seven CAMAC crates were used for TDC's, ADC's, coincidence registers, and blind scalers. Ten TKO crates were used for the drift chamber TDC's. A memory module called the Memory Partner (MP) was assigned to each crate to process the front-end modules in parallel. A crate controller called the Control Head (CH) scanned the data of the modules in each crate and sent them to the corresponding MP event by event during the 0.5 *second* beam pulse. The MP had enough memory size (32bit × 8K) to record all events in a beam pulse. These intermediate buffers made the online computer free from the trigger handling.

Fig. 3.3 shows a time sequence of the data acquisition. The master trigger signal inhibited subsequent triggers and started the conversion of all TDC's and ADC's. When the hardware-filter veto signal (Section 2.3) came within the decision time of 750 $nsec$, a clear signal was sent to all the modules and after 12 μsec, which is necessary to clear the ADC's and TDC's, the trigger inhibition was removed. Otherwise, the system waited for the 60 μsec conversion time, and commanded the MP's to start storing data. It took 200 ~ 400 μsec to store all the data into the MP's (MP busy). Then the system cleared all the modules and waited for the next trigger.

The ratio of the number of the inhibited triggers to that of the master triggers was regarded as the dead-time ratio. During data taking subsequent triggers were inhibited and not taken, but the preset scaler of the $K_L^0 \to \pi^+\pi^-$ trigger continued counting with-

out inhibition. The $K_L^0 \to \pi^+\pi^-$ trigger after prescaling had a periodic time-structure, and the period (20 ~ 30 $msec$) was much longer than the event taking time. Thus the $K_L^0 \to \pi^+\pi^-$ trigger was not inhibited by the previous $K_L^0 \to \pi^+\pi^-$ trigger. The other triggers came at random and could be inhibited by any of the triggers. Hence, a dead-time difference between the dilepton decay modes and the $K_L^0 \to \pi^+\pi^-$ decay occurred. The correction for it will be discussed in Section 5.6. For the last quarter period the preset scaler was inhibited during event taking, and no such difference occurred.

The MP's were installed in two CAMAC crates, which were connected with the online computer μVAX II using two Kinetic K3922 crate controllers and a Kinetic K2922 interface board. At the end of the beam pulse, blind scaler data were sent to the MP and the data in all MP's were transferred to the computer. During about 2 sec of the beam interval the computer processed all the data. First it sorted the data in an event by event structure. After that a process called the software-filter checked the hodoscope H1 meantimer TDC's of each event. The events which satisfied the following requirement were accepted and recorded on 6250 BPI magnetic tape : at least one H1 counter signal in the left arm and one H1 counter signal in the right arm were within 5 $nsec$ of each other. The average data size of one event was about $2K$ bytes and one 2400-foot tape was filled in about one hour. For the last quarter of the running period 2 $Gbyte$ 8 mm video tape was used instead. It could be used for more than eight hours.

As an online monitor of the data, at least one event per beam pulse was sampled and accumulated into histograms by the computer. The high voltages and the currents of the drift chambers and the power supply voltages and the field strength of each spectrometer magnet were also monitored, and when an accident happened it sounded an alarm and paused data-taking.

3.3 Running Condition

KEK E137 experiment was proposed in February 1985 [25] and approved in December 1985. A survey of a neutral beam line and the construction of the detector system were started in October 1986, and preliminary data-taking[1] was performed from December 1987 to March 1988 [26]. A physics data-taking run (*lepton run*) began in June 1988 and lasted until July 1989. It was called the PHASE-1. After the modifications of the beam line described in Section 2.1 we restarted data-taking in December 1989. It was called the EXTENSION, and was completed in May 1990.

The KEK-PS was operated in a two-week mode or a three-week mode. Each running period was called a *cycle* and labeled by an alphabet letter : K $\sim$ Y (PHASE-1)[2] and ZA $\sim$ ZF (EXTENSION). The length of each cycle was counted in units of *shift*, which was equal to eight hours. A two-week mode had typically 25 shifts (200 hours) and a three-week mode had 40 shifts (320 hours). The number of lepton runs and the beam line condition for each cycle are shown in Table C.1 and Table C.2. One run contained $80K \sim 100K$ events. We did data-taking for $\sim$ 6400 hours and collected $\sim 4 \times 10^8$ events including $\sim 54 \times 10^6$ $K_L^0 \to \mu^+\mu^-$ trigger events and $\sim 80 \times 10^6$ $K_L^0 \to \pi^+\pi^-$ trigger events. The data from a special run for the detector calibration were not used in the analysis.

The K_L^0 momentum spectrum for various production angles was calculated by two methods. One method was to use Sanford-Wang's empirical formula [29] (Fig. 3.4), which assumed that the K_L^0 production cross section was equal to the the average of the K^+ and K^- production cross sections, and the other[3] was to extrapolate the K_L^0 production data at Fermilab (300 GeV protons) [30] to our momentum range using Feynman scaling (Fig. 3.5 [31]). In the latter method there was only a slight difference between the 0 *degree* spectrum and 2 *degrees* spectrum unlike Sanford-Wang's formula. The number of K_L^0 in the beam solid angle at 0 *degree* production was 2×10^7 per 10^{12} protons in the momentum range from 2 GeV/c to 8 GeV/c using Sanford-Wang's formula. The neutral particle background was measured by the beam line survey prior to data-taking with the following results : the flux of neutrons ($\geq$ a few MeV) was

[1]Since the detector tuning had not been completed at this stage, this data sample is not used in this thesis.

[2]The first result of E137 [13] was based on the data from K to T. M. Kuze's thesis [27] and [28] were based on the PHASE-1 data sample.

[3]This method was also used in the proposal of BNL E791 experiment [32].

$(4 \sim 10) \times 10^8$ per 10^{12} protons and that of photons ($\geq$ 100 MeV) was 4×10^8 per 10^{12} protons in the beam region.

The single counting rate of each detector under the normal running condition in the PHASE-1 and EXTENSION is summarized in Table C.3. The rate was much larger than that expected from the K_L^0 secondary particles, which suggested that the interactions of neutrons and photons with the material (residual gas atoms in the decay chamber and beam pipe, detector elements, etc.) dominated the single counting rate. The counting rates at various trigger stages are summarized in Table C.4 Changing the production angle from 0 *degree* to 2 *degrees* reduced the rate to be less than about 60 % of PHASE-1. Consequently the single counting rates under the EXTENSION condition ($2.5 \times 10^{12}ppp$, 2 *degrees*) were less than those under the PHASE-1 condition ($1.5 \times 10^{12}ppp$, 0 *degree*). The number of events per pulse recorded on tape was less than 100, and the dead-time ratio of data-taking was less than 7 %.

The recorded data were processed by an offline computer HITAC M-280H/680H. The events of each run were analyzed by a fast tracking program COND [27] just after the online data-taking in order to monitor the running condition and to get the preliminary result as soon as possible. After the physics data-taking was completed, we reanalyzed all the data using a common, more refined track finding program RECOND and reduced them to 250 magnetic tapes, $\sim 16 \times 10^6$ events including $\sim 0.7 \times 10^6$ $K_L^0 \to \mu^+\mu^-$ trigger events and $\sim 6.9 \times 10^6$ $K_L^0 \to \pi^+\pi^-$ trigger events. To those events a second tracking program RESP, which searched for more adequate drift chamber hits and made a track fitting to determine the kinematic quantities, was applied. Next, a particle identification routine selected true dileptonic decay events, and finally several selection criteria were applied to these events in order to select good events and eliminate background events from the sample. This procedure is described in the next chapter. Concurrently with the RECOND process an offline monitor program checked whether the TDC and ADC distributions of each counter remained the same.

Some changes in the detector running conditions occurred during the experiment which are summarized as follows.

- In the K and L cycles, drift chamber tuning and helium bag installation were not completed and the Čerenkov mirrors were not properly shaped. As a result the mass resolution of the spectrometer and the electron identification efficiency of the Čerenkov counter were worse than those of the other cycles.

- The first 9 % of the cycle X data was not used in the analysis because some cables were misconnected in the trigger system.
- From the offline monitor check it was found that for 54 lepton runs the peak positions of the muon identifier TDC distribution were unstable. These runs were not used in the analysis.
- From the ZA to ZF runs, only the Čerenkov counter signal was required for the electron trigger, because the trigger rate was reduced.
- The preset scaler of the $K_L^0 \rightarrow \pi^+\pi^-$ trigger was inhibited from the ZC to ZF runs.

Chapter 4

Tracking and Event Selection

4.1 Track Finding (RECOND)

The task of the track finding program RECOND was to find tracks in the spectrometer and obtain crude estimates of the track parameters and kinematical quantities. Only the information from the hodoscopes and drift chambers was used. The calibration parameters of these detectors were calculated cycle by cycle using a COND-analysis event sample.

The algorithm is somewhat intricate and can be divided into a sequence of six steps. Details of this program are described in Appendix B.

1. **Hit selection**
   First, the algorithm selected pairs of H1 and H2 counters in which an H1 counter and the corresponding H2 counter or a neighboring counter are hit. From H1 hit positions a " road " for track finding in the spectrometer was set in each view. The road width was wide enough not to lose the track hits at this stage. In the Y-view the hodoscope hit positions were estimated from the difference between the up and down TDC's of H1. The drift chamber hits within this road were used for the track finding.
2. **Y-view track finding**
   Since the magnetic field was oriented in the $-Y$ direction, the Y-view of the track,

as shown in Fig. 4.1 (a), was approximately fitted to a straight line. The track finding started from W5 and W4, and directed to the upstream chambers. At most 100 combinations of wire hits were stored for the next step.

3. **X-view track finding**
   The X-view of the track, as shown in Fig. 4.2 (a), was fitted to a bent line. A change in the track direction in the upstream magnet was nearly equal to that in the downstream magnet, because two magnets had the same magnetic field. The bent line consisted of three straight line segments with the same bending angle at the centers of of two magnets. As in the case of the Y-view track, the track finding started from the downstream chambers and at most 100 combinations were stored for the next step.

4. **Three-dimensional track finding and selection of candidates**
   From all the combinations of the X-view tracks and the Y-View tracks, at most 50 combinations were stored in each arm. Then all three-dimensional tracks in the left arm were combined with all tracks in the right arm, and the following kinematical quantities were calculated.

   vertex distance Extrapolating the left and right arm tracks in the upstream direction, the minimum distance between the two tracks, also known as the distance of closest approach, was defined as the vertex distance, *Dist*.

   vertex point The vertex point (V_X, V_Y, V_Z) was taken to be the midpoint of the vertex distance.

   kaon momentum The vector sum of the momenta[1] of the left and right arm tracks, $\vec{p_K} = \vec{p_L} + \vec{p_R}$, was taken to be the kaon momentum, and $|\vec{p_K}| \equiv p_K$.

   invariant mass Each of the trigger bits (μe, $e\mu$, $\mu\mu$, ee, $\pi\pi$) of the coincidence register module was checked[2], and the invariant masses of the two particles were calculated assuming the mass hypotheses of the trigger bit for the two tracks ($M_{\mu e}$, $M_{e\mu}$, $M_{\mu\mu}$, M_{ee}, $M_{\pi\pi}$).

   collinearity angle The angle between the kaon momentum and the line which connected the vertex point and the target, as shown in Fig. 4.3, was defined as the *collinearity angle, θ*. For two-body decays this angle should be zero, while for three-body decays with a missing neutral particle it should not be zero in general.

   Rough cuts on these quantities were applied to reject wrong combinations. Using the fitting χ^2's in the left and right arms and the vertex distance, 10 combinations or less were selected for the next step.

5. **Spline fitting of the track**
   For each wire hit combination, the left-right ambiguity of the wires in each chamber was resolved. When wires in staggered planes had hits there were four combinations of the drift lengths as shown in Fig. 4.4. The slope of each combination was calculated and compared with the slope of the track determined previously, and the drift length combination with the most closely matched slope was chosen. The quantity *Dsum* :

$$\begin{aligned} Dsum &\equiv (\, the\ sum\ or\ difference\ of\ drift\ lengths\,) \\ &- 0.45\,cm \\ &+ (\, the\ correction\ for\ the\ slope\ of\ track\,) \end{aligned}$$

   should be zero in the absence of the measurement error of the drift lengths, and the left-right ambiguity resolved combination gave a minimum for *Dsum*.[3] The left-right ambiguity of a single-hit wire was not resolved.

   Then the track was reconstructed using the spline method [33]. A quintic (fifth order) spline function was used as the track model, and a track ($X = X(Z)$, $Y = Y(Z)$) was represented by a linear combination of five track parameters. They were

[1] At this stage the momentum of the track was calculated from the total integrated field strength and the difference in the slopes of the bent line.

[2] For example, when the $\mu\mu$ trigger bit was on the event was regarded as a $\mu\mu$ event.

[3] The root mean-squared of the distribution of $Dsum \times 1/\sqrt{2}$, as shown in Fig. 2.5, was regarded as the intrinsic spatial resolution of the chamber.

determined from the least squares fit of the coordinates in the chamber planes , X_c's and Y_c's, which were calculated from the drift length and wire position, to this track model. 46 planes, which included the 20 chamber planes, were used for spline interpolation in each arm. Z coordinate of each plane is shown in Table D.1. The fitting χ^2 was defined as :

$$\chi^2_{track} \equiv \sum_{j=1}^{N_X} \frac{(X_c - X(Z))_j^2}{\sigma_j^2} + \sum_{j=1}^{N_Y} \frac{(Y_c - Y(Z))_j^2}{\sigma_j^2}$$

where N_X and N_Y were the numbers of the chamber planes used for fitting, and σ was the spatial resolution. It must be noted that only the diagonal terms of the error matrix were used to calculate this χ^2.

6. **Selection of the candidate**
Using the track parameters in each arm, the kinematical quantities were calculated once again. The final selection χ^2 was calculated, and the wire hit combination which gave the minimum χ^2 was chosen as the track candidate of the event.

Events were written on magnetic tape together with their tracking information and kinematical information calculated in this analysis for the subsequent analysis. However, $\mu\mu$ or μe events whose invariant masses were less than $475\,MeV/c^2$ were discarded at this stage.

4.2 Second Tracking (RESP)

The second tracking program RESP analyzed all the surviving events, and determined the track parameters and kinematical quantities. This program had two procedures : hit wire search and track fitting.

4.2.1 Hit wire search

As shown in Table C.3, the single counting rates of the chambers were high and there were extra hits in one plane. A hit wire selected by the RECOND program might be a fake hit that was not due to a real track. Furthermore, the chamber TDC did not

contain proper time information when an accidental hit had already started the TDC. In these cases tracking results gave false kinematic information. In this program, with some selection criteria, more adequate hits in each chamber were locally searched for around the spline track.

First, for the wires selected by the RECOND program the left-right ambiguity was resolved once more using a fitted track. *Dsum* defined in Section 4.1. and the absolute value of the residual of a fit, *Rdev* :

$$Rdev \equiv |X_c - X(Z)|, \quad or \quad |Y_c - Y(Z)|$$

were calculated. The ambiguity for single-hit wires was resolved from the (positive or negative) sign of the residual.

Another combination of staggered-hit wires was searched for around it, as shown in Fig. 4.5, for the staggered-hit wires which satisfied the following criteria : *Dsum* was greater than $0.12\,cm$, or *Dsum* was greater than $0.09\,cm$ and one of their *Rdev*'s was greater than $0.12\,cm$. The combination which gave a minimum *Dsum* was regarded as the proper staggered-hit wires. *Dsum* had to be less than $0.15\,cm$, which is about 3.5 times their resolution. Otherwise, only the wire which had the smaller *Rdev* was accepted as the single-hit wire.

For single-hit wires, only the wire which had a maximum *Rdev* in the X-view or Y-view was checked. When *Rdev* was greater than $0.17\,cm$ another hit wire was searched for within two cells in the direction of the track, as shown in Fig. 4.6, and the wire which gave a minimum *Rdev* was taken. Furthermore, when an adjacent wire, shown in Fig. 4.7, was also hit, then *Dsum* was calculated with this combination. Staggered-hit wires were taken instead of a single-hit wire when the following criteria were satisfied : *Dsum* was less than $0.10\,cm$, or *Dsum* was less than $0.15\,cm$ and both *Rdev*'s were less than $0.13\,cm$.

In each arm, when the wire hit combination was changed by the search described above, the spline fitting was performed with a new combination, and the chamber hit search was made again using a new track. The spatial resolution of $400\,\mu m$ was assigned to staggered-hit wires and of $0.10\,cm$ to single-hit wires for this fitting procedure. When the combination was not changed or the search was iterated three times, the combination was sent to the track fitting process.

4.2.2 Track fitting

The purpose of the track fitting was to get the best set of the track parameters [34]. The definition of the track χ^2 was generalized as follows.

$$\chi^2_{TF} \equiv \sum_{i=1}^{N_X}\sum_{j=1}^{N_X}(X_c - X(Z))_i \cdot (E^X_{ij})^{-1} \cdot (X_c - X(Z))_j + \sum_{i=1}^{N_Y}\sum_{j=1}^{N_Y}(Y_c - Y(Z))_i \cdot (E^Y_{ij})^{-1} \cdot (Y_c - Y(Z))_j$$

The matrix E_{ij} was the *error (covariance)* matrix and its inverse matrix $(E_{ij})^{-1}$ was the *weight* matrix. The error matrix represented the correlation of the position measurement errors between chamber planes. The matrix element was defined as follows :

$$E^X_{ij} \equiv \langle\, (X^{err}_c - X^{noerr}_c)_i \cdot (X^{err}_c - X^{noerr}_c)_j \,\rangle \ .$$

X^{err}_c and X^{noerr}_c were the coordinates with and without errors in the measurement and $\langle \ldots \rangle$ meant the expectation value. (It was the same for E^Y_{ij}.)

The error matrix was divided into two parts.

$$E_{ij} = \sigma^2_i \cdot \delta_{ij} + E^{M.S.}_{ij}(p)$$

The first part, a diagonal matrix, was due to the measurement error of each drift chamber. It had no correlation with another plane. σ_i was the intrinsic spatial resolution of the chamber : $330\,\mu m$ for W1 $\sim$ W2, and $280\,\mu m$ for W3 $\sim$ W5. The second part was due to multiple scattering (M.S.) . An upstream scatter changed the position and direction of the charged particle and affected all the downstream measurements. Here " error in the measurement " means the deviation of the position of the *real* track from that of the *ideal* track, i.e., the track which would have been formed if there had been no multiple scattering. It must be emphasized that, in the track fitting, the coordinates were fitted not to the real track but to the ideal track, and from them the proper estimates of the track parameters at the entrance to the spectrometer were obtained. The matrix elements were assumed to be scaled by a square-inverse of the momentum, $\frac{1}{p^2}$, because in the standard formula the width of a projected multiple scattering distribution was

proportional to $\frac{1}{p}$ [18,35]. Since it was impossible to calculated these matrix elements analytically, they were determined using a Monte Carlo simulation. This method is explained in Appendix C.

In each arm, two error matrices, E^X_{ij} and E^Y_{ij}, were calculated using the intrinsic chamber resolutions[4] and the momentum obtained from the RECOND-analysis. These two 10×10 matrices were then inverted to obtain the weight matrices. Then the spline fitting was performed and the track parameters were calculated. The spline track in the previous step was used as a starting fit, and the magnetic field at each plane was calculated along it. Using the track parameters in each arm, the kinematical quantities were determined finally.

Track fitting was also performed in the upstream (W1, W2, and W3) and downstream (W3, W4, and W5) halves of the spectrometer independently. In this case 6×6 error matrices were used. The upstream momentum, p_{up}, and the downstream momentum, p_{dn}, were used to check the consistency of the track in subsequent analysis.

4.3 Particle Identification

The track obtained from the RESP-analysis was extrapolated straight to the downstream detectors. Along the track, a particle identification routine searched for the proper hits of each detector which satisfied the following criteria.

Čerenkov counter Signals of the cells that the particle passed through were examined : The timing[5] was in the range from $-2.5\,nsec$ to $3.0\,nsec$ of the time expected from the hodoscope. The time resolution was $0.56\,nsec$.

Shower counter The X position of the center of the counter section was within $12\,cm$ (EMF) or $16\,cm$ (EMB) of the track X position.[6]

[4] For a single-hit wire, (intrinsic resolution) $\times\sqrt{2}$ was used.

[5] The pulse-height correction using the ADC value was included.

[6] The timing information was not used for the shower counter.

Muon identifier For MU1 and MU2, the X position of the center of the scintillator was within 33 cm of the track position and the timing of the meantime was within 2.5 $nsec$ of the expected time. For MU3 and MU4, the Y position was within 42 cm and the timing[7] was within 3 $nsec$. The time resolutions were 0.74 $nsec$(MU1), 0.63 $nsec$(MU2), 0.83 $nsec$(MU3), and 0.87 $nsec$(MU4).

The routine then identified the particle as follows.

Muon identification was based on the range-momentum relation using the four layers MU1, MU2, MU3, and MU4 of the muon identifier. First, the track whose momentum was less than 1.0 GeV/c or which had no proper hit in MU1 and MU2 was rejected, because it did not really satisfy the muon trigger requirement. Next, the "last plane" that the track reached, which represented the range of the particle in this detector, was determined as follows.

1. The track which had a proper hit in MU4 and in at least two layers of MU1, MU2, and MU3 was regarded to have penetrated four iron blocks and reached MU4. The last plane was MU4.
2. The track which had a proper hit in MU3 and in at least one layer of MU1 and MU2 was regarded to have penetrated the first three block and reached MU3, and stopped in the fourth block. The last plane was MU3.
3. The track which had a proper hit in MU2 was regarded to have reached MU2 but not MU3. The last plane was MU2.
4. The track which had a proper hit in MU1 was regarded to have penetrated only the first block. The last plane was MU1, but it was not regarded as a muon track.

Fig. 4.8 shows the momentum distributions of the muon sample[8] for the first three cases. From the distributions the following muon momentum ranges were obtained for the three "last planes."[9]

[7] The signal propagation-time correction using the track X position was included, since the scintillator of MU3 and MU4 was viewed at one end.

[8] The selection criteria for the muon and electron samples are described in Appendix E.

[9] The muon ADC information was not used for muon identification.

1. Last plane was MU4 $\Longrightarrow 1.7\,GeV/c \le p$.
2. Last plane was MU3 $\Longrightarrow 1.3\,GeV/c \le p \le 2.6\,GeV/c$.
3. Last plane was MU2 $\Longrightarrow 1.0\,GeV/c \le p \le 1.7\,GeV/c$.

The above requirements reduced the punchthrough of pions.

The ADC information of the Čerenkov counter and the shower counter was used to identify the electron from $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays, and to confirm that the muon track identified by the above algorithm was not an electron track. The relative gain of each photomultiplier and the signal attenuation from the propagation in the EM scintillators were corrected for. Čerenkov counter The ADC values of the Čerenkov counter were summed, and the sum less than the lower limit, which corresponded to 1.5 photoelectrons, was rejected. The ADC values of the shower counter of the proper hits were summed separately for the forward (E_{EMF}) and the backward (E_{EMB}) sections. The total energy deposit E was obtained as

$$E = \frac{E_{EMF} + 0.8 \times E_{EMB}}{(1.0 + 0.8)} ,$$

where the empirical factor of 0.8 was determined so as to minimize the energy resolution. For electron identification, the lower limits for E (0.5 GeV), E_{EMF} (0.139 GeV), and E_{EMB} (0.044 GeV) were applied to discard minimum ionizing particles. It was further required that E was more than 0.7 times its track momentum p. Fig. 4.9 shows the distributions of the energy-momentum ratio, E/p, for the electron and muon samples. When E of a muon track was greater than 0.5 GeV and more than 0.6 times its momentum, it was suspected to an electron track and discarded. A scatter plot between p and E for the electron sample is shown in Fig. 4.10, and that for the muon sample is shown in Fig. 4.11.

A muon track without the proper ADC information in both the Čerenkov counter and the shower counter and with the muon trigger bit was identified as a muon in the analysis. A track which passed both the Čerenkov counter and shower counter requirements and was not identified as a muon track was, with the electron trigger bit on, identified as an electron. Finally, when the $\mu\mu$ trigger bit of the coincidence registor was on and both tracks were identified as a muon, the event was identified as the $K_L^0 \to \mu^+\mu^-$ event. The same electron and muon identification requirements were used for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ events.

To select $K_L^0 \to \pi^+\pi^-$ decays it was necessary to reject $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ decays from the $K_L^0 \to \pi^+\pi^-$ trigger events. The pion track was identified by the requirement that it was not identified as a muon track by the muon identifier and was not identified as an electron track by the Čerenkov counter. Only the $\pi\pi$ trigger bit was required. A $K_L^0 \to \pi^+\pi^-$ event sample was used to determine the final selection criteria described in the next section.

4.4 Selection Criteria

All the events were further checked by the criteria described in the following subsections. The criteria were the same for all decay modes. In the figures in these subsections, the histograms with the solid line represent the distributions of real $K_L^0 \to \pi^+\pi^-$ events, and the open squares represent the distributions of Monte Carlo[10] $K_L^0 \to \pi^+\pi^-$ events. The distributions are normalized[11] to contain the same numbers of events in the ranges of the cuts imposed by the criteria.

4.4.1 Track quality

Fig. 4.12 shows the probability distribution of the track χ^2 defined in Section 4.2, χ^2_{TF}, and Fig 4.13 shows the distribution of the reduced track χ^2,

$$\chi^2_{TF\,red} \equiv \frac{1}{N_X + N_Y - 5} \times \chi^2_{TF} ,$$

where N_X and N_Y were the numbers of the chamber planes used for fitting and $(N_X + N_Y - 5)$ was the degree of freedom. The probability distribution was nearly flat for large probabilities (small χ^2 values), which meant that the error matrix used for the χ^2 calculation was correctly estimated. However, the distribution disagreed for large χ^2

[10]Details of the Monte Carlo simulation are described in Appendix A.

[11]The number of Monte Carlo $K_L^0 \to \pi^+\pi^-$ events was scaled by the factor of 2.90.

values : the real data had much longer tails. This discrepancy suggested that the Monte Carlo simulation did not perfectly reproduce the performance of the drift chambers such as the effect of the electronics noise or the tail of the resolution function. Since the χ^2 distribution was not understood completely, a looser cut was applied :

- a track whose reduced χ^2 was less than 4.0

was accepted.

Furthermore, in order to check the consistency of the track and to reject a pion which had decayed in flight during the passage through the spectrometer, the following quantity called *up-down* was calculated :

$$UD \equiv \frac{|p_{up} - p| + |p_{dn} - p|}{p} ,$$

where p_{up}, p_{dn}, and p were the momenta in the upstream and downstream halves of the spectrometer and in the whole spectrometer, respectively (Section 4.2). The distribution of UD is shown in Fig 4.14. In order to reject the backgrounds, a relatively tight cut was made on UD :

- a track whose UD was less than 0.06

was accepted. The loss of Monte Carlo tracks by this cut was about 7 %.

As for the numbers of the chamber planes (N_X, N_Y),

- $N_X \geq 8$ and $N_Y \geq 7$

were required. In the case of $N_X = 8$, there were two drift chambers which had only one hit plane in the X-view of the track. And when these two chambers were adjacent to each other the constraint on the track was not stringent through them. The requirement that

- when $N_X = 8$, the chambers which had only one hit plane in the X-view should *not* be adjacent to each other.

was used.

4.4.2 Track momentum

- A track momentum less than $4.5\,GeV/c$ [12]

was required for all tracks. And the momentum balance between the left and right arm tracks,

$$PB \equiv \frac{p_L - p_R}{p_L + p_R},$$

where p_L (p_R) was the momentum of the left (right) arm track, was checked to confirm the two-body decay.

- $-0.5 \leq PB \leq 0.5$

was required, which meant that the ratio of the left-arm momentum to the right-arm momentum, p_L/p_R, was within the range between 1/3 and 3.

The distribution of the track momentum in each arm is shown in Fig. 4.15, and that of the momentum balance *PB* is shown in Fig. 4.16. The cuts hardly eliminated events. Fig. 4.17 shows the distribution of the reconstructed kaon momentum p_K, which was the absolute value of the vector sum of the momenta of the two tracks. The distribution was well reproduced by the Monte-Carlo simulation. Fig. 4.18 shows the kaon momentum distributions in the PHASE-1 ($0\,degree$ production) and EXTENSION ($2\,degrees$ production). Both distributions were quite similar, and there was no apparent difference.

4.4.3 Decay vertex

Fig. 4.19 shows the distribution of the vertex distance, *Dist*. The resolution of *Dist* was slightly worse than that of the Monte Carlo events. Fig. 4.20 shows the distribution of the Z coordinate of the vertex point, V_Z. The Z coordinate of the decay chamber was between $-1000\,cm$ and $0\,cm$. Events whose V_Z was less than $-1000\,cm$ came from the kaon decay upstream of the chamber. Fig. 4.21 shows the distribution of the direction angle squared, θ_V^2, between the beam line and the line which connected the vertex point and the target. The spread due to the first collimator was $(\,7\,mrad\,)^2$, but it was smeared

[12] It is equal to the threshold momentum of the Čerenkov counter for muons.

by the size of the target. The smearing was seen on both the real events and Monte Carlo events. The following requirements were used for the decay vertex quantities.

- $Dist \leq 1.5\,cm$
- $-1000\,cm \leq V_Z \leq 0\,cm$
- $\theta_V^2 \leq (\,9\,mrad\,)^2$

4.4.4 Aperture cuts

Fig. 4.22 and Fig. 4.23 show the distributions of the X coordinates of the track at the exit of the decay chamber and at the entrance to the upstream magnet, and Fig. 4.24 shows the distribution of the Y coordinate of the track at the exit of the downstream magnet. To make sure that the track had passed the exit window of the decay chamber and also cleared the magnet pole faces or coils,

- $28.0\,cm \leq X \leq 127.2\,cm$ at the exit of the decay chamber
- $X \geq 42.0\,cm$ at the entrance to the upstream magnet
- $-49.5\,cm \leq Y \leq 49.5\,cm$ at the exit of the downstream magnet

were required. These cuts were fairly loose for these distributions.

4.4.5 Track-counter association

For the hodoscopes,

- the X position of the center of the hit H1 counter was within $4.0\,cm$ of the track X position
- the meantime difference between the left and right arm hit H1 counters was within $2.5\,nsec$.

were required, and to confirm that the track reached the effective region covered by the shower counters and the muon identifier,

- $X \geq 48.43\,cm$ (left arm), $49.27\,cm$ (right arm), and $|Y| \leq 72\,cm$ at the shower counter
- $45\,cm \leq X \leq 177\,cm$ and $|Y| \leq 80\,cm$ at the muon identifier MU1.

were required.

4.5 Results on $K_L^0 \to \pi^+\pi^-$ Events

Fig. 4.25 shows a scatter plot of the invariant mass ($M_{\pi\pi}$) vs the collinearity angle squared (θ^2) for 10^3 $K_L^0 \to \pi^+\pi^-$ events. An event cluster can be seen clearly at the K_L^0 mass ($497.7\,MeV/c^2$) with θ^2 near zero. From this plot the fiducial region for acceptable $K_L^0 \to \pi^+\pi^-$ events was defined as :

- $493\,MeV/c^2 < M_{\pi\pi} < 502\,MeV/c^2$ *and*
- $\theta^2 < 3\,(mrad)^2$.

It was the same for all two-body decay modes.

Figs. 4.26 and 4.27 are the $M_{\pi\pi}$ distribution of events for which $\theta^2 < 3\,(mrad)^2$ and the θ^2 distribution of events in the mass range $493\,MeV/c^2 < M_{\pi\pi} < 502\,MeV/c^2$, respectively, for all the $K_L^0 \to \pi^+\pi^-$ events. The number of $K_L^0 \to \pi^+\pi^-$ events in the fiducial region, $N_{\pi\pi}^0$, was 158776. The solid curves in the figures are the distributions obtained from the Monte Carlo $K_L^0 \to \pi^+\pi^-$ events, which are normalized to contain the same number of events in the fiducial region. The mass-resolution for the $K_L^0 \to \pi^+\pi^-$ decay, $\sigma_{M_{\pi\pi}}$, was $1.28\,MeV/c^2$, and the θ^2-resolution was $0.92\,(mrad)^2$. The acceptable mass and angular ranges were about three times their resolutions, and the spectra of $M_{\pi\pi}$ and θ^2 were well reproduced by a Monte Carlo calculation. The fraction of the Monte Carlo $K_L^0 \to \pi^+\pi^-$ events which fell in the fiducial region was $94.7 \pm 0.1\,\%$.

There exists a small excess of the real $K_L^0 \to \pi^+\pi^-$ events on the lower side of the $M_{\pi\pi}$ distribution (" non-Gaussian tail "). It will be discussed in Section 5.2.

4.6 Results on $K_L^0 \to \mu^+\mu^-$ Events

Fig. 4.28 shows a scatter plot of the invariant mass ($M_{\mu\mu}$) vs θ^2 for the $K_L^0 \to \mu^+\mu^-$ events. The box in the figure indicates the boundary of the fiducial region. The $M_{\mu\mu}$ and θ^2 distributions are shown in Figs. 4.29 and 4.30. The number of $K_L^0 \to \mu^+\mu^-$ events in the fiducial region, $N_{\mu\mu}^0$, was 179. In Fig. 4.29 a peak centered at the K_L^0 mass is clearly separated from the steeply falling background events in the region $M_{\mu\mu} < 490\,MeV/c^2$. The mass-resolution for the $K_L^0 \to \mu^+\mu^-$ decay was $1.5\,MeV/c^2$, which was also well reproduced by a Monte Carlo calculation as illustrated by the solid curve in the figure. The fraction of the Monte Carlo $K_L^0 \to \mu^+\mu^-$ events which fell in the fiducial region was $98.2 \pm 0.1\,\%$.

4.7 Results on $K_L^0 \to \mu e$ and $K_L^0 \to ee$ Events

Fig. 4.31 shows a scatter plot of the invariant mass [13] ($M_{\mu e}$) vs θ^2 for the $K_L^0 \to \mu e$ events. There are no events in the fiducial region. Thus the upper limit of the $K_L^0 \to \mu e$ branching ratio at 90% confidence level was calculated as 2.3 events times the single-event sensitivity for the $K_L^0 \to \mu e$ decay.

There is *one* event in the fiducial region in the $K_L^0 \to ee$ event plot as shown in Fig. 4.32. Fig. 4.33 shows a plot of M_{ee} vs p_t over an extended mass region, where p_t is the K_L^0 transverse momentum with respect to the target-to-vertex direction :

$$p_t \equiv p_K \times \theta \,.$$

There are several high mass events above a cluster of events which are actually $K_L^0 \to \pi e \nu$ events whose pions are misidentified as electrons. Fig 4.34 shows a scatter plot of the background events expected from a Monte Carlo calculation which used the decays $K_L^0 \to \pi e \nu$, $K_L^0 \to ee\gamma$, and $K_L^0 \to eeee$ [14] as the background sources for $K_L^0 \to ee$ events at ten times the sensitivity of this experiment. The general shape of the Monte Carlo generated event distribution is quite similar to Fig 4.33, and the

[13] $M_{\mu e}$ and $M_{e\mu}$ will not be distinguished.

[14] The decay $K_L^0 \to eeee$ is theoretically predicted [36], and two events have been observed recently [37].

high mass events in the real data seem to come mainly from the decays $K_L^0 \to ee\gamma$ and $K_L^0 \to eeee$, the latter decay mode being mostly responsible for the events in the $M_{ee} > 485\,MeV/c^2$ region. The one event in the fiducial region is at the tail end of the background distribution and cannot be distinguished from it at the level of the present statistics. Therefore, the upper limit of the $K_L^0 \to ee$ branching ratio at 90% confidence level was calculated as 3.9 events times the single-event sensitivity for the $K_L^0 \to ee$ decay.

The details of the analysis for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays are described elsewhere [38].

Chapter 5

Calculation of the $K_L^0 \to \mu^+\mu^-$ Branching Ratio

5.1 Outline

The numbers of $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \pi^+\pi^-$ decays expected in the fiducial region, $N_{\mu\mu}$ and $N_{\pi\pi}$, are written as follows[1] :

$$N_{\mu\mu} = N_{K_L} \times B.R.(K_L^0 \to \mu^+\mu^-) \times A_{\mu\mu} \times \epsilon_{\mu\mu} \ , \tag{5.1}$$

$$N_{\pi\pi} = N_{K_L} \times B.R.(K_L^0 \to \pi^+\pi^-) \times A_{\pi\pi} \times \epsilon_{\pi\pi} \ , \tag{5.2}$$

where

N_{K_L}	: number of K_L^0 's which have decayed in the decay chamber,
$A_{\mu\mu}$, $A_{\pi\pi}$	: acceptances, and
$\epsilon_{\mu\mu}$, $\epsilon_{\pi\pi}$	: event identification efficiencies.

As the direct measurement of the K_L^0 flux (N_{K_L}) is difficult, the $K_L^0 \to \pi^+\pi^-$ decay, which is kinematically similar to and simultaneously accumulated with the $K_L^0 \to \mu^+\mu^-$ decay, is used for normalization. By taking the ratio of the above two equations to remove N_{K_L} , which is the same for both modes, and regarding the numbers of $\mu\mu$ and

[1] The subscripts $\mu\mu$, $\pi\pi$ mean the decay modes $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \pi^+\pi^-$.

$\pi\pi$ events detected in the fiducial region as $N_{\mu\mu}$ and $N_{\pi\pi}$, the branching ratio for the $K_L^0 \to \mu^+\mu^-$ decay $B.R.(K_L^0 \to \mu^+\mu^-)$ is given as follows :

$$B.R.(K_L^0 \to \mu^+\mu^-) = \frac{N_{\mu\mu}}{N_{\pi\pi}} \times B.R.(K_L^0 \to \pi^+\pi^-) \times \frac{A_{\pi\pi}}{A_{\mu\mu}} \times \frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \times C_{\pi\pi\,/\,\mu\mu} , \quad (5.3)$$

where

$C_{\pi\pi\,/\,\mu\mu}$: correction factor that represents the differences between the $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \mu^+\mu^-$ decays.

For each cycle n (n = K $\sim$ Y, ZA $\sim$ ZF) equations (5.1) and (5.2) are rewritten as follows[2] :

$$N_{\mu\mu}^n = N_{K_L}^n \times B.R.(K_L^0 \to \mu^+\mu^-) \times A_{\mu\mu} \times \epsilon_{\mu\mu}^n , \quad (5.4)$$

$$N_{\pi\pi}^n = N_{K_L}^n \times B.R.(K_L^0 \to \pi^+\pi^-) \times A_{\pi\pi} \times \epsilon_{\pi\pi}^n , \quad (5.5)$$

and, by taking the ratio the following equation is obtained :

$$N_{\mu\mu}^n = N_{\pi\pi}^n \times \frac{B.R.(K_L^0 \to \mu^+\mu^-)}{B.R.(K_L^0 \to \pi^+\pi^-)} \times \frac{A_{\mu\mu}}{A_{\pi\pi}} \times \frac{\epsilon_{\mu\mu}^n}{\epsilon_{\pi\pi}^n} \times \frac{1}{C_{\pi\pi\,/\,\mu\mu}^n} .$$

By summing up over all the cycles and using $N_{\pi\pi} = \sum_n N_{\pi\pi}^n$ and $N_{\mu\mu} = \sum_n N_{\mu\mu}^n$, equation (5.3) is rewritten as follows :

$$B.R.(K_L^0 \to \mu^+\mu^-) = \frac{N_{\mu\mu}}{N_{\pi\pi}} \times B.R.(K_L^0 \to \pi^+\pi^-) \times \frac{A_{\pi\pi}}{A_{\mu\mu}} \times \overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \cdot C_{\pi\pi\,/\,\mu\mu}} , \quad (5.6)$$

where

$$\overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \cdot C_{\pi\pi\,/\,\mu\mu}} \equiv \left[\sum_n \frac{N_{\pi\pi}^n}{N_{\pi\pi}} \times \frac{\epsilon_{\mu\mu}^n}{\epsilon_{\pi\pi}^n} \cdot \frac{1}{C_{\pi\pi\,/\,\mu\mu}^n} \right]^{-1} .$$

The $K_L^0 \to \pi^+\pi^-$ branching ratio,

$$B.R.(K_L^0 \to \pi^+\pi^-) = (\, 2.03 \pm 0.04 \,) \times 10^{-3} \ [18] ,$$

contains an uncertainty of 2.0 %.

The single-event sensitivity for the $K_L^0 \to \mu^+\mu^-$ decay, $S.E.S.(K_L^0 \to \mu^+\mu^-)$, is defined as follows:

$$S.E.S.(K_L^0 \to \mu^+\mu^-) = \frac{1}{N_{\pi\pi}} \times B.R.(K_L^0 \to \pi^+\pi^-) \times \frac{A_{\pi\pi}}{A_{\mu\mu}} \times \overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \cdot C_{\pi\pi\,/\,\mu\mu}} .$$

[2] The superscript n designates a cycle.

5.2 Number of $K_L^0 \to \pi^+\pi^-$ Events

There were background events for the $K_L^0 \to \pi^+\pi^-$ events around the fiducial region, which are seen clearly in the semi-log $M_{\pi\pi}$- and θ^2-distributions (Figs. 5.1 and 5.2). The background events for the $K_L^0 \to \pi^+\pi^-$ decay are due to $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ decays whose leptons were not rejected by the muon identifier and the Čerenkov counter. Since the event distribution outside the fiducial region is almost flat in the $M_{\pi\pi}$ distribution, a simple method was used for the background estimation. The average number of events in the 9 MeV/c^2 wide regions adjacent to the mass fiducial boundaries :

- $484\,MeV/c^2 \le M_{\pi\pi} \le 493\,MeV/c^2$, $\theta^2 < 3\,(mrad)^2$

and

- $502\,MeV/c^2 \le M_{\pi\pi} \le 511\,MeV/c^2$, $\theta^2 < 3\,(mrad)^2$

was regarded as the number of background events in the fiducial region, and was subtracted from the number of events in the fiducial region.

For each cycle the above procedure was made. The number of $K_L^0 \to \pi^+\pi^-$ events in the fiducial region ($N_{\pi\pi}^{0\ n}$), the number of background events ($N_{\pi\pi}^{b.g.\ n}$), and the number of events after background subtraction ($N_{\pi\pi}^{s.\ n}$) :

$$N_{\pi\pi}^{s.\ n} = N_{\pi\pi}^{0\ n} - N_{\pi\pi}^{b.g.\ n}$$

are summarized in Tables E.1 and E.2. The total number of $K_L^0 \to \pi^+\pi^-$ events was determined to be 157281 by subtracting the background events (1495) from the events in the fiducial region (158776). After correcting for the prescaling factor of the $K_L^0 \to \pi^+\pi^-$ trigger (Section 3.1), the total number of $K_L^0 \to \pi^+\pi^-$ events detected in the fiducial region, $N_{\pi\pi}$, was determined to

$$\begin{aligned} N_{\pi\pi} &= 6.435 \times 10^7 - 0.060 \times 10^7 \\ &= (6.374 \pm 0.017 \pm 0.060) \times 10^7 . \end{aligned}$$

The first uncertainty is the statistical fluctuation (0.26 %), and the second one is the systematic uncertainty from the background subtraction (0.94 %). For the latter uncertainty the number of background events (0.060×10^7) itself was used.

Fig. 5.3 shows the *proper time* distribution of the $K_L^0 \to \pi^+\pi^-$ events. It agreed well with that of the Monte Carlo simulation, which used the kaon lifetime of $51.7\,nsec$ [18]. The K_S^0 contamination in the $K_L^0 \to \pi^+\pi^-$ events due to the the *nuclear interaction* was estimated to be less than 10^{-5} and that due to the K_S^0 *regeneration*[3] to be less than 10^{-8}. They are both completely negligible.

As described in Section 4.5, the lower side of the $M_{\pi\pi}$ distribution was not reproduced by the Monte Carlo $K_L^0 \to \pi^+\pi^-$ events only. One possibility to explain it is the effect of the $K_L^0 \to \pi^+\pi^-\gamma$ decay.[4] It is impossible to distinguish $K_L^0 \to \pi^+\pi^-\gamma$ decays from $K_L^0 \to \pi^+\pi^-$ decays, because photons from the kaon decay were not seen at all in this experiment. The Monte Carlo $M_{\pi\pi}$ distribution which included both the $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \pi^+\pi^-\gamma$ decays is shown in Fig. 5.4. The effect of $K_L^0 \to \pi^+\pi^-\gamma$ decay is not sufficient to explain this non-Gaussian tail. The error of $N_{\pi\pi}$ from this discrepancy, however, is within the systematic uncertainty described above.

To confirm the correctness of the above estimation, $N_{\pi\pi}$ was calculated by another method. The $K_L^0 \to \pi^+\pi^-$ trigger events without pion track identification were called the " minimum-bias " $K_L^0 \to \pi^+\pi^-$ events. They were checked by the same criteria and the invariant masses for these events were calculated assuming that both tracks were pions. The mass distribution of the minimum bias $K_L^0 \to \pi^+\pi^-$ events for which $\theta^2 < 3\,(mrad)^2$ is shown in Fig. 5.5. They consisted of the $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \pi^+\pi^-\gamma$ decays (a peak centered at the K_L^0 mass) and the three-body $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ decays (background around the peak). There were 180873 events in the fiducial region. Fig. 5.6 shows the background distribution from a Monte Carlo Calculation : the distributions of the Monte Carlo $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ events[5] were normalized[6] to contain the same number of events in the region $470\,MeV/c^2 \le M_{\pi\pi} \le 525\,MeV/c^2$ excluding the region around the peak : $485\,MeV/c^2 \le M_{\pi\pi} \le 510\,MeV/c^2$. The

[3]In this estimation $|\rho/\eta_{+-}| = 21.2$ [39] and $|\eta_{+-}| = 2.3 \times 10^{-3}$ [18] were used, where ρ is the regeneration amplitude and η_{+-} the CP violation parameter.

[4]Details of the $K_L^0 \to \pi^+\pi^-\gamma$ decay are described in Appendix D.

[5]The acceptances for these three-body decay modes are summarized in Table E.3.

[6]The branching ratios[18] : $B.R.(K_L^0 \to \pi\mu\nu) = (27.0 \pm 0.4) \times 10^{-2}$ and $B.R.(K_L^0 \to \pi e\nu) = (38.7 \pm 0.5) \times 10^{-2}$ were used.

region $M_{\pi\pi} \ge 510\,MeV/c^2$ consisted mainly of $K_L^0 \to \pi e\nu$ decay, while the region $M_{\pi\pi} \le 485\,MeV/c^2$ consisted of both $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ decays. When the number of background events in the fiducial region was estimated using this Monte Carlo background distribution (12042.6), the total number of $K_L^0 \to \pi^+\pi^-$ events detected in the fiducial region, $N_{\pi\pi}^{M.B.}$, was found to be

$$\begin{aligned} N_{\pi\pi}^{M.B.} &= 7.332 \times 10^7 - 0.488 \times 10^7 \\ &= (6.844 \pm 0.023) \times 10^7. \end{aligned}$$

As will be described in Section 5.4, the average identification efficiency of the $K_L^0 \to \pi^+\pi^-$ decay, $\bar{\epsilon}_{\pi\pi}$, was 0.937 ± 0.003. Then, $N_{\pi\pi}^{M.B.} \times \bar{\epsilon}_{\pi\pi}$ was $(6.412 \pm 0.029) \times 10^7$, which is within the systematic uncertainty of $N_{\pi\pi}$.

In order to check the effect of background subtraction, $N_{\pi\pi}$ was calculated by changing the mass region for the estimation of the background events. For example, when the mass region for the minimum-bias $K_L^0 \to \pi^+\pi^-$ events normalization was used, $N_{\pi\pi}$ was 6.408×10^7. All the results agreed with $N_{\pi\pi}$ to within the above systematic uncertainty. Therefore we conclude that the number of $K_L^0 \to \pi^+\pi^-$ events, $N_{\pi\pi}$, is correct to within the error quoted earlier.

5.3 Acceptance

In this thesis the word " acceptance " means the ratio of the number of events detected, after tracking and event selection, *in the fiducial region* to that of K_L^0 decays. The acceptance is formally written as the product of the following quantities :

detector acceptance : the efficiency of the decay particles to be accepted in the detector system,

tracking efficiency : the percentage of the events which were reconstructed by the tracking program (RECOND) with respect to the number of accepted decay candidates, and

selection efficiency : the percentage of the reconstructed events which, after all the cuts had been applied, fell in the fiducial region.

The acceptance calculation was made using a Monte Carlo simulation. For each decay mode, a Monte Carlo program generated 5 *million* decay events and made track generation, data simulation, and superposition of accidental hits. These procedures are described in Appendix A. The Monte Carlo data were reconstructed by the RECOND and RESP programs, and were selected by the same criteria as those of the real data. It was required that the muon momentum was greater than $1.0\,GeV/c$, which was in the muon identification routine. Then the number of events in the fiducial region was counted. The results are summarized in Table E.4. The acceptance *ratios* were

$$\begin{aligned} A_{\pi\pi} \,/\, A_{\mu\mu} &= 1.188 \pm 0.011\,, \\ A_{\pi\pi} \,/\, A_{\mu e} &= 1.084 \pm 0.009\,, \text{ and} \\ A_{\pi\pi} \,/\, A_{ee} &= 1.107 \pm 0.010\,. \end{aligned}$$

The absolute value of the acceptance depends on the various experimental conditions such as the K_L^0 flux, single-counting rate, and inefficiencies of the detectors. For example, the acceptance (tracking efficiency) of $K_L^0 \to \pi^+\pi^-$ decay decreased with increasing beam intensity, which was simulated by accidental hits superposed onto the Monte Carlo events as shown in Fig. 5.7. However, the ratio of acceptance rather than the absolute value is needed, which reduces the systematic error of the branching ratio. The effects on the Monte Carlo acceptance calculation of the beam intensity and dead sense wires of the drift chambers were examined, and one example, the results of the acceptance calculation *without* accidental hits superposition, is shown in Table E.5. For all cases the change in the value of the acceptance ratio was within 1 % and was consistent with the statistical uncertainty.

The acceptance depends on the various cut variables imposed by the criteria. When a certain cut value is changed, not only the acceptance but also the number of real events in the fiducial region is changed. The effects on the branching ratio due to the selection cuts will be discussed in Section 5.8.

5.4 Event Identification Efficiency

The particle identification efficiency (ε) for a particular particle such as the muon was defined as the ratio of the number of tracks (n) identified by the identification routine to the total number of the sample tracks (N) :

$$\varepsilon \pm \sigma_\varepsilon = \frac{n}{N} \pm \sqrt{\frac{\varepsilon \cdot (1-\varepsilon)}{N-1}}\,.$$

Here the error for the binomial distribution was used. The selection criteria of the muon, pion, and electron samples and the definition of each efficiency are described in detail in Appendix E. The efficiency for muon identification (ID) and that for misidentifying pions as muons in the muon identifier and the efficiency for electron ID in the shower counter,

$$\varepsilon_\mu^{MU}(p_i)\,,\ \varepsilon_{\pi\to\mu}^{MU}(p_i)\,,\ \text{ and }\ \varepsilon_e^{EM}(p_i)\,,$$

were determined as functions of momentum p_i. The efficiency for electron ID in the Čerenkov counter,

$$\varepsilon_e^{CH}(X_j, Y_k)\,,$$

was determined as a function of the hit position of the track, X_j and Y_k.[7] Fig. 5.8 shows the momentum dependence of the muon ID efficiency. The efficiency was almost flat for momenta greater than $1.3\,GeV/c$. Fig. 5.9(a) shows that of the efficiency of pions misidentified as muons. The dips at 1.7 and $2.6\,GeV/c$ in Figs. 5.8 and 5.9(a) were due to the cuts imposed on the momentum range in the ID routine. Fig. 5.10 shows the momentum dependence of the electron ID efficiency of the shower counter, and Fig. 5.11 shows the X and Y position dependence of the Čerenkov counter. The shower counter efficiency increased with momentum, and the dips in the Čerenkov counter efficiency were due to the boundaries of the mirrors.

The efficiencies were measured for each cycle. The efficiencies for a typical cycle are shown in Tables E.6, E.7, and E.8. The pion contamination effect on ε_μ^{MU}, which was about 0.5% as described in Appendix E, is shown for each momentum bin in Table E.6.

[7] For the other efficiencies the overall efficiency of the sample was used, because n in each bin was small.

Since the tracks of the $K_L^0 \to \pi^+\pi^-$ events in the fiducial region consisted of pions and muons mainly from the pion decay in flight *after the spectrometer*, the effect of the pion decay before the spectrometer[8], which was estimated using the $K_L^0 \to \pi e \nu$ Monte Carlo events as shown in Fig. 5.9(b), was subtracted from $\varepsilon^{MU}_{\pi\to\mu}$ obtained with the pion sample. The results are shown in Table E.7.

The Monte Carlo events were corrected for these efficiencies by weighting each Monte Carlo event by the product of the particle ID efficiencies of the left and right arms. The fraction (f) of events falling in a certain bin was determined as follows :

$$f\{bin\} \pm \sigma_{f\{bin\}} = \frac{N_{bin}}{N_{tot}} \pm \frac{\sqrt{N_{bin}}}{N_{tot}} ,$$

where N_{bin} was the number of events in the bin, N_{tot} was the total number of events in the fiducial region, and the error of the Poisson distribution was used. For example, the left-arm muon efficiency in the muon identifier was expressed as follows :

$$\epsilon_\mu^{MU\,L} = \sum_i f\{i\} \cdot \varepsilon_\mu^{MU\,L}(p_i) ,$$

where the superscript L (R) means the Left (Right) arm. The statistical uncertainty of the efficiency, $\sigma_{\epsilon_\mu^L}$, was given as

$$(\sigma_{\epsilon_\mu^{MU\,L}})^2 = \sum_i \left[\left(\sigma_{f\{i\}} \cdot \varepsilon_\mu^{MU\,L}(p_i)\right)^2 + \left(f\{i\} \cdot \sigma_{\epsilon_\mu^{MU\,L}(p_i)}\right)^2 \right] .$$

An example of the calculation is also shown in Tables E.6 and E.7. The efficiencies ϵ_μ^{MU}, ϵ_e^{CH}, and ϵ_e^{EM} in each arm [9] are shown in Table E.9.

The event identification efficiency for the $K_L^0 \to \mu^+\mu^-$ decay, $\epsilon_{\mu\mu}$, was expressed as follows :

$$\begin{aligned}\epsilon_{\mu\mu} &= (1 - \varepsilon^{CH\,L}_{\mu\to e}) \cdot (1 - \varepsilon^{EM\,L}_{\mu\to x}) \cdot (1 - \varepsilon^{CH\,L}_{\mu\to e}) \cdot (1 - \varepsilon^{EM\,R}_{\mu\to x}) \\ &\times \sum_{i,l} f\{i;l\} \cdot \varepsilon_\mu^{MU\,L}(p_i) \cdot \varepsilon_\mu^{MU\,R}(p_l) ,\end{aligned}$$

[8]The effect of the decay in the spectrometer was negligible because most of the tracks did not survive the selection criteria.

[9]In this table, the Monte Carlo $K_L^0 \to \mu^+\mu^-$ events were used for the muon ID efficiency and $K_L^0 \to \mu e$ events were used for the electron ID efficiencies.

and that for the $K_L^0 \to \pi^+\pi^-$ decay, $\epsilon_{\pi\pi}$, was expressed as follows

$$\begin{aligned}\epsilon_{\pi\pi} &= (1 - \varepsilon^{CH\,L}_{\pi\to e}) \cdot (1 - \varepsilon^{CH\,R}_{\pi\to e}) \\ &\times \sum_{i,l} f\{i;l\} \cdot (1 - \varepsilon^{MU\,L}_{\pi\to\mu}(p_i)) \cdot (1 - \varepsilon^{MU\,R}_{\pi\to\mu}(p_l)) .\end{aligned}$$

The efficiencies for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays could be expressed similarly.

The event ID efficiencies for each cycle, $\epsilon^n_{\mu\mu}$, $\epsilon^n_{\mu e}$, ϵ^n_{ee}, and $\epsilon^n_{\pi\pi}$, are summarized in Table E.10. The average PID efficiency :

$$\bar{\epsilon} \equiv \sum_n \frac{N^n_{\pi\pi}}{N_{\pi\pi}} \times \epsilon^n$$

were :

$$\begin{aligned}\overline{\epsilon_{\mu\mu}} &= 0.763 \pm 0.006 , \\ \overline{\epsilon_{\mu e}} &= 0.744 \pm 0.005 , \\ \overline{\epsilon_{ee}} &= 0.730 \pm 0.004 , \textit{ and} \\ \overline{\epsilon_{\pi\pi}} &= 0.937 \pm 0.003 .\end{aligned}$$

Even if, in Appendix E, the value of $\varepsilon^{MU}_{\mu\to PION}$ or the number of $\pi\pi$ (and $\pi\pi\gamma$) events contained in the sample was changed within a reasonable range, the change in the average event ID efficiency was within 0.5 % for the $K_L^0 \to \mu^+\mu^-$ decay and 0.25 % for the $K_L^0 \to \mu e$ decay. These deviations are included in the above errors.

5.5 Nuclear Interaction Loss

Pions undergo strong interaction with nuclei in material while muons and electrons do not. Since nuclear interaction was not included in the simulation, its effect on the $K_L^0 \to \pi^+\pi^-$ events had to be estimated with another method. The most important effect on the sensitivity was the loss of $K_L^0 \to \pi^+\pi^-$ events at the trigger level, which was called the *nuclear interaction loss*. It is one of the correction factors that represent the difference between the $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \mu^+\mu^-$ decays.

As described in Section 2.6, The most dense part of the detector system before H2 was the hodoscope H1 and Čerenkov counter regions. When a nuclear interaction occurred in this region the pion from the $K_L^0 \to \pi^+\pi^-$ decay might not reach the corresponding H2 counter. The efficiency of the nuclear interaction loss of the pion track in this region, $\epsilon^{N.I.}_{\pi\to X}$, was studied by both the data from a special run and a simulation.

A special run was made under a low beam intensity (1.0×10^{12} ppp) in the last cycle (ZF) of the experiment. The trigger requirements were changed so that

- at least one hit on the H1 in one arm (PI arm), and
- the semi-parallel signal of the hodoscope and an electron signal from the Čerenkov counter in the other arm (EL arm)

were required. By taking the electron from the $K_L^0 \to \pi e \nu$ decay in the EL arm, a sample of pions which reached the H1 counter was obtained in the PI arm. The analysis method was similar to that of the particle ID efficiency measurement. The data of this run were reconstructed by the RECOND program excluding the requirements for H2, and were selected by the standard selection criteria. The track in the EL arm was identified as an electron track by the Čerenkov counter and the muon identifier, and the track in the PI arm was extrapolated straight from H1 to H2. The nuclear interaction loss, $\epsilon^{N.I.}_{\pi\to X}$, was estimated from the ratio of the number of the tracks which did not produce signals in the corresponding H2 counters in each arm. In the calculation of the nuclear interaction loss as a function of momentum, the effects of pion decay, multiple scattering, hodoscope selection cut and the H2 counter efficiency were taken into account. The H2 counter efficiency was measured by another special run, for which the shower counter signals were used as triggers. It was found to be (99.5 $\pm$ 0.3) % which was also included to the correction.

A simulation for nuclear interaction loss was made using the FLUKA program [40], which is a Monte Carlo simulation program package developed at CERN for low-energy nuclear interaction. The results of the momentum dependence of $\epsilon^{N.I.}_{\pi\to X}$ from the special run data and from the FLUKA simulation are shown in Fig 5.12. The FLUKA results

reproduced well the momentum dependence of the data. Though the total interaction rate from the values of cross sections used in the FLUKA program was about 10 %[10] and had no momentum dependence, the results in Fig 5.12 were 2 $\sim$ 5 % and showed clear dependence for lower momenta. It was because the probability that the scattered pion or the secondary particles were accepted by the corresponding H2 counters was high and was further enhanced by the effect of Lorentz boost as the momentum increased.

The overall rate of the loss of the $K_L^0 \to \pi^+\pi^-$ events by nuclear interaction, $\epsilon^{N.I.}_{\pi\pi\to X}$, was calculated using the Monte Carlo $K_L^0 \to \pi^+\pi^-$ events and $\epsilon^{N.I.}_{\pi\to X}(p_i)$ obtained from the *special run data* :

$$1 - \epsilon^{N.I.}_{\pi\pi\to X} = \sum_{i,l} f\{i;l\} \cdot (1 - \epsilon^{N.I.\,L}_{\pi\to X}(p_i)) \cdot (1 - \epsilon^{N.I.\,R}_{\pi\to X}(p_l)) .$$

The result was :

$$\epsilon^{N.I.}_{\pi\pi\to X} = 0.0571 \pm .0058 .$$

5.6 Correction for the Dead-time Difference

We investigated other correction factors that had to be included in the sensitivity calculation. As described in Section 3.2, for the cycles from K to ZB the dead time for the $K_L^0 \to \pi^+\pi^-$ trigger was smaller than that for the dileptonic triggers. The correction factor, $C^{d.t.}_{\pi\pi/\mu\mu}$, depended on the beam condition and the trigger rate. It was determined cycle by cycle.[11]

The following terms are needed for obtaining the correction factor:

T : duration of the beam pulse (= 0.5 *second*),

n : number of triggers (in the beam pulse),

n_p : number of the $K_L^0 \to \pi^+\pi^-$ triggers,

n_l : number of the dileptonic triggers,

[10] This number comes from elastic scattering ($\sim$ 3 %) and inelastic scattering ($\sim$ 7 %). It is nearly equal to the rate expected from the value of nuclear *collision* length (9.39×10^{-2}) of H1 and the Čerenkov counter.

[11] For the cycles from ZC to ZF, this factor was essentially 1.0.

t : average dead time necessary for event taking,

The dead time for the $K_L^0 \to \pi^+\pi^-$ triggers comes only from the dileptonic triggers : $n_l\, t$, and the dead time for the dileptonic triggers comes both from the dileptonic and $K_L^0 \to \pi^+\pi^-$ triggers : $(n_l + n_p)\, t$. Then the number of $K_L^0 \to \pi^+\pi^-$ triggers inhibited is $n_p \times n_l t / T = n^2 r (1 - r) t / T$ and the number of dileptonic triggers inhibited is $n_l \times (n_l + n_p) t / T = n^2 (1 - r) t / T$, where $r = n_p / n = 1 - n_l / n$. The sum of them is the total number of inhibited triggers : $n^2 (1 - r^2) t / T$, and the ratio of the sum to n is referred to as the dead-time ratio, R : $R = n(1 - r^2) t / T$. Using the above results we obtain

$$R_p = \frac{n_l t}{T} = \frac{n(1 - r) t}{T} = \frac{R}{1 + r}, \; and$$
$$R_l = \frac{n t}{T} = \frac{R}{1 - r^2},$$

where subscripts p and l refer to the $\pi^+\pi^-$ and dileptonic modes, and

$$C^{d.t.}_{\pi\pi/\mu\mu} \equiv \frac{1 - R_p}{1 - R_l} = \frac{1 - \frac{R}{1+r}}{1 - \frac{R}{1-r^2}}.$$

For each lepton run of a cycle, R and r were obtained from the blind scaler data.[12] The correction factor was calculated for each run. Typical example values are : $R = 7.4\,\%$, $r = 0.28$, $R_p = 5.8\,\%$, $R_l = 8.0\,\%$, and $C^{d.t.}_{\pi\pi/\mu\mu} = 1.024$. The factors were then averaged for each cycle. The mean value was regarded as the correction factor of a cycle, $C^{d.t.\,n}_{\pi\pi/\mu\mu}$, with the standard deviation, $\sigma_{C^{d.t.\,n}_{\pi\pi/\mu\mu}}$. The results are summarized in Table E.10. The average correction factor for the entire experiment,

$$\overline{C^{d.t.}_{\pi\pi/\mu\mu}} \equiv \sum_n \frac{N^n_{\pi\pi}}{N_{\pi\pi}} \times C^{d.t.\,n}_{\pi\pi/\mu\mu},$$

was[13]

$$1.0125 \pm 0.0029 .$$

[12] R was obtained from the scalers of the counts of Secondary Emission Chamber (used for the beamline monitor) with and without the trigger gate.

[13] Since the factor 1.0 was used for the cycles from ZC to ZF, the value was lower than those for each cycle.

Using the event identification efficiencies ($\epsilon^n_{\mu\mu}$, $\epsilon^n_{\pi\pi}$) and the correction factor ($C^{d.t.\,n}_{\pi\pi/\mu\mu}$) for each cycle summarized in Table E.10, we get

$$\overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \cdot C^{d.t.}_{\pi\pi/\mu\mu}} = 1.243 \pm 0.009 .$$

The factors for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays are calculated similarly[14] :

$$\overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu e}} \cdot C^{d.t.}_{\pi\pi/\mu e}} = 1.274 \pm 0.010 , \; and$$
$$\overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{ee}} \cdot C^{d.t.}_{\pi\pi/ee}} = 1.300 \pm 0.007 .$$

5.7 Number of $K_L^0 \to \mu^+\mu^-$ Events

In addition to the true $K_L^0 \to \mu^+\mu^-$ events there were also some background events around the fiducial region. For example, in Fig 4.28 the number of $K_L^0 \to \mu^+\mu^-$ events in the region $493\, MeV/c^2 < M_{\mu\mu} < 502\, MeV/c^2$ and $3\,(mrad)^2 < \theta^2 < 9\,(mrad)^2$, $N^{b.g.}_{\mu\mu}$, was 5, while the number of real $K_L^0 \to \mu^+\mu^-$ events in the same region was expected to be 2.6 from the Monte Carlo calculation.

Four sources of background for the $K_L^0 \to \mu^+\mu^-$ decay were considered from the semileptonic kaon decays. For each case we estimated the upper boundary of the reconstructed $\mu^+\mu^-$ mass. For all but the third case, the track momenta were assumed to be well-measured.

case 1 $K_{\mu 3}$ decay with the pion decaying in flight into a muon <u>before</u> the spectrometer :

Since $M_{\mu\mu}$ is invariant in this case, we may work in the kaon rest frame. Let P_π and P_μ be the four-momentum vectors of the pion and muon from the $K_{\mu 3}$ decay, and $P_{\mu'}$ be the vector of the decay muon. Then the invariant masses are :

$$\begin{aligned} M^2_{\pi\mu} &= (P_\pi + P_\mu)^2 , \; and \\ M^2_{\mu\mu} &= (P_\mu + P_{\mu'})^2 \\ &= M^2_{\pi\mu} - m^2_\pi + m^2_\mu - 2\,P_\mu \cdot (P_\pi - P_{\mu'}) . \end{aligned}$$

[14] The same value of $C^{d.t.\,n}_{\pi\pi/\mu\mu}$ was used for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays.

$M_{\mu\mu}$ is maximum when $M_{\pi\mu}$ is equal to M_K and the pion decay in flight in the forward direction :

$$M_{\mu\mu} \ < \ 488.8\, MeV/c^2 \ = \ M_K - 8.9\, MeV/c^2 \ .$$

case 2 $K_{\mu 3}$ decay with the pion decaying <u>after</u> the spectrometer or being misidentified as a muon :

The invariant masses have to be calculated in the laboratory frame, because a wrong mass is assigned to the pion track.

$$\begin{aligned} M^2_{\pi\mu} &= \left(\sqrt{m_\pi^2 + p_\pi^2} + \sqrt{m_\mu^2 + p_\mu^2} \right)^2 - (\vec{p_\pi} + \vec{p_\mu})^2 \\ &\sim m_\pi^2 \left(1 + \frac{p_\mu}{p_\pi} \right) + m_\mu^2 \left(1 + \frac{p_\pi}{p_\mu} \right) + p_\pi\, p_\mu\, \theta^2_{\pi\mu} \ , \ and \\ M^2_{\mu\mu} &\sim m_\mu^2 \left(1 + \frac{p_\mu}{p_\pi} \right) + m_\mu^2 \left(1 + \frac{p_\pi}{p_\mu} \right) + p_\pi\, p_\mu\, \theta^2_{\pi\mu} \ , \end{aligned}$$

where $\vec{p_\pi}$ and $\vec{p_\mu}$ are the momentum vectors ($|\vec{p_\pi}| \equiv p_\pi$ and $|\vec{p_\pi}| \equiv p_\pi$) and $\theta_{\pi\mu}$ is the opening angle between the pion and muon.

$$M^2_{\mu\mu} - M^2_{\pi\mu} \ = \ \left(m_\mu^2 - m_\pi^2 \right) \left(1 + \frac{p_\mu}{p_\pi} \right)$$

and, when $M_{\pi\mu} \sim M_K$,

$$M_{\mu\mu} \ < \ M_K + \left(m_\mu^2 - m_\pi^2 \right) \left(1 + \frac{p_\mu}{p_\pi} \right) \frac{1}{2\, M_K} \quad .$$

Since the momentum ratio was restricted to be within the range between 1/3 and 3 (Section 4.4),

$$M_{\mu\mu} \ < \ 486.5\, MeV/c^2 \ = \ M_K - 11.1\, MeV/c^2 \ .$$

case 3 $K_{\mu 3}$ decay with the pion decaying <u>in</u> the spectrometer :

A wrong momentum is obtained from the tracking program. When the measured momentum is greater than the actual one, the calculated invariant mass can be above the kaon mass. However, this background is expected to be rejected by the *UD* cut of the track and, since the track reconstruction is not correct, θ^2 is expected to be large.

case 4 K_{e3} decay with both the pion and electron being misidentified as muons :

Using the same formula as in **case 2**,

$$M^2_{\mu\mu} - M^2_{\pi e} \ = \ \left(m_\mu^2 - m_\pi^2 \right) \left(1 + \frac{p_e}{p_\pi} \right) + \left(m_\mu^2 - m_e^2 \right) \left(1 + \frac{p_\pi}{p_e} \right)$$

where p_π and p_e are the momenta of the pion and electron from the K_{e3} decay. $M_{\mu\mu}$ can exceed the kaon mass due to the mass difference between the muon and electron. This background is reduced by the requirements on the Čerenkov counter and the shower counter in the muon identification process.

The background events in the region $M_{\mu\mu} < 490\, MeV/c^2$ are mainly due to cases **1** and **2**. The mass-resolution of $1.5\, MeV/c^2$ was good enough to reject these background events in the fiducial region. The background in the region $M_{\mu\mu} > 490\, MeV/c^2$ are expected to be explained by **cases 3** and **4**.

For the estimation of the number of background events in the fiducial region, it was assumed that

- the θ^2 distribution of the background events was *flat*

in the mass range $493\, MeV/c^2 < M_{\mu\mu} < 502\, MeV/c^2$. Then the number of background events in the region $3\,(mrad)^2 < \theta^2 < 9\,(mrad)^2$ would be $5 - 2.6 \ = \ 2.4$, and therefore

$$\begin{aligned} N_{\mu\mu} &= 179 \ - \ 2.4\,/\,2 \\ &= 177.8 \pm 13.4 \pm 1.2 \ . \end{aligned}$$

The first uncertainty is the statistical fluctuation (7.5 %), and the second one is the systematic uncertainty from the background subtraction (0.67 %), for which the number of background events itself was used. The numbers of $K_L^0 \to \mu^+\mu^-$ events and $K_L^0 \to \pi^+\pi^-$ events in the fiducial region for each cycle are shown in Table E.11.

The Monte Carlo simulation for each background source was made to understand the $K_L^0 \to \mu^+\mu^-$ background events qualitatively and quantitatively. The Monte Carlo $K_L^0 \to \pi\mu\nu$ and $K_L^0 \to \pi e\nu$ events whose reconstructed $\mu^+\mu^-$ mass was greater than $480\, MeV/c^2$ were generated by restricting the three-body decay kinematics. The efficiency of the pion to be misidentified as a muon was $1.65 \pm 0.30\,\%$ and that of the electron to be misidentified was $(2.6 \pm 0.2) \times 10^{-3}$ %, which were calculated from the pion and electron samples and were used to calculate the sensitivity.[15]

case 1 Fig. 5.13 ($\times 1.2$)

The background events are in the region $M_{\mu\mu} < 490\,MeV/c^2$. They are nearly flat in the θ^2 distribution.

case 2 Fig. 5.14 ($\times 5.1$)

The backgrounds are concentrated in the small θ^2 region. The events whose $M_{\mu\mu}$ is greater than $487\,MeV/c^2$ are events falsely reconstructed with some of the superposed chamber hits.

case 3 Fig. 5.15 ($\times 2.0$)

The events $M_{\mu\mu} > 490\,MeV/c^2$ are due to the pion decay in the spectrometer region between the third and the fourth drift chambers, and are not rejected by the *UD* cut. The number of background events expected in the fiducial region (and in the region $3\,(mrad)^2 < \theta^2 < 9\,(mrad)^2$) is 0.5 ± 0.5.

case 4 Fig. 5.16 [16] ($\times 2.2 \cdot 10^4$), and Fig. 5.17 [17] ($\times 1.6 \cdot 10^3$)

The events are distributed in the region $M_{\mu\mu} < 525\,MeV/c^2$. The θ^2 distributions of these events are shown in Figs. 5.18 and 5.19. The distributions is not flat, but when they were used for the background subtraction the number of $K_L^0 \to \mu^+\mu^-$ events in the fiducial region is within the systematic uncertainty. The number of background events expected from this decay is less than 0.1.

Fig. 5.20 shows the $M_{\mu\mu}$ distribution of the Monte Carlo background events, which were normalized to the sensitivity of this experiment.

The Monte Carlo simulation of the semileptonic kaon decays did not reproduced the backgrounds in the region $M_{\mu\mu} > 490\,MeV/c^2$ perfectly. Figs. 5.21 and 5.22 show the scatter plots of $M_{\mu\mu}$ vs θ^2 of the $K_L^0 \to \mu^+\mu^-$ events in the PHASE-1 and EXTENSION.

[15]The sensitivities for these Monte Carlo generated data are greater than the experimental sensitivity by the factors indicated in the parentheses.

[16]the pion decay after the spectrometer or being misidentified as a muon

[17]the pion decay before or in the spectrometer

In the region $M_{\mu\mu} > 502\,MeV/c^2$ there is no event in Fig. 5.22, while several events are seen in Fig. 5.21. Since the events whose $M_{\mu\mu}$ is greater than $525\,MeV/c^2$ could not be explained from the kaon decay and the difference between Fig.s 5.21 and 5.22 is the beamline condition, these background events are considered to be induced by neutrons and photons in the neutral beam.

In consequence of the estimation above we concluded that the assumption for the background estimation was correct within the above uncertainty.

5.8 Sensitivity and Branching Ratio

In order to study the effects of the selection criteria on the branching ratio, the quantity called the *double ratio* :

$$\begin{aligned} DR &\equiv \frac{(N_{\mu\mu,data} / N_{\mu\mu,M.C.})}{(N_{\pi\pi,data} / N_{\pi\pi,M.C.})} \\ &\propto \frac{N_{\mu\mu}}{N_{\pi\pi}} \times \frac{A_{\pi\pi}}{A_{\mu\mu}} \end{aligned}$$

was calculated as a function of each selection cut[18] . The dependences of *DR* on χ^2, *Dist*, and *UD* are shown in Figs. 5.23, 5.24, and 5.25. Over reasonable ranges of values of χ^2 and *Dist* the variation of *DR* was less than 1 %, which is smaller than the statistical uncertainty. As for *UD*, when the cut was loosened *DR* changed by at most 3 %. It is the largest deviation due to changes in the selection criteria.

The effect of *UD* was studied in detail. Since the $K_L^0 \to \mu^+\mu^-$ event sample was small it was difficult to distinguish a systematic bias, if it exists, from the effect of the statistical fluctuation. We thereby used, for both the real and Monte Carlo data, large and contamination-free samples of muon and pion tracks from the semileptonic kaon decays (Appendix E). We calculated the double ratio of the muon and pion *tracks* :

$$DR_{L(R)arm} \equiv \frac{\left(N_{\mu\,L(R)arm,data} / N_{\mu\,L(R)arm,M.C.}\right)}{\left(N_{\pi\,L(R)arm,data} / N_{\pi\,L(R)arm,M.C.}\right)}$$

as a function of *UD*, which is shown in Fig. 5.26. The distribution was nearly flat for $UD > 0.05$, and when the cut of *UD* was changed form 0.20 to 0.06 $DR_{L(R)arm}$ changed

[18]The other cut values remained unchanged.

- 1.0004 ± 0.0036 for the left arm and
- 1.0030 ± 0.0037 for the right arm,

with the value of $DR_{L(R)arm}$ at 0.20 set equal to 1.0. This suggests that the variation of DR is mainly due to a statistical fluctuation rather than systematic uncertainty. We included the above values as the correction for the acceptance ratio due to the effect of UD cut. The same study was made for the electron and pion tracks, and the results were : 0.9803 ± 0.0030 for the left arm and 0.9820 ± 0.0031 for the right arm.

For the $K_L^0 \to \mu^+\mu^-$ decay, the effect of changes in the upper limit of θ^2 by $\pm 0.5\,(mrad)^2$ from $\theta^2 = 3\,(mrad)^2$ on the acceptance ratio was about 1 %, It was also included in the uncertainty of the acceptance ratio. The final values of the acceptance ratio were :

$$
\begin{aligned}
A_{\pi\pi} \,/\, A_{\mu\mu} &= 1.192 \pm 0.018 \ , \\
A_{\pi\pi} \,/\, A_{\mu e} &= 1.065 \pm 0.010 \ , \ and \\
A_{\pi\pi} \,/\, A_{ee} &= 1.064 \pm 0.010 \ .
\end{aligned}
$$

The single-event sensitivity (S.E.S.) for the $K_L^0 \to \mu^+\mu^-$ decay is now given as follows :

$$
\begin{aligned}
S.E.S.(K_L^0 \to \mu^+\mu^-) &= \frac{1}{N_{\pi\pi}} \times B.R.(K_L^0 \to \pi^+\pi^-) \\
&\times \frac{A_{\pi\pi}}{A_{\mu\mu}} \times (\,1 - \epsilon^{N.I.}_{\pi\pi\to x}\,) \times \overline{\frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \cdot C^{d.t.}_{\pi\pi\,/\,\mu\mu}} \ .
\end{aligned}
$$

The sensitivities for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays are obtained similarly. All the values necessary for the calculation are summarized in Table E.12. Then the sensitivities are

$$
\begin{aligned}
S.E.S.(K_L^0 \to \mu^+\mu^-) &= (\,4.45 \pm 0.18\,) \times 10^{-11} \ , \\
S.E.S.(K_L^0 \to \mu e) &= (\,4.08 \pm 0.12\,) \times 10^{-11} \ , \ and \\
S.E.S.(K_L^0 \to ee) &= (\,4.16 \pm 0.13\,) \times 10^{-11} \ .
\end{aligned}
$$

The sensitivities of this experiment for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays are better than those previously achieved [18,12,13,14].

The branching ratio for the $K_L^0 \to \mu^+\mu^-$ decay is

$$
B.R.(K_L^0 \to \mu^+\mu^-) \;=\; N_{\mu\mu} \times S.E.S.(K_L^0 \to \pi^+\pi^-) \ .
$$

In this thesis only the statistical fluctuation of $N_{\mu\mu}$ (7.5 %) is defined as the ***statistical*** error, which is the main uncertainty of this measurement. The ***systematic*** error was determined as follows : all the other uncertainties of the values obtained in this experiment are combined in quadrature and then the uncertainty of the $K_L^0 \to \pi^+\pi^-$ branching ratio (2.0 %) is *added*. The experimental result of the $K_L^0 \to \mu^+\mu^-$ branching ratio is

$$
B.R.(K_L^0 \to \mu^+\mu^-) \;=\; (\,7.9 \pm 0.6 \pm 0.3\,) \times 10^{-9} \ ,
$$

where the first uncertainty is the statistical error and the second one is the systematic error, which is about 4 % and smaller than the statistical error. This result is

- above the unitarity limit (Section 1.2),
- consistent with the previous result of our experiment[13] , and
- also consistent with the *recent* result of the BNL E791 experiment[15].

It is shown together with the results of all the previous experiments in Fig. 5.27.

We also obtained the upper limits for the $K_L^0 \to \mu e$ and $K_L^0 \to ee$ decays as [38,41]:

$$
\begin{aligned}
B.R.(K_L^0 \to \mu e) &< 2.3 \times S.E.S.(K_L^0 \to \mu e) \\
&= 9.4 \times 10^{-11} \ , \ and \\
B.R.(K_L^0 \to ee) &< 3.9 \times S.E.S.(K_L^0 \to ee) \\
&= 1.6 \times 10^{-10} \ .
\end{aligned}
$$

5.9 Checks on the Result

The following studies were performed to support the correctness of our analysis.

1. **PHASE-1 and EXTENSION :**

   $M_{\mu\mu}$ vs θ^2 scatter plots are already shown in Figs. 5.21 and 5.22. The number of $K_L^0 \to \mu^+\mu^-$ events in the fiducial region was 111 in the PHASE-1 and 68 in the EXTENSION. Based on each sample the $K_L^0 \to \mu^+\mu^-$ branching ratio[19] of $(7.8 \pm 0.7) \times 10^{-9}$ in the PHASE-1 and $(8.0 \pm 1.0) \times 10^{-9}$ in the EXTENSION were obtained. They are within the systematic error of the result based on the total data sample.

The ratio

$$\frac{N^{0n}_{\mu\mu}}{N^{n}_{\pi\pi}} \times \frac{\epsilon^{n}_{\pi\pi}}{\epsilon^{n}_{\mu\mu}} \times C^{m}_{\pi\pi/\mu\mu}$$

in each cycle is shown in Table E.11. These values are consistent with the value based on the total sample (3.49×10^{-6}) within one standard deviation, except for those of the cycles K, M, O, and W. These variations are regarded as statistical fluctuations. This can be seen by the fact that if we omit, for example, the K cycle the branching ratio becomes $(7.7 \pm 0.6) \times 10^{-9}$. The branching ratio in each cycle is shown in Fig. 5.28(a) and, as a function of proton beam intensity, in Fig. 5.29.

2. **Para and Spara triggers :**
   For the $K_L^0 \to \mu^+\mu^-$ decays Para trigger of the hodoscope signals was used, while for the $K_L^0 \to \pi^+\pi^-$ decays Spara trigger was used. When the branching ratio was calculated using only the $K_L^0 \to \pi^+\pi^-$ events that satisfied the parallel requirements of the hodoscopes, the results was $(8.0 \pm 0.6) \times 10^{-9}$ and was within the systematic error.

3. **Kaon momentum, vertex point, and X coordinate :**
   The branching ratio was calculated as functions of kaon momentum, vertex point, and the X coordinate of the track at the exit of the decay chamber. The results, which were all consistent within the statistical uncertainty, are shown in Fig. 5.28(b),(c),(d).

4. **$K_L^0 \to \mu^+\mu^-$ event Identification :**
   To confirm that none of the counters used for the muon identification biased the

[19]Only the statistical error will be quoted in this section.

$K_L^0 \to \mu^+\mu^-$ events, the branching ratio was calculated by changing the muon identification process. The results are summarized in Table E.13.

(a) Only the muon identifier was used :
The background events came in because the electron from the $K_L^0 \to \pi e \nu$ decay was not rejected. When the θ^2 distribution of the $K_L^0 \to \pi e \nu$ events mentioned in Section 5.7 was used, however, the branching ratio was $(8.1 \pm 0.6) \times 10^{-9}$.

(b) Only the Čerenkov counter and the shower counter were used :
In contrast to the above case the muon identifier was used only for the trigger, and the pion that punched through the first and second iron blocks was not rejected.

(c) The muon identifier and the Čerenkov counter or the shower counter were used :
These are for a check on the muon veto effect on the electron identification counters.

They are all consistent with the value obtained by the normal muon identification process.

5. **Magnetic field :**
   The magnetic field dependence of the acceptance for each decay mode is shown in Fig. 5.30.
   Since the peak of the invariant mass distribution of the $K_L^0 \to \pi^+\pi^-$ events using the measured field map was $497.49\,MeV/c^2$, which was only $0.18\,MeV/c^2$ lower[20] than the kaon mass, we concluded that the effect of the magnetic field on the acceptance ratio was much less than the uncertainty quoted above. Fig. 5.31 shows the distribution of the track angle of the $K_L^0 \to \pi^+\pi^-$ events at the hodoscope H2 in the X-Z plane for three sets of the magnetic field : the normal field used for the experiment ($B = 1.0$), and the field reduced by 5% ($B = 0.95$) and

[20]It corresponded to the momentum shift of about 0.06%.

by 10 % (B = 0.90). Because the neighboring H2 counters are overlapped by 4.8 cm, the track which satisfied the semi-parallel coincidence passed one of five counter regions shown in the figure. When the field changed the track angle distribution also changed, but the data (bars) agreed well with the Monte Carlo result (histogram) for each magnet field set.

At the end of these studies we concluded that we had correctly determined the branching ratio from our data sample.

Chapter 6

Theoretical Discussion

6.1 Top Quark Mass and Mixing Parameters

Before a theoretical discussion on the $K_L^0 \to \mu^+\mu^-$ branching ratio, what we know about the mass of the top quark and the quark-mixing parameters from other experiments are briefly summarized in this section.

The top quark unfortunately has not been discovered yet. The present lower limit of the top quark mass (m_t) is $89\,GeV/c^2$ at 95 % confidence level [42] from the CDF experiment at the Tevatron, a proton-antiproton collider ($\sqrt{s} = 1.8\,TeV$). The constraint from a fit to the electroweak parameters precisely measured by the LEP electron-positron collider experiments is $140 \pm 40\,GeV/c^2$ [1] .

The quark mixing in the charged weak currents is represented in the Standard Model by a 3 × 3 unitary matrix called the Kobayashi-Maskawa (KM) matrix [44] :

$$
\begin{aligned}
V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\
&\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} ,
\end{aligned}
$$

[1]This value is quoted from [43]. The error includes experimental and theoretical uncertainties, and the uncertainty of the Higgs mass from 50 to 1000 GeV/c^2.

where in the second matrix the Maiani-Wolfenstein parametrization [45] is used. The elements of the third-row of this matrix, V_{td}, V_{ts}, and V_{tb}, have to be determined by the *indirect* measurement of the loop-induced processes of the kaon and B-meson.

There are four independent parameters in the above parametrization :

$$\lambda\,,\ A\,,\ \rho\,,\ \textit{and}\ \ \eta\ .$$

$\lambda = 0.22$ is the sine of the Cabibbo angle [46]. The B-meson lifetime and the semileptonic $b \to c$ decay rate give [47] $|\,V_{cb}\,| = 0.044 \pm 0.007$ or $A = |\,V_{cb}\,|\,/\,\lambda^2 = 0.91 \pm 0.14$. The constraints on ρ and η come from

1. $b \to u$ transition in the semileptonic B decay ($|\,V_{ub}\,/\,V_{cb}\,|$),
2. CP-violation parameter ϵ in the $K^0 - \bar{K}^0$ transition, and
3. $B_d^0 - \bar{B}_d^0$ mixing parameter x_d ,

and they depend on the following parameters : m_t, B-meson decay constant (f_B), and bag constants of kaon (B_K) and B-meson (B_B). One example of the theoretical calculation[2] is :

1. $\sqrt{\rho^2 + \eta^2} = 0.50 \pm 0.18\,,$
2. $A^2\eta\,[0.8 + 1.43A^2(1-\rho)B(x_t)] = (0.525 \pm 0.006)\,/\,B_K\,,$ *and*
3. $A^2\,[(1-\rho)^2 + \eta^2]\,B(x_t) = (3.1 \pm 0.6)\left(130\,MeV\,/\,f_B\sqrt{B_B}\right)^2\,,$

where

$$B(x_t) \equiv \frac{x_t}{4}\left[1 + \frac{3 - 9x_t}{(x_t - 1)^2} + \frac{6x_t^2 \ln x_t}{(x_t - 1)^3}\right]$$

is a box-diagram function calculated by Inami and Lim [49] with $x_t \equiv m_t^2/M_W^2$. The following values are used for the above formula : $|\,V_{ub}\,/\,V_{cb}\,| = 0.11 \pm 0.04$, $|\,\epsilon\,| = (2.272 \pm 0.022) \times 10^{-3}$, and $x_d = 0.71 \pm 0.11$. Normally the values $B_K = 0.8 \pm 0.2$ and $B_B = 1$ are assumed. It is pointed out [43,47] that recent lattice calculations favor

[2]In this thesis I use the formula in [47], which is an up-to-date version of [48]. They are essentially the same as the formula in [43].

$f_B = 250 \pm 50\,MeV$, which is larger than the value $f_B = 130 \pm 40\,MeV$ favored in the previous KM parameter calculations [50].

The allowed regions in the ρ-η parameter space for m_t of 100, 140, 180, and $250\,GeV/c^2$, and for f_B of $130 \pm 40\,MeV$ and 250 ± 50 are shown in Figs. 6.1 and 6.2. A is assumed to be equal to 0.91 in this analysis. The solid semicircles with centers at $(\rho,\eta) = (0,0)$ indicate the constraints from $|\,V_{ub}\,/\,V_{cb}\,|$, the dotted hyperbolas of the approximate form $\eta(1-\rho)m_t^2$ indicate the constraints from $|\,\epsilon\,|$, and the dashed circular arcs with centers at $(1,0)$ indicate the constraints from $B_d^0 - \bar{B}_d^0$ mixing. As m_t increases the the allowed range of ρ becomes narrower. There are two sets of allowed region : $\rho < 0$ and $\rho > 0$. The region $\rho > 0$ is favored when $f_B = 250 \pm 50\,MeV$ is used.

Once m_t is measured the parameters ρ and η will also be determined more accurately. If, after more precisely measurements, the boundaries of the above constraints *do not overlap*, it will be a signal of a new physics beyond the Standard Model.

6.2 Constraints from $K_L^0 \to \mu^+\mu^-$ Branching Ratio

For the theoretical discussion I combine the statistical and systematic errors of the branching ratio in quadrature and use as the branching ratio

$$B.R.(K_L^0 \to \mu^+\mu^-) \;=\; (\,7.9 \pm 0.7\,) \times 10^{-9}\ .$$

The difference between this branching ratio and the *unitarity limit* described in Section 1.2 : $U.L.(K_L^0 \to \mu^+\mu^-) = (6.83 \pm 0.32) \times 10^{-9}$, is

$$\begin{aligned} \Delta B.R. &\equiv B.R.(K_L^0 \to \mu^+\mu^-) - U.L.(K_L^0 \to \mu^+\mu^-) \\ &= (\,1.1 \pm 0.7\,) \times 10^{-9}\ . \end{aligned}$$

It is the contribution from the *real part* of the $K_L^0 \to \mu^+\mu^-$ decay amplitude, $Re\,A_{\mu\mu}$, which consists of

- the long-distance part from the 2γ exchange diagram, A_{EM}, and

- the short-distance part from the Z^0 exchange diagrams and the W box diagram, A_W.

These diagrams are already shown in Fig.1.1. I define the decay amplitude $A_{\mu\mu}$ with the convention in [51] :

$$\begin{aligned}
\frac{B.R.(K_L^0 \to \mu^+\mu^-)}{B.R.(K_L^0 \to \gamma\gamma)} &= \frac{\Delta B.R. + U.L.(K_L^0 \to \mu^+\mu^-)}{B.R.(K_L^0 \to \gamma\gamma)} \\
&\equiv \frac{2\beta}{\pi^2}\left(\frac{m_\mu}{m_K}\right)^2 |A_{\mu\mu}|^2 \\
|A_{\mu\mu}|^2 &\equiv |Re\, A_{\mu\mu}|^2 + |\, Im\, A_{\mu\mu}|^2 \ , \ and \\
Re\, A_{\mu\mu} &= A_{EM} + A_W \ ,
\end{aligned}$$

where

$$R \equiv \frac{2\beta}{\pi^2}\left(\frac{m_\mu}{m_K}\right)^2 = 8.27 \times 10^{-3} \ .$$

The short-distance part is calculated from the standard electroweak theory. It depends on the quark masses and the KM matrix element as follows [48] :

$$|A_W| \propto \left| Re\left(\sum_{i=c,t} \eta_i V_{is}^* V_{id} C(x_i)\right)\right| \ ,$$

where [49]

$$C(x_i) \equiv \frac{4x_i - x_i^2}{4(1-x_i)} + \frac{3x_i^2 \ln x_i}{4(1-x_i)^2}$$

and Re means that only the *CP-conserving* decay

$$K_L^0(\,CP-\,) \to \mu^+\mu^-(\,^1S_0\,, CP-\,)$$

contributes to the $K_L^0 \to \mu^+\mu^-$ decay. $Re\,(V_{cs}^* V_{cd}) = \lambda(1-\lambda^2/2) \sim 0.22$ is larger than $Re\,(V_{ts}^* V_{td}) = A^2\lambda^5(1-\rho) \sim 5 \times 10^{-4}$ but $C(x_c) \sim 6 \times 10^{-4}$ is much smaller than $C(x_t) > 1$, and the charm-quark contribution is negligible. $\eta_t \simeq 1$ is the QCD correction factor. $|A_W|$ can be written as :

$$|A_W| = 9.28 \times 10^{-3} \times A^2\,|1-\rho|\,C(x_t) \ .$$

The $K_L^0 \to \mu^+\mu^-$ branching ratio gives constraints on m_t and ρ (or $Re\, V_{td}$) and, when m_t is assumed, gives a *vertical line* in the ρ-η space.

The estimate of the long-distance part, i.e., the 2γ exchange diagram with either of the two photons is off-shell, is ambiguous and model dependent. There are three models which were used for the previous theoretical analyses.

model A [52]

It assumes that no accidental cancellation occurs between A_{EM} and A_W, and neglects the contribution from A_{EM}. As a consequence this model gives only an upper limit of $|A_W|$ from the experimental value.

model B [53]

A_{EM} is estimated from the decay $\eta \to \mu^+\mu^-$, in which the contribution from the *short-distance* part (weak interaction) can be ignored. It is assumed that the ratio of the real to imaginary parts of the amplitude is the same for $K_L^0 \to \mu^+\mu^-$ as for $\eta \to \mu^+\mu^-$, and the ratio is calculated using $B.R.(\eta \to \mu^+\mu^-) = (6.5 \pm 2.1) \times 10^{-6}$ [18] and $U.L.(\eta \to \mu^+\mu^-) = (4.31 \pm 0.06) \times 10^{-6}$ from $B.R.(\eta \to \gamma\gamma) = (38.9 \pm 0.5) \times 10^{-2}$ [18] :

$$\left|\frac{A_{EM}}{Im\, A_{\mu\mu}}\right|^2 = 0.51 \pm 0.49$$

model C [54,51]

It is based on the *vector meson dominance* model. At first the 2γ diagram with one photon *on-shell* and the other photon *off-shell* is considered. It is assumed that its amplitude has two contributions (Fig. 6.3) :

- $|\Delta S| = 1$ transition between pseudoscaler mesons $K_L^0 \to \pi, \eta, \eta'$ followed by $\pi, \eta, \eta' \to \gamma\gamma^*$, and
- $K_L^0 \to K^*\gamma$ transition followed by $|\Delta S| = 1$ transition between vector mesons $K^* \to \rho, \omega, \phi$ and $\rho, \omega, \phi \to \gamma^*$.

The latter contribution[3] vanishes when the virtual photon is saturated by the real photon, and its ratio with respect to the former contribution is represented by the parameter α_K. The recent measurements of the form factor of the K_L^0 Dalitz

[3]Model B is regarded as a model that neglects this contribution (" pole dominance ").

decay $K_L^0 \to ee\gamma$ give *nonzero* values of α_K : -0.28 ± 0.13 [55] and $-0.280 \pm 0.083^{+0.054}_{-0.034}$ [56].

The estimate of the 2γ amplitude with ***both photons off-shell*** is more ambiguous. Using $\alpha_K = -0.28$ it was estimated [51][4] that

$$-0.6 \times 10^{-2} \leq A_{EM} = -A_W \pm \sqrt{\frac{\Delta B.R.}{B.R.(K_L^0 \to \gamma\gamma) \cdot R}} \leq 0.3 \times 10^{-2} \ ,$$

where the minimal and maximal values arise from saturating one virtual photon by vector mesons and both virtual photons, respectively, and

$$\left| \frac{A_{EM}}{Im\, A_{\mu\mu}} \right|^2 \leq 0.025 \ ,$$

which suggests that the contribution of the real part of the 2γ exchange diagram is small with respect to the imaginary part.

I adopt **model C** for theoretical discussion in this thesis. The allowed range of $|A_W|$ is finally rewritten as follows [5] :

$$\begin{aligned} & \left(\sqrt{\frac{\Delta B.R.}{B.R.(K_L^0 \to \gamma\gamma) \cdot R}} - 0.6 \times 10^{-2} \right) \\ < \quad & 7.7 \times 10^{-3} \times |1 - \rho|\, C(x_t) \ < \ \left(\sqrt{\frac{\Delta B.R.}{B.R.(K_L^0 \to \gamma\gamma) \cdot R}} + 0.6 \times 10^{-2} \right) \ . \end{aligned}$$

Fig. 6.4 shows the allowed range for m_t as a function of $\Delta B.R.$, where $-0.30 < \rho < 0.38$ is assumed. $\Delta B.R. = (1.1 \pm 0.7) \times 10^{-9}$ in this experiment indicates that m_t is less than $310\, GeV/c^2$. It is consistent with the results obtained from the precise Z^0 measurements at LEP. Though this m_t range is quite large with an upper bound which is not very stringent, it is obtained from the *low-energy* kaon rare decay and is complementary to the *very-high-energy* Z^0 physics. Furthermore, it does not depend on the Higgs sector.

The *lower* bounds on ρ for m_t of 100, 140, 180, and $250\, GeV/c^2$ are also shown in Figs. 6.1 and 6.2 as vertical lines[6] . The $K_L^0 \to \mu^+\mu^-$ branching ratio gives more

[4] A more stringent limit is expected to be obtained by future experiments of $K_L^0 \to \mu^+\mu^-\gamma$ and $K_L^0 \to e^+e^-e^+e^-$ decays.

[5] Both the positive and negative interferences between A_{EM} and A_W are taken into account.

[6] The solid lines indicate the bounds when $\Delta\, B.R. = 1.1 \times 10^{-9}$, the dotted lines indicate the bounds when $\Delta\, B.R. = 1.8 \times 10^{-9}$, and the dashed lines indicate the bounds when $\Delta\, B.R. = 0. \times 10^{-9}$.

important constraints, compared with the constraints from $B_d^0 - \bar{B}_d^0$ mixing, on ρ as m_t becomes heavier, and especially for the case of $f_B = 130 \pm 40\, MeV$. The top quark mass greater than $200\, GeV/c^2$ is barely consistent with the constraints from the kaon and B-meson decays.

6.3 Prospects for other Rare Kaon decays

At the end of this thesis I briefly mention two kaon decay modes which occur via almost the same processes as the $K_L^0 \to \mu^+\mu^-$ decay but, in contrast to it, the long-distance contributions are estimated to be negligible.

$K^+ \to \pi^+\nu\bar{\nu}$:

The diagrams of this decay are shown in Fig 6.5. In this decay there is no contribution involving photons. The branching ratio was calculated only from the short-distance contributions [47,48] :

$$\begin{aligned} B.R.(K^+ \to \pi^+\nu\bar{\nu}) \ &= \ 10^{-6} \left| D(x_c) + 3.3 \times 10^{-3} A^2 (1 - \rho) D(x_t) \right|^2 \\ &\quad + 1.08 \times 10^{-11} A^4 \eta^2\, |\, D(x_t)\, |^2 \\ &\propto \ \left| \sum_{i=c,t} \eta_i V_{is}^* V_{id} D(x_i) \right|^2 \ , \end{aligned}$$

where[49]

$$D(x_i) \equiv \frac{x_i}{4} \left[\frac{3(x_i - 2)}{(x_i - 1)^2} \ln x_i + \frac{x_i + 2}{x_i - 1} \right] \ ,$$

and is expected to be in the range of 10^{-10}. The charm-quark contribution is not negligible. The present upper limit of the branching ratio is 3.4×10^{-8}[18], and will be improved by E787 experiment at BNL [57]. When this decay is observed and the branching ratio is measured, it will give new constraints on the ρ-η space.

$K_L^0 \to \pi^0\nu\bar{\nu}$:

To this decay only the CP-violating amplitude contributes and, furthermore, the *direct CP-violating* amplitude dominates over the *indirect* one[58,47,48].

$$B.R.(K_L^0 \to \pi^0\nu\bar{\nu})_{indirect} \ \sim \ 5 \times 10^{-15} \ , \ \textit{and}$$

$$
\begin{aligned}
B.R.(K_L^0 \to \pi^0 \nu \bar{\nu})_{direct} &= 4.61 \times 10^{-11} A^4 \eta^2 \,|\, D(x_t) \,|^2 \\
&\propto \left| Im \left(\sum_{i=c,t} \eta_i V_{is}^* V_{id} D(x_i) \right) \right|^2 .
\end{aligned}
$$

The branching ratio is expected to be in the range of 10^{-11}. This decay gives constraints on η (*horizontal line* in the ρ-η space), and provides a clean test for CP violation in the Standard Model. It is very difficult to detect the $K_L^0 \to \pi^0 \nu \bar{\nu}$ decay because of the lack of kinematical constraints on the events, and there has been no dedicated search for it.[7] However, some people have started a feasibility study for an experiment to search for this decay [59,60].

[7]An upper limit of $B.R.(K_L^0 \to \pi^0 \nu \bar{\nu})$ of $\sim 1\,\%$ is obtained [58] from precious data.

Chapter 7

Conclusions

A *precise* measurement of the branching ratio of the decay of the long-lived neutral kaon into two muons has been made at the KEK 12-GeV Proton Synchrotron. 179 $K_L^0 \to \mu^+\mu^-$ events were observed in the fiducial region. Normalizing to the $K_L^0 \to \pi^+\pi^-$ events, which were simultaneously accumulated, the single-event sensitivity was 4.45×10^{-11} and the result was :

$$
B.R.(K_L^0 \to \mu^+\mu^-) \;=\; (\,7.9 \pm 0.6 \pm 0.3\,) \times 10^{-9} \;.
$$

The first uncertainty (7.5 %) is the *statistical* error and the second one (4 %) is the *systematic* error. This value of the branching ratio is above the theoretical lower bound (unitarity limit), consistent with the previous result of our experiment, and also consistent with the recent result of the BNL E791 experiment. In the Standard Model the $K_L^0 \to \mu^+\mu^-$ branching ratio gives constraints on the top-quark mass and the Kobayashi-Maskawa matrix parameter ρ. Using a theoretical model, the above result indicates that the top-quark mass is less than $310\,GeV/c^2$ and the heavier the mass of the top quark is the more important lower bounds are given on ρ.

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Appendix A

E137 Monte Carlo Simulation

A Monte Carlo program simulated events from K_L^0 decays in the E137 detector system. It consisted of five steps : kaon generation, two-body or three-body decay generation, track generation, drift chamber and hodoscope data simulation, and superposition of accidental hits. Monte Carlo data, whose format was the same as that of the real data, were also reconstructed by the RECOND and RESP programs, and both sets of data were compared with the same selection criteria.

A.1 Kaon generation

The point where a kaon was generated was determined randomly within the target ($10\,mm$ in diameter and $120\,mm$ long). The flight direction was chosen so that the kaon was projected in a cone of the first collimator ($7\,mrad$ half-cone angle). The kaon momentum was chosen from a "modified" spectrum of Sanford-Wang's formula (see below) between 1 and $10\,GeV/c$, and its lifetime was chosen from the exponential distribution with the K_L^0 mean life. The momentum spectrum was weighted with the probability of decaying within the decay chamber ($-1000\,cm \leq Z \leq 0\,cm$), and the sampling range of the lifetime was limited so as to make a kaon with the given momentum decay in it. The weighted spectrum is shown in Fig A.1(a).

The kaon momentum spectrum calculated by Sanford-Wang's formula for $0\,degree$ production, $f_0(p)$, was modified by comparing the Monte Carlo $K_L^0 \to \pi^+\pi^-$ events and

the real $K_L^0 \to \pi^+\pi^-$ events obtained in a *very low* intensity beam run ($\sim 1 \times 10^{11}$ ppp, $7\,K$ events). The momentum distributions of both sets of events were compared, and the Monte Carlo spectrum, $f(p)$, was modified until two distributions agreed with each other. The correction factor, $r(p)$, was parameterized as follows :

$$\begin{aligned} r(p) &\equiv \frac{f(p)}{f_0(p)} \\ &= \exp(-2\,\alpha \cdot p) \\ &\quad \times (0.9823 + 0.0766p - 0.0327p^2 + 0.0033p^3) \\ &\quad \times (0.9139 + 0.0730p - 0.0182p^2 + 0.0013p^3)\,, \end{aligned}$$

where p was the momentum (in GeV/c) and α was a fitting parameter[1] : $\alpha = 0.136$. The modified kaon spectrum is shown in Fig. A.1(b).

The data and Monte Carlo results are compared in Section 4.4. The observed momentum distribution of the $K_L^0 \to \pi^+\pi^-$ events in the EXTENSION ($2\,degrees$ production) was quite similar to the distribution in the PHASE-1 ($0\,degree$ production). Thus the momentum spectrum was not changed in the simulation of the EXTENSION data.

A.2 Decay generation

The decay of the kaon was simulated in the rest frame, and then the momentum vectors of the secondary particles were Lorentz-transformed to the laboratory frame using the kaon momentum vector. For each of the unstable secondary particles (pion or muon) its lifetime was chosen using the mean life as done for the kaon.

The two-body decay occurred isotropically. For three-body decays, the energies of the pion and the lepton were chosen using the standard parameterization of the V-A Dalitz plot density[18].

[1] The two polynomial terms were the result of fine tuning.

A.3 Track generation

Each secondary (charged) particle was projected straight from the kaon decay point to the exit window, and was projected through the spectrometer, hodoscope H1, Čerenkov counter, and hodoscope H2. When both particles traversed H1 and H2 in each arm with a parallel requirement (for the dilepton decays) or a semi-parallel requirement (for the $K_L^0 \rightarrow \pi^+\pi^-$ decay and the three-body decays), this event was accepted.

In the spectrometer, where the magnetic field was calculated from the same field map used for the tracking, the projection was performed using a fourth-order Runge-Kutta method [34]. 46 planes for the spline interpolation in the tracking program (Section 4.1) were also used for the Runge-Kutta projection, and the step lengths are shown in Table D.1. At each step the X and Y positions and directions of the particle were changed due to multiple scattering in the material between two planes. The standard multiple scattering formula [18,35] for the width of the projected angular distribution was used :

$$\theta_0 = \frac{13.6MeV}{\beta c p} z \sqrt{x/X_0}\, [1 + 0.038 \ln(x/X_0)] ,$$

where θ_0, p, βc and z were the width, momentum (in MeV/c), velocity, and charge number of the incident particle (in this case $z = 1$), and x/X_0 was the thickness of the medium in radiation lengths. Furthermore, a *non*-Gaussian distribution, which was numerically calculated by Marion and Zimmerman[61], was used because large-angle scatters could not be simulated from the Gaussian distribution. This distribution was scaled so that the 1/e point was equal to the above θ_0. The distribution curves are shown in Fig. A.2, where B is a parameter which depends on the material. The position displacement due to scattering was calculated as if the deflection all occurred at the depth $\delta x \times 2/\sqrt{3}$, where δx was a step length.

After the spectrometer, the particle was projected straight through the counters with the effect of multiple scattering taken into account. The projection through the shower counter and the muon identifier was not performed.

Before the projection to the next plane, the lifetime of the unstable particle was checked using the total path length. The muon track was discarded if it decayed. If the

pion decayed within the next step, it was projected back to its decay position and the secondary particle, muon, was generated and projected from the decay point.

A.4 Data simulation

From the X and Y positions of the projected track at each chamber plane the hit wire number and the drift length were calculated. The drift length was smeared by a Gaussian distribution with the spatial resolution of the chamber and was converted to the drift time using the space-drift time relation. The time was then digitized to the TDC channel number. The actual chamber parameters in the experiment and the surveyed value of the displacement of each chamber position were used for these calculations, and the effect of the signal propagation time in the wire was included[2]. Chamber efficiencies and dead wires were also simulated. The track not within each chamber's fiducial region was discarded.

The X position of the track in the hodoscope was transformed to the hit counter number, and hodoscope TDC channel numbers were smeared to reproduce the meantime distributions.

The detector simulation was not performed for the Čerenkov counter, the shower counter, and the muon identifier. The performance of these detectors was included in the Monte Carlo simulation using the particle-identification efficiencies measured from the real data of the semileptonic decay sample (Section 5.4). However, this simulation was sufficient to process the Monte Carlo events by the RECOND and RESP programs.

A.5 Superposition of accidental hits

As shown in Table C.3 there were extra hits in the chamber planes and hodoscopes, while in the Monte Carlo data there was only one hit at the track position. Thus, by the simple detector simulation described above, it was impossible to simulate the effect of accidental hits on the tracking process such as the reduction of the track reconstruction

[2]The velocity of the signal in the wire was $25\,cm/nsec$ from the measurement.

efficiency.

To include this effect the technique of superposing "accidental[3]" hits on Monte Carlo events was used. A special run, called the "pulser-run," which was randomly triggered by a pulse-generator signal, was made under various beam conditions in the PHASE-1 and EXTENSION. The chamber and hodoscope TDC values of a pulser-run events were superposed on those of a Monte Carlo event. When the same wire or counter had both a real hit and an accidental hit, the TDC value which started earlier was taken.

The TKO-standard TDC used for the drift chamber TDC accepted only the signal which came after the trigger gate opened. If a proper signal came immediately after an accidental signal overlapping the leading edge of the gate signal, the two signals could not be discriminated, and as a result the proper signal was "lost." It is called the *occupation* effect, which occurs under a high counting-rate environment. To simulate the occupation effect the chamber TDC values of another event in the same pulser-run were used. When the same wire had two such hits, the TDC value of the Monte Carlo event and that of the pulser-run event, which was shifted to simulate the accidental chamber signal, was compared. If it was found that two signals were not separated, the Monte Carlo TDC hit of this wire was discarded.

[3]Here "accidental" means "not due to the hodoscope trigger".

Appendix B

Details of the RECOND Program

B.1 Hit selection

The timing requirements for the drift chamber hits and hodoscope hits were as follows.

drift chamber The drift time calculated from the TDC was in the range from $-20\,nsec$ to $130\,nsec$.

hodoscope H1 The difference of the meantime (the average of the up and down TDC's) from that of at least one H1 counter in the other arm was within $3\,nsec$.

At least one properly hit counter in each hodoscope, at least one hit wire in the X or X' plane and one hit wire in the Y or Y' plane in each chamber were required. In addition, it was required that more that eight chamber planes in the X- and Y-views of each arm had at least one hit wire. Otherwise, the event was discarded at this step.

B.2 Y-view track finding

The Y-view hit positions in the five chambers, Y_1, Y_2, Y_3, Y_4, and Y_5, were calculated from the mid-point of two adjacent hit wires in the staggered planes (Y and Y') or the wire position when the adjacent wire had no hit. Then the algorithm was as follows.

- Take any hit in the chamber W5 and any hit in W4 within $7.2\,cm$ (= wire spacing $\times 8$) of each other.

- Construct a straight line from Y_5 to Y_4. For each line,
  - Project the line to W3, take any hit in W3 within 3.6 cm of the projected position (Fig. 4.1 (b)), and construct a new line from Y_5, Y_4, and Y_3 using the least-square method.
  - For each line project it to W2, take any hit in W2 within 3.6 cm of the projected position (Fig. 4.1 (c)), and construct a new line from Y_5, Y_4, Y_3, and Y_2.
  - Project each line to W1, and take any hit in W1 within 5.4 cm of the projected position (Fig. 4.1 (d)).
  - Require that the wires of at least 7 planes were hit, and calculate[1] the χ^2 of fitting $Y_1 \sim Y_5$ to a line[2], referred to as χ^2_Y.

  In each combination of Y_5 and Y_4, store only one combination of hit wires in W1, W2, and W3 that gave the minimum χ^2.
- Arrange the wire hit combinations in the order of ascending χ^2, and store at most 100 combinations for the next step.

B.3 X-view track finding

The X-view hit positions in the five chambers, X_1, X_2, X_3, X_4, and X_5, were calculated in the same way as the Y-view positions.

- Take any hit in the W5 and any hit in W4 within 8.1 cm of each other.
- Construct a line from X_4 to X_5, and project it to the hodoscope H1 and H2. Pick an H1 counter whose center position was within 5.4 cm of the projected position, and an H2 counter within 10.8 cm of the projected position.
- Project the line to the center of the downstream magnet ($Z = Z_{dm}$) and calculate the intersection position X_{dm} (Fig. 4.2 (b)).

[1] In the track finding procedures a reduced χ^2, *i.e.*, χ^2/(degree of freedom) was used.

[2] Only in this case a bent line defined in the next section was used instead of a straight line.

- Take any hit in W3 within 17.1 cm of X_5, project a line from X_{dm} through X_3 to the center of the upstream magnet ($Z = Z_{um}$), and calculate the intersection position X_{um}.

  Here the X-view of the track was assumed to be a bent line with three parameters aa, bb, and tt.

$$X = \begin{cases} aa \times Z + bb - tt \times (Z - Z_{um}) & Z \le Z_{um} & [X_1 \to X_2 \to X_{um}] \\ aa \times Z + bb & Z_{um} \le Z \le Z_{dm} & [X_{um} \to X_3 \to X_{dm}] \\ aa \times Z + bb + tt \times (Z - Z_{dm}) & Z_{dm} \le Z & [X_{dm} \to X_4 \to X_5] \end{cases}$$

  For each combination of X_5, X_4, and X_3, from which a bent line was determined exactly, :
  - Project the bent line from X_{um} to W2[3], take any hit in W2 within 9.0 cm of the projected position (Fig. 4.2 (c)), and construct a new bent line from X_2, X_3, X_4, and X_5 using the least-square method.
  - Project each new line to W1 and take any hit within 4.5 cm of the projected position (Fig. 4.2 (d)).
  - Require that the wires of at least 7 planes were hit, and fit $X_1 \sim X_5$ to a bent line. Make sure that the center positions of the H1 and H2 counters were within 10.0 cm of the projected positions of the bent line, and calculate the fitting χ^2, χ^2_X.

  Store only one combination of W1 and W2 hit wires that gave the minimum χ^2.
- Arrange the wire hit combinations in the order of ascending χ^2, and store at most 100 combinations for the next step.

The criteria of acceptable hits in each chamber were chosen from a study of a COND-analysis event sample.

[3] A correction for the 50 $mrad$ tilt of W1 and W2 with respect to the beam line was included.

B.4 Three-dimensional track finding

- Calculate the average of χ^2_X and χ^2_Y referred to as χ^2_{XY}, for all the combinations of the X-view tracks and the Y-view tracks. Arrange and store at most 50 combinations in the order of ascending χ^2_{XY} in each arm.

- Combine all three-dimensional tracks in the left arm with all tracks in the right arm, and calculate the **vertex distance**, **vertex point**, **kaon momentum**, **invariant mass**, and **collinearity angle** defined in Section 4.1.

- Apply the following rough cuts on these quantities.

  - $Dist \leq 30\,cm$
  - $-1500\,cm \leq V_Z \leq 100\,cm$
  - $M_{\mu e}, M_{e\mu}, M_{\mu\mu} \geq 430\,MeV/c^2$ [4]
  - $p_K \times \theta^2 \leq 0.55\,MeV/c \cdot (\,rad\,)^2$ [5]

- Calculate the average of the χ^2's of tracks of the left and right arms, χ^2_{LR}, and the vertex χ^2, $\chi^2_V \equiv (Dist/1.0\,cm)^2$. Then calculate the averages of χ^2_{LR} and χ^2_V. Arrange the combinations in the ascending order of this quantity and store at most 10 combinations for the next step.

B.5 Spline fitting of the track

For each wire hit combination the algorithm made the following analysis.

- Solve the left-right ambiguity of the wires in each chamber. When wires in staggered planes had hits, $Dsum$ defined in Section 4.1 was calculated. The combination with $-0.23\,cm \leq Dsum \leq 0.23\,cm$ was regarded as the proper adjacent-hit pair with the spatial resolution of $400\,\mu m$ for each wire. If not, the adjacent wires were regarded as uncorrelated single-hit wires. The left-right ambiguity of a

[4]For ee and $\pi\pi$ events no invariant mass cut was applied.

[5]This quantity was used because its distribution had no dependence on the kaon momentum.

single-hit wire was not resolved, and the wire position was used as a hit position. Its spatial resolution was assumed to be $0.35\,cm$.

- Reconstruct a track using the spline method. A track was represented by a linear combination of five track parameters :

  - two spatial coordinates, X_0 *and* Y_0,
  - two direction tangents, $X'_0 \equiv \left.\frac{dX}{dZ}\right|_0$ *and* $Y'_0 \equiv \left.\frac{dY}{dZ}\right|_0$, and
  - the inverse of the momentum with proper sign, $\frac{1}{p}$,

  at the entrance to the spectrometer.

$$X_i = X(Z_i) = X_0 + X'_0 \cdot Z_i + \frac{1}{p} \cdot \bar{X}(Z_i)$$

$$Y_i = Y(Z_i) = Y_0 + Y'_0 \cdot Z_i + \frac{1}{p} \cdot \bar{Y}(Z_i)$$

$\bar{X}(Z)$ and $\bar{Y}(Z)$ were the double integrated cubic spline function. Track parameters were determined from the least squares fit of the coordinates in the chamber planes to this track model. The bent line obtained in the previous step was used as a starting fit, and the magnetic field at the line in each plane was calculated from the field map using the third order Bessel interpolation formula. The spline fitting procedure was iterated four times in each arm to determine the track parameters.

B.6 Selection of the candidate

Using the track parameters in each arm, the kinematical quantities and the vertex χ^2 were calculated once again, and the final selection χ^2, χ^2_{final}, was defined as :

$$\chi^2_{final} \equiv \frac{1}{f} \cdot \left(\chi^2_{track,L\,arm} + \chi^2_{track,R\,arm} + \chi^2_V\right)$$

where

$$f \equiv (N_{X,L\,arm} + N_{Y,L\,arm} - 5) + (N_{X,R\,arm} + N_{Y,R\,arm} - 5)$$

was the degree of freedom, and the wire hit combination which gave the minimum χ^2 was selected as the track candidate of the event.

Appendix C

Estimation of the Error Matrix by a Monte Carlo Method

The definition of the error matrix E_{ij} was as follows[1] (Section 4.2).

$$E_{ij}^{X} \equiv \langle\, (X_c^{err} - X_c^{noerr})_i \cdot (X_c^{err} - X_c^{noerr})_j \,\rangle$$

$$E_{ij}^{X} = \sigma_i^2 \cdot \delta_{ij} + E_{ij}^{X\ M.S.}(p)$$

$$E_{ij}^{X\ M.S.}(p) \equiv \left\langle\, (X_c^{M.S.} - X_c^{noM.S.})_i \cdot (X_c^{M.S.} - X_c^{noM.S.})_j \,\right\rangle$$

$E_{ij}^{X\ M.S.}(p)$ was assumed to be scaled by a square-inverse of the momentum, $\frac{1}{p^2}$, because $(X_c^{M.S.} - X_c^{noM.S.})_i$ was proportional to $\frac{1}{p}$, and was written as follows.

$$E_{ij}^{X\ M.S.}(p) \equiv \frac{E_{ij}^{X\ 0}}{p^2}$$

$E_{ij}^{X\ 0}$ was determined using a Monte Carlo simulation described below.

$100\,K$ $K_L^0 \to \pi^+\pi^-$ decays were generated by a Monte Carlo program (Appendix A). Pions were transported through the spectrometer twice, first with multiple scattering and second without multiple scattering. When one of the pions decayed or did not traverse the semi-parallel pairs of the hodoscopes this Monte Carlo event was not used. The track positions in the chamber planes for each case, $(X_{mc}^{M.S.})_i$'s and $(X_{mc}^{noM.S.})_i$'s, were stored, and $E_{ij}^{X\ 0}$ was calculated by summing over many events and taking an average :

$$E_{ij}^{X\ 0} \equiv \left\langle\, p^2 \cdot (X_{mc}^{M.S.} - X_{mc}^{noM.S.})_i \cdot (X_{mc}^{M.S.} - X_{mc}^{noM.S.})_j \,\right\rangle .$$

[1] An X-view of the track is used here as an example.

p was a pion momentum of each Monte Carlo track, and p^2 was the weight to compensate for the momentum dependence of the track position deviations.

In the track fitting, the error matrix for a track of momentum p was calculated with the following formula :

$$E_{ij}^{X} = \sigma_i^2 \cdot \delta_{ij} + \frac{E_{ij}^{X\ 0}}{p^2} ,$$

and inverted to obtain the weight matrix $(E_{ij}^{X})^{-1}$. Fig. A.3 is the error matrix and the weight matrix elements for an X-view of the left arm, when $p = 1.0\,GeV/c$. It is clear that the off-diagonal elements, i.e., the effects of multiple scattering, should not be neglected.

To confirm that this estimation of the error matrix was correct, the following test statistics were used :

$$q_i = \frac{(X_c - X(Z))_i}{s_i}$$

$$s_i \equiv \sqrt{\mathbf{E}_{ii}^{\mathbf{X}} - [\mathbf{A}^{\mathbf{X}} \cdot ((\mathbf{A}^{\mathbf{X}})^{\mathrm{T}} \cdot (\mathbf{E}^{\mathbf{X}})^{-1} \cdot \mathbf{A}^{\mathbf{X}})^{-1} \cdot (\mathbf{A}^{\mathbf{X}})^{\mathrm{T}}]_{ii}}$$

These quantities are called normalized residuals or *pulls*. $\mathbf{E}^{\mathbf{X}}$ was the error matrix, $(\mathbf{E}^{\mathbf{X}})^{-1}$ was the weight matrix, and $\mathbf{A}^{\mathbf{X}}$ was the *derivative* matrix, which was a constant matrix in the spline method and calculated for each track during the process of the least-square fitting. $\mathbf{A}^{\mathbf{X}}$ and $\mathbf{A}^{\mathbf{Y}}$ were as follows :

$$\mathbf{A}^{\mathbf{X}} = \begin{pmatrix} 1 & Z_1 & 0 & 0 & \bar{X}(Z_1) \\ 1 & Z_2 & 0 & 0 & \bar{X}(Z_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & Z_{10} & 0 & 0 & \bar{X}(Z_{10}) \end{pmatrix} , \text{ and } \mathbf{A}^{\mathbf{Y}} = \begin{pmatrix} 0 & 0 & 1 & Z_1 & \bar{Y}(Z_1) \\ 0 & 0 & 1 & Z_2 & \bar{Y}(Z_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & Z_{10} & \bar{Y}(Z_{10}) \end{pmatrix} .$$

s_i was regarded as the standard deviation of the residual, and the pulls should follow the standard normal distribution with mean 0 and variance 1. The pull distribution for each chamber plane was monitored during the RESP-analysis, and it was confirmed that the distributions were not deviated from the expected normal distribution and were the same for muon and electron tracks.