研 究 速 報

Damage Investigation on Structure Using Laser Doppler Vibrometer (LDV), Time Domain System Identification Algorithm and Model Strain Energy (MSE)

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1. Introduction

In recent years, civil engineers are giving much attention and concern on durability of structures. As the matter of fact, many of the structures are ageing and due to some improper method of working during construction period, each structure is having partial deterioration level in the sense that there are differences in level of deterioration in each part of a structure that some may collapse first and followed by the others in sequence. Then, the impact is on difficulties to detect and localize the damage since we are dealing with a structure that sometimes not just big but also huge in size. This need is even added by another requirement to quantify the degree of damage itself. Therefore, as summary we can say that we need to develop a Non-Destructive Testing method that both can capture overall strength information of a structure and also can detect, localize, and quantify the degree of damage.

As the answer to the need, we are developing such a method that can meet both the needs and here, we are going to present this to the reader. The framework of this method starts from the utilization of *Laser Doppler Vibrometer* (*LDV*) that is used to capture (ambient) vibration information of a structure from which, in combination with *Time-Domain System Identification Algorithm*, strength property information of the structure can be retracted. Afterwards, remembering that a change in the strength level of a structure resembles in a change of mode shape and stiffness matrix, a method based on *Modal Strain Energy* (*MSE*) is utilized that leads us to ability to detect, localize, and quantify the damage.

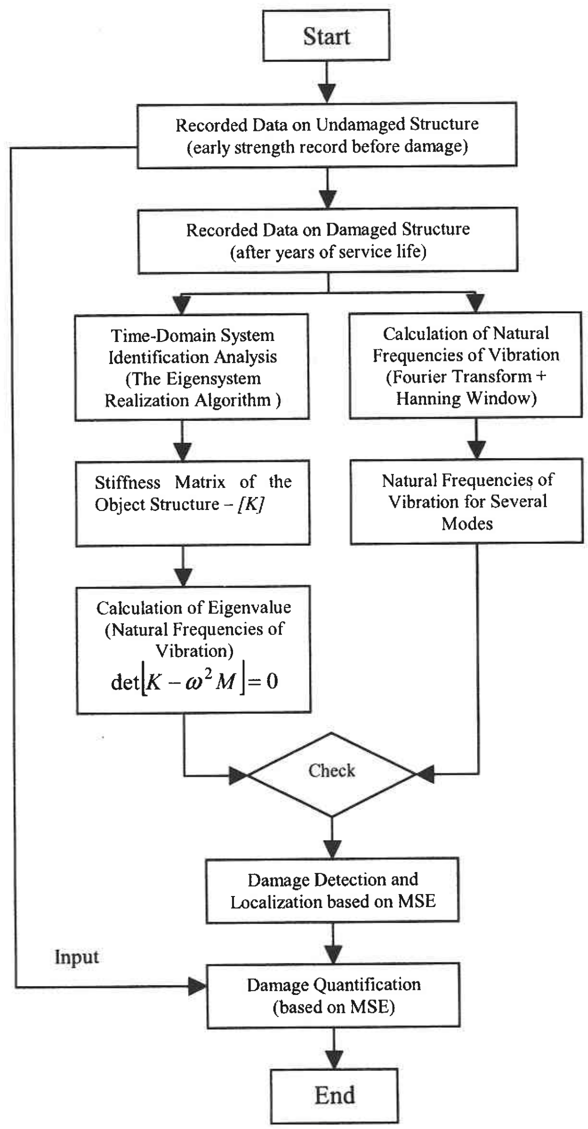

Fig. 1　Framework of Damage Investigation-Analysis

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2. Theories

2.1 The Time Domain System Identification Algorithm

Structure's strength and structure's stiffness are inseparable items that when we are trying to examine the strength part, we always have to obtain the stiffness part first. Ambient vibration of several designated points of structure is first recorded. The vibration of the points follows the ordinary vibration function as the following:

$$\textit{Ordinary from:} \quad M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = Ju(t) \quad \cdots\cdots (1)$$

which can be rewritten in state-space form as follows:

$$\textit{State-space from:} \quad \begin{aligned} \dot{x}(t) &= F_c x(t) + H_c u(t) \\ y(t) &= D_c x(t) + v(t) \end{aligned} \quad \cdots\cdots (2)$$

where u (t) is the unknown excitation; M, C, and K are mass, damping, and stiffness matrix respectively; q is the lateral structural displacement and the dot over it represents degree of differentiation of q over time. In state-space form, x (t), F_c, H_c, and D_c can be described more detail into:

$$\begin{aligned} x(t) &= \begin{Bmatrix} q(t) \\ \dot{q}(t) \end{Bmatrix}, & F_c &= \begin{bmatrix} 0_{mxm} & I_{mxm} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \\ H_c &= \begin{bmatrix} 0_{mxm} \\ M^{-1}J \end{bmatrix}, & D_c &= \begin{bmatrix} I_{mxm} & 0_{mxm} \end{bmatrix} \end{aligned} \quad \cdots\cdots (3)$$

When we are dealing with time history record of ambient vibration, the recorded data will be discretized by a fixed interval of time so that the continuous-time state-space form of equation (2) can be rewritten in discrete-time form for the kth time step as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k = Ax_k + w_k \\ y_k &= Dx_k + v_k \end{aligned} \quad \cdots\cdots (4)$$

and

$$\begin{aligned} A &= e^{F_c \Delta t} = \sum_{j=0}^{\infty} \frac{(F_c \Delta t)^j}{j!}, \\ B &= -\int_0^{\Delta t} e^{F_c \tau} H_c d\tau = F_c^{-1}(A-1)H_c \end{aligned} \quad \cdots\cdots (5)$$

Eq. (4) and (5) relates discrete time-series (ambient) vibration of the structure (x_k and y_k) with matrix Fc in which information about structure's stiffness ($-M^{-1}K$) is available. Therefore, our aim should be to put together time-history (ambient) vibration in such a manner that matrix F_c can be extracted and this is the first step in our method of analysis.

2.2 The Modal Strain Energy (MSE) and Modal Strain Energy Change Ratio (MSECR).

Structural damage often causes a loss of stiffness in one or more elements of a structure, but not a loss in the mass. When damage occurs in the structure, it can be represented as a small change in the original system that the stiffness matrix K^d, the ith modal eigenvalue λ_d, and the ith mode shape Φ_i^d of the damaged system can be expressed as:

$$K^d = K + \sum_{j=1}^{L} \Delta K_j = K + \sum_{j=1}^{L} \alpha_j K_j \quad (-1 < \alpha_j \leq 0) \quad \cdots\cdots (6)$$

$$\lambda_i^d = \lambda_i + \Delta\lambda_i ; \quad \Phi_i^d = \Phi_i + \Delta\Phi_i \quad \cdots\cdots (7)$$

where the damage in an element L is expressed as a fractional change of the elemental stiffness matrix; the superscript d denotes damage; α_j is a coefficient defining a fractional reduction of the jth elemental stiffness matrix; and L is the total number of elements in the system.

The elemental MSE is defined as the product of the elemental stiffness matrix and the second power of the mode shape component. The MSE of the jth element in ith mode for both before and after damage occurrence is expressed as:

$$MSE_{ij} = \Phi_i^T K_j \Phi_i ; \quad MSE_{ij}^d = \Phi_i^{d^T} K_j \Phi_i^j \quad \cdots\cdots (8)$$

where MSE_{ij} and MSE_{ij}^d are undamaged and damaged MSE of the jth element in ith mode shape, respectively. K_j is the undamaged stiffness of the structure and it is used in both damaged and undamaged MSE. Considering the case of un-acknowledgment of undamaged stiffness due to missing proper early examination/record on the structure's strength, the values of K_j can be assumed from the damaged stiffness K^d since localization of damage using MSE is taken on relative basis across elements of the structure. MSE_{ij} and MSE_{ij}^d are the basis for localizing the damage on structure by calculating the *Modal Strain energy Change Ratio* (*MSECR*) as defined below:

$$MSECR_{ij} = \frac{\left| MSE_{ij}^d - MSE_{ij} \right|}{MSE_{ij}} \quad \cdots\cdots (9)$$

Therefore, in examining position of damage using *MSECR*, the damaged eigenvectors Φ_i^d are calculated from damaged stiffness values K^d, which is the result of stage one of analysis. Afterwards, we assume values of undamaged stiffness of the structure K_j and continue with calculation of *MSECR* of each element for all modes of vibration. The final value of *MSECR* representing each element is the summation of all modes normalized by the maximum *MSECR* value as defined below:

$$MSECR_j = \sum_{i=1}^{Dof} \frac{MSECR_{ij}}{MSECR_{i,\max}} \quad \cdots\cdots (10)$$

The position of damage based on $MSECR_j$ value can be localized by pointing out the element (s) having high value of $MSECR_j$.

Foregoing is the explanation about the application of MSE for localizing damage on structure in which we do not need exact values of undamaged stiffness. Nevertheless, in order to quantify the degree of damage itself, the exact values of undamaged stiffness are indispensable that we need to start our NDT investigation by taking initial undamaged (vibration) record of the structure. We are still developing our application program for this purpose and leave it for the next discussion.

3. The Field Investigation

Our recent application of our NDT method was on a tunnel structure located in Kyobashi, Ginza, Tokyo. It is a concrete structure upon which an 8.5-meter width of roadway is passing through. Special attention is given to the middle part of the tunnel since after preliminary investigation using bare eyes, one can easily find cold joint on the left wall and several crack patterns on the roof that altogether make it a good "specimen" for our research. Since the structure is by-nature a 3-dimentional structure, discretization of it follows the system pictured in Figure 2. Simply, each wall was divided into 5 elements and the roof was divided into 4 elements

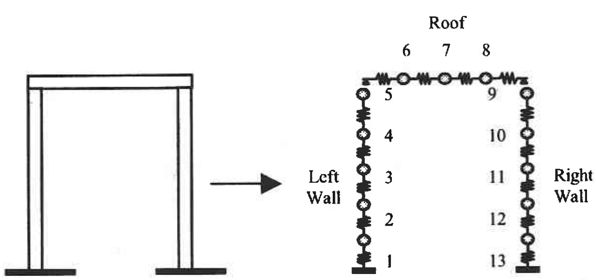

Fig. 2 Discretization of the tunnel

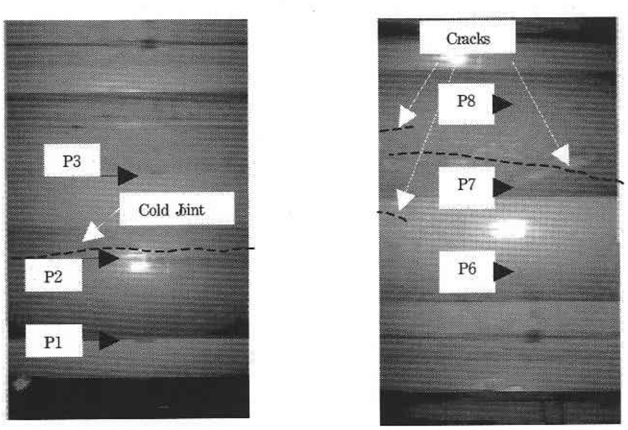

Fig. 3 Deterioration of the Tunnel

The location of cold joint in left wall was in the lower boundary of section 3 (adjacent to section 2) and the crack patterns on the roof was mainly existing in section 8 as illustrated in Figure 3.

On site, we recorded ambient vibration of all 13 points at 5000 Hz sampling rate for 10 minutes. Several points were designated to be points of reference, which were P1 for left wall, P7 for roof, and P13 for right wall. Then, on site investigation was finished at this stage and followed by in-house analysis using our developed application program built in *Mathematica* environment.

4. Analysis and Discussion

From all records of (ambient) vibration of all 13 points, we started the analysis with both time-domain system identification analysis to capture the stiffness values of the tunnel and Fourier transformation (mixed with Hanning window up to 100 degree) of the time-history data to capture the frequencies of vibration of the system. For the first goal, we started by construction of Hankel matrix, which put together all the time-history data. After several stages of analysis, we could find the matrix F_c from which we could extract stiffness values K^d. Afterwards, we calculated the damaged eigenvalues λ_d and eigenvectors Φ_i^d where the former were used for verification and the latter were used for damage localization. Then, we assumed certain values of undamaged stiffness which are by-nature equal for each element of each tunnel's segment due to similar design properties and assumption that all elements were initially good/perfect in condition.

Following it, we continued by calculating the *MSECR* values of all elements and the result of it are exhibited in Figure 4. In examining Figure 4, the criterion for determining damaged elements is Relative MSECR Value that equals to or exceeds 0.8. Applying this criterion, we can see that our framework of analysis can localize the bare-eye identified damage elements on the tunnel consistently. The cold joint line that exists in the lower boundary of section 3 (close to section 2) in left wall is shown by a tall bar stemming for section 2 (refer to Figure 4 for Left Wall). Moreover, the existence of several crack patterns on the roof is

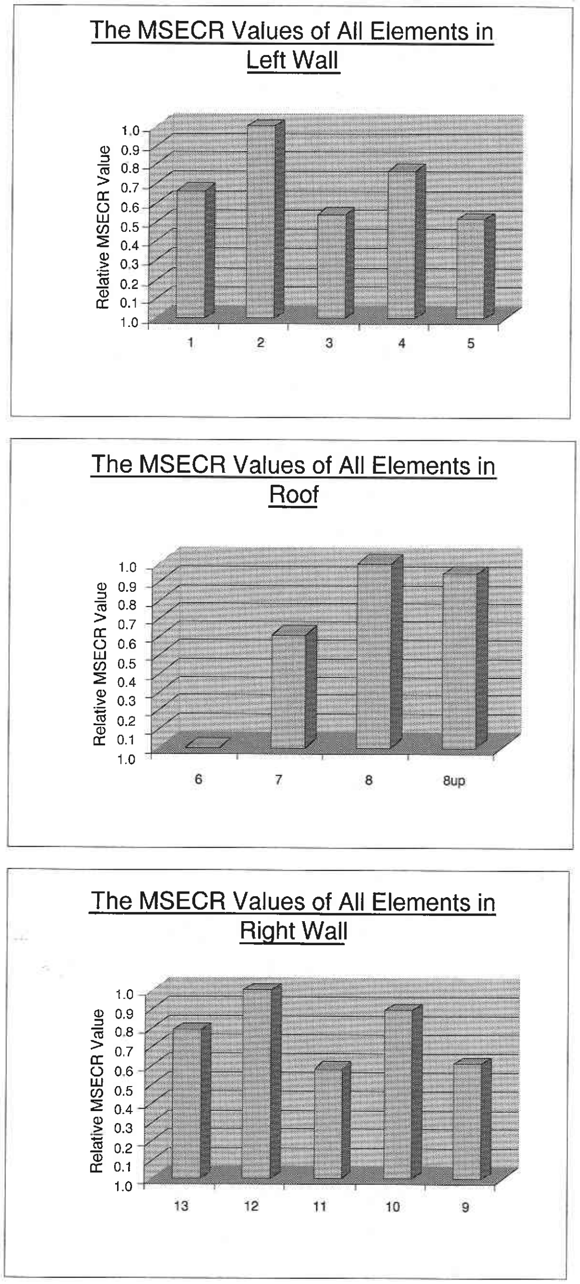

Fig. 4 The MSECR Values for Each Element

well shown by tall bars stemming for element number 8 and 8_{up} (please refer to Figure 4 for Roof).

Further analysis on Figure 4 leads us to question about the other tall bars stemming for element no. 10 & 12 (for the right wall). Our previous preliminary bare-eye investigation on the tunnel did not say anything about damage on those elements but our NDT investigation has shown something in addition and we believe that these are the elements having hidden damage. In order to have assurance about these hidden-damaged elements, we have come to conclusion to complement our NDT method using Laser Doppler Vibrometer (LDV) with regular NDT techniques since our NDT method using LDV can capture the aggregate strength information of a structure without having ability to know about the detail of it. Therefore, as our analysis showed additional information about locations of hidden damage, we should treat this additional information as an input for further test using regular NDT method.

5. Conclusion

A NDT method utilizing Laser Doppler Vibrometer, Time-Domain System Identification Analysis, and Modal Strain Energy has been presented. The real application of it on a tunnel structure has shown promising result which respect to ability to capture stiffness values of the structure and to detect and localize the damage on it. Moreover, the system is found able to detect and localize hidden damage as well with complementary need to verify the location of hidden damage by using regular NDT techniques. Future development of our method will enable us to quantify the degree of damage.

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