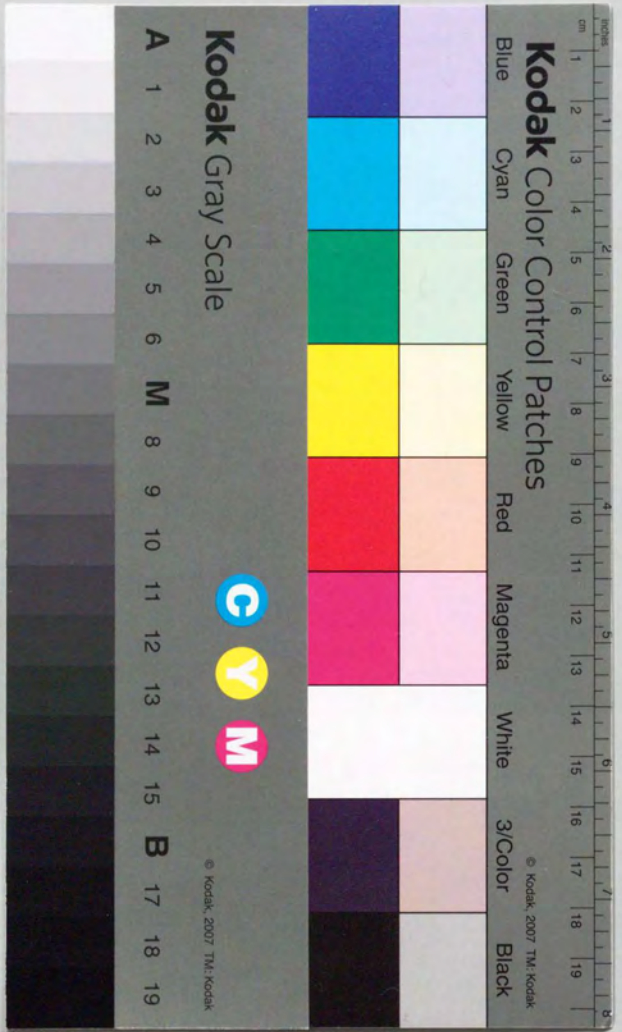

減衰する宇宙項と宇宙論

西　岡　　毅

Thesis

Decaying Cosmological Constant and Cosmology

Tsuyoshi Nishioka

Institute of Physics, University of Tokyo-Komaba, Tokyo, Japan

Supervised by Professor Yasunori Fujii

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Abstract

A decaying cosmological constant is one of many approaches to solve the cosmological constant problem. The approach tells us that the effective cosmological constant decays like $\sim t^{-2}$ asymptotically. In accordance with this, the present cosmological constant is extraordinary small due to the oldness of our universe. The crucial ingredients are a generalized scalar-tensor theory and a careful analysis of conformal transformations. After a conformal transformation to make the gravitational constant strictly constant, the cosmological constant turns into the potential of the gravitational scalar. The model, hence, is regarded as one of the new inflationary models. The model reproduces successful cosmology by exploiting degrees of freedom of a generalized scalar-tensor theory. Sufficient reheating is obtained with introducing couplings of the gravitational scalar to another scalar field like a Higgs scalar. As it turns out, non-linear effects, like a relaxation oscillation, play important roles in the cosmological solutions. This behavior is expected to provide a key feature to explain the recent analysis indicating the presence of a small but nonzero cosmological constant.

Contents

Chapter 1

Introduction

The cosmological constant has brought some desirable and undesirable consequences to both cosmology and particle physics ever since it was introduced by Einstein for the first time. The "cosmological constant problem"[1] is an extremely serious and fundamental problem but has been poorly understood for a long time. Many efforts have been attempted from different points of view. But none of them has reached a complete success. The idea of a decaying cosmological constant[2][3][4][5], the main issue in this paper, is just one of such approaches with only partial success so far. We still believe that the approach is viable as a natural solution of the problem, and deserves further development.

Historically Einstein introduced the cosmological constant first to obtain the solution of a static universe. In those days most of the people, including himself, were obsessed by a belief that our universe is static and does not expand nor shrink. Then Hubble discovered that our universe is in fact expanding, making it no longer necessary to have a static solution of the universe. Einstein finally withdrew his own proposal to introduce the cosmological constant.

Friedmann discovered a class of solutions expanding with only matter distribution and with no cosmological constant. This model is called Friedmann model or FRW model, for Robertson and Walker contributed to the studies of its line element. Fried-

mann model is still of central importance in modern cosmology. With the success of the Big Bang cosmology which embodies Friedmann model, the potential importance of a cosmological constant has been almost forgotten. The cosmological constant recently reemerged, however, in an entirely new context through the effort to combining particle physics and cosmology.

When we apply particle physics to cosmology, the vacuum energy begins to have an important meaning. In Minkowski space-time vacuum energy is not a physical observable; only the difference in energy between a state and the vacuum state is observable. The vacuum energy, hence, can be ignored. In curved space-time, however, the vacuum energy is not simply ignored and behaves like a cosmological constant. If a theory is applicable to an energy scale Λ_E, we may naturally expect the vacuum energy density with a scale of Λ_E^4. Since general relativity may be probably applicable to the Planck scale, it is naturally expected that $\rho_v \sim m_{\rm Pl}^4 \sim 10^{76}{\rm GeV}^4$ where $m_{\rm Pl}$ is the Planck mass $\sim 10^{19}{\rm GeV}$.

On the other hand, the observation gives a very stringent upper bound of the cosmological constant. Since any energy density must be smaller than the critical density $\rho_{\rm critical} = 3H_0^2/8\pi G \sim 10^{-46}{\rm GeV}^4 \sim 10^{-120} m_{\rm Pl}^4$, the present vacuum energy density or the present cosmological constant has an upper bound equal to $\rho_{\rm critical}$. The expectation for the cosmological constant from particle physics, hence, is larger than the observational limit by some 120 orders of magnitude. This requires an extreme fine-tuning or cancellation to the unimaginable order of 120 or so. This is called the cosmological constant problem. Many approaches to solve this problem have been tried; for example, supersymmetry, supergravity, superstring, anthropic consideration, changing gravity, quantum cosmology and a decaying cosmological constant (adjustment mechanism).

Particle physics brings not only the disgusting cosmological constant problem but also blessed inflationary paradigms to cosmology. Inflation[6] was originally born by combining the grand unified theories with cosmology and can solve many cosmological puzzles which the Big Bang cosmology fails to explain. An effective cosmological constant in the early universe leads naturally to an inflationary expansion, so inflation seems to be almost a common phenomenon of theories with phase transition. However the end of inflation is far more difficult to achieve in a natural way than in its onset; this is called the "graceful exit" problem. Any mechanism for ending the inflationary era which does not entail a reasonable solution to the cosmological constant problem would seem to be seriously incomplete at best. Because inflation solves some important cosmological problems, including the horizon problem, it is desirable if we have a solution of the cosmological constant problem which allows for inflation. In this sense the cosmological constant problem is closely connected to the inflationary model. This means that we need to know the behavior of the effective cosmological constant on the cosmic time. But most of the approaches to the cosmological constant problem have not answered the time behavior of the effective cosmological constant saying only that the cosmological constant becomes zero. Moreover these answers are far from perfect. A decaying cosmological constant, however, seems to be a more promising possibility than other approaches because it describes dynamical decay of the cosmological constant. The purpose of the present paper is to investigate a decaying cosmological constant in considerable detail based on our own work[2][4][5].

A decaying cosmological constant assumes an effective Lagrangian in four dimensional space-time as the starting point. This Lagrangian should be determined by more fundamental theories, like superstring theory, for example. Unfortunately, few of such fundamental theories have resulted in low-energy effective Lagrangians of sufficient details. We notice, however,many of these theories suggest some generalization of Einstein gravity[7]. We, hence, investigate a decaying cosmological constant from a rather "phenomenological" view point. The mechanism of a decaying cosmological constant is formulated usually in terms of a scalar field called the gravitational scalar field $\phi(t)$. A fine-tuning to an unimaginable accuracy of 120 orders or so can be avoided because the

effective cosmological constant behaves like t^{-2} asymptotically and the present age of the universe is very old ($t_{pr} \simeq 14\mathrm{Gyr} \sim 10^{60} t_{Pl}$ where t_{Pl} is the Planck time $\sim 10^{-43}$sec). In addition, Λ is expected to be large enough in the past to cause inflation. In spite of these advantages it has been criticized that, among other reasons, it is not known how the mechanism applies to successive cosmological phase transitions, each of which may create a new cosmological constant. Certainly yet another novel idea will be needed for the expected repetition to be realized. Nevertheless, as a basis of future developments, theoretical attempts for a single occurrence as they stand seem promising enough to deserve further studies. If we stand on this point of view, we find that the effective gravitational constant $G_{\rm eff}$ also decays in this mechanism. One may thereby criticize that obviously there is no need to worry about a nonzero cosmological constant in a world with the gravitational interactions turned off. Even accepting that $G_{\rm eff}$ is not really zero at the present time, the calculated rate of the change of $G_{\rm eff}$ is beyond the observational upper bound[8]. We point out, however, that G by itself is not subject to direct physical measurements; it is (almost) always multiplied by masses: Gm^2 or Gm. This observation raises a possibility that the decrease of G can be compensated by an increase of masses of elementary particles.

It is also important to notice that we can always go to a conformal frame (CF) in which G is a true constant. Analysis will often be simpler in this CF. In addition, we point out that any nontrivial theory of gravitation is not invariant under a conformal transformation: $g_{\mu\nu} \to \Omega(x)^2 g_{\mu\nu}$. Different CFs describe the same physics in different manners. In the CF with G constant, the effective cosmological constant is no longer constant and becomes a potential of the gravitational scalar field. The effective cosmological constant, hence, decays like t^{-2} asymptotically. This solution of the cosmological constant problem is almost satisfactory from the view point of particle physics, but successful cosmology requires various restrictions because the effective cosmological constant still remains to some extent. This feature may prevent us from obtaining much success of the Big Bang cosmology. Primordial nucleosynthesis is particularly sensitive to this feature and may restrict our model stringently.

Our model in new CF can be regarded as one of many inflationary models. Prescription for inflationary models is, hence, applicable to ours. But our model has two serious different features from usual inflationary models. The gravitational scalar interacts with usual matter fields far weaker than an usual inflaton does. If the gravitational scalar has a Yukawa coupling to a fermion representing usual matter, the magnitude of this coupling constant is $O(10^{-19})$. The other feature is a shape of the potential of the gravitational scalar. This potential cannot have any stable points to avoid fine-tuning of the cosmological constant. The gravitational scalar field does not oscillate around a stable point whereas a usual inflaton does. We wonder if our model has sufficient reheating, because we cannot expect that the energy of the gravitational scalar converts matter energy sufficiently. In this paper we consider an interaction between the gravitational scalar and a scalar field like a Higgs scalar. We introduce a dissipative term based on this interaction as quantum effect.

Solving the cosmological equations, we find that non-linear effect plays an important role. We, hence, investigate these equations with numerical calculations. We found very interesting behaviors of solutions, which is generally called a relaxation oscillation. A relaxation oscillation which is a universal phenomenon in nature, has not been well understood yet because of the non linearity. In our model this phenomenon occurs rather commonly.

Recent analysis about the cosmological parameters[9] reports that the cosmological constant may be nonzero. If this is true, the cosmological constant has a lower bound and the cosmological constant problem becomes much more difficult. We have to explain the nonzero and extraordinary small cosmological constant without fine-tuning. At the present time no approach has been tried to solve this "new" cosmological constant problem. We apply a decaying cosmological constant model to this "new" problem in

two ways. One way is "soft" fine-tuning of parameters in our simple model. This way gives rather interesting time behavior of the effective cosmological constant. The other way is introducing another scalar field which has a artificial potential to mimic the "suspending" cosmological constant for a time[10].

In Chap.2 we discuss the cosmological constant problem following mainly Weinberg [1]. In Chap.3 we outline the basic idea of inflation and the cosmological puzzles which inflation elegantly solves. Chap.4 is a main part of this paper where we present a decaying cosmological constant model. We apply our model to cosmology. We derive a dissipative term which plays an important role in the reheating epoch. We find interesting behaviors of the solutions. In Chap.5 we discuss the model of nonzero Λ from a point of view of a decaying cosmological constant.

Chapter 2

The cosmological constant problem

Einstein equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \frac{1}{8\pi G} T_{\mu\nu}, \tag{2.1}$$

where $R_{\mu\nu}$ is a Ricci tensor and R is scalar curvature. The third term of the left-hand side is called a cosmological term. On right-hand side, $T_{\mu\nu}$ is an energy-momentum tensor of matter fields. Quantum field theory tells us that the occurrence of the cosmological term is almost inevitable. In quantum field theory the vacuum in itself has energy in general with its density acting just like a cosmological constant. In Minkowski space-time the energy-momentum tensor of the vacuum is given by

$$\langle T_{\mu\nu} \rangle = \rho_v \eta_{\mu\nu}, \tag{2.2}$$

where ρ_v is the energy density of the vacuum. The occurrence of $\eta_{\mu\nu}$ is a consequence of Lorentz invariance. In curved space-time (2.2) is expected to be generalized to

$$\langle T_{\mu\nu} \rangle = \rho_v g_{\mu\nu}. \tag{2.3}$$

Before we estimate the vacuum energy, we study a crude experimental upper bound on Λ or ρ_v provided by measurements of cosmological redshifts z as a function of distance, the program initiated by Hubble in the late 1920's. We assume Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right), \tag{2.4}$$

where $k = -1, 0, 1$ represents the sign of spatial curvature. The behavior of the cosmic scale factor a is governed by the 00-component of (2.1)

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{1}{3}\Lambda, \tag{2.5}$$

where ρ is the energy density of matter and $\cdot$ means time derivative. The expansion rate today is estimated as

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)_{\rm pr} &\equiv H_0 = 100h \text{km/sec/Mpc} \\ &\simeq 1.04h \times 10^{-10}/\text{yr},\end{aligned} \tag{2.6}$$

with $h \cong 0.5 \sim 1$. Since no gross effects of curvature is observed we find roughly

$$\frac{|k|}{a_{\rm pr}^2} \lesssim H_0^2. \tag{2.7}$$

The energy density of matter is not much greater than its critical value

$$|\rho| \lesssim 3H_0^2/8\pi G = \rho_{\rm critical}. \tag{2.8}$$

Hence (2.5) shows

$$|\Lambda| \lesssim H_0^2, \tag{2.9}$$

or in physics units,

$$|\rho_v| \lesssim 10^{-29}\text{g/cm}^3 \approx 10^{-47}\text{GeV} \sim 10^{-120} m_{\rm Pl}^4. \tag{2.10}$$

The observational upper bound 10^{-47}GeV^4 is extraordinary small compared with any theoretical estimates of ρ_v. For example, summing the zero-point energies of all normal modes of some field of mass m up to a wave number cut-off $\Lambda_{\rm E} \gg m$ yields a vacuum energy density (with $\hbar = c = 1$)

$$\rho_v = \int_0^{\Lambda_{\rm E}} \frac{4\pi k^2 dk}{(2\pi)^3}\left(\frac{1}{2}\right)\sqrt{k^2+m^2} \approx \frac{\Lambda_{\rm E}^4}{16\pi^2}. \tag{2.11}$$

If we believe general relativity up to the Planck scale, then we might take $\Lambda_E \approx (8\pi G)^{1/2}$, which would give

$$\rho_v \approx 2^{-10}\pi^{-4}G^{-2} = 2 \times 10^{71}\mathrm{GeV}^4. \tag{2.12}$$

The difference of energy scale between observational upper bound and theoretical estimate of ρ_v is as large as 120 orders or so. Moreover our universe has experienced many phase transitions since its birth. Whenever a phase transition occurred, ρ_v is expected to have changed by the value with different Λ_E substituted in (2.11). It is quite unnatural to expect that the vacuum energy has reached the present extraordinary small value as a final result of many phase transitions. It appears as if nature requires an extreme fine-tuning or cancellation to the unimaginable order of 120 or so. This is called the cosmological constant problem.

The situation shows that our understanding of field theory or particle physics is far from complete. Many proposals have been tried to solve this problem. None of them has solved the problem perfectly. In this paper we focus upon one of the proposals; a decaying cosmological constant.

Recently the observational cosmology suggests that the cosmological constant may have a lower bound. Recent analysis on the mass density and dark matter indicates strongly that the mass parameter $\Omega_0 = \rho/\rho_{\mathrm{critical}}$ at the present time is considerably smaller than 1. A detailed analysis of the number count of faint galaxies taking the evolution effect into account suggests that the universe of nonzero Λ ($\lambda = (\Lambda/(8\pi G)/\rho_{\mathrm{critical}} = 0.9, \Omega_0 = 0.1, k = 0$) is even better than the open universe of $\Lambda = 0$ and $\Omega_0 = 0.1$. Galaxy formation based on cold dark matter (CDM) also indicates that a nonzero Λ is desirable to loosen the restriction from the isotropy of cosmic microwave background radiation (CMBR)[11]. Another support for a nonzero Λ comes from the age of the universe determined from Hubble parameter is comparable with the age of some celestial bodies, for example, a globular cluster. We are close to a danger that the age of the universe based on a H_0 is shorter than the age of the

globular cluster. A nonzero Λ can avoid this contradiction because the cosmological constant can lengthen the age of the universe as inferred from H_0 alone[12]. All these circumstantial evidences point to a suggestion that the cosmological constant is nonzero having a lower bound. If the cosmological constant should have a lower bound, the cosmological constant problem might be far more difficult than the one without a lower bound. Most of the approaches have tried to make the cosmological constant strictly zero; They are not prepared to confront with the "new" problem. On the other hand, a decaying cosmological constant allows some nonzero value, making it better suited for the "new" problem.

Chapter 3

Inflation

3.1 Big Bang cosmology

The standard Big Bang cosmology[13][14] has achieved great success. It explains primordial nucleosynthesis and CMBR among other things. We hence start with accepting Big Bang cosmology. There are, however, many cosmological puzzles[6][13] which cannot be explained by Big Bang cosmology. The horizon problem[15], the flatness problem[16] and the monopole problem are main puzzles of theirs. Before we discuss these problems, we mention Big Bang cosmology briefly.

The standard Big Bang cosmology is based on a homogeneous and isotropic Robertson Walker metric (2.4). The matter energy-momentum tensor is given by

$$T_{\text{matter}}{}^{\mu}{}_{\nu} = \text{diag}(-\rho, p, p, p), \tag{3.1}$$

where ρ is the energy density and p is the pressure. The $\mu = 0$- component of the conservation of the stress energy ($T^{\mu\nu}{}_{;\nu} = 0$) gives

$$\dot{\rho} = -3H(\rho + p), \tag{3.2}$$

where H is Hubble parameter $\dot{a}/a$. For the simple equation of state $p = \zeta\rho$, where ζ is independent of time, the energy density evolves as $\rho \propto a^{-3(1+\zeta)}$. For example, the radiation energy density behaves $\rho_r \propto a^{-4}$, the non-relativistic matter energy density behaves $\rho_m \propto a^{-3}$ and the vacuum energy density behaves $\rho_v \sim$ const.

We solve (2.5) with $\Lambda = 0$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{1}{3}\rho, \tag{3.3}$$

where we use the unit system $8\pi G = c = \hbar = 1$ (see Appendix A). When the energy density ρ, satisfying $p = \zeta\rho$, is dominant in the universe with spatially flat $k = 0$, the scale factor a evolves like $a \sim t^{2/[3(1+\zeta)]}$.

The standard cosmology says that the radiation energy had been dominant initially and the non-relativistic energy caught up with the radiation energy. Since then ρ_m, meaning the non-relativistic matter energy density, has been dominant. The scale factor, hence, behaves like

$$a \sim \begin{cases} t^{1/2} & t < t_{\text{eq}} \\ t^{2/3} & t > t_{\text{eq}} \end{cases}, \tag{3.4}$$

where t_{eq} is "the equal time" meaning $\rho_r \simeq \rho_m$ and is given by

$$t_{\text{eq}} = \left(\frac{\pi^2}{45}\frac{T_0^4}{H_0^2\Omega_0}\right)^{3/2} t_{\text{pr}} \simeq 4.4 \times 10^{10}(\Omega_0 h^2)^{-2}\text{sec} \sim 1.6 \times 10^{53}(\Omega_0 h^2)^{-2}, \tag{3.5}$$

where T_0 is the present temperature of CMBR $\simeq 2.7\text{K} \sim 1.0 \times 10^{-31}$ and $H_0 \sim 8.6h \times 10^{-61}$ and $\Omega = 0.01 \sim 1$. The relation between the radiation energy density and the temperature of the universe is given by

$$\rho_r = \frac{\pi^2}{30} g_*(T) T^4, \tag{3.6}$$

where $g_*(T)$ is the total number of effectively massless degrees of freedom, given by

$$g_* = \sum_{i=\text{boson}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermion}} g_i \left(\frac{T_i}{T}\right)^4, \tag{3.7}$$

where g_* is a function of T and the sum runs over only those species with mass $m_i \ll T$, for example $g_* \sim 100$ at GUT scale. Incidentally the entropy density is also given by

$$s = \frac{2\pi^2}{45} g_{*S} T^3, \tag{3.8}$$

where

$$g_{*S} = \sum_{i=\text{boson}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\text{fermion}} g_i \left(\frac{T_i}{T}\right)^3. \tag{3.9}$$

The redshift at t_{eq} is given by

$$1+z_{eq}=\frac{a_{pr}}{a_{eq}}=2.3\times10^4\Omega_0 h^2. \qquad (3.10)$$

In the early universe the matter and radiation were in good thermal contact, but eventually the density of free electrons became too low to maintain thermal contact and matter and radiation decoupled. Roughly speaking this occurs when $\Gamma_\gamma\simeq H$ where Γ_γ is the interaction rate of the photon, given by

$$\Gamma_\gamma=n_e\sigma_T \qquad (3.11)$$

where n_e is the number density of free electrons and σ_T is the Thomson cross section $\sim 6.65\times10^{-25}\mathrm{cm}^2$. We know the redshift at the photon decoupling is given by

$$1+z_{dec}=\frac{a_{pr}}{a_{dec}}\simeq 1100. \qquad (3.12)$$

Consequently the CMBR carries information at z_{dec}.

3.2 The horizon problem

In this way Big Bang cosmology is based upon the assumption of homogeneity and isotropy of the universe. Moreover the CMBR which is uniform to $\delta T/T\lesssim 10^{-4}$ on angular scales from $10''$ to $180°$ shows that the universe has been unbelievably isotropic and homogeneous since the time of photon decoupling.

If the entire observable universe were in causal contact at the epoch of photon decoupling, microphysical processes like Compton scattering, could have smoothed out the whole observable universe and then the CMBR could have been uniform very much. However, within the standard cosmology it is very difficult to imagine that this has happened because of the existence of particle horizons. The particle horizon is defined by

$$d_H(t)=a(t)\int_0^t\frac{dt}{a}. \qquad (3.13)$$

At the radiation dominant era, $d_H(t)$ is

$$d_H(t)=2t. \qquad (3.14)$$

We discuss the horizon in the two ways, One way uses the entropy within a horizon volume[6][13] and the other way uses conformal diagrams where the light cone is at $\pm45°$[15].

The size of the particle horizon at a given epoch is conventionally expressed in terms of the entropy within a horizon volume:

$$S_{\mathrm{HOR}}=s\frac{4\pi}{3}t^3\simeq\begin{cases}6.30g_*^{-1/2}(m_E/T)^3 & t\lesssim t_{eq}\\ 3\times10^{87}(\Omega_0h^2)^{-3/2}(1+z)^{-3/2} & t\gtrsim t_{eq}\end{cases}. \qquad (3.15)$$

Note that the entropy contained within the horizon at early times was much less than that today, about 10^{88}. The entropy within the horizon at decoupling, when typical photons in the CMBR last scattered, was $S_{\mathrm{HOR}}(z_{dec}\simeq1100)\simeq10^{83}$. We thus find that the present Hubble volume consisted of about 10^5 causally disconnected regions at the decoupling. Causal processes could not have effected the smoothness consequently. The causally connected region at the decoupling, thereby, occupies only an angle of only about $0.8°$ on the sky today though the CMBR is uniform across the sky. There is no physical explanation for why the universe on very large scale is so very smooth. This is called the Horizon problem.

We discuss the horizon problem in another approach using conformal diagrams. The maximum coordinate distance which any signal could have traveled up to now is equal to the conformal time at the present time;

$$u_{ph}=\int_0^{t_{pr}}\frac{dt}{a}, \qquad (3.16)$$

because the light cone is at $\pm45°$ in the conformal diagram. u_{ph}, giving the particle horizon $d_{ph}=a(t_{pr})u_{ph}$, is divided into two parts: $u_{ph}=u_{vh}+u_{pph}$, where

$$u_{vh}=\int_{t_{dec}}^{t_{pr}}\frac{dt}{a}, \qquad (3.17)$$

and

$$u_{\rm pph} = \int_0^{t_{\rm dec}} \frac{dt}{a}. \tag{3.18}$$

$u_{\rm vh}$ is called the coordinate distance of visual horizon because it is a maximum coordinate distance from here to the farthest place we can see through light. $u_{\rm pph}$ is, so to speak, the primeval particle horizon. If the coordinate distance between two different events at decoupling is smaller than $u_{\rm pph}$, these two events are considered to be causally connected. Therefore the number of causally disconnected regions of the universe at decoupling is given by $(u_{\rm vh}/u_{\rm pph})^3$.

Although, strictly speaking, $t_{\rm eq} \neq t_{\rm dec}$ ($z_{\rm eq} \sim 10^4$ and $z_{\rm dec} \simeq 1100$), we assume $t_{\rm eq} = t_{\rm dec}$ here for simplicity. The scale factor is given by

$$a = \begin{cases} a_1 t^{1/2} & t < t_{\rm dec} \\ \left(\frac{3}{4}\right)^{2/3} a_1 t_{\rm dec}^{-1/6} \left(t + \frac{1}{3} t_{\rm dec}\right)^{2/3} & t > t_{\rm dec} \end{cases}. \tag{3.19}$$

where a and H is continuous at $t_{\rm dec}$ and a_1 is a constant. Then $u_{\rm pph}$ and $u_{\rm vh}$ is

$$u_{\rm pph} = 2\frac{a_{\rm dec}}{a_1^2}, \tag{3.20}$$

and

$$u_{\rm vh} = 4\frac{a_{\rm dec}}{a_1^2}\left[\left(\frac{a_{\rm pr}}{a_{\rm dec}}\right)^{1/2} - 1\right] \tag{3.21}$$

This gives finally

$$u_{\rm vh}/u_{\rm pph} = 2[(1 + z_{\rm dec})^{1/2} - 1] \simeq 64. \tag{3.22}$$

This result is consistent with the previous result using entropy.

3.3 The flatness problem

For FRW models the quantity Ω_0, along with H_0, specify our present cosmological model, in that Ω_0 and H_0 determine the radius of curvature $a_{\rm curv}^2 = H_0^{-2}/|\Omega_0 - 1|$ and the observation says Ω_0 lies in the interval $[0.01, \text{few}]$. This fact implies that $a_{\rm curv} \sim H_0^{-1}$ and the matter energy density ρ_0 is nearly equal to $\rho_{\rm critical}$.

This may not seem so remarkable, but when one takes account of the fact that Ω changes with time

$$\Omega(t) = \frac{1}{1 - x(t)}, \tag{3.23}$$

where

$$x(t) = \frac{k}{a^2}/\frac{\rho}{3} \propto \begin{cases} a^2(t) & t \lesssim t_{\rm eq} \\ a(t) & t \gtrsim t_{\rm eq} \end{cases}, \tag{3.24}$$

Ω implies quite unacceptable initial "flatness". Since Ω varies as $1/(1-x)$, earlier Ω was close to unity: at the Planck time

$$|\Omega(t_{\rm Pl}) - 1| \lesssim O(10^{-60}), \tag{3.25}$$

which implies that the radius of curvature of the universe was enormous compared to the Hubble radius:

$$a_{\rm curv}(t_{\rm Pl}) \gtrsim 10^{30} H^{-1}(t_{\rm Pl}). \tag{3.26}$$

Though no law of physics precludes such fantastic initial data, this suggests that our FRW model was very special indeed, characterized by the following initial data as the Planck time

$$|\Omega - 1| \lesssim O(10^{-60}), \tag{3.27}$$

$$a_{\rm curv} \gtrsim O(10^{30}) H^{-1}, \tag{3.28}$$

$$|(k/a^2)|/(\rho/3) \lesssim O(10^{-60}), \tag{3.29}$$

If all of the above quantities were given by order unity at the Planck time, the universe would have either recollapsed in a "jiffy" (few $\times$ 10^{-43}sec) for $k = +1$, or reached a temperature of 3K at the age of 10^{-11}sec for $k = -1$. We, hence, are thus today due to quite extraordinary flat initial condition. This is called the flatness problem.

3.4 Inflation

Inflationary scenarios[6][13][17] are capable of avoiding the horizon and flatness problems in a sophisticated manner. At present there are many different versions of the inflationary scenario. The main feature of all these versions are common and sometimes called the inflationary paradigm. It requires the existence of an epoch when the effective cosmological constant was the dominant component of the energy density of the universe. The scale factor, hence, grew exponentially (or quasiexponentially) during this epoch. Such an expansion is called inflation. The key point of inflation to solve some cosmological puzzles is that a small, smooth, and causally coherent patch of size less than H^{-1} can grow to such a size it easily encompasses the comoving volume that becomes the entire observable universe today during an epoch of inflation. The important point of inflationary scenario to take notice is that most of energy of the effective cosmological constant converts energy of matter after inflation. This nonadiabatic process is called "reheating", implying the enormous entropy has been produced because the vacuum energy hardly decays during inflation. As a result the universe becomes hot like a "big bang" after inflation (again).

This enormous entropy increase thereby solves some cosmological puzzles. First it can solve the horizon problem since the smooth patch, which contained only a small fraction of the entropy of the presently observed universe before inflation, after inflation contains an entropy that is many times greater than that of the observable universe due to reheating. The comoving volume that encompasses all that we can see today fits easily within the smooth patch after inflation.

We would like to see this situation with the conformal diagram[15]. We consider the primordial inflation for simplicity. During inflation the scale factor a evolves from a_i to $a_f = Za_i$ and the cosmic time passes from $t_i = 0$ to t_f, where Z means the size of the inflation. We may neglect space curvature term k/a^2 of (3.3) for simplicity and assume the flat space ($k = 0$) where we regard the effective cosmological constant as the energy density of vacuum.

During inflation the scale factor behaves like

$$a = a_i \exp(H_i t) \qquad (t_i = 0 < t < t_f), \tag{3.30}$$

with the Hubble parameter $H = H_i = \sqrt{\rho_v/3}$. After inflation, the universe becomes radiation dominant because of reheating. During the radiation dominant era the scale factor is given by

$$a = a_f \left(\frac{t - t_a}{t_f - t_a}\right)^{1/2} \qquad (t_f < t < t_{eq}), \tag{3.31}$$

and the Hubble parameter is

$$H = \frac{1}{2}\frac{1}{t - t_a} \qquad (t_f < t < t_{eq}), \tag{3.32}$$

where we choose $t_a = t_f - (1/2)H_i^{-1}$ in order that a and H are continuous at t_f. After t_{eq}, the universe becomes non-relativistic matter dominant;

$$a = a_{eq} \left(\frac{t - t_b}{t_{eq} - t_b}\right)^{2/3} \qquad (t > t_{eq}), \tag{3.33}$$

and

$$H = \frac{2}{3}\frac{1}{t - t_b} \qquad (t > t_{eq}), \tag{3.34}$$

where we choose

$$a_{eq} = a_f \left(\frac{t_{eq} - t_a}{t_f - t_a}\right)^{1/2} = a_f [2H_i(t_{eq} - t_a)]^{1/2}, \tag{3.35}$$

and $t_b = -(1/3)t_{eq} + (4/3)t_a$ in order that a and H are continuous at t_{eq}.

We assume $t_{eq} = t_{dec}$ for simplicity again. In this case u_{pph} consists of two parts: u_{infl} and $u_{noninfl}$ where

$$u_{infl} \equiv \int_0^{t_f} \frac{dt}{a} = \frac{1}{a_i H_i}\left(1 - \frac{1}{Z}\right), \tag{3.36}$$

and

$$u_{noninfl} \equiv \int_{t_f}^{t_{dec}} \frac{dt}{a} = \frac{1}{a_i H_i}\frac{1}{Z}\left(\frac{a_{dec}}{a_f} - 1\right). \tag{3.37}$$

Therefore u_{pph} is given by

$$u_{pph} = u_{infl} + u_{noninfl} = \frac{1}{a_i H_i}\left(1 - \frac{2}{Z} + \frac{a_{dec}}{a_f}\frac{1}{Z}\right). \tag{3.38}$$

And u_{vh} is given by

$$u_{vh} = \int_{t_{dec}}^{t_{pr}} \frac{dt}{a} = \frac{2}{Z^2} \frac{a_{dec}}{a_i^2 H_i} \left[\left(\frac{a_{pr}}{a_{dec}} \right)^{1/2} - 1 \right]. \tag{3.39}$$

Note that (3.39) is much smaller than (3.21) with $a_1, a_i, H_i \sim O(1)$ because of inflation. In this case the ratio of u_{vh} and u_{pph} is given by

$$\frac{u_{vh}}{u_{pph}} = \frac{2}{Z} \frac{a_{dec}}{a_f} \frac{(a_{pr}/a_{dec})^{1/2} - 1}{1 - \frac{2}{Z} + \frac{1}{Z} \frac{a_{dec}}{a_f}}. \tag{3.40}$$

If the rate of expansion during inflation is sufficient large ($Z \gg 1$), u_{vh}/u_{pph} becomes

$$\frac{u_{vh}}{u_{pph}} \simeq \frac{2}{Z} \frac{a_{dec}}{a_f} \left(\frac{a_{pr}}{a_{dec}} \right)^{1/2}, \tag{3.41}$$

where a_{dec}/a_f is

$$\begin{aligned} \frac{a_{dec}}{a_f} &\simeq \left(\frac{t_{dec}}{t_f} \right)^{1/2} = \left(\frac{t_{pr}}{t_f} \right)^{1/2} \left(\frac{t_{dec}}{t_{pr}} \right)^{1/2} \\ &\sim \frac{a_{dec}}{a_{pr}} \times 10^{30}, \end{aligned} \tag{3.42}$$

because we may reasonably assume that $t_f \sim O(1)$ and $t_{pr} \sim H_0^{-1} \sim 10^{60}$. As the final result we obtain

$$\begin{aligned} \frac{u_{vh}}{u_{pph}} &\simeq \frac{2}{Z}(1 + z_{dec})^{-1/2} \left(\frac{t_{pr}}{t_f} \right)^{1/2} \\ &\sim \frac{2}{Z}(1 + z_{dec})^{-1/2} \times 10^{30}. \end{aligned} \tag{3.43}$$

If all the region we can see today through light fits within an unique causally connected region at decoupling, u_{vh}/u_{pph} is smaller than 1. This inequality is the necessary condition for sufficient inflation, giving $Z > 3 \times 10^{28} \sim e^{66}$ in this case.

Next inflation solves the flatness problem: Right after inflation the energy density of the universe was reheated to comparable temperature with that before inflation, while

the curvature radius grew exponentially during inflation. The ratio $x = (k/a^2)/(\rho/3)$ decreased by a factor of about Z^2 accordingly. Thus, the radius of curvature of the universe today thereby should still be much greater than the present Hubble radius. Consequently inflation explains the flatness of the universe and also may predict that Ω_0 should be close to 1.

Inflation also solves the problem of unwanted relics, for example, monopole: An unwanted relic that was produced before inflation, with an abundance given by (n_X/s), where n_X means number density of an unwanted relic X is reduced exponentially by the same factor that the entropy increased like $(n_X/s)_f = Z^{-3}(n_X/s)_i$ after inflation. Note that the initial entropy of the universe is irrelevant because the heat we see today was all produced during reheating. Needless to say, the baryon asymmetry of the universe must be produced after inflation, which requires that the reheat temperature is enough high for baryogenesis to occur in the usual way.

Inflationary models having such merits are formulated usually in terms of a scalar field, proposed originally by Sato and Guth in 1981[6]. In the original model, referred to as "old inflation", inflation is driven by a Higgs scalar field at the GUT scale. This model has a difficulty, a graceful exit problem. In this scenario inflation ends with completion of a first-order phase transition. But the first-order phase transition never catches up with expansion of the universe because the expansion is too fast; inflation never ends and the universe becomes very inhomogeneous. New inflationary model is proposed in order to avoid this problem by Linde[18], and by Albrecht and Steinhardt[19]. The essence of this model is a second-order phase transition instead of the first-order phase transition. The field driving inflation which is also a Higgs scalar rolls over the potential slowly. This field is often called an inflaton. This model has, however, another problem that density perturbation requires fine-tuning of coupling constants. Chaotic inflation, proposed by Linde[20], shows, on the other hand, that a general scalar field which is not necessary a Higgs scalar can also drive inflation at the Planck scale. In the more recent model of extended inflation two scalars are introduced[21]. One is a Higgs scalar driving a inflation in the same way as in the old model of inflation that ends with completion of the first-order phase transition. Another scalar field is a JBD field and makes an expansion slower because this field allows the effective gravitational constant vary with time. As a consequence the first-order phase transition now catches up with the expansion of the universe, thus solving the graceful exit problem. The JBD scalar field interacts with ordinary matter fields as weakly as gravitation does and may be called a gravitational scalar field.

In any case inflationary models demand the effective large cosmological constant in the very early universe and require that the effective cosmological constant becomes almost zero after inflation. Any inflationary models, hence, need fine-tuning of the cosmological constant; confronting with the cosmological constant problem has been always avoided. We would like to consider a model in which the cosmological constant is sufficiently large to cause inflation in the very early universe and naturally becomes extremely small in the late universe without fine-tuning. The cosmological constant must be time-dependent.

Chapter 4

Decaying cosmological constant

4.1 Mechanism of a decaying cosmological constant

In order to implement the idea of a cosmological constant that decays like t^{-2}, we consider a generalized scalar-tensor theory of gravity with a positive cosmological constant Λ[4]:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}F_G(\phi)R - \frac{1}{2}F_K(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \Lambda\right] + \mathcal{L}_{\text{matter}} + \mathcal{L}_c. \qquad (4.1)$$

We use the unit system $c = \hbar = 8\pi G = 1$. $F_G(\phi)$ and $F_K(\phi)$ are arbitrary functions of ϕ. We assume that the size of Λ is Planck scale and that this Lagrangian has no potential of ϕ for simplicity. Later we would like to discuss the effect of a potential of ϕ produced by backreaction of quantum fluctuations. The gravitational scalar field ϕ having a nonminimal gravitational coupling is decoupled from ordinary matter fields in $\mathcal{L}_{\text{matter}}$. The last term $\mathcal{L}_c$ represents a very weak matter coupling of ϕ as will be specified shortly.

Introducing nontrivial functions of $F_G(\phi)$ and $F_K(\phi)$ is not an unnatural assumption because many of the unified theories like superstring or supergravity theories suggest the possible presence of such functions, although no complete details have been worked out on the explicit forms of $F_G(\phi)$ and $F_K(\phi)$ in the low-energy effective action in four dimensional space-time[7]; if more fundamental theories or principles were at hand, we would have been in a better position to determine $F_G(\phi)$ and $F_K(\phi)$. The best we can do at present is to choose $F_G(\phi)$ and $F_K(\phi)$ from a phenomenological point of view in order that the way the cosmological constant decays is consistent with realistic cosmology.

We conventionally choose

$$F_G(\phi) = 1 + \xi\phi^2, \tag{4.2}$$

where ξ is a positive constant naturally chosen as $O(\xi) \sim 1$. Since we have yet to find a clue on $F_K(\phi)$ and also the choice of $F_K(\phi) = 1$ is not consistent with realistic cosmology, as will be shown later, we choose conveniently[4]

$$F_K(\phi) = (1 + \chi\xi\phi^2)/F_G. \tag{4.3}$$

Here χ is a constant and the key feature of this $F_K(\phi)$ is $F_K = 1$ when $\xi\phi^2 \ll 1$ and $F_K = \chi$ when $\xi\phi^2 \gg 1$. Of course $\chi = 1$ corresponds to $F_K = 1 = \text{const}$. Note that a degree of freedom of $F_K(\phi)$ can be eaten into a degree of freedom of $F_G(\phi)$ with redefining ϕ to become $F_K = 1$ if F_K is positive definite. Nontrivial $F_K(\phi)$ is, hence, meaningful if and only if F_K is not positive definite. When F_K is negative, ϕ looks like a ghost field in appearance. This statement is not necessarily correct because ϕ is not a canonical field. Later we choose $\chi < 0$ phenomenologically but the ensuing canonical field (later called σ) can be a normal field, not a ghost.

In cosmological applications we expect that ϕ is spatially homogeneous depending only on the cosmic time t. Then the effective gravitational constant $G_{\text{eff}} = (8\pi F_G)^{-1}$ depends on t.

We apply a conformal transformation $g_{\mu\nu} \to g_{*\mu\nu}$:

$$g_{\mu\nu} = F_G^{-1} g_{*\mu\nu}, \tag{4.4}$$

in order that the effective gravitational constant remains strictly constant in this new CF. There are some reasons to favor this CF. First, theoretical analysis is more transparent than in the original CF in which G is not constant. Second, the result will be in conformity with the experimental situation that ruled out the time variability of

G to the accuracy better than 10^{-11} per year[8]. Notice also $\sqrt{-g} = F_G^{-2}\sqrt{-g_*}$ and $g^{\mu\nu} = F_G g_*^{\mu\nu}$. The term $\sqrt{-g}F_G R$ is then expressed as $\sqrt{-g_*}R_*$ plus terms containing the derivative $\partial_\mu F_G = (dF_G/d\phi)\partial_\mu\phi$ (see Appendix B). The extra terms add up to the kinetic term of ϕ yielding

$$-\frac{1}{2}\sqrt{-g_*}g_*^{\mu\nu}\left[\frac{3}{2}\frac{(dF_G/d\phi)^2}{F_G^2} + \frac{F_K}{F_G}\right]\partial_\mu\phi\partial_\nu\phi, \tag{4.5}$$

which can be brought to a canonical form $-\frac{1}{2}\sqrt{-g_*}g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma$ if the new field σ is defined by

$$\frac{d\sigma}{d\phi} = \pm F_G^{-1}\sqrt{D}, \tag{4.6}$$

where

$$D = F_K F_G + \frac{3}{2}\left(\frac{dF_G}{d\phi}\right)^2. \tag{4.7}$$

For the choice (4.2) and (4.3), D is given by

$$D = 1 + (\chi + 6\xi)\xi\phi^2. \tag{4.8}$$

If this D, the inside in the square brackets in (4.5), is negative, then σ is a ghost. We avoid this by imposing a condition

$$\chi \geq -6\xi, \tag{4.9}$$

to ensure that σ is a normal canonical field. Notice that (4.9) allows a negative χ. Then $\mathcal{L}$ becomes

$$\mathcal{L} = \sqrt{-g_*}\left[\frac{1}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \Lambda F_G^{-2}\right] + \mathcal{L}_{*\text{matter}} + \mathcal{L}_{*c}. \tag{4.10}$$

The third term ΛF_G^{-2} acts as a potential V of ϕ (or σ):

$$V = \Lambda F_G^{-2}. \tag{4.11}$$

It behaves as shown in Fig.1. The scalar field $\phi(t)$ as a function of the cosmic time t rolls down the potential hill, being driven eventually to infinity. This is in fact an origin to cause the effective cosmological constant to decay.

The matter system composed of massless fermions and gauge fields is known to be conformally invariant, leaving the corresponding portion of $\mathcal{L}_{\rm matter}$ the same as in the original CF. This is not the case with scalar fields, like Higgs fields. This is the main issue which will be discussed in connection with reheating.

The simplest example of $\mathcal{L}_c$ is

$$\mathcal{L}_c = -\sqrt{-g} f \bar{\psi}\psi\phi, \tag{4.12}$$

for a spinor field ψ[4]. In the new CF, $\mathcal{L}_c$ becomes

$$\mathcal{L}_{*c} = -\sqrt{-g_*} f F_G^{-1/2} \phi \bar{\psi}_* \psi_*. \tag{4.13}$$

For a slowly varying $\phi(t)$, (4.13) gives a mass of the spinor field: $m = f\phi F_G^{-1/2}$, which tends asymptotically to a constant $m_\infty = f\xi^{-1/2}$ as $\phi \to \infty$. In fact this mass becomes almost constant after inflation. This is consistent with the time non-variability of G as measured by using atomic clocks. If the interaction (4.13) gives masses of ordinary light elementary particles like quarks and leptons in this way, m_∞ is expected to be order 1 Gev or less. Accordingly the dimensionless coupling constant f is as small as $\sim 10^{-19}$ or less since ξ turns out roughly of the order one.

One may wonder at this point why one should not start with (4.10) in the new CF, forgetting about (4.1) in the original CF. We could do so if we find a good reason to accept (4.10) as it stands. One of the crucial ingredients in (4.10) is an association of Λ with ϕ. The potential V as given by (4.11) will be shown to assume a particularly simple and suggestive form in the limit of large σ :$V \sim e^{-\sigma/\kappa}$, though its real meaning is yet to be explored. Moreover, we know that more fundamental theories, e.g. superstring theories, suggest that the gravitational scalar has a nonminimal coupling to gravity and nontrivial kinetic factor. This may suggest that it is meaningful to start from the original CF. On the other hand, the measurement of $\dot{G}/G$ is carried out by using atomic clocks. In accordance with this, we must compare the theory with the experiment by using a CF in which m stays constant. This is precisely the new starred CF.

4.2 Cosmological applications

4.2.1 Cosmological equations

On the basis of (4.10) we now consider cosmology. We simplify the analysis by assuming a spatially flat Robertson-Walker metric ($k = 0$):

$$g_{*00} = -1, \qquad g_{*ij} = a_*(t_*)^2\delta_{ij}, \tag{4.14}$$

where t_* is the cosmic time and a_* is the scale factor in the new starred CF. The relation to the quantities in the original CF is given by $dt = F_G^{-1/2}dt_*$ and $a = F_G^{-1/2}a_*$ because of $ds^2 = F_G^{-1}ds_*^2$ The scalar field $\sigma(t_*)$ also evolves as a function of t_*. Matter is represented by a general radiation energy density $\rho_r(t_*)$. Notice that most part of the realistic matter is dominated by spinor fields and gauge fields which are conformally invariant. In what follows we suppress the symbol $*$ to simplify the notations.

The three independent equations (the Einstein equation, the σ equation, and the covariant conservation of the radiation energy) are

$$3H^2 = \frac{1}{2}\dot{\sigma}^2 + V + \rho_r = \rho_v + \rho_r, \tag{4.15}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV}{d\sigma} = -F_d(\sigma)\dot{\sigma}, \tag{4.16}$$

$$\dot{\rho}_r + 4H\rho_r = F_d(\sigma)\dot{\sigma}^2, \tag{4.17}$$

where $H = \dot{a}/a$, the overdot meaning a derivative with respect to t. We included a dissipative term $F_d(\sigma)\dot{\sigma}$ in conformity with the Bianchi identity. The radiation energy density obeys the same equation as in the conventional calculations; this is permissible because of conformal invariance of matter, ignoring mass terms and the kinetic terms of spinless fields. Later we derive $F_d(\sigma)$ as a quantum effect. This dissipation term is appreciable only during the reheating epoch. In the reheating epoch the σ energy, which might be called the vacuum energy, is transferred to the matter energy ρ_r due to this dissipation.

It is convenient to express the right-hand side of (4.15) as

$$\rho = \rho_v + \rho_r, \tag{4.18}$$

where ρ_v is the vacuum energy:

$$\rho_v = \frac{1}{2}\dot{\sigma}^2 + V. \tag{4.19}$$

We apply (4.15)-(4.17) to the epochs after some time (chosen to be $t = 1$) around the Planck time. With the potential $V(\phi)$, as shown in Fig.1, suppose the classical field ϕ starts from a small initial value moving slowly toward infinity. The potential is rather flat for $\xi\phi^2$, falling off like $\sim \phi^{-4}$ for $\phi \to \infty$. The solution of (4.15)-(4.17) will then show different types of behavior according to the value of $\xi\phi^2$ in comparison with unity. We thus consider the three eras separately: (i) primordial inflation, (ii) the transient era, (iii) the asymptotic era of a power-law expansion. The first and the last eras permit analytic solutions while only the numerical solution is available for the second era in which the major effect is reheating. No detailed analysis is attempted for the later epochs of the matter-dominated era because we choose parameters to make the asymptotic era (iii) sufficiently close to the standard radiation-dominated expansion. One may also place , if one wishes, another radiation-dominated era prior to inflation, as in the GUT inflationary scenario, though the result remains essentially unchanged.

4.2.2 Inflation

The first era (i) is characterized by $\xi\phi^2 \ll 1$ so that $F_G \approx 1$ and $F_K \approx 1$. Then σ may be identified with ϕ because of $d\sigma/d\phi \approx 1$ from (4.6). For $\sigma \approx \phi$, (4.11) gives

$$V \approx \Lambda(1 - 2\xi\sigma^2) \approx \Lambda = \text{const.}, \tag{4.20}$$

$$V' \equiv \frac{dV}{d\sigma} \approx -4\Lambda\xi\sigma. \tag{4.21}$$

It may also be reasonable to choose the initial conditions such that $\sigma \sim O(1)$ and $\dot{\sigma} \sim O(1)$. Equation (4.15) suggests an exponentially increasing a. Dissipation may not

be important, and ρ_r will rapidly fall off according to (4.17), thus justifying a posteriori to set the right-hand side of (4.15) equal to Λ. In this way we find

$$a(t) = \exp{(Ht)}, \tag{4.22}$$

with $H = \sqrt{\Lambda/3}$. By using this in (4.16) (ignoring the dissipative term), we obtain

$$\ddot{\sigma} + \sqrt{3\Lambda}\dot{\sigma} - 4\Lambda\xi\sigma = 0. \tag{4.23}$$

This linear equation can be solved in the form

$$\sigma(\approx \phi) = \sigma_i \exp{(\gamma t)}, \tag{4.24}$$

with

$$\gamma = \frac{1}{2}\sqrt{3\Lambda}\left(\sqrt{1 + \frac{16}{3}\xi} - 1\right), \tag{4.25}$$

where we have discarded a decreasing solution. Such behaviors of a and $\sigma(\approx \phi)$ are the same as the results in the original CF because the difference between the new starred CF and the original CF is almost negligible in the case of $\xi\phi^2 \ll 1$.

We want a sufficient inflation with $Z \equiv \exp\sqrt{\Lambda/3}\tau \gg 1$ where τ is the duration of inflation, as defined by

$$\xi\phi^2(\tau) \sim 1. \tag{4.26}$$

This can be further put into the form

$$Z = (8x/\sigma_i^2)^x, \tag{4.27}$$

where $x \equiv \frac{1}{8}\xi^{-1}$. The desired value $Z \gtrsim e^{60}$ can be obtained (with $\sigma_i \sim 1$) for

$$x \gtrsim 13 \quad \text{or} \quad \xi \lesssim 1 \times 10^{-2}. \tag{4.28}$$

This crude estimate of the size of inflation gives almost the same result as the technique using the slow roll-over condition[13].

Next we discuss the density perturbation produced by inflation[13]. In the standard cosmology all the comoving scales λ cross the "horizon" H^{-1} for microphysics only once, whereas λ can cross it twice in the inflationary scenario. All cosmologically interesting scales which is initially sub-horizon sized, cross outside H^{-1} during inflation because H is constant. Such escaped scales cross back inside the "horizon" again after inflation. Due to this cosmological "good bye" and "hello again" feature of inflation in ref.[13] density perturbations is kinematically imprinted. Roughly speaking, quantum fluctuations on a given scale arisen when that scale is sub-horizon sized, "freeze in" as classical metric perturbations after the scale crosses outside the horizon and finally becomes density perturbations when the scale re-enter the horizon.

In order to analyze the spectrum of fluctuations, we, hence, need to know when a given scale crosses outside the horizon during the inflationary epoch. A convenient way of specifying when a given scale crossed outside the horizon is by the number of e-folds between horizon-crossing and the end of inflation N_λ. For example, when $\lambda \sim H_0^{-1}$, $N_\lambda \sim 50$.

At horizon crossing, the amplitude of a density perturbation when it crosses back inside the horizon, $(\delta\rho/\rho)_{\rm HOR}$, is known to nearly equal to the gauge invariant quantity ζ being constant for super-horizon sized perturbations[22]. At horizon crossing, ζ has a particularly simple form: $\zeta = \delta\rho/(\rho + p)$. Then $(\delta\rho/\rho)_{\rm HOR}$ becomes

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm HOR} \simeq \zeta_{N_\lambda} = \left(\frac{\delta\phi V'}{\dot\sigma^2}\right)_{N_\lambda} \simeq \left(\frac{H^2}{\dot\sigma}\right)_{N_\lambda}, \tag{4.29}$$

where we have used the fact that $\rho_v \simeq V$, $\rho_v + p_v = \dot\sigma^2$, $\delta\sigma \simeq H/2\pi$ and the slow-roll equation of motion, $V' = -3H\dot\sigma$.

The number of e-folds from the onset of inflation is given by

$$\begin{aligned} N(\sigma_i \to \sigma) &= \int_{t_i}^{t} H dt = \int_{\sigma_i}^{\sigma} \frac{V(\sigma)}{-V'(\sigma)} d\sigma \\ &= \frac{1}{4\xi} \ln\left(\frac{\sigma}{\sigma_i}\right), \end{aligned} \tag{4.30}$$

from (4.20), (4.21) and slow-roll equation. The slow-roll approximation is valid until $|V''(\sigma_e)| \simeq 9H^2$, or for

$$\xi\sigma_e^2 \simeq \frac{1}{2}, \qquad \sigma_e \simeq (2\xi)^{-1/2}. \tag{4.31}$$

This condition is, of course, compatible with (4.26).

Using slow-roll equations it is a simple matter to evaluate $(\delta\rho/\rho)_{\rm HOR}$,

$$N(\sigma \to \sigma_e) = \frac{1}{4\xi} \ln\left(\frac{\sigma_e}{\sigma}\right) = -\frac{1}{4\xi} \ln\left[(2\xi)^{1/2}\sigma\right], \tag{4.32}$$

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm HOR} \simeq \left(\frac{H^2}{\dot\sigma}\right)_{N_\lambda} \simeq \left(\frac{3H^3}{-V'}\right)_{N_\lambda} \simeq \sqrt{\frac{\Lambda}{24\xi}} \exp(4\xi N_\lambda), \tag{4.33}$$

which for $N_\lambda \simeq 50$ leads to density perturbations of order 10^1 with $\xi = 1.0 \times 10^{-2}$ and $\Lambda = 1$. In order to seed the observed structure in the universe, perturbations of amplitude 10^{-5} to 10^{-4} or so are probably required. On the basis of the measured isotropy of the CMBR, the amplitude can be no larger than 10^{-4}. Unfortunately we find it impossible to obtain density perturbations of this small in our simple model from (4.33). However, since our model with F_K easily allows power-law inflation to occur, we reasonably expect to obtain sufficiently small density perturbations by assuming somewhat generalized forms of F_G and F_K[23]. We will discuss this in detail in a future work.

4.2.3 Asymptotic solutions

We now move on to the asymptotic solution in the era (iii) for which $\xi\phi^2 \gg 1$ and hence

$$F_G \approx \xi\phi^2, \qquad F_K \approx \chi. \tag{4.34}$$

Equation (4.6) is now

$$\frac{d\sigma}{d\phi} \approx (6 + \chi\xi^{-1})^{1/2}\phi^{-1}, \tag{4.35}$$

which is integrated to give

$$\phi = \phi_1 \exp\left(\frac{1}{4\kappa}\sigma\right), \tag{4.36}$$

where the coefficient ϕ_1 is likely to be of the order one and

$$\kappa = \frac{(6+\chi\xi^{-1})^{1/2}}{4}. \tag{4.37}$$

Notice that κ depends not only on F_G but also on F_K. The potential (4.11) now takes the form

$$V \sim \exp\left(-\frac{\sigma}{\kappa}\right). \tag{4.38}$$

Taking also $\rho_r \sim a^{-4}$ into consideration, we find two different analytic solutions depending on κ:

i) $0 < \kappa < 1/2$, namely $-6\xi < \chi < -2\xi$

$$a(t) \sim t^{1/2}, \tag{4.39}$$

$$\sigma(t) = 2\kappa \ln t, \tag{4.40}$$

$$\rho_r(t) = \frac{3}{4}(1-4\kappa^2)t^{-2}, \tag{4.41}$$

$$\rho_v(t) = 3\kappa^2 t^{-2}, \tag{4.42}$$

and

ii) $\kappa > 1/2$, namely $\chi > -2\xi$

$$a(t) \sim t^{2\kappa^2}, \tag{4.43}$$

$$\sigma(t) = 2\kappa \ln t, \tag{4.44}$$

$$\rho_r(t) \sim t^{-8\kappa^2}, \tag{4.45}$$

$$\rho_v(t) = 12\kappa^4 t^{-2}, \tag{4.46}$$

where we have ignored dissipation. In both solutions ϕ evolves like $\sim t^{1/2}$ from (4.36) and (4.40) or (4.44).

The solution (ii) fails to evolve into the radiation-dominant universe, because the exponent $2\kappa^2$ in (4.43) is larger than 1/2 for $\kappa > 1/2$. Hence we must choose the solution (i) for successful cosmology. Note the solution (i) requires a negative χ. If we

naively chose $F_K = 1$, the result would have been the solution (ii) with χ replaced by 1. Then follows $\kappa^2 > 3/8$ from (4.37) and the scale factor expands faster than $t^{3/4}$, deviating dangerously away from the standard expansion $\sim t^{1/2}$. This is the reason why we consider a generalized scalar-tensor theory with $F_K \neq 1$. Of course, we do not claim the uniqueness of (4.3); it is only one of the convenient candidates. One might think of an even simpler choice $F_K \equiv \chi < 0$. This brings in a complication, however: for $\xi(6\xi+\chi)\phi^2 < \chi$, D given by (4.8) is negative, and hence σ becomes a ghost. In the following discussion we confine ourselves to the solution (i).

From (4.18),(4.41) and (4.42) we also obtain

$$\rho_v/\rho = 4\kappa^2 = \frac{1}{4}(6+\chi\xi^{-1}), \tag{4.47}$$

which should be constrained $\lesssim 0.1$ for the successful analysis of the primordial nucleosynthesis[24][25].

As the effective cosmological constant ρ_v behaves like $\sim t^{-2}$ from (4.42), the present effective cosmological constant becomes $\sim 10^{-120}$ due to the present age of universe $t_{pr} \sim 10^{10}\text{y} \sim 10^{60}$. This explains the present extraordinary small cosmological constant naturally without fine-tuning.

4.2.4 Dissipation

During the transient era between primordial inflation and the asymptotic era of a power-law expansion, various non-linear effects including reheating are expected to have taken place. As compared with ordinary new-inflationary models, we encounter two difficulties on the reheating process.

One is that the gravitational scalar field σ interacts with ordinary matter fields very weakly: if the gravitational scalar has a Yukawa coupling to a spinor field representing ordinary matter fields, the coupling constant is $f \sim 10^{-19}$. The ordinary inflaton couples to other fields rather strongly, hence producing sufficient reheating, whereas the gravitational scalar field has a much weaker coupling, nearly as weak as gravity.

The other is that the potential of σ has no minimum point to cause the cosmological constant to decay. If it had a minimum point, we would have to fine-tune the height of the potential at the minimum point[1]. In ordinary inflationary models, in contrast, the potential of the inflaton has a minimum point and the inflaton oscillates many times around the minimum point during the reheating epoch. The energy stored in the inflaton can, hence, be converted effectively to the energy of ordinary matter fields. But the gravitational scalar σ does not oscillate and has only one chance that the energy stored in the gravitaional scalar is converted to ordinary matter.

One may wonder if a decaying cosmological constant model is unable to reheat the universe sufficiently. As a possible remedy, we consider couplings of the gravitational scalar to a scalar field like a Higgs scalar. The couplings is produced rather naturally under the conformal transformation. Note that the couplings are strong enough to cause sufficient reheating as, we will show in detail later. The dissipative term will be derived as a quantum effect.

Dissipation may arise from particle creation due to the temporal change of the mass of σ; the squared mass as defined by the second derivative of the potential decreases as σ goes down the potential. The question is that the rate of reheating might be too weak as the above-mentioned. It is easy to see that the effect would be important only in the early epoch, more specifically only during the period of reheating. We derive a dissipative term based on this process by following the Morikawa-Sasaki-Ringwald recipe[26][27].

Let the field σ be divided into the classical background σ_c and the quantum fluctuation σ_q:

$$\sigma = \sigma_c + \sigma_q, \tag{4.48}$$

with the condition $\langle\sigma_q\rangle = 0$. We substitute (4.48) into (4.10) and derive the equation

of motion for σ_c:

$$-\Box\sigma_c + \frac{\partial V(\sigma_c)}{\partial\sigma} + \frac{1}{2}V^{(3)}(\sigma_c)\langle\sigma_q^2\rangle + O(\sigma_q^3) = 0, \tag{4.49}$$

and the Lagrangian of σ_q:

$$\mathcal{L}_q = \sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma_q\partial_\nu\sigma_q - \frac{1}{2}V^{(2)}(\sigma_c)\sigma_q^2 + O(\sigma_q^3)\right], \tag{4.50}$$

where

$$V^{(n)}(\sigma_c) = \frac{\partial^n V(\sigma_c)}{\partial\sigma^n}. \tag{4.51}$$

The third term $\frac{1}{2}V^{(3)}(\sigma_c)\langle\sigma_q^2\rangle$ and higher terms of σ_q in (4.49) represent the backreaction of the particle production, causing dissipation. We focus upon the lowest-order terms, neglecting the last term of (4.49).

In cosmological applications of quantum field theory, it seems most convenient to use the conformal time η ($d\eta = dt/a$) and introducing the redefined field $\varphi = a\sigma_q$. Then (4.50) is put into the form

$$\mathcal{L}_q = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}M^2\varphi^2, \tag{4.52}$$

where

$$M^2 \equiv a^2V^{(2)}(\sigma_c) - a''/a = a^2V^{(2)}(\sigma_c) - \frac{1}{6}R, \tag{4.53}$$

the prime meaning a derivative with respect to η. It appears as if we were in Minkowski space-time and the effect of space-time curvature is packed into the mass term M^2. It is noticed that M^2 is not always positive. In the following discussion, we limit ourselves to σ_c giving the positive M^2.

We define the creation and annihiration operators at time η by

$$\varphi(\eta, x) = \int \frac{dk}{(2\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{2\omega_k(\eta)}}\left[a_k(\eta)e^{ikx} + a_k^\dagger(\eta)e^{-ikx}\right], \tag{4.54}$$

where $\omega_k^2(\eta) = k^2 + M^2(\eta)$[26]. The Hamiltonian is then instantaneously diagonalized;

$$H(\eta) = \frac{1}{2}\int dk\omega_k(\eta)\left[a_k(\eta)a_k^\dagger(\eta) + a_k^\dagger(\eta)a_k(\eta)\right]. \tag{4.55}$$

The vacuum state $|0\rangle$ may be defined by

$$a_k(\eta_0)|0\rangle = 0, \tag{4.56}$$

where η_0 is a conformal time chosen appropriately. The operators $a_k(\eta)$ and $a_k^\dagger(\eta)$ are related to $a_k(\eta_0)$ and $a_k^\dagger(\eta_0)$ by a Bogoliubov transformation.

We define diagonal and off-diagonal elements of the density matrix by

$$x_k(\eta) = \langle a_k^\dagger(\eta) a_k(\eta)\rangle, \tag{4.57}$$

and

$$y_k(\eta) = \langle a_k(\eta) a_k(\eta)\rangle. \tag{4.58}$$

These variables $x_k(\eta)$ and $y_k(\eta)$ can be represented by Bogoliubov coefficients, satisfying the following equations[28]:

$$x_k'(\eta) = \frac{\omega_k'(\eta)}{\omega_k(\eta)} \Re e\, y_k(\eta), \tag{4.59}$$

$$y_k'(\eta) = \frac{\omega_k'(\eta)}{\omega_k(\eta)}\left(x_k(\eta) + \frac{1}{2}\right) - 2i\left(\omega_k(\eta) - \frac{i}{2}\Gamma_k(\eta)\right) y_k(\eta), \tag{4.60}$$

where we have explicitly introduced the the decay width $\Gamma_k(\eta)$ of σ_q as the imaginary part of M. By using formal solutions of these equations, $\langle\varphi^2(\eta)\rangle$ is expressed as a non-local function of time. Sufficient dissipation, however, will destroy long-time correlations, hence leaving $\langle\varphi^2(\eta)\rangle$ as a local function of $\sigma_c, \dot\sigma_c, a$ and time derivatives of a.

We solve these two equations in an adiabatic order expansion[27]. We replace η temporarily by $\eta_1 \equiv \eta/T$ where T is called the adiabatic parameter. We perform an expansion of (4.59) and (4.60) in the inverse power of T:

$$\frac{1}{T}x_k' = \frac{1}{T}\frac{\omega_k'}{\omega_k}u_k, \tag{4.61}$$

$$\frac{1}{T}u_k' = \frac{1}{T}\frac{\omega_k'}{\omega_k}\left(x_k + \frac{1}{2}\right) + 2\omega_k v_k - \Gamma_k u_k, \tag{4.62}$$

$$\frac{1}{T}v_k' = -2\omega_k u_k - \Gamma_k v_k, \tag{4.63}$$

where $u_k = \Re e\, y_k$ and $v_k = \Im m\, y_k$, and

$$x_k = x_k^{(0)} + T^{-1}x_k^{(1)} + T^{-2}x_k^{(2)} + \cdots, \tag{4.64}$$

$$u_k = u_k^{(0)} + T^{-1}u_k^{(1)} + T^{-2}u_k^{(2)} + \cdots, \tag{4.65}$$

$$v_k = v_k^{(0)} + T^{-1}v_k^{(1)} + T^{-2}v_k^{(2)} + \cdots. \tag{4.66}$$

To the lowest order, $\Re e\, y_k(\eta)$ is given by

$$\Re e\, y_k(\eta) = \frac{\omega_k'(\eta)}{\omega_k(\eta)}\left(x_k(\eta) + \frac{1}{2}\right)\frac{\Gamma_k}{4\omega_k^2}, \qquad \text{for } \Gamma_k \ll \omega_k. \tag{4.67}$$

We express $\langle\varphi^2(\eta)\rangle$ in terms of $x_k(\eta)$ and $y_k(\eta)$ as follows

$$\langle\varphi^2(\eta)\rangle = \int \frac{dk}{(2\pi)^3 2\omega_k(\eta)}[2\Re e\, y_k(\eta) + 2x_k(\eta) + 1]. \tag{4.68}$$

The portion of $\langle\varphi^2(\eta)\rangle$ responsible for dissipation is now given by

$$\langle\varphi^2(\eta)\rangle_{\rm dis} = \int \frac{dk}{(2\pi)^3 2\omega_k(\eta)} 2\Re e\, y_k(\eta) = \int \frac{dk}{4(2\pi)^3}\frac{\omega_k'}{\omega_k^4}\left(x_k + \frac{1}{2}\right)\Gamma_k. \tag{4.69}$$

For simplicity, we may neglect $x_k(\eta)$. ω_k' causes dissipative effect because ω_k' includes $\dot\sigma_c$:

$$\begin{aligned}
2\omega_k\omega_k' &= \frac{d}{d\eta}\omega_k^2 && (4.70)\\
&= a\frac{d}{dt}(k^2 + M^2) && (4.71)\\
&= a^3\left[V^{(3)}(\sigma_c)\dot\sigma_c + 2HV^{(2)}(\sigma_c) - \frac{\dddot a}{a} - 3H\frac{\ddot a}{a}\right]. && (4.72)
\end{aligned}$$

The decay width Γ_k is roughly the square of the coupling constant. We find that the dissipative term would be too small if the coupling of φ is as weak as the gravitational interaction. Suppose, however, Higgs fields are present as natural ingredients of the universe. Moreover Higgs scalar fields couple to other matter fields rather strongly, decaying into other matter fields rather quickly. It may suffice to discuss only the decaying of σ_q into a Higgs scalar without going into details of the decay of a Higgs scalar[4]. To simplify the argument let us consider a single neutral scalar field Φ. In the original CF, the kinetic term of Φ is given by

$$\mathcal{L}_{\Phi\mathrm{kin}} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right). \tag{4.73}$$

We apply the conformal transformation: $\Phi = F_G^{\frac{1}{2}}\Phi_*$. Then $\mathcal{L}_{\Phi\mathrm{kin}}$ becomes (again dropping $*$)

$$\mathcal{L}_{\Phi\mathrm{kin}} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}D_\mu\Phi D_\nu\Phi\right), \tag{4.74}$$

where $D_\mu = \partial_\mu + (1/2)\partial_\mu(\ln F_G)$. Notice the emergence of the derivative couplings. No similar term is present for gauge fields or spinor fields. We also point out that the noninvariant terms appear even if the exponent $1/2$ in the conformal transformation of Φ is replaced by some other (noncanonical) value, unless Φ is left unchanged.

We pick up interaction terms for ϕ which gets sufficiently large toward the end of inflation:

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = \sqrt{-g}\Big[&-\frac{1}{4\kappa}g^{\mu\nu}\partial_\mu\left(\frac{1}{2}\Phi^2\right)\partial_\nu\sigma_q - \left(\frac{1}{4\kappa}\right)^2 g^{\mu\nu}\partial_\mu\sigma_c\partial_\nu\sigma_q\Phi^2 \\ &-\frac{1}{2}\left(\frac{1}{4\kappa}\right)^2 g^{\mu\nu}\partial_\mu\sigma_q\partial_\nu\sigma_q\Phi^2\Big]\end{aligned} \tag{4.75}$$

All the terms have dimension 5 or higher, being suppressed by constants of mass dimensions, of the order of m_{Pl} though not shown explicitly. For this reason the interaction (4.75) is as weak as gravity at late epochs. We use the conformal time η and introduce ψ by $\psi = a\Phi$. Then the interaction terms become

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} &= -\frac{1}{4\kappa}a^{-1}\left[\eta^{\mu\nu}\partial_\mu\left(\frac{1}{2}\psi^2\right)\partial_\nu\varphi + h\psi^2\varphi' + h\psi\psi'\varphi - h^2\psi^2\varphi\right] \\ &\quad - \left(\frac{1}{4\kappa}\right)^2\dot{\sigma}_c\left[-\varphi'\psi^2 + h\psi^2\varphi\right],\end{aligned} \tag{4.76}$$

where $h = a'/a$ and $\partial_0 = \partial/\partial\eta$. We then compute Γ_k which represents the decay rate of φ into two ψ to the one-loop order. First we compute the invariant amplitude $\mathcal{M}$ of

$\varphi \to \psi\psi$:

$$\begin{aligned}\mathcal{M} &= -\frac{1}{4\kappa}a^{-1}\left[ik\cdot(-i(p_1+p_2)) + ih(-2k^0 + (p_1^0+p_2^0)) - 2h^2\right] \\ &\quad - \left(\frac{1}{4\kappa}\right)^2\dot{\sigma}_c\left[2ik^0 + 2h\right],\end{aligned} \tag{4.77}$$

where a particle φ with momentum k decays into two ψs with momentum p_1 and p_2. Using momentum conservation: $k = p_1 + p_2$, $\mathcal{M}$ becomes

$$\mathcal{M} = -\frac{1}{4\kappa}a^{-1}\left[k^2 - 2h^2 + \frac{1}{2\kappa}a\dot{\sigma}_c h - i(h - \frac{1}{2\kappa}a\dot{\sigma}_c)k^0\right]. \tag{4.78}$$

Note that $\mathcal{M}$ depends on momentum k because of the derivative couplings. Then $|\mathcal{M}|^2$ becomes

$$|\mathcal{M}|^2 = \left(\frac{1}{4\kappa}\right)^2 a^{-2}\left[\left(k^2 - 2h^2 + \frac{1}{2\kappa}a\dot{\sigma}_c h\right)^2 + (h - \frac{1}{2\kappa}a\dot{\sigma}_c)^2(k^0)^2\right], \tag{4.79}$$

at $k = p_1 + p_2$. Using this $|\mathcal{M}|^2$, the decay rate Γ_k is formally written by

$$\begin{aligned}\Gamma_k &= N\frac{(2\pi)^4}{2\omega_k}\iint\frac{dp_1}{(2\pi)^3 2\tilde{\omega}_{p_1}}\frac{dp_2}{(2\pi)^3 2\tilde{\omega}_{p_2}}\delta^{(4)}(k - p_1 - p_2)|\mathcal{M}|^2 \\ &= \frac{N}{32\pi^2}\frac{1}{\omega_k}\int\frac{dp}{\tilde{\omega}_p\tilde{\omega}_{k-p}}\delta(\omega_k - \tilde{\omega}_p - \tilde{\omega}_{k-p})|\mathcal{M}|^2,\end{aligned} \tag{4.80}$$

where $\tilde{\omega}_p = \sqrt{p^2 + m^2}$ and m is Higgs mass and $N = 1/2$ because φ decays into two identical particles ψ[29]. In the rest frame $\Gamma_{k=0}$ is calculated simply giving

$$\Gamma_{k=0} = \frac{N}{16\pi}\frac{1}{M}|\mathcal{M}|^2_{k=0}, \tag{4.81}$$

with

$$|\mathcal{M}|^2_{k=0} = \left(\frac{1}{4\kappa}\right)^2 a^{-2}(M^2 + h^2)\left[M^2 + 4h^2 - \frac{2}{\kappa}ah\dot{\sigma}_c\right] + O(\dot{\sigma}_c^2), \tag{4.82}$$

where we assumes $m \ll M$. From Lorentz invariance, we obtain

$$\begin{aligned}\Gamma_k(\eta) &= \frac{M}{\omega_k}\Gamma_{k=0} = \frac{N}{16\pi}\frac{1}{\omega_k}|\mathcal{M}|^2_{k=0} \\ &= \frac{1}{32\pi}\frac{1}{\omega_k}\left(\frac{1}{4\kappa}\right)^2 a^{-2}(M^2 + h^2)\left[M^2 + 4h^2 - \frac{2}{\kappa}ah\dot{\sigma}_c\right] + O(\dot{\sigma}_c^2),\end{aligned} \tag{4.83}$$

where we assume that $M^2 > 0$ and that mass of ψ is negligible. Picking out the term proportional to $\dot{\sigma}_c$ from $\frac{1}{2}V^{(3)}(\sigma_c)\langle\sigma_q^2\rangle$ in (4.49), we obtain the dissipative term $F_d(\sigma_c)\dot{\sigma}_c$ as follows

$$F_d(\sigma_c)\dot{\sigma}_c = \frac{1}{2}V^{(3)}(\sigma_c)\langle\sigma_q^2\rangle_{\rm dis} = \frac{1}{2}V^{(3)}(\sigma_c)a^{-2}\langle\varphi^2\rangle_{\rm dis} \tag{4.84}$$

$$= \frac{1}{32768\pi^2}\left(\frac{1}{4\kappa}\right)^2 V^{(3)}(\sigma_c)\left(\frac{a}{M}\right)^3\left[\left(\frac{M}{a}\right)^2 + H^2\right]$$

$$\times \left[V^{(3)}(\sigma_c)\left\{\left(\frac{M}{a}\right)^2 + 4H^2\right\} - \frac{2}{\kappa}H\left(2HV^{(2)} - \frac{\dddot{a}}{a} - 3H\frac{\ddot{a}}{a}\right)\right]\dot{\sigma}_c, \tag{4.85}$$

where we neglected higher-order term of $\dot{\sigma}_c$.

4.2.5 Numerical example

We solve (4.15)-(4.17) numerically by using (4.85), which, due to the condition $M^2 > 0$, enters only after certain time roughly corresponding to the point of inflection of $V(\sigma)$. We show an example in Fig.2, plotting a, ϕ, ρ_r/ρ, p_v/ρ_v, ρ_r and ρ_v vs. t for $\Lambda = 1$, $\xi = 0.8 \times 10^{-2}$, $\chi = -4.48 \times 10^{-2}$ [hence $4\kappa^2 = 0.1$] and the initial value $\phi = \dot{\phi} = 1$ at $t = 1$ where $p_v = \dot{\sigma}^2/2 - V$ is the "pressure" of the vacuum. We have chosen this ξ to obtain sufficient inflation and this χ to obtain successful primordial nucleosynthesis.

We find: (i) An exponential growth of $a(t)$ to $\gtrsim e^{60}$ ending at $t_1 \sim 10^3$ (in units of the Planck time) emerges naturally from the potential $V = \Lambda F_G^{-2}$ with (4.2) having a "plateau", with no need to design the potential of Φ specifically, an advantage of this model in which a gravitational scalar plays a role of the inflaton; we have ignored the classical Φ in the present example. From t_1 to $t_2 \sim 10^{17}$ the scale factor a evolves according to a power-law $\sim t^{1/3}$ and expands like $\sim t^{1/2}$ after t_2. By the way, primordial nucleosynthesis occurs at about $t \sim 10^{45}$ and non-relativistic matter energy becomes dominant at $t = t_{\rm eq} \sim 10^{54}$. This example, hence, reproduces successful cosmology.

(ii) The scalar field ϕ also grows exponentially until t_1 obeying (4.24). The asymptotic solution $\phi \sim t^{1/2}$ begins at $t_4 \sim 10^{40}$. Note that ϕ stays nearly constant from t_2 to

t_4 owing to non-linear effects.

(iii) The curve of ρ_r/ρ shows that the radiation energy becomes dominant after t_2 and begins to converge to the asymptotic value $(1 - 4\kappa^2 = 0.9)$ around t_4. Notice the corresponding behaviors of a and ϕ at t_2 and t_4. The ratio p_v/ρ_v, so to say, representing an equation of state of the vacuum, shows a very interesting behavior due to non-linear effects. This behavior is often called a relaxation oscillation and is one of common phenomena in nature. Of most interests the behavior between $t_3 \sim 10^{32}$ and t_4 showing that ρ_v behaves like an "suspending" cosmological constant. Around t_4, p_v/ρ_v begins to damped-oscillate as a function of $\ln t$ and finally settles to $1/3$.

(iv) After the end of inflation, dissipation quickly pushes up ρ_r which had supercooled. The dissipative interaction, however, begins to dwindle, as was expected. Also the growth of $a(t)$ is still quite fast. As a consequence ρ_r shows a rapid decrease, leaving a spike-like behavior. It then starts decreasing slowly like $\sim t^{-4/3}$ at t_1. Combining this with the power-law expansion $a(t) \sim t^{1/3}$ from t_1 to t_2, we find that entropy production is no longer appreciable. We may define the reheating temperature $T_{\rm reh}$ by ρ_r at the onset of this power-law behavior. According to (3.6) with $g_* \sim 100$ we find $T_{\rm reh} \sim 10^{14}$GeV. This shows that sufficient reheating can be expected in this mechanism despite the absence of an oscillating phase.

(v) ρ_v behaves like a "cosmological constant" at the beginning and decays like $\sim t^{-2}$ from t_1 to t_2 and like $\sim t^{-8/3}$ from t_2 to t_3. Again it behaves like a "cosmological constant" from t_3 to t_4 and the asymptotic solution $(\sim t^{-2})$ begins after t_4. This behavior may provide a natural explanation of the present cosmological constant which is 10^{-120} times as small as $m_{\rm Pl}^4$.

Many features sketched above are quite general, insensitive to the choice of parameters and initial conditions, given the rate of the inflationary expansion and the asymptotic value of ρ_v/ρ.

4.2.6 Transient era

Although the behavior of solutions during the transient era is very anomalous as shown in Fig.2, Such a behavior is rather common in the system of the cosmological equations (4.15)-(4.15) with an exponential potential $V \sim e^{-\sigma/\kappa}$. A detailed analysis will follow.

Right after inflation, $K(=\frac{1}{2}\dot\sigma^2)$, representing the energy density of the kinetic part of σ, becomes dominant because σ is going down a steep slope of the potential. We reasonably assume $K \gg V \gg \rho_r$ at t_1. Then (4.15)-(4.17) become

$$3H^2 = \frac{1}{2}\dot\sigma^2 (= K), \tag{4.86}$$

$$\ddot\sigma + 3H\dot\sigma = 0, \tag{4.87}$$

$$\rho_r \sim a^{-4}, \tag{4.88}$$

where dissipation is neglected. The potential is already $V \sim e^{-\sigma/\kappa}$ because σ has grown exponentially like (4.24) until t_1. Then a solution is analytically given by

$$a \sim t^{1/3}, \qquad \sigma = \sqrt{2/3}\ln\left(\frac{t}{t_1}\right), \qquad \rho_r = \rho_{r1}\left(\frac{t}{t_1}\right)^{-4/3}, \tag{4.89}$$

where ρ_{r1} is the radiation energy at t_1 depending on the reheating.

Note that the potential as a function of t is given by

$$V = V_1\left(\frac{t}{t_1}\right)^{-\sqrt{2/3}/\kappa}, \qquad O(V_1) \sim t_1^{-2}, \tag{4.90}$$

whereas $K = (1/3)t^{-2}$. If $\kappa < \kappa_c = 1/\sqrt{6}$, V decays faster than t^{-2} and $V/K \to 0$. As we must choose $\kappa < 0.158$ to obtain successful cosmology, the example in Fig.2 satisfies this condition $\kappa < \kappa_c$. The plateau $p_v/\rho_v = 1$ from t_1 to t_3 in Fig.2 corresponds to this situation. From (4.89) we find that the universe becomes radiation-dominant at $t_2 \simeq (3\rho_{r1})^{-3/2}t_1^{-2}$ ($K \simeq \rho_r$ at t_2). Until t_2 the potential V has reduced to the value V_2 which is much smaller than K or ρ_r

$$V_2 = V_1\left(\frac{t_2}{t_1}\right)^{-\sqrt{2/3}/\kappa} \ll t_2^{-2}. \tag{4.91}$$

We define a time scale t_V with $t_V \sim V_2^{-1/2}$. Roughly speaking, σ has never felt the existence of the potential until t_V; it is as if σ flew over the potential freely. This feature may be peculiar to an exponential potential. Surely if V were $\sim \sigma^{-n}$ and K were dominant satisfying (4.89), V/K could not have decreased; rather it would have increased. In the case of a scalar field with an exponential potential, K and V may behave independently of each other, and the condition $\kappa < \kappa_c$ is, so to say, "flying-over condition".

We find that $H = (1/2)t^{-1}$ after t_2 since the universe is radiation-dominant. Moreover $3H\dot\sigma \gg -V^{(1)} = V/\kappa$. For these reasons the behavior of σ is given by

$$\dot\sigma = \dot\sigma_2\left(\frac{t}{t_2}\right)^{-3/2}, \qquad \sigma = \sigma_2 + 2\dot\sigma_2\left(1 - \left(\frac{t}{t_2}\right)^{-1/2}\right), \tag{4.92}$$

where $\sigma_2 = \sqrt{2/3}\ln(t_2/t_1)$ and $\dot\sigma_2 = \sqrt{2/3}t_2^{-1}$. For $\sigma_2 \gg \dot\sigma_2$, σ is nearly constant $\approx \sigma_2$. This explains why $\ln\phi$ is almost constant from t_2 to t_4 in Fig.2. From $K \sim t_2 t^{-3}$ and $V \simeq V_2$, t_3 is given by $t_3 \sim (t_2/V_2)^{1/3}$ as $K \sim V$ at t_3 and $K/V \to 0$ after t_3. This shows the behavior of the "second cosmological constant" from t_3 to t_4. The value of σ has been frozen around σ_2 until t_V because of $V \sim t_V^{-2}$ although V becomes comparable to $3H\dot\sigma$ between t_3 and t_V from (4.92). This tells us that $t_4 \sim t_V$.

After t_4 the behavior of σ is expected to approach the analytic solution (4.40) since the potential V begins to participate in the evolution of σ again. The difference between σ and (4.40) is defined by

$$\sigma(t) = 2\kappa(\ln t + \varepsilon(t)), \tag{4.93}$$

where ε is dimensionless difference function of t. We substitute (4.93) into (4.16) obtaining

$$\ddot\varepsilon + \frac{3}{2}t^{-1}\dot\varepsilon + t^{-2}\varepsilon = 0, \tag{4.94}$$

where $V = \kappa^2 e^{-\sigma/\kappa}$, and higher-order terms of ε have been neglected. Since this is a homogeneous equation, we obtain $\varepsilon \sim t^\delta$ with $\delta = -\frac{1}{4} \pm i\frac{\sqrt{15}}{4}$. We thus find ε is giving by

$$\varepsilon \sim t^{-1/4} \sin\left(\frac{\sqrt{15}}{4}\ln t\right), \tag{4.95}$$

which represents a damped oscillation as a function of $\ln t$. This shows how σ approaches the asymptotic solution (4.40). This behavior is hardly seen in the plot of $\ln\phi$ but is seen clearly in p_v/ρ_v in Fig.2.

We show two examples in Fig.3 and Fig.4. In Fig.3 χ is chosen to be $\kappa = \kappa_c$ and the other parameters and initial values are the same in Fig.2. This example does not have an anomalous behavior like Fig.2, as might have been expected. In Fig.4 we appropriately choose $\chi = -0.04692$ to make the "suspending cosmological constant" V_2 about 10^{-120} and the other parameters and initial values are the same in Fig.2. This "suspending cosmological constant"$\sim 10^{-120}$ may be related with the recent report on the very small but nonzero cosmological constant. We, of course, fine-tune χ to obtain this result (cf. if $\chi = -0.04690$, $V_2 \sim 10^{-110}$). Still this fine-tuning is much better than the notorious fine-tuning to the order of 120. With this "soft" fine-tuning accepted, the approach, however, is not suitable for the recent analysis because the cosmological constant is not dominant today as shown in the curve ρ_r/ρ in Fig.4. This simple model cannot make the vacuum energy dominant again satisfying the recent analysis.

4.3 An exceptional choice

If the condition (4.47) should hold true to a good approximation, one might be tempted to speculate that there is a yet-to-be-known "symmetry" which forces $\chi = -6\xi$ giving $\kappa^2 = 0$. Obviously, however, this is a singular limit which requires a separate analysis starting from $F_K = (1 - 6\xi^2\phi^2)/F_G$, to find a surprisingly simple result[4].

Equation (4.8) now simplifies to $D = 1$, yielding

$$\frac{d\sigma}{d\phi} = -\frac{1}{1+\xi\phi^2}, \tag{4.96}$$

where we have chosen the minus sign in (4.6). We thus obtain

$$\sigma = \frac{1}{\sqrt{\xi}}\,\mathrm{arccot}\,(\sqrt{\xi}\phi), \tag{4.97}$$

by choosing the integration constant such that $\phi \to \infty$ corresponds to $\sigma = 0$. The domain of σ is limited to $|\sigma| \le (\pi/2)\xi^{-1/2}$, unlike in the previous analysis with $\kappa^2 \neq 0$. The potential (4.11) is then

$$V = \Lambda \sin^4(\sqrt{\xi}\sigma). \tag{4.98}$$

In our model this is an exceptional case in which $V_{\rm min} = 0$ occurs at a finite value of σ.

For $\sigma \approx 0$, V behaves like

$$V \approx \lambda_1 \sigma^4, \tag{4.99}$$

with $\lambda_1 = \Lambda\xi^2$. As one notices, this is reminiscent to the potential assumed in the model of chaotic inflation. In ref.[20], however, a question remains why the minimum of the potential should vanish. We, on the other hand, derive (4.98) and (4.99) without the additional cosmological constant from the cosmological constant without additional potentials in the original CF.

4.4 Quantum effect

As we have already discussed dissipation as a .quantum effect, we would like to discuss the radiative correction because almost of all our analysis has been limited to the classical field equations; quantum effects may not be important after the Planck time. Taking aside suspected quantum effects of ϕ before the end of inflation, we discuss the possible radiative correction to the starting Lagrangian (4.1).

For the asymptotic era, we notice a simplification for radiative corrections to $V(\sigma_c)$. In view of (4.48), $V(\sigma_c)$ will be modified to

$$\tilde{V}(\sigma_c) = \tilde{B}e^{-\sigma_c/\kappa}, \tag{4.100}$$

where

$$\tilde{B} \sim \langle e^{-\sigma_q/\kappa} \rangle. \qquad (4.101)$$

The vacuum on the right-hand side depends on $\sigma_c(t)$, and does $\tilde{B}$. We may reasonably expect, however, that $\tilde{B}$ depends on σ_c only weakly not to affect the original dependence as indicated in $(V \sim e^{-\sigma/\kappa})$ in any seriously manner.

There is, however, another effect which could be more problematic. One might ask how the results are affected if one adds renormalization counterterms to the starting Lagrangian. In the original CF, the most important will be the quartic term $\sim f^4\phi^4$ due to ψ loop if we choose the simple example (4.12) as $\mathcal{L}_c$. During early epochs this term is unimportant because of the smallness of $f \sim 10^{-19}$. At later times smallness may not be enough, in fact this term survives the reduction due to $F_G^{-2} \sim \phi^{-4}$ after the conformal transformation, giving a constant energy density $\sim f^4 \sim 10^{-76}$ which is damagingly too large, about 44 orders larger than the upper bound of the present cosmological constant. We point out, however, that a more careful analysis is needed in choosing a CF before the quantum theory is applied.

In the original unstarred CF, the nonminimal coupling $\sqrt{-g}\frac{1}{2}\xi\phi^2 R$ in (4.1) acts effectively as a mass term of ϕ, giving a mass-squared $-\xi R$ which is negative in the inflationary era. One then introduce a vacuum expectation value ϕ_0. Substituting $\phi = \phi_0 + \tilde{\phi}$ in the above nonminimal coupling term yields a mixing term between the fluctuating part $\tilde{\phi}$ and some components of the gravitational fields. For example, in the weak-field approximation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the nonminimal coupling term gives $-\xi\phi_0(\partial_\mu\tilde{\phi})[\partial_\nu h^{\nu\mu} - \partial^\mu(\eta^{\rho\sigma}h_{\rho\sigma})]$. A diagonalization procedure must be applied to obtain a correct particle picture as a basis of quantization. This is done precisely by the conformal transformation, thus providing another indication that the starred CF with the canonical field σ is preferred to the original CF.

On using (4.36) the coupling term (4.13) is approximated by (again dropping $*$)

$$\mathcal{L}_c = -\sqrt{-g} f \xi^{-1/2}(1 - \frac{1}{2}\xi^{-1}\phi^{-2})\bar{\psi}\psi, \qquad (4.102)$$

for $\xi\phi^2 \gg 1$. According to (4.36),(4.48) and $\phi = \phi_a t^{1/2}$ where ϕ_a is a constant, we obtain

$$\phi = \phi_a t^{1/2} \exp\left(\frac{1}{4\kappa}\sigma_q\right). \qquad (4.103)$$

substituting this into (4.102) and expanding into a power series with respect to σ_q yields

$$\begin{aligned} \mathcal{L}_c &= -\sqrt{-g} f \xi^{-1/2} \\ &\times \left[1 - \frac{1}{2}\xi^{-1}\phi_a^{-2}t^{-1} \times \left\{1 + \frac{1}{4\kappa}\sigma_q + \frac{1}{32\kappa^2}\sigma_q^2 + \cdots\right\}\right]\bar{\psi}\psi. \end{aligned} \qquad (4.104)$$

The Yukawa coupling "constant" of $\bar{\psi}\psi\sigma_q$ is then ft^{-1}, which is smaller than the naive expectation f by $t^{-1} \sim 10^{-61}$ at the present time. As a consequence the force mediated by a single-σ exchange is ~ 120 orders weaker than what is expected as a fifth force. Because of this weak coupling suppressed by the age of the universe, no sizable effects arise from ψ loops in contrast with what has been suspected otherwise in the original CF.

It appears as if the quantization and conformal transformation are two mutually non-comutable procedures. This might be acceptable in view of the conformal noninvariance in our model theory, though it is yet to be shown in more detail how the quantization program is carried out in each CF. In this sense, admittedly, our conclusion of the small cosmological constant is still tentative.

4.5 Decaying Λ in the original CF

In some models of a decaying cosmological constant, calculation is carried out in the original CF. As mentioned before, however, the field ϕ in this CF is not a canonical field also having a mixing with part of the metric field through the nonminimal coupling. This prevents us from fully understanding the physical situation in the original CF.

Moreover the effective gravitational constant changes as a function of t. It is interesting to study the difference between the original CF and the new starred CF in which our analysis has been done in detail. In this section we discuss the same subject in the original CF on the basis of the works of Dolgov and Ford.

The equation of motion of ϕ is derived from (4.1):

$$-\Box\phi + \frac{1}{2}\frac{1}{F_K}\frac{dF_K}{d\phi} - \frac{1}{2}\frac{1}{F_K}\frac{dF_G}{d\phi}R = 0, \tag{4.105}$$

and the Einstein equation $\delta\mathcal{L}/\delta g^{\mu\nu} = 0$ reads as

$$F_G G_{\mu\nu} + \Lambda g_{\mu\nu} = F_K(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi) + F_{G;\mu;\nu} - g_{\mu\nu}g^{\rho\sigma}F_{G;\rho;\sigma} + T_{\mathrm{matter}\mu\nu}, \tag{4.106}$$

where $T_{\mathrm{matter}\mu\nu}$ is the energy-momentum tensor of ordinary matter fields for which we assume the radiation energy. The effective gravitational constant is $(8\pi F_G)^{-1}$ and the cosmological term acts effectively as Λ/F_G.

Strictly speaking, we do not know how to define the energy-momentum tensor of a noncanonical field with a nonminimal coupling to gravity. We still define it following Ford:

$$\tilde{T}_{\phi\mu\nu} = F_K(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi) + F_{G;\mu;\nu} - g_{\mu\nu}g^{\rho\sigma}F_{G;\rho;\sigma} - (F_G - 1)G_{\mu\nu}. \tag{4.107}$$

Then (4.106) becomes

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \tilde{T}_{\phi\mu\nu} + T_{\mathrm{matter}\mu\nu}. \tag{4.108}$$

It looks as if the gravitational constant were a constant, but the true effective gravitational constant which is subject to observation is still $(8\pi F_G)^{-1}$.

From our conventional choice (4.2) and (4.3), the 00-component of (4.108) is equivalent to

$$3H^2 = \Lambda + \tilde{\rho}_\phi + \rho_r, \tag{4.109}$$

where $H = \dot{a}/a$ and

$$\begin{aligned}\tilde{\rho}_\phi &= \tilde{T}_{\phi 00} \\ &= \frac{1}{2}F_K\dot{\phi}^2 - 3H^2\xi\phi^2 - 6H\xi\dot{\phi}\phi.\end{aligned} \tag{4.110}$$

Assuming $\phi = \phi(t)$, the Klein-Gordon equation (4.105) takes the form

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\xi}{F_G F_K}(\chi - F_K)\dot{\phi}^2\phi - \frac{6\xi}{F_K}\left(\frac{\ddot{a}}{a} + H^2\right)\phi = 0. \tag{4.111}$$

Equations (4.109) and (4.111) are a pair of nonlinear coupled equations which determines the evolution of the homogeneous scalar field $\phi(t)$ and the scale factor $a(t)$.

For the early time ($\xi\phi^2 \ll 1$), (4.109) and (4.111) become

$$3H^2 = \frac{1}{2}\dot{\phi}^2 - 3H^2\xi\phi^2 - 6H\xi\dot{\phi}\phi + \Lambda, \tag{4.112}$$

and

$$\ddot{\phi} + 3H\dot{\phi} - 6\xi\left(\frac{\ddot{a}}{a} + H^2\right)\phi = 0, \tag{4.113}$$

where higher-order terms of ϕ and ρ_r are neglected. The solution of these equations are approximately

$$a(t) = a_i e^{H_i t}, \qquad H_i = \sqrt{\frac{\Lambda}{3}}, \tag{4.114}$$

and

$$\phi(t) = \phi_i e^{\gamma t}, \tag{4.115}$$

$$\gamma = \frac{3}{2}\sqrt{\frac{\Lambda}{3}}\left[\left(1 + \frac{16}{3}\xi\right)^{1/2} - 1\right] \simeq 4\sqrt{\frac{\Lambda}{3}}\xi. \tag{4.116}$$

The metric is here a de Sitter metric determined by the cosmological constant. Because ϕ is unstable, it initially grows exponentially. Its energy density $\tilde{\rho}_\phi$ is negative and increasing in magnitude:

$$\tilde{\rho}_\phi \approx -\Lambda\left(1 + \frac{16}{3}\xi\right)\xi\phi^2. \tag{4.117}$$

The negative sign reflects the strange nature of a noncanonical field with a nonminimal coupling. It is this feature, however, that serves to decrease the positive contribution coming from the cosmological constant.

At late times ϕ is expected to be sufficiently large ($\xi\phi^2 \gg 1$) and then $F_G \approx \xi\phi^2$, $F_K \approx \chi$. Equations (4.109) and (4.111), hence, become

$$3H^2 = \frac{1}{2}\chi\dot{\phi}^2 - 3H^2\xi\phi^2 - 6H\xi\dot{\phi}\phi + \Lambda + \rho_r, \tag{4.118}$$

and

$$\ddot{\phi} + 3H\dot{\phi} - 6\chi^{-1}\xi\left(\frac{\ddot{a}}{a} + H^2\right)\phi = 0. \tag{4.119}$$

From these equations the late-time behaviors of the solutions are giving by the following asymptotic forms depending on χ:

i)$-6\xi < \chi \leq -2\xi$

$$a(t) \sim \text{const.}, \tag{4.120}$$

$$\phi(t) \sim \lambda t, \qquad \lambda = \sqrt{\frac{4}{6\xi + \chi}\Lambda}, \tag{4.121}$$

$$\rho_r(t) \to -3\frac{2\xi + \chi}{6\xi + \chi}\Lambda > 0, \tag{4.122}$$

$$\tilde{\rho}_\phi(t) \to -\frac{2\chi}{6\xi + \chi}\Lambda (= \Lambda + \rho_r), \tag{4.123}$$

and

ii)$\chi > -2\xi$

$$a(t) \sim t^{\alpha}, \qquad \alpha = \frac{1}{2} + \frac{1}{4}\chi\xi^{-1}, \tag{4.124}$$

$$\phi(t) \sim \lambda t, \qquad \lambda = \sqrt{\frac{16}{3\xi}\Lambda}\left[(\chi\xi^{-1} + 6)\left(\chi\xi^{-1} + \frac{10}{3}\right)\right]^{-1/2}, \tag{4.125}$$

$$\rho_r(t) \sim t^{-2-\chi\xi^{-1}}, \tag{4.126}$$

$$\tilde{\rho}_\phi(t) = -\Lambda + O(t^{-2}). \tag{4.127}$$

In both case ϕ grows linearly in time and the scalar field energy density $\tilde{\rho}_\phi$ asymptotically cancels the original cosmological constant.

The solution (i), corresponding to the solution (4.39)-(4.42) which serves satisfactory results to cosmology in the new starred CF, has rather strange behaviors: The scale factor a stops the growth and the radiation energy density ρ_r does not decay. One may

then conclude that different CFs have different physics. This is not necessarily correct if the time dependence of particle masses is taken into consideration. Particle mass m is expected to behave like $\sim t$ asymptotically in the original CF, for example, mass derived from (4.12). If one in the original CF used "Bohr radius" ($\hbar^2/(me^2)$) as a measure of a length, he might feel as if the universe expanded because the measure would be shorter and shorter as time would go. This suggest that different CFs might have the same physics, though we need further study.

The solution (ii) corresponds to (4.43)-(4.46) in the new starred CF. From (4.37),(4.43),(4.124) the exponent α in (4.124)is related to the exponent $\alpha_* = 2\kappa^2$ in (4.43) in the new starred CF through

$$\alpha_* = \frac{1}{2} + \frac{1}{2}\alpha. \tag{4.128}$$

The solution (ii) also satisfies this relation which is consistent with $dt = F_G^{-1/2}dt_*$ and $a = F_G^{-1/2}a_*$.

Chapter 5

The "suspending" cosmological constant

Recent analyses on the mass density and dark matter indicate strongly that Ω_0 at the present time is considerably smaller than 1[30]. This is apparently in conflict with the prediction of the scenario of inflation, unless the parameters in the theory are fine-tuned to some extent. The deficit may be made up by a cosmological constant Λ. A detailed analysis[9] of the number count of faint galaxies taking the evolution effect into account shows in fact that including Λ ($\lambda = \Lambda/8\pi G\rho_{\mathrm{critical}} = 0.9, \Omega_0 = 0.1, k = 0$) fits the observed result even better than $\Lambda = 0$ and $\Omega_0 = 0.1$. These two fits will be referred to as the "Λ fit" and "the open fit", respectively. A possible occurrences of Λ of the same order of magnitude has been discussed also in connection with the CMBR anisotropy[31].

From a theoretical point of view, however, one encounters a two-fold unnaturalness if the cosmological constant is a true constant. First, the value itself of the required cosmological constant is about 120 orders too small compared with the theoretical natural value $\Lambda_0 \sim m_{\mathrm{Pl}}^4$; a well-known problem. Second, given the value of Λ, the near coincidence $\Lambda \sim \rho \sim t^{-2}$ occurs once and for all only in an extremely short period in the entire history of the universe extending to the enormous length of future to come; it is hard, if not impossible, to believe that such a rare event is happening in front of us.

In this section we propose a more natural way of reconciling a small Ω_0 with inflation by extending a scalar-tensor theory that has been developed to implement the scenario of a decaying cosmological constant. We show that the goal is achieved if we introduce two scalar fields, one for the decay of the effective cosmological constant, the scalar field energy, in the overall time scale of the universe and the other for a temporary deviation that makes the effective cosmological constant imitate a true constant. We call such a deviation a "suspending" cosmological constant. Notice that this second scalar field is still a gravitational scalar different from the Higgs-type scalar field which played a role for reheating. We find it necessary also to introduce a special potential, an example of which is proposed from a phenomenological point of view. As one of the ensuing consequences we expect a nearly periodic but sporadic large-scale structure of the universe somewhat reminiscent of (but not exactly the same as) the recently reported periodic structure of the universe[32].

In the Λ fit in ref.[9] the whole energy density ρ consists of ρ_m and ρ_Λ: $\rho = \rho_m + \rho_\Lambda$ where $\rho_\Lambda = \Lambda/8\pi G$ and ρ_m for the density obeying $a^3\rho_m =$ const. (nonrelativistic matter whether it is visible or not). The behavior of the scale factor $a(t)$ for the Λ fit is shown in Fig.5a. We notice a somewhat faster rise as compared with the conventional behavior $a \sim t^{2/3}$ for $\Omega_0 = 1$ and $\Lambda = 0$, a behavior that is excluded as strongly disfavored in ref.[9]. The same feature is also shared by the open fit. The difference among these fit is even more pronounced if we plot the deceleration parameter $q = -\ddot{a}a/\dot{a}^2$ as shown in Fig.5b, indicating that the Λ fit agrees with observation better than the open fit because of its stronger acceleration towards the present time. We ask ourselves if this extra acceleration can be understood in a reasonable manner.

It may appear that the required faster growth of $a(t)$ is simply due to the dominant "dark matter" part which, not necessarily ρ_Λ, obeys an equation of state different from the one that applies to the ordinary part ρ_m. According to the relation $\alpha = 2/[3(1+\zeta)]$ with $\zeta = p/\rho$ for $a(t) \sim t^\alpha$, however, $\alpha > 2/3$ implies $\zeta < 0$; a negative pressure is hardly expected from the ordinary thermodynamical origin. An equivalent effect still ensues if the dark matter consists largely of a spatially uniform scalar field with kinetic energy K and the potential V; one finds

$$\zeta_s = \frac{K-V}{K+V}, \tag{5.1}$$

which may vary from -1 to $+1$.

If ζ_s remains negatively constant, the energy ρ_s of the scalar field would grow faster than and surpass ρ_m at some epoch in the same way as the nonrelativistic matter does compared with the radiation energy. In this scenario, nearly the same two unnatural aspects as mentioned above with a purely constant Λ ($\zeta = -1$) surface again. It would be more natural if the scalar field energy exhibits a combination of two behaviors; a globally decaying pattern $\sim t^{-2}$ and local derivations simulating a nearly constant energy density for some duration; the latter for the extra acceleration as indicated by the behavior of the deceleration parameter.

The first behavior, which assures that ρ_s is not too small or too large compared with ρ_m, is precisely the behavior which has been shown to emerge in Chap.4. that yielded $\Lambda_{\text{eff}} = \rho_s \sim t^{-2}$ up. Obviously, however, the model lacks enough complexity accounting for the second behavior, local deviations. We expect an improvement by introducing another massive scalar field which is different from the scalar field Φ for reheating; its oscillation would likely result in desired behavior. This would be a favored scenario, because the local deviation expected for the scalar field energy is then not a single and isolated event but has repeated and will repeat itself, certainly appealing to our view of naturalness. From these considerations we try to offer a simple model theory. We do this mainly from a phenomenological point of view trying to see what the theory should be like for the reconciliation scenario to be implemented.

In the new starred CF we add a new field $\tilde{\Phi}$ in (4.10) (dropping $*$)

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g^{\mu\nu}\partial_\mu\tilde{\Phi}\partial_\nu\tilde{\Phi} - V(\sigma,\tilde{\Phi}) + L_m\right]. \tag{5.2}$$

In addition to ΛF_G^{-2} we also include a mass term of $\tilde{\Phi}$, giving a potential

$$V(\sigma,\tilde{\Phi}) = \exp\left(-\frac{\sigma}{\kappa}\right)\left[\Lambda + \frac{1}{2}m^2\tilde{\Phi}^2 U(\sigma)\right], \tag{5.3}$$

with m and κ in (4.37) parameters basically of the order of one in Planck units. The explicit form of $U(\sigma)$ will be specified later. Terms other than those of $\tilde{\Phi}$ were designed originally to implement the decaying cosmological constant in (4.10).

Although the $\tilde{\Phi}$ terms in (5.2) should be also understood to have been derived from those in (4.1) before the conformal transformation, we assume the mass term in (5.2) is multiplied by $\exp(-\sigma/\kappa)$ without entering into details for the following reason.

Without this factor (and with a constant U), $\tilde{\Phi}$ would have oscillated with a constant period $\sim m^{-1}$, with the cosmological redshift ignored for the moment. From Fig.5, however, we find that the expected local derivation has a period, if there is any, nearly comparable with the present age itself; the universe has witnessed barely one cycle at most. If m^2 is multiplied by $\exp(-\sigma/\kappa) \sim t^{-2}$ due to (4.40) , on the other hand, one finds that $\tilde{\Phi}$ behaves liked a damped harmonic oscillator as a function of $\ln t$, as will be shown shortly. This allows the oscillation to have occurred some number of times since the beginning of the universe.

From (5.2) and (5.3) follow the cosmological equations with $k=0$:

$$3H^2 = \rho_s + \rho_m = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\tilde{\Phi}}^2 + V + \rho_m, \tag{5.4}$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \exp\left(-\frac{\sigma}{\kappa}\right)\left[\frac{\Lambda}{\kappa} + \frac{1}{2}m^2\tilde{\Phi}^2\left(\frac{U}{\kappa} - \frac{dU}{d\sigma}\right)\right] = 0, \tag{5.5}$$

$$\ddot{\tilde{\Phi}} + 3H\dot{\tilde{\Phi}} - \exp\left(-\frac{\sigma}{\kappa}\right)m^2U\tilde{\Phi} = 0, \tag{5.6}$$

$$\dot{\rho_m} + 3H\rho_m = 0. \tag{5.7}$$

If U is constant, one finds asymptotic solutions:

$$a(t) \sim t^{2/3}, \tag{5.8}$$

$$\sigma(t) = 2\kappa \ln\left(\sqrt{\frac{\Lambda}{2}\frac{t}{\kappa}}\right), \tag{5.9}$$

$$\tilde{\Phi}(t) = At^{-1/2}\sin(\tilde{m}\ln t), \tag{5.10}$$

$$\rho_m(t) = \frac{4}{3}(1-3\kappa^2)t^{-2}, \tag{5.11}$$

where A is an integration constant and

$$\tilde{m} = \sqrt{\frac{2\kappa^2 m^2}{\Lambda}U - \frac{1}{4}}. \tag{5.12}$$

Obviously the energy

$$\rho_{\tilde{\Phi}} = \frac{1}{2}\dot{\tilde{\Phi}}^2 + \frac{1}{2}\exp\left(-\frac{\sigma}{\kappa}\right)m^2\tilde{\Phi}^2 U(\sigma), \tag{5.13}$$

decays like $\sim t^{-3}$, hence no contribution to the total energy in the asymptotic era. (The corresponding σ energy ρ_σ decays like $\sim t^{-2}$.)

An interesting result may follow, however, if we allow U to depend on σ. Choose

$$U(\sigma) = 1 + B\sin(\omega\sigma), \tag{5.14}$$

for example. The whole potential $V(\sigma, \tilde{\Phi})$ is illustrated in Fig.6. One may choose the "initial" values of the classical fields $\sigma(t)$ and $\tilde{\Phi}(t)$ at the epoch t_a somewhat after the end of the reheating period following inflation. The "imaginary particle" in the $\sigma - \tilde{\Phi}$ plane will begin to roll down the valley toward $\sigma \to \infty$ with a meandering behavior. Depending on the parameters in V as well as the initial conditions, we find two distinct types of behavior:

(i) $\tilde{\Phi}$ shows a nearly simple damped oscillation as with a constant U. The energy $\rho_{\tilde{\Phi}}$ decrease like $\sim t^{-3}$.

(ii) $\tilde{\Phi}$ shows some "undamped" quasi-oscillation with its (approximate) period with respect to $\ln t$ quite different from $\tilde{m}^{-1}$ or any other time-scale derived directly from ω prepared in (5.14), with $\rho_{\tilde{\Phi}}$ decaying globally according to $\sim t^{-2}$. With the same period the scale factor $a(t)$ shows a sporadic behavior.

The behavior (ii) seems to serve our favored scenario, as will be explained below. In an example shown in Fig.7a, we observe a flip-flop behavior of $\tilde{\Phi}(t)$ repeated nearly periodically if measured in $\ln t$. Somewhat unexpectedly, $\tilde{\Phi}(t)$ is stuck onto the potential slope for some time much longer $\tilde{m}^{-1}$. This is due to a competition between the weak restoring force and the friction $3H\dot{\tilde{\Phi}}$. During the period prior to an almost complete halt of $\tilde{\Phi}$, another field σ moves ahead overriding many (more than 20 in this example) crests of the sinusoidal potential. As σ slows down, the force due to this potential finally traps the movement giving it a slight "backward kick". This triggers a sudden "free fall" of $\tilde{\Phi}$ toward $\tilde{\Phi} = 0$, causing a sudden change of ζ_s. It is around this time when $a(t)$ is pushed away from a smooth behavior, thus giving the desired behavior of an extra acceleration (followed by deceleration). The result is in fact a succession of deviations of $a(t)$ from the standard background $a \sim t^{1/2}$ or $a \sim t^{2/3}$ occurring in coincidence with the free fall of $\tilde{\Phi}$. These deviations might be called "anomalous" behaviors, showing up as small bumps in $\ln a$ plotted against $\ln t$, as in Fig.7a. Each bump lasts for $\Delta \log_{10} t \sim 1$, which is basically the scale $\sim \tilde{m}^{-1}$. We modestly tuned the parameters such that one of the bumps occurs around the present time chosen to be $t_0 = 1.6 \times 10^{60}$, corresponding to 14Gyr. The deceleration parameter $q(t)$ is shown in Fig.7b in a magnified scale around the present time. Comparing this with Fig.5b we find the agreement with the Λ fit rather remarkable. This can be understood reasonably because $\rho_s (= \Lambda_{\rm eff}) = \rho_\sigma + \rho_{\tilde{\Phi}}$ remains nearly constant, as expected, as also shown in Fig.7b. This is essentially the same suspending cosmological constant as we observed in Fig.5.

In this way we arrive at the conclusion that our model, which "improves" the naive decaying cosmological constant scenario, allows solutions in which the contribution from the scalar fields mimics precisely the cosmological constant supposed to make up the energy density deficit, hence saving the inflation scenario. We must accept, however, that our present epoch is in one of the anomalous eras of deviation from the global background behavior, each lasting for a relatively short duration if viewed in $\ln t$ (but by no means short in terms of t), which is not too unnatural. One of the underlying mechanisms is the same nonlinear effect, known as a "relaxation oscillation", as in the simpler model. Some comments will follow.

We obtain $\Omega_0 = 0.092$ and $h = 0.89$ for the example in Fig.7, showing that these parameters and the age (chosen to be 14Gyr) are less constrained than they are in the standard cosmology with $\Lambda = 0$.

We included both relativistic matter ρ_r and nonrelativistic matter ρ_{nr} to demonstrate that the model can be applied to epochs much earlier than one can access by direct observation like galaxy counting. We first notice that alternating dominance between the vacuum energy (the energy of the scalar fields ρ_s) and real matter $\rho_m = \rho_r + \rho_{nr}$ is a common place, though both ρ_s and ρ_m fall globally like $\sim t^{-2}$. This makes it possible that the small Ω_0 at present is fully compatible with the almost purely radiation-dominated ($\rho_s \ll \rho_m \sim \rho_r$) era around the epoch of the nucleosynthesis. (The dominance of ρ_r for $41 \lesssim \log_{10} t \lesssim 47$, i.e. $10^{-2} \lesssim t \lesssim 10^4$sec as well as the coincidence $\rho_r \sim \rho_{nr}$ at $\log_{10} t \sim 54$ in Fig.7a are a result of our choice of parameters.) On the other hand, the vacuum energy might be dominant in some relatively short periods (in $\ln t$), like in the present epoch. It is most important to study whether these past anomalous behaviors have left any trace that can be observed at present. The recently reported periodic large-scale structure of the universe[32] might be a phenomenon to which our theory applies, as will be discussed again at the end of the chapter. If, however, the nearest past bump occurred before the recombination era as in the example of Fig.7a, the effect ought to be more indirect. It is also worth recalling in this connection that the process of baryogenesis is known to be rather insensitive to the presence of the vacuum energy[24]. Notice that the overall behavior does not depend much on whether matter is relativistic or nonrelativistic.

In another example of Fig.8 a transition takes place from type (ii) to type (i) behavior. It is interesting to find that $\rho_{\tilde{\Phi}}$ oscillates nearly with intrinsic period $2\pi/\omega$.

Except in this kind of examples, no direct effect of ω is visible. In the behavior (ii), the only role of $U(\sigma)$ is to give a "backward kick"; for the rest of the time this potential is dormant. The kicking force, however, must be "ready" to act whenever the time is ripe. For this reason the oscillating potential seems favored, if not absolutely necessary. In fact the sinusoidal form, which may look somewhat unusual, is by no means unique; it was discovered rather accidentally in our trial-and-error approach to arrive at the desired result anyway. In further searches we may meet with other more familiar forms producing similar results. No attempt is made therefore to find a theoretical basis of $U(\sigma)$, at least for the time being, although (5.14) combined with (4.36) suggests a term of the form $c\phi^{i\gamma}\tilde{\Phi}^2$ + h.c., with the constants c and γ chosen to be complex and real, respectively, in the original Lagrangian (4.1) before the conformal transformation. We also emphasize that all the constants in our model are "natural", not being much away from the Planckian order of magnitude, nevertheless resulting in effects of scales larger by many orders.

We integrated our classical equations starting at an epoch t_a after reheating. In principle nothing prevents us from starting at the Planck time; the whole history of classical evolution from inflation to the present time can be analyzed in terms of a full-fledged two-scalar model. In practice, however, some complication is inevitable particularly in connection with dissipative interactions as was discussed in Chap.4. In this chapter we have conveniently split the entire period into two, focusing on our interest in late times. (We borrowed some of the initial values at t_a from the result in Chap.4 in which we started from the Planck time.)

We assumed the occurrence of the conformal factor $\exp(-\sigma/\kappa)$ in front of the mass term of $\tilde{\Phi}$. This should be derived by choosing a suitable conformal property of $\tilde{\Phi}$, related also to a possible choice of the ϕ-dependence of the $\tilde{\Phi}$ kinetic term in the original CF. A detailed discussion on point is left for future studies.

In this connection it is interesting to note that the effective mass at the present time is $\exp(-\sigma/2\kappa)m \sim t_0^{-1}m \sim 10^{-32}$eV which is close to but about two orders smaller than the value suggested by Morikawa[33] who tries to explain an oscillatory structure of the universe. Although this difference is within the range of adjustable parameters, we were motivated primarily by the behavior in a longer time scale corresponding to $z \lesssim 5$ in ref.[9] in contrast to $z \lesssim 0.5$ in ref.[32][33]. Also unlike the almost sinusoidal oscillation of the scalar field (and eventually part of the Hubble parameter) in ref.[33], we obtain sporadic changes due to a nonlinear effect. We nevertheless observe that the two different calculations share a crucial ingredient; behaviors in one cycle shown in ref.[33] are in fact the same as those in one of the anomalous eras. Both are essentially acceleration followed by deceleration (vice versa) of the scale factor due to the alternating dominance of K and V of the scalar field. We may reasonably expect then that each of the anomalous eras, if it occurs at a sufficiently late time, would result in an isotropic clustering structure. Our future program naturally includes the question whether the short-period oscillation can be accommodated in our model.

Chapter 6

Conclusion

According to the model of a decaying cosmological constant, the effective cosmological constant decays asymptotically like $\sim t^{-2}$. This provides a natural understanding of the cosmological constant which is sufficient large to cause inflation in the early epochs but is smaller today by 120 orders. In other words, the present cosmological constant is extremely small simply because our universe is very old.

To inplement such an idea, the model has two crucial ingredients. One is a generalized scalar-tensor theory with a nonminimal gravitational coupling and an extended form of the kinetic term. The other is the proper selection of a CF in which the gravitational constant is strictly constant and particle masses are asymptotically constant. Of course it is still an open question in which CF we are in reality. However, the question is closely related to quantum theory. Then we expect that quantum theory will bring us some principles to choose a CF in the future. Even if we obtain such a principle, we need further study to find if we can deal with cosmological constants created by successive cosmological phase transitions.

Our model in the new CF may be regarded as one of the new inflationary models. Our detailed calculations demonstrated that both sufficient inflation and sufficient reheating occur in our model. In the calculations, sufficient reheating is owed to couplings derived from a conformal transformation, of the gravitational scalar to a Higgs scalar.

We found, however, that the calculated density perturbation is too large to be consistent with the observed isotropy of CMBR. We still hope that better results may emerge by choosing a modified form of the nonminimal grvitational coupling.

We have also found that the gravitational scalar field evolves in an unexpected but very interesting manner. Such a behavior is generally called a relaxation oscillation. Trying to explain the recent analysis on a nonzero cosmological constant by exploiting this behavior, we found that the model with a single gravitational scalar is too simple to fit the required value of Λ. Pursuing the scenario further, we introduce another scalar field to solve this "new" cosmological constant problem and we need some special potential to cause a relaxation oscillation. We somehow discovered an example on a try-and-error basis. It is yet to be shown how unique this potential is and if there are other more natural examples.

Appendix A

We list some numbers from conversion table to the Planck unit system ($G = c = \hbar = 1$).

$$m_{\rm Pl} = 1.22 \times 10^{19}{\rm GeV}, \qquad 1{\rm GeV} = 8.2 \times 10^{-20}{\bf P}$$

$$r_{\rm Pl} = 1.61 \times 10^{-33}{\rm cm}, \qquad 1{\rm cm} = 6.2 \times 10^{32}{\bf P}$$

$$t_{\rm Pl} = 5.4 \times 10^{-44}{\rm sec}, \qquad 1{\rm sec} = 1.85 \times 10^{43}{\bf P}$$

$$1{\rm y} = 3.2 \times 10^{7}{\rm sec} = 5.9 \times 10^{50}{\bf P}$$

$$1{\rm ly} = 9.6 \times 10^{17}{\rm cm} = 5.9 \times 10^{50}{\bf P}$$

It is also convenient to use the "modified" Planck unit system(**E**) in which $8\pi G = c = \hbar = 1$. We find

$$m_{\rm E} = m_{\rm Pl}/\sqrt{8\pi} = 0.1995 m_{\rm Pl}$$

$$r_{\rm E} = \sqrt{8\pi} r_{\rm Pl} = 5.0133 r_{\rm Pl}$$

$$t_{\rm E} = \sqrt{8\pi} t_{\rm Pl} = 5.0133 r_{\rm Pl}$$

and hence

$$m_{\rm E} = 2.44 \times 10^{18}{\rm GeV}, \qquad 1{\rm GeV} = 4.12 \times 10^{-19}{\bf E}$$

$$r_{\rm E} = 8.09 \times 10^{-33}{\rm cm}, \qquad 1{\rm cm} = 1.24 \times 10^{32}{\bf E}$$

$$t_{\rm E} = 2.71 \times 10^{-43}{\rm sec}, \qquad 1{\rm sec} = 3.69 \times 10^{42}{\bf E}$$

$$1{\rm y} = 3.2 \times 10^{7}{\rm sec} = 1.2 \times 10^{50}{\bf E}$$

$$1{\rm ly} = 9.6 \times 10^{17}{\rm cm} = 1.2 \times 10^{50}{\bf E}$$

$$H_0 = 100h\text{km/sec/Mpc} = 1.04h \times 10^{-10}\text{y}^{-1} = 8.6h \times 10^{-61}\mathbf{E}$$

$$T_0 = 2.7\text{K} = 1.0 \times 10^{-31}\mathbf{E}.$$

Appendix B

In this appendix we mention the conformal transformation, assuming torsionless.

i) metric

We apply conformal transformation $g_{\mu\nu} \to g_{*\mu\nu}$:

$$g_{\mu\nu} = \Omega^{-2}(x) g_{*\mu\nu}, \tag{B.1}$$

which is interpreted as changing a length scale (or unit) locally: $ds^2 \to \Omega^2(x) ds^2$. This transformation is said to bring one from a CF to another CF.

The Cristoffel connection transforms into

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{*\mu\nu} - \delta^\lambda_\nu \partial_\mu f - \delta^\lambda_\mu \partial_\nu f + g_{*\mu\nu} g_*^{\lambda\sigma} \partial_\sigma f, \tag{B.2}$$

in the $*$ CF with $f = \ln \Omega$.

Riemann tensor transforms into

$$\begin{aligned} R^\lambda_{\mu\nu\kappa} \;=\; & R^\lambda_{*\mu\nu\kappa} \\ & + \delta^\lambda_\nu \nabla_{*\mu} \nabla_{*\kappa} f - \delta^\lambda_\kappa \nabla_{*\mu} \nabla_{*\nu} f \\ & + g_{*\mu\kappa} \nabla^\lambda_* \nabla_{*\nu} f - g_{*\mu\nu} \nabla^\lambda_* \nabla_{*\kappa} f \\ & + \delta^\lambda_\nu (\nabla_{*\mu} f \nabla_{*\kappa} f - g_{*\mu\kappa} \nabla^\sigma_* f \nabla_{*\sigma} f) \\ & - \delta^\lambda_\kappa (\nabla_{*\mu} f \nabla_{*\nu} f - g_{*\mu\nu} \nabla^\sigma_* f \nabla_{*\sigma} f) \\ & + g_{*\mu\kappa} \nabla^\lambda_* f \nabla_{*\nu} f - g_{*\mu\nu} \nabla^{*\lambda} f \nabla_{*\kappa} f, \end{aligned} \tag{B.3}$$

where $\nabla_{*\mu}$ is covariant derivative in the $*$ CF.

From (B.3), Ricci tensor transforms into

$$\begin{aligned} R_{\mu\nu} &= R_{*\mu\nu} \\ &+ 2\nabla_{*\mu}\nabla_{*\nu}f + g_{*\mu\nu}\nabla_*^{\lambda}\nabla_{*\lambda}f + 2\nabla_{*\mu}f\nabla_{*\nu}f - 2g_{*\mu\nu}\nabla_*^{\lambda}f\nabla_{*\lambda}f, \end{aligned} \quad \text{(B.4)}$$

in four dimensional space-time.

Finally scalar curvature transforms into

$$R = \Omega^2[R_* + 6\nabla_{*\mu}\nabla_*^{\mu}f - 6\nabla_{*\mu}f\nabla_*^{\mu}f]. \quad \text{(B.5)}$$

ii) scalar field Φ

Applying the conformal transformation (B.1), Φ also transforms into Φ_*:

$$\Phi = \Omega\Phi_*. \quad \text{(B.6)}$$

Then the kinetic part of the Lagrangian of Φ transforms into

$$\begin{aligned} \mathcal{L}_{\Phi\mathrm{kin}} &= -\frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \\ &= -\frac{1}{2}\sqrt{-g_*}g_*^{\mu\nu}D_\mu\Phi_*D_\nu\Phi_*, \end{aligned} \quad \text{(B.7)}$$

with $D_\mu = \partial_\mu + \partial_\mu f$. The derivative coupling emerges in the $*$ CF because $\mathcal{L}_{\Phi\mathrm{kin}}$ is not conformal invariant.

iii) spinor field ψ

Applying the conformal transformation (B.1), ψ also transforms into ψ_*:

$$\psi = \Omega^{3/2}\psi_*. \quad \text{(B.8)}$$

We introduce the vierbein $b^i{}_\mu$ satisfying

$$g_{\mu\nu} = b^i{}_\mu b_{i\nu}, \qquad \eta_{ij} = b_{i\mu}b_j^{\mu}, \quad \text{(B.9)}$$

where $b_{i\mu} = \eta_{ij}b^j{}_\mu$, $b_i^{\mu} = g^{\mu\nu}b^i{}_\nu$ and the Greek index means a space-time index and the Latin index means a local Lorentz frame index.

Let us to see conformal invariance of the kinetic part of the Lagrangian of ψ:

$$\mathcal{L}_{\psi\mathrm{kin}} = -\frac{1}{2}b\bar{\psi}(\not{D} - \overleftarrow{\not{D}})\psi, \quad \text{(B.10)}$$

with $b = \det b^i{}_\mu$ and

$$\begin{aligned} \not{D}\psi &= \gamma^k b_k^{\mu} D_\mu\psi \\ &= \gamma^k b_k^{\mu}\left(\partial_\mu + \frac{1}{4}\omega^{ij}{}_{,\mu}\gamma_{ij}\right)\psi, \end{aligned} \quad \text{(B.11)}$$

and

$$\bar{\psi}\,\overleftarrow{\not{D}} = \bar{\psi}\left(\overleftarrow{\partial}_\mu - \frac{1}{4}\omega^{ij}{}_{,\mu}\gamma_{ij}\right)\gamma^k b_k^{\mu}, \quad \text{(B.12)}$$

where $\omega^{ij}{}_{,\mu}$ is spin connection and γ^i is 4×4 Dirac matrix satisfying

$$\{\gamma^i, \gamma^j\} = 2\eta^{ij}, \quad \text{(B.13)}$$

and $\gamma_{ij} = \frac{1}{2}[\gamma_i, \gamma_j]$.

(B.10) is divided into two parts:

$$\mathcal{L}_{\psi\mathrm{kin}} = -\frac{1}{2}b\bar{\psi}(\not{\partial} - \overleftarrow{\not{\partial}})\psi - \frac{1}{8}b\omega_{ij,k}\bar{\psi}\{\gamma^k, \gamma^{ij}\}\psi. \quad \text{(B.14)}$$

Under the conformal transformation (B.1), (B.8), the first part in the right-hand of (B.14) is trivially invariant. We see that the second part is also conformal invariant with torsionless. Assuming torsionless, the spin connection $\omega_{ij,k}$ is given by

$$\omega_{ij,k} = \frac{1}{2}(\Delta_{k,ij} - \Delta_{i,jk} + \Delta_{j,ik}), \quad \text{(B.15)}$$

where $\Delta_{i,jk}$ is Ricci's rotation coefficient defined by

$$\Delta_{k,ij} = (b_i^{\mu}b_j^{\nu} - b_j^{\mu}b_i^{\nu})\partial_\nu b_{k\mu}. \quad \text{(B.16)}$$

Using (B.15), (B.16), we apply the conformal transformation to $\omega_{ij,k}$

$$\omega_{ij,k} = \Omega[\omega_{*ij,k} + \eta_{ik}\partial_{*j}f - \eta_{jk}\partial_{*i}f], \quad \text{(B.17)}$$

where ∂_{*i} means $b^{\mu}_{*i}\partial_{\mu}$. One may wonder the additional terms $\Omega[\eta_{ik}\partial_{*j}f-\eta_{jk}\partial_{*i}f]$ breaks conformal invariance. But these terms disappears in $\mathcal{L}_{\psi\text{kin}}$ because

$$\begin{aligned}(\eta_{ik}\partial_{*j}f-\eta_{jk}\partial_{*i}f)\{\gamma^{ij},\gamma^{k}\} &= 2\partial_{*i}f\{\gamma^{ji},\gamma_{j}\} \\ &= 0,\end{aligned} \tag{B.18}$$

using (B.13). In this way $\mathcal{L}_{\psi\text{kin}}$ is conformal invariant with torsionless.

vi) gauge field A_{μ}

Applying the conformal transformation (B.1), A_{μ} transforms into $A_{*\mu}$:

$$A_{\mu}=A_{*\mu}. \tag{B.19}$$

Then the kinetic part of the Lagrangian of A_{μ} is trivially conformal invariant because

$$\begin{aligned}\mathcal{L}_{A_{\mu}} &= \sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right] \\ &= \sqrt{-g_{*}}\Omega^{-4}\left[-\frac{1}{4}\Omega^{4}g_{*}^{\mu\rho}g_{*}^{\nu\sigma}F_{*\mu\nu}F_{*\rho\sigma}\right] \\ &= \sqrt{-g_{*}}\left[-\frac{1}{4}g_{*}^{\mu\rho}g_{*}^{\nu\sigma}F_{*\mu\nu}F_{*\rho\sigma}\right],\end{aligned} \tag{B.20}$$

where $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$.

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Figure Captions

Fig.1: The potential $V = \Lambda(1+\xi\phi^2)^{-2}$ given by (4.11) with (4.2).

Fig.2: An example of the numerical solutions a, ϕ, ρ's and p_v plotted against $\log_{10} t$, with in units of the Planck time. (The epoch of nucleosynthesis, the end of the radiation-dominated universe and the present age correspond roughly to 45, 54 and 60, respectively, on the abscissa.) Parameters chosen are $\Lambda = 1$, $\xi = 0.8 \times 10^{-2}$ and $\chi = -4.48 \times 10^{-2}$ in the unit system of $8\pi G = 1$. Initial conditions are given by $\phi = 1$, $\dot{\phi} = 1$ and $\rho_r = 1$ at $t = 1$.

Fig.3: Another example with $\chi = -2.67 \times 10^{-2}$ with the same values of the other parameters as in Fig.2. $\chi = -2.67 \times 10^{-2}$ corresponds to $\kappa = \kappa_c = 1/\sqrt{6}$.

Fig.4: An example with $\chi = -4.692 \times 10^{-2}$ with the same values of the other parameters as in Fig.2. The example shows that the "suspending cosmological constant" equals about 10^{-120}.

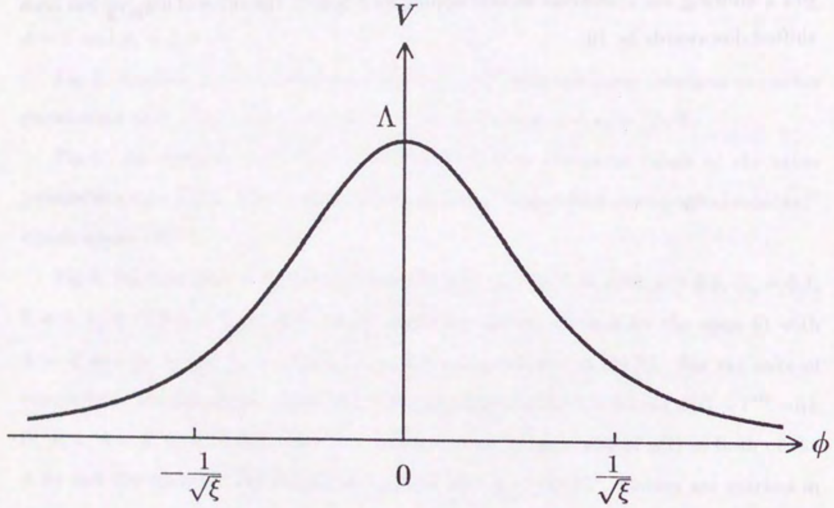
Fig.5: (a) The thick solid curve shows the $a(t)$ for the Λ fit with $\lambda = 0.9$, $\Omega_0 = 0.1$, $k = 0$, $t_0 = 12.5$Gyr, $h = 1.0$ in ref.[9], while the dashed curve is for the open fit with $\Lambda = 0$ and $\Omega_0 = 0.1$, $t_0 = 12.6$Gyr,$h = 0.7$ discussed also in ref.[9]. For the sake of comparison, the thin curve represents the (excluded) standard behavior $a(t) \sim t^{2/3}$ with $\Omega_0 = 1$, $\Lambda = 0$, $t_0 = 13.0$Gyr, $h = 0.5$. Notice an accelerated rise of $a(t)$ in both of the Λ fit and the open fit. (b) Deceleration parameter $q = -\ddot{a}a/\dot{a}^2$. Curves are marked in the same way as in (a).

Fig.6: A bird's-eye view of the potential $V(\sigma, \tilde{\Phi})$ as given by (5.3) and (5.14).

Fig.7: (a) An example of our cosmological solutions exhibiting a desired anomaly, a bump in the scale factor $a(t)$ around the present epoch chosen to be $t_0 = 1.6 \times 10^{60} = 14$Gyr. Also shown in figure are $\tilde{\Phi}$, σ, q, relativistic matter energy ρ_r (dotted curve), non-relativistic matter energy ρ_{nr} (dashed), the sum of the scalar field energies $\rho_s = \rho_\sigma + \rho_{\tilde{\Phi}}$ (solid). The parameters are $\Lambda = 1$, $\kappa = 0.158$, $m = 4.75$. The

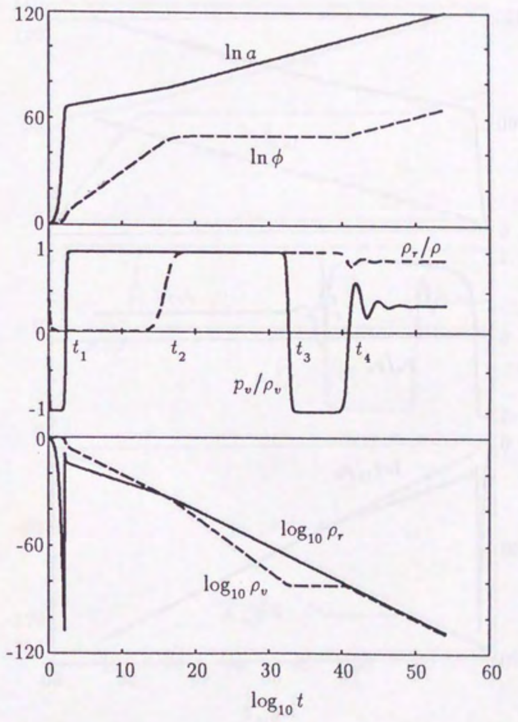
initial condtions chosen are $\sigma = 9.067$, $\dot{\sigma} = 0$, $\tilde{\Phi} = 1.837$, $\dot{\tilde{\Phi}} = 0$, $\rho_r = 2.04 \times 10^{-21}$, $\rho_{nr} = 4.46 \times 10^{-44}$ at $t = t_a = 10^{10}$. The overall scale of a is arbitrary. (b) Magnified plots of q, $\Omega = \rho_m/3H^2 = \rho_m/(\rho_m + \rho_s)$ and ρ' around the present time, indicated by the vertical line. The Hubble parameter is $h = 0.89$, and $\Omega_0 = 0.092$.

Fig.8: An example showing a transition from the type (ii) to the type (i) behavior for $\omega = 10.0$. Notice an oscillation of $\rho_{\tilde{\Phi}}$ (thick solid curve) and the overall decay according to t^{-3} after $t \gtrsim 10^{40}$. To avoid an overlapping for $t \lesssim 10^{40}$, the curve of $\log_{10} \rho_{\tilde{\Phi}}$ has been shifted downwards by 10.



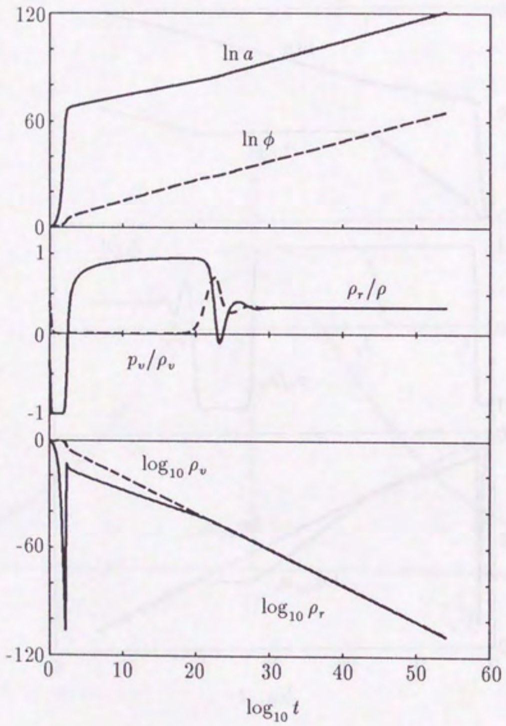
V
Λ
φ
$-\frac{1}{\sqrt{\xi}}$
0
$\frac{1}{\sqrt{\xi}}$

Fig. 1



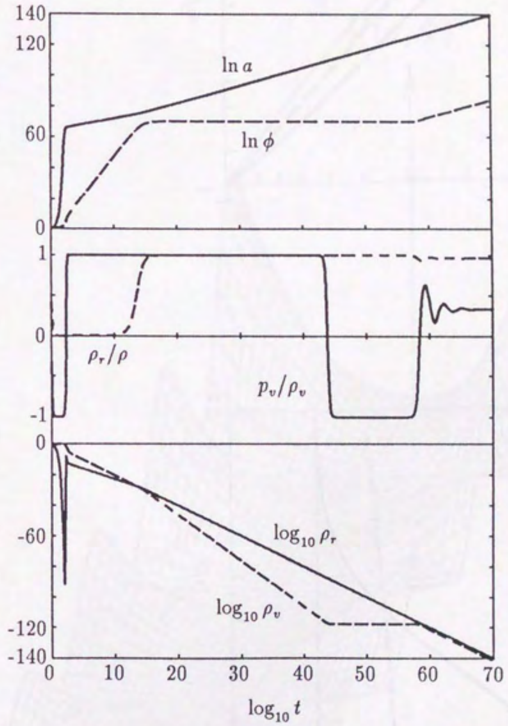
120
60
0
ln a
ln φ
1
0
-1
ρ_r/ρ
p_v/ρ_v
t_1
t_2
t_3
t_4
0
-60
-120
$\log_{10}\rho_r$
$\log_{10}\rho_v$
0
10
20
30
40
50
60
$\log_{10} t$

Fig.2



ln a
ln ϕ
ρ_r/ρ
p_v/ρ_v
$\log_{10}\rho_v$
$\log_{10}\rho_r$
$\log_{10} t$

Fig.3



ln a
ln ϕ
ρ_r/ρ
p_v/ρ_v
$\log_{10}\rho_r$
$\log_{10}\rho_v$
$\log_{10} t$

Fig.4

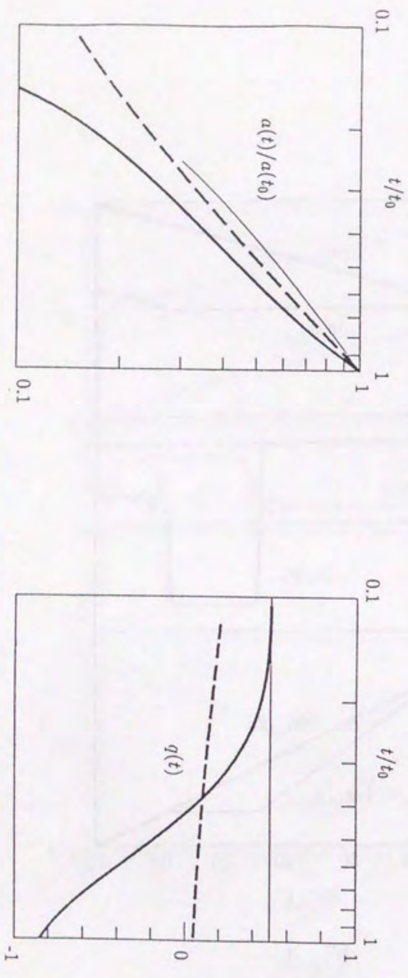


Fig.5

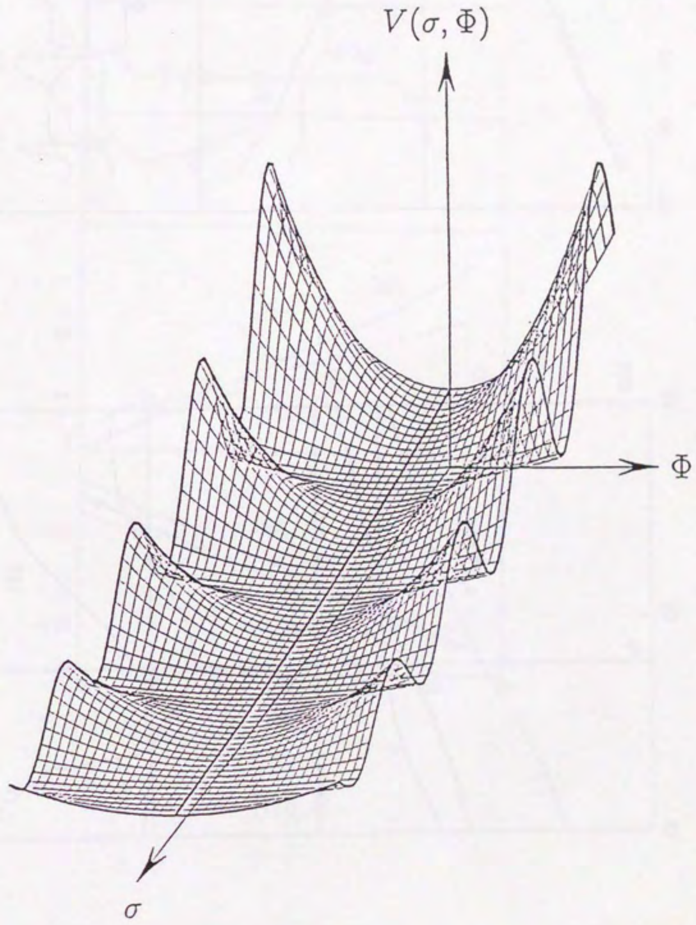


Fig.6

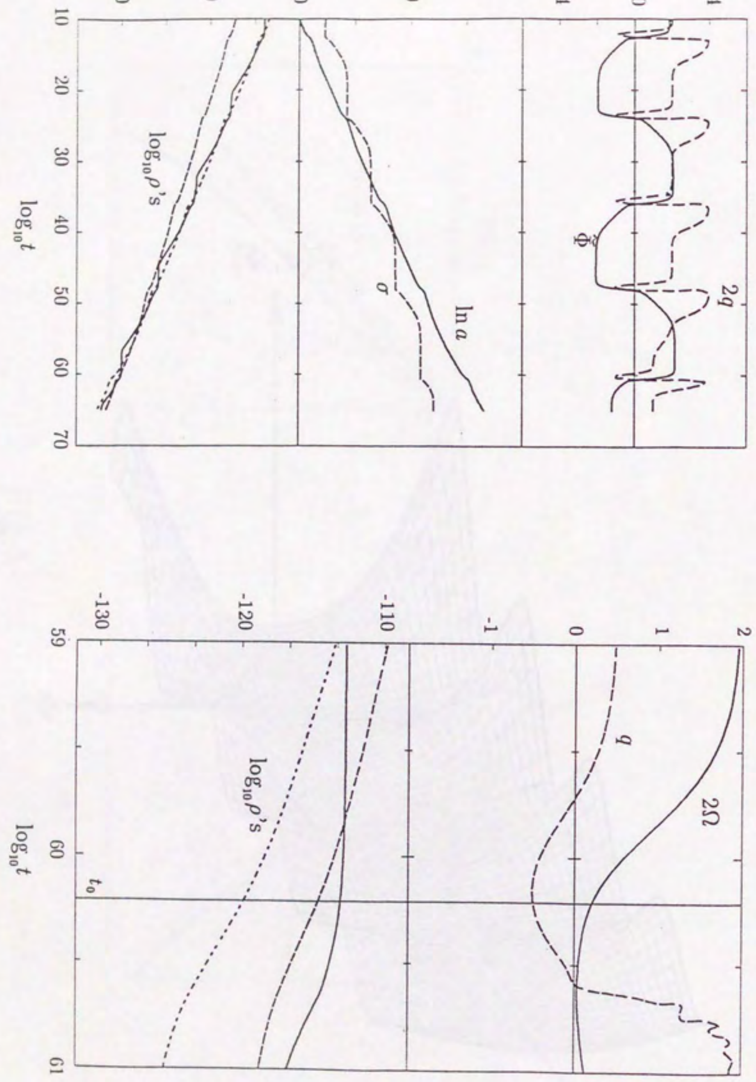

2q
$\tilde{\Phi}$
ln a
σ
$\log_{10}\rho$'s
$\log_{10} t$
2Ω
q
t_0
(a) (b)

Fig.7

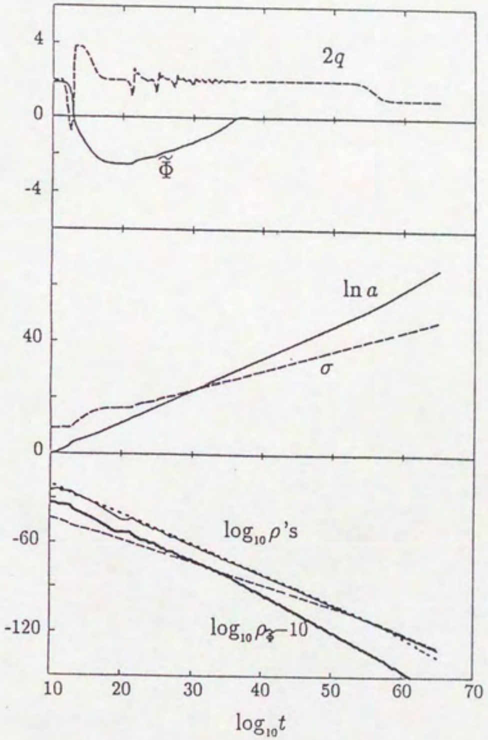

2q
$\tilde{\Phi}$
ln a
σ
$\log_{10}\rho$'s
$\log_{10}\rho_{\tilde{\phi}}-10$
$\log_{10} t$

Fig.8