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Prediction Intervals for Day-Ahead Photovoltaic Power Forecasts with Non-Parametric and Parametric Distributions

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Abstract – The objective of this study is to compare the suitability of a non-parametric and 3 parametric distributions in the characterization of prediction intervals of photovoltaic power forecasts with high confidence levels. The prediction intervals of the forecasts are calculated using a method based on recent past data similar to the target forecast input data, and on a distribution assumption for the forecast error. To compare the suitability of the distributions, prediction intervals were calculated using the proposed method and each of the 4 distributions. The calculations were done for one year of day-ahead forecasts of hourly power generation of 432 PV systems. The systems have different sizes and specifications, and are installed in different locations in Japan. The results show that, in general, the non-parametric distribution assumption for the forecast error yielded the best prediction intervals. For example, with a confidence level of 85% the use of the non-parametric distribution assumption yielded a median annual forecast error coverage of 86.9%. This result was close to the one obtained with the Laplacian distribution assumption (87.8% of coverage for the same confidence level). Contrasting with that, using a Gaussian and Hyperbolic distributions yielded median annual forecast error coverage of 89.5% and 90.5%.

Keywords: Photovoltaic power, Day-ahead forecasting, Forecast error, Prediction intervals, Maximum likelihood estimation, Parametric versus non-parametric distributions.

1. Introduction

Photovoltaic, PV, power generation is reaching high levels of participation in the energy mix of several power markets around the world. Due to this trend, and the fact that PV power generation can vary strongly according to weather conditions, increasing attention is being given to the development of methods that can mitigate or anticipate strong PV power fluctuations. One of these methods is forecasting of PV power generation. In the search for better forecasts, often deterministic methods are developed, as the single value they yield as output can be easily used to assess the error of the forecasts [1-3]. In many cases, however, besides a single forecast value, it is useful to have information about its uncertainty. For example, power utilities and stakeholders in power markets are interested in having not only a forecast for the PV generation of a system or region, but also have information, with high confidence, of how much the real generation can deviate from such forecast. In this regard, there are a few studies available. For example, Lorenz et al [4] assumed the PV forecast error in Germany to follow a Gaussian distribution, and proposed to obtain prediction intervals based on the standard error of solar irradiance forecasts. The standard error of the forecasts was calculated according to solar zenith angle and cloudiness of target time of the forecasts. They concluded that for ensembles their approach yields intervals covering measured values with a ratio slightly lower than expected (91% coverage for intervals with confidence of 95%). Bacher et al [5] discarded the Gaussian distribution assumption and proposed to calculate prediction intervals for PV power forecasts using quantile regression applied according to the forecast horizon. With their method a specific prediction interval is obtained per forecasted value, regardless weather conditions and hour of day for which the forecast is done. Marquez and Coimbra [6], assumed a Gaussian distribution for the error of forecasts of solar irradiance in the U.S., determining the standard deviation with an ANN based regression model. They found good agreement between the coverage of the intervals and their confidence level. Nevertheless, they also noted systematic departures of the expected behavior in a few clustered days of the period of analysis.

In previous studies, we proposed a method to yield prediction intervals with high confidence level values for one-day ahead forecasts of PV power in Japan based on the assumptions of 2 distributions for the forecast error [7]. The method yielded good prediction intervals, but it had the limitation of being validated with data of only 2 PV systems. In this study, we extend the analysis to a large number of PV systems with a variety of PV cell and

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installation conditions. Furthermore, we evaluate the performance of the proposed method using also the hyperbolic distribution and a non-parametric distribution assumption to characterize the forecast error. All the evaluations were done with one year of data. The performance of the proposed method and of the suitability of each of the 4 distributions assumptions were evaluated with the normalized size of the intervals, and the effective annual coverage of the forecast error that the intervals provided. Four high confidence level values, from 85% to 97.5%, were used in the evaluations.

In the following sections the description of the proposed method and data are presented, as well as its validation in the calculation of prediction intervals with 4 confidence levels. This paper is an extended and modified version of one originally presented at the International Conference of Electrical Engineering, ICEE 2016, which was focused on the application of the hyperbolic distribution in the prediction interval problem [8].

2. Description of the Set of PV Systems

One year of hourly power generation data of 432 PV systems were used to compare the effect of the different distributions to characterize prediction intervals of forecasts of PV power generation. This set of PV systems is part of the Field Test Project funded by the New Energy and Industrial Technological Development Organization, NEDO, in Japan. One year of hourly power generation data available for 2010 of each PV system studied was used. The PV systems are mostly of the non-residential type, and their capacity vary widely (from 10 kW to more than 100 kW). A heat map is presented in Table 1 to indicate the concentration of PV systems according to the region and capacity.

Regarding the PV cell type, around 80% of the systems employ polycristalline silicon cells and near 10% of all systems use cells of the heterojunction with intrinsic thin layer type. Finally, the most common installation conditions were 20 degrees for the module tilt angle and south orientation, with more than 20% and 38% of PV systems of

Table 1. Ratio of PV Systems according to their region in Japan and their nominal capacity

Capacity (kW)	≤11	11<x≤21	21<x≤51	51<x≤101	>101
Hokkaido	4	1	-	1	-
Tohoku	6	2	4	-	-
Kanto	48	20	30	7	4
Chubu	37	27	29	11	4
Kansai	26	21	13	11	4
Shikoku	9	9	1	1	1
Chugoku	18	12	4	1	3
Kyushu	32	14	7	5	3
Okinawa	-	1	1	-	-
Total	180	107	89	37	19
% of total	42%	25%	21%	8%	4%

the set. Regardless, the set of systems includes examples with a variety of installation conditions, including vertical angles (systems installed in building walls), bifacial modules etc. The total PV power capacity of the set is near to 15 MW. Further information about the PV systems from which this subset was selected is available on [9].

3. Methodology

To calculate prediction intervals for a day-ahead forecast of PV power, first, the forecast must be done. Such forecast method is described in section 3.1, followed by the description of the method used to calculate prediction intervals in section 3.2.

3.1 PV power forecasts

To forecast PV power generation for each PV system, one day ahead of time, in hourly fashion, a method proposed in previous studies was used [10]. Thus, only a brief description of it is provided. With this method a machine learning technique called ν support vector regression, SVR, is used to build a forecast model which uses as input data numerical weather prediction from the grid-point meso-scale model, GPV-MSM, data set of the Japan Meteorological Agency. For each day of forecasts a new model is built by training the SVR algorithm with the equivalent in hours of the 60 days of input-output data (860 hours after excluding night periods) preceding the target data. In this fashion, the target day data are separated of the training data set, and the most recent data are always used to train the forecast model. The algorithm was set with an ensemble based approach, and the Gaussian kernel was used as kernel function in the SVR formulation.

Regarding the input data, the version of the GPV-MSM used yields hourly numerical weather prediction for several variables 8 times a day with a forecast horizon going up to 33 hours ahead of time. For the day-ahead forecasts, input data from just one release time were used, the one released at 12h in Japan Standard Time (JST). For any given system, each hour of PV power generation was forecasted using as input variables predictions of air temperature, relative humidity, and cloudiness in three levels provided by the GPV-MSM. Besides these variables, the extraterrestrial solar radiance was also used as input. The PV power generation of each hour is regarded as the output data of the forecasts. One should note that in this configuration, predictions of solar radiance are not used at all. Finally, the forecasts were done for each day and each PV system, from 6h to 19h, for one year (namely the year of 2010).

3.2 Methods to calculate prediction intervals

The forecasts done with the method described in section 3.1 are deterministic. Only a single value for the PV power generation in each target hour is provided. The prediction intervals for the forecast errors are calculated based on two assumptions. The first one is with respect to the relation between past and future forecast errors. We assumed that there is similarity between past and future forecast errors that occurs at the same or similar weather, input data and location conditions. The second assumption regards the distribution of the forecast error. We assumed two cases. In the first case it was assumed that the forecast error follows a known distribution whose parameters can be obtained through the maximum likelihood estimation method. This approach to characterize prediction intervals for forecasts of PV power was preliminary evaluated with data of 2 PV systems in [7]. In the second case, it was assumed that the distribution is non-parametric and that it can be derived directly from the past forecast errors.

Based on these two assumptions and two cases, from 60 days of forecast errors preceding the target hour (the target hour not included), those with the input data most similar to the input data of the target hour are selected to characterize the forecast error distribution of the latter. Sixty days of data were selected based on trial and error tests done in a previously [7]. The selection of the most similar past hours is done using as metric the Euclidean distance between the input data of the target hour and of the past data. Any hour with a distance higher than a predetermined threshold d is excluded from the set. The remaining points are used to characterize the forecast error distribution for the target hour. The threshold d was determined according to the distribution assumption and trial and error evaluations with test data. For the Laplacian and Gaussian distribution assumption d was set as the 10^{th} percentile of the set of past data. For the hyperbolic distribution and non-parametric assumptions d was set as the 15^{th} percentile.

Using this method a subset of past forecasts errors is obtained and used to characterize the prediction interval according to the target hour. When a parametric distribution is assumed, the best distribution that fits the subset of data is calculated with the maximum likelihood estimation method and the distribution probability density. For the Laplacian distribution, the probability density is as showed in Eq. 1. Applying the maximum likelihood estimation, MLE, σ in Eq. 1 becomes the mean absolute error of the forecasts. Once σ is found, the percentile p_s, of a forecast error following the Laplacian distribution and with a probability 1- 2s can be calculated as showed in Eq. 2.

$$\mathrm{p(z)} = \frac{1}{2\sigma}e^{\frac{|z|}{\sigma}} \tag{1}$$

$$p_s = -\sigma \ln(2s) \tag{2}$$

For the Gaussian distribution the application of the MLE yields the p_s described in Eq. (3).

$$p_s = \sigma\Phi^{-1}(1-s) \tag{3}$$

Regarding the hyperbolic distribution, the distribution that fits the data with maximum likelihood is calculated numerically, using the Generalized Hyperbolic library available in R language. Once the hyperbolic distribution that fits the data and its parameters are determined, corresponding p_s values can be directly obtained from the corresponding quantile function.

In the non-parametric case, the percentiles are calculated directly from the subset containing the past data similar to the target hour. In a previous study we evaluated the non-parametric approach still considering the forecast error distribution symmetric [7]. In this study the non-parametric assumption does not assume any symmetry regarding the distribution of the forecast error.

3.3 Evaluation of the prediction intervals

In applications of PV power forecasts, users are most interested in the coverage of forecast errors with high confidence levels. This happens because having to deal with a forecast error that greatly exceed its corresponding prediction interval may imply economical losses or strong imbalances between power demand and supply in a scenario of high PV power penetration. Thus, we focused on the performance of the proposed method regarding 4 high confidence levels, 85%, 90%, 95% and 97.5%.

Another way to verify the validity of the prediction intervals and of the distribution assumptions used to calculated them, is to estimate their cost in energy. The prediction intervals can be used to estimate the amount of reserve power required to deal with the forecast errors of PV power generation. It can also be used to provide an idea of the necessary grid flexibility to deal with variable renewable energy. It is desired to have intervals that are as small as possible, while providing the expected forecast error coverage indicated by the confidence levels. Thus, to also compare the prediction intervals obtained with the different distributions, regarding their size, they were expressed in energy normalized by each PV system maximum capacity value.

4. Results and Discussion

4.1 Analysis of the forecast error coverage

If the modeling and assumptions are properly made, the observed frequency with which the measured PV power generation is within the intervals, the forecast error coverage of the intervals, should approximate or match the confidence level used to calculate the intervals. The forecast error coverage obtained for these confidence levels, with the 4 distributions assumptions and for the 432 PV systems, during one year are in Fig. 1.

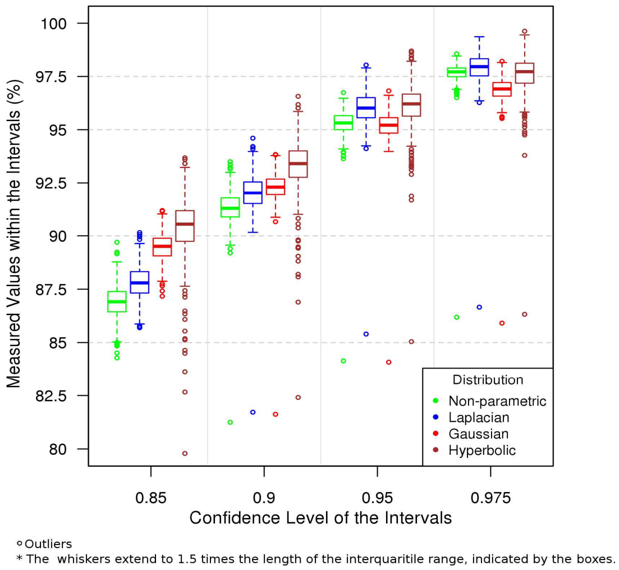


Fig. 1. Observed frequency of measured PV power generation within the prediction intervals for 432 PV systems, 1 year of data, and 4 confidence levels

The results in Fig. 1 show that, in general, the median value of the forecast error coverage provided with the prediction intervals correlated well with the confidence levels. Nevertheless, it is clear from Fig. 1 that the non-parametric assumption yielded the best prediction intervals. Its median value approximated better the confidence levels for all values tested. Considering only the parametric distributions, the Laplacian distribution assumption yielded the best results. In spite of the good results achieved with the non-parametric and Laplacian distribution assumptions, for confidence levels of 85% and 90% the achieved forecast error coverage values were higher than expected, indicating a trend of underestimation of the forecast error coverage for lower confidence levels.

Regarding the Gaussian distribution assumption, its use was only effective when calculating prediction intervals with a confidence level of 95%. For other confidence levels, its use underestimated (for 85%, 87.5%, 90%) or overestimated (for 97.5%) the forecast error coverage.

Finally, the hyperbolic distribution assumption yielded the worst results. In spite of having median values close to the ones obtained with the other distribution assumptions for all confidence levels, the dispersion of its forecast error coverage was considerably higher than the ones provided with intervals calculated with the other 3 distributions assumptions.

The forecast error coverage of the proposed method to calculate prediction intervals with any of the 4 distributions' assumptions per region is plotted in Fig. 2a, 2b, 2c and 2d. Besides the plots, a regression spline with 3 degrees of freedom was plotted to indicate general trends of the variation of the results with the regions.

Looking at the results, the prediction intervals had tendency to yield forecast error coverage values higher than expected, particularly at confidence levels of 85% and 90%. Regarding regional characteristics, the intervals were in average larger than necessary in the south of Japan (Kyushu and Okinawa areas). This second trend is related with the fact that forecasts of PV power generation in the south of Japan tended to present higher errors than in other regions due to differences in weather and insolation conditions. These characteristics affects the overall PV power forecast error distribution, decreasing the suitability of the assumptions made to calculate the prediction intervals. This makes more difficult to obtain subsets of similar data for each target hour from just 60 days of past forecasts. Still, the overall effect was small when the distribution was non-parametric or Laplacian.

The best results were achieved when the non-parametric distribution was used. Its use yielded the best prediction intervals at low confidence levels and also less overestimation on south regions. When the prediction intervals are calculated with the Gaussian or Hyperbolic distribution assumptions, besides the tendency to yield higher prediction intervals and excessive forecast error coverage going southwards, the regression lines in Fig. 2c and 2d show that, the size of the intervals were also more affected by the location of the PV systems than when the non-parametric or Laplacian distributions were used. This behavior indicates that both distribution assumptions are not suitable to characterize the PV power forecast error in a general way.

4.2 Analysis of the interval's sizes

To evaluate the size of the prediction intervals, they were expressed in energy normalized by each PV system maximum capacity in Fig. 3. Starting with the results obtained with the assumption of hyperbolic distribution, it yielded unnecessarily large intervals, regardless the confidence level. This happened even when the achieved forecast error coverage had, in average, good correspondence with confidence level used to calculate the intervals. For example, with a confidence level of 97.5% in Fig. 3d, the size of the prediction intervals were larger than the ones obtained with the non-parametric and Laplacian distributions assumptions, 27.4% and 16.4% respectively. Moreover, with the hyperbolic distribution assumption, the prediction intervals had a tendency to increasingly overestimate the forecast error coverage with the reduction of the confidence level as noted also in Fig. 2.

Comparing now the prediction intervals obtained with the 3 remaining distributions, their average sizes was strongly related with the forecast error coverage they provided. Furthermore, the relation was different according to the distribution assumption. For example, whereas for prediction intervals obtained with the Laplacian distribution assumption required in average 0.41 kWh/ kW_{cap} to yield 97.9% of forecast error coverage, the prediction intervals calculated with non-parametric distribution assumption required 6.1%

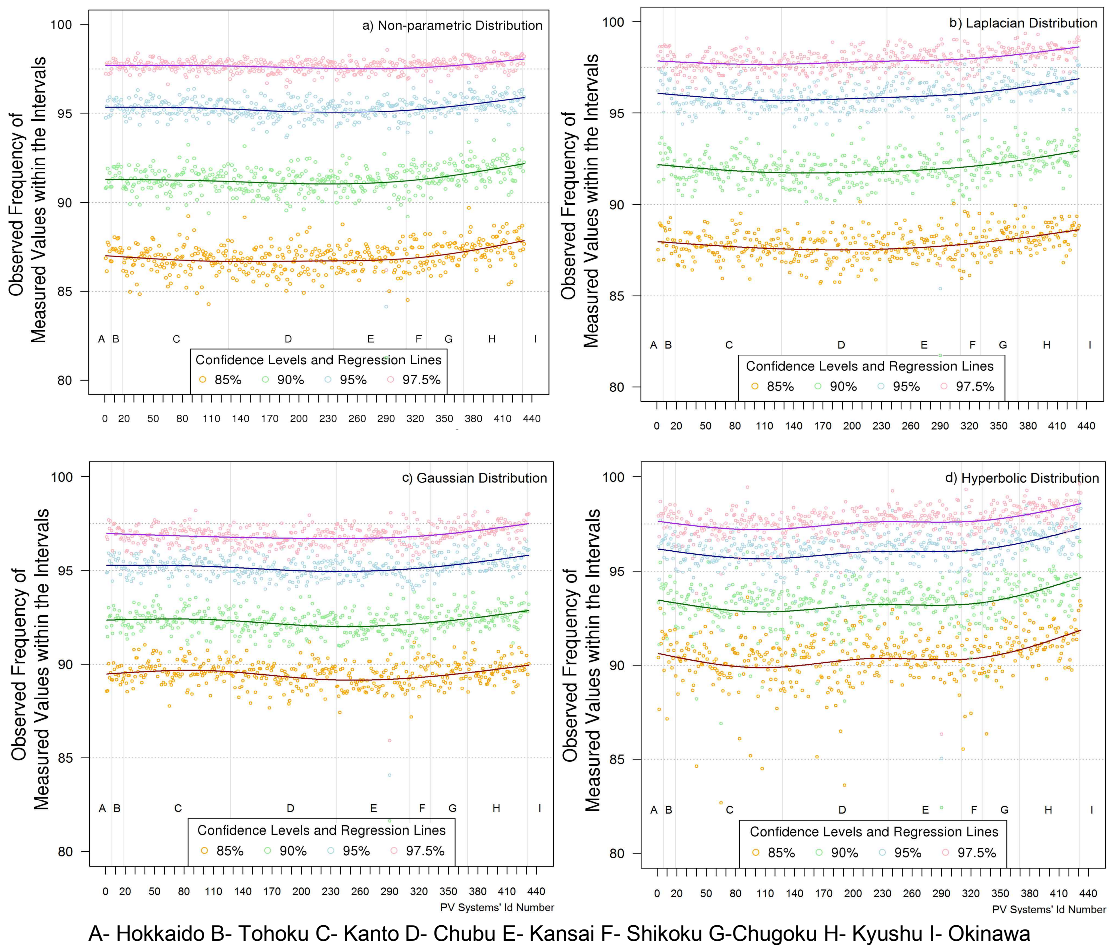


Fig. 2. Observed frequency of measured PV power generation within the prediction intervals (forecast error coverage) calculated with the Non-parametric (a), Laplacian (b), Gaussian (c), Hyperbolic (d) distributions from Hokkaido to Okinawa

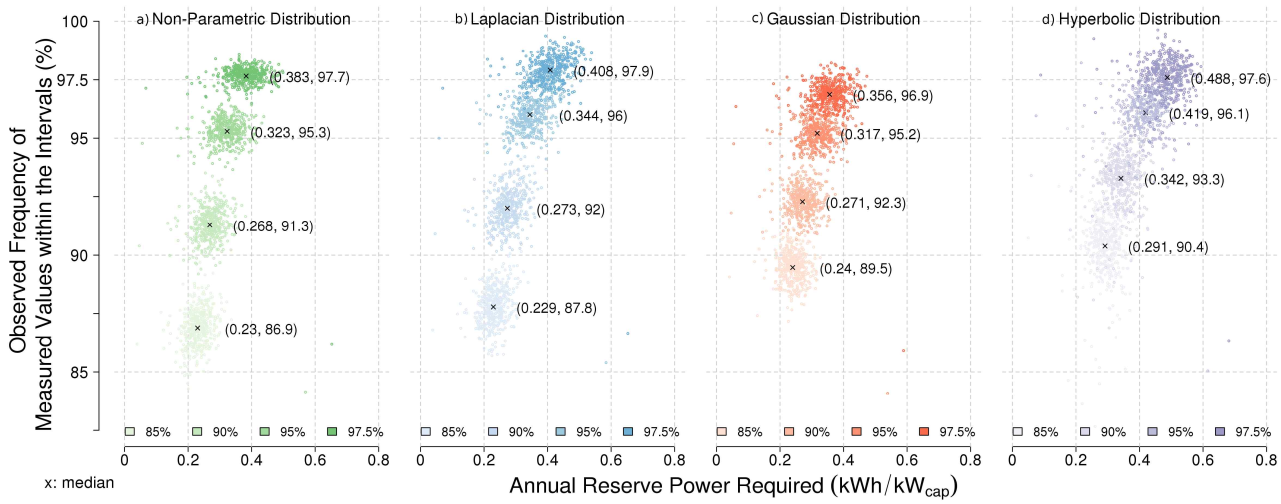


Fig. 3. Observed frequency of measured PV power generation within the prediction intervals versus their average sizes expressed in terms of annual reserve power

less to yield 97.7% of forecast error coverage.

Fig. 3 also shows the cost in flexibility and reserve power to increase the forecast error coverage, according to the distribution assumption. For example, to increase the forecast error coverage from 86.9% to 97.7% using intervals calculated with the non-parametric distribution assumption, it was necessary to increase the average reserve power (the size of the prediction intervals) by more than 66%. With Laplacian distribution assumption the average reserve power had to be increased by more than 78% to increased the forecast error coverage from 87.8% to 97.9%. From this point of view too, the prediction intervals calculated with the non-parametric distribution assumption were better than those calculated with the Laplacian distribution.

Finally, a discussion regarding the meaning of the size of the intervals is made. Using the non-parametric distribution related results in Fig. 3a as reference, for the 432 PV systems studied in Japan, the average reserve power required to cover the forecast error approximately 87% of the time, was 23% of PV systems' capacity in terms of energy. This value goes up to almost 40% if the required forecast error coverage target is 97.7%. Such values are high due to the fact that they are for single PV systems forecasts. In regional scale, and considering other power grid flexibility measures as interconnections, curtailment, battery, etc, the required reserve power should be smaller.

4.3 Performance in different weather conditions

Another important aspect of the intervals is their ability to characterize the forecast error properly in different weather conditions and hours during the day. Depending on these and other conditions, such as the installation and specification of a PV system for example, the causes of the forecast error may change. Thus, a method to calculate prediction intervals for PV power forecasts should be flexible enough to capture this characteristic of the forecast error.

To show how the distribution assumption regarding the forecast error affects the ability of the proposed method to yield proper prediction intervals in different conditions, two days of calculations of a typical PV system are presented in Fig. 4. The first day in Fig.4 contains the PV power generated and forecasted one day ahead of time in the case of a day with clear sky in summer. The second day in Fig. 4 contains the same variables for a cloudy to sunny day in winter.

Starting with the winter day, Fig. 4e to Fig. 4h, regardless the distribution assumption, the proposed method correctly identified the hour where high forecast errors were expected. Nevertheless, the size of the intervals and their shapes varied according to the error distribution assumption. In this case, the hyperbolic distribution assumption was the worst one, as it yielded intervals larger than the other ones. Still regarding the winter day, the remaining 3 assumptions yielded similar results although, as expected, the Laplacian and Gaussian distribution assumptions yielded symmetric intervals, which contrasts with the results of the non-parametric assumption.

Such difference regarding the symmetry of the prediction interval in some cases, depending on the days conditions and hour of power generation, are not always meaningful. However, in days of high PV power generation as the one showed in Fig. 4a to Fig. 4d the sizes of the intervals

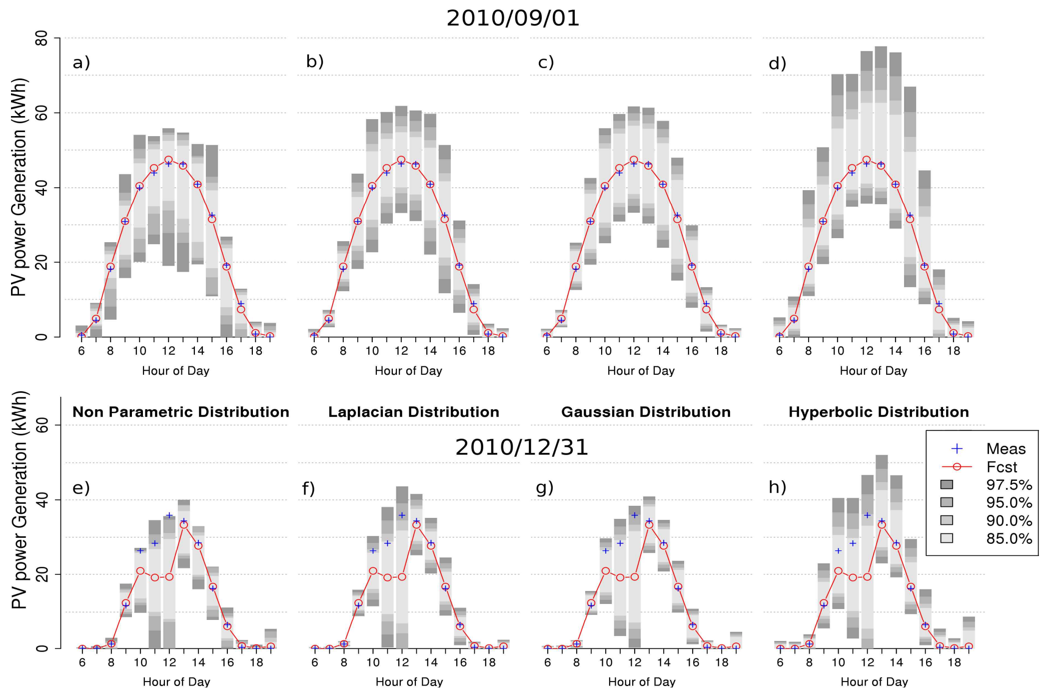


Fig. 4. Prediction intervals calculated (with different distribution assumptions) for two days of PV power forecasts of a single system. (Forecast error distributions: a), e): non-parametric; b), f): Laplacian; c), g): Gaussian, d), h): Hyperbolic)

and their symmetry are relevant. On the day showed in Fig. 4a to Fig. 4d, at 12h near to the maximum PV power generation was forecasted. Consequently, if any forecast error occur it is likely it will be more an overestimation error than an underestimation one. Moreover, at 12h in Fig. 4a to Fig. 4d the prediction interval, due to their sizes, should not be symmetric as it would mean PV power generation higher than the maximum possible.

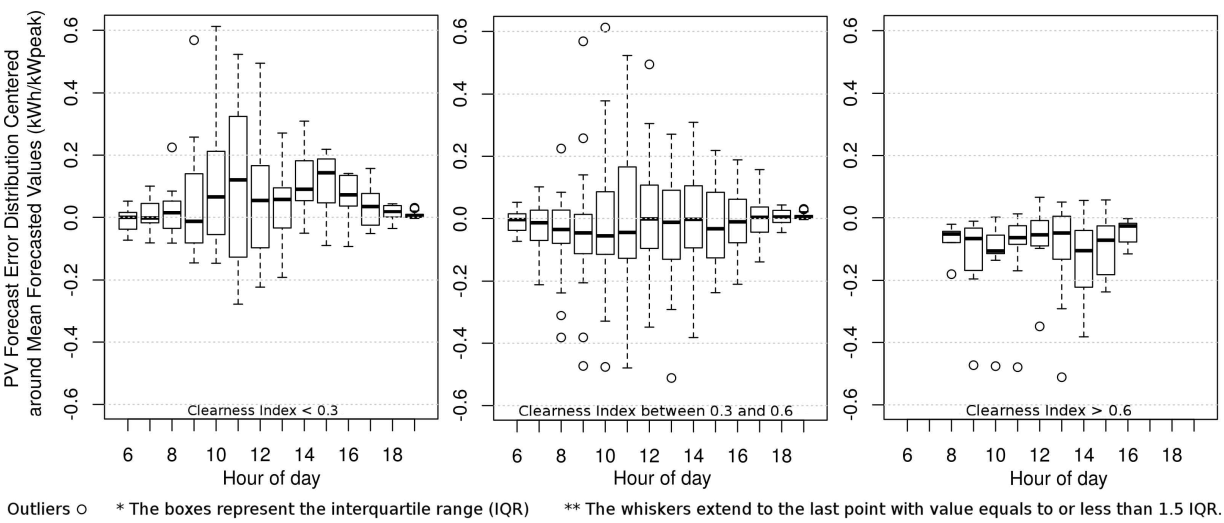
Thus, the physics of the problem has an important effect on the symmetry and on the tail of the PV power forecast error distributions. As a further example, in the beginning and end of the day, the low solar elevation cause low PV power generation. Thus, forecast errors will usually be in the direction of underestimating the PV power, if the forecast is cloudy for a sunny day (there can be no negative PV power). On the other hand, at peak hours if the peak PV power is forecasted and the forecast has an error, the only possible error is on the side of overestimation. Finally, as showed in Fig. 4, there will be also hours where error symmetry can also happen as in the middle of morning or afternoon.

4.4 Results discussion

In cases of distributions that can be asymmetric, the hyperbolic and the non-parametric should have yielded the best results. However, with the hyperbolic distribution assumption the proposed method presented poor performance. A few reasons explain such result. First, the algorithm used to calculate the intervals with the hyperbolic distribution automatically decides which direction the longest tail of the distribution should face, and in many cases the selected direction was not the correct one.

Checking the results of PV system's samples, we also noted that depending of the weather conditions, season and hour of day, the type of forecast error and its corresponding distribution change; and that many times the proposed algorithm could not find a suitable hyperbolic distribution to fit the error distribution. Two factors can cause this problem, limitations of the library used, and the fact that in many cases the error distribution, although asymmetric, was not hyperbolic at all. A third reason, has to do with the shape of the hyperbolic distribution. To to obtain longer tails for such distribution, with the library used, its peak was often reduced (there is a trade-off between long tails and sharp peak). Annually, the forecast errors for single PV systems in Japan frequently have low values, and in a few cases it had high values. Thus, it tends to have high peak and heavy tails, although in some hours asymmetric ones due to the physics of the problem.

In Fig. 5 we plotted the forecast error distribution in June for the power generation of a typical residential PV system, 10 kW_{rated}, installed in the Kanto region. The forecast errors in Fig. 5 are grouped per range of clearness index to provide a simple distinction of weather patterns. As the results in Fig. 5 show, depending on the kind of weather and hour of day the error distribution changes. In Fig. 5(a) at 10 h for example, the error distribution is asymmetric toward positive values indicating that at such hour, when the weather tends to the overcast type, if a forecast error occurs it was on the side of overestimating the real PV power generation (forecasting a sunny hour for a hour that actually was cloudy). On the other hand, for sunny weather on Fig. 5(c), if a forecast error occur it will often be toward underestimating the real PV power generation (forecasting a cloudy hour for a hour that was sunny). In this example, the kind of asymmetry and its location varied widely even though the errors were restricted to 30 days of June. If seasonal weather patterns



(a) Tending to overcast Weather (b) Partial cloudy to partial sunny (c) Partial sunny to sunny

Fig. 5. Hourly distribution of the forecast error for 3 kinds of days in June of a residential PV system (10 kW_{rated})

are considered more variation of the error distribution occurs. This characteristic makes the task of fitting a single distribution shape to all such error patterns difficult. The plots in Fig. 5 also show that in many cases outliers are considerably higher than the extension of the whisker, which indicates the general heavy tail shape of the error distribution.

The results in Fig. 5 are of just one PV system in one month. To show the general forecast error and outliers' characteristics, in Fig. 6 we plotted the annual error distribution for each of the 432 PV systems studied. As it is the annual error distribution, the asymmetry caused by different hours, weather patterns and seasons disappears.

The outliers, large forecast errors, however, remain. The results in Fig. 6 indicate that in general near to 80 % of the forecast errors were within the whiskers, and 20 % of the errors in the year were outliers. This happens regardless the PV system. The whiskers cover errors between approximately -0.2 kWh/kW_{peak} to 0.2 kWh/kW_{peak}. The extreme outliers, forecasts errors, reached values as high as 1 kWh/kW_{peak}.

In reality, what is desired from the point of view of power grid operation is the distribution of error for each possible season, weather and hour. Such distributions will be different of the annual one.

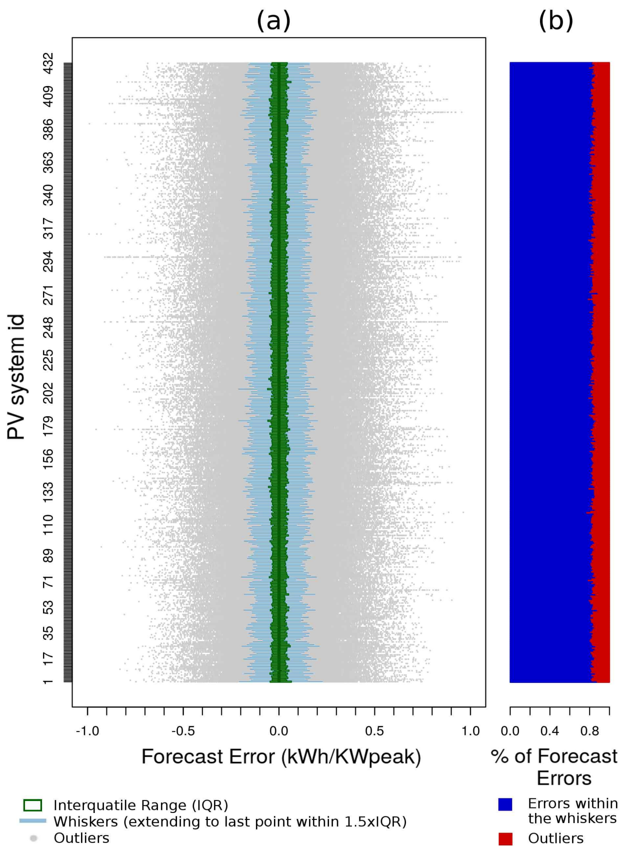


Fig. 6. Annual forecast error distribution of each PV system studied using box plots (a) and ratio of forecast errors regarded as outliers (b)

Nevertheless, the annual distribution of forecast errors show that the frequency of outliers and their magnitude may yield in many cases, localized forecast error distribution with heavy tails.

Given the variety of error distributions being modeled, it should be noted that depending on the weather, season, hour and installation conditions of the PV system, any of the tested distributions may be suitable to be used in the calculation of prediction intervals for a few particular hours. However, based on the results presented, only with the non-parametric and the Laplacian distributions assumptions it was possible to characterize properly the prediction intervals for a variety of PV systems and conditions throughout the year. This conclusion is valid for the 4 high confidence levels targeted. Still, whenever the asymmetry of the intervals became important, only the non-parametric distribution provided results consistent with the physical phenomena.

5. Comparison with Other Studies in Literature

Although the research on methods to forecast PV power generation are abundant in literature, just recently attention to the uncertainty of PV forecasts started to grow. Thus, there are only a few peer-reviewed studies about this topic. Additionally, in each study the methods to yield prediction intervals are evaluated in different periods, for different confidence levels and with different data. This fact makes impossible to perform a direct comparison of methods. Still, a simple assessment of results reported by different groups with their methods can be done. Such assessment can show typical results achieved by different approaches in the problem of prediction intervals for PV power forecasts.

The results in Table 2, summarize the results of 6 studies we found in technical literature. We included in the comparison, methods to yield uncertainty of irradiance forecasts, as they are closely related to PV power forecasts.

Although no direct comparisons can be done, a few trends are noted. Firstly, as currently there is no recommended metric to evaluate the performance of such methods, each group of researchers evaluated its methods in different ways. Some report good agreement with the 95% confidence level as Marquez and Coimbra [6] and no specific value. Other researchers checked the performance for different confidence levels, such as 92% or 96% Hirata et al [11]. And a few others evaluated the average size of the intervals as Ohtake at al [12].

Secondly, despite the different confidence levels used to evaluate the intervals, all of them were higher than 80% (95% was a common one). This reflects the fact that the main concern regarding PV power forecasts uncertainty is with coverage of high deviations from forecasted values. For a power utility for example, it is interesting to know how big the forecast error can be with high confidence levels. In this way, the power utility can be

Table 2. Comparison of methods available in literature to obtain prediction intervals for PV power of solar irradiance

Method	Confidence level	Effective Error Coverage	Data Set	Data Set Size and Resolution
Lorenz et al [4]: Normal distribution assumption and standard deviation of the based on solar zenith and clear sky index.	95%	Achieved (No specific value reported)	Point irradiance measurements of 200 weather stations in Germany	Hourly data measured from January to October of2007 (up to 72h ahead forecasts).
	95%	91%	Regional irradiance measurements in Germany.	
Bacher et al [5]: Used quantile regression assuming that the nornal distribution is not adequated in this problem.	95%	Yielded intervals but did not reported their effective coverage.	Data of 21 PV systems located in Jutland, Denmark.	One year of data with 15m resolution. 3h NWP data used were resampled to 15m resolution (up to 36h ahead forecasts).
Marquez and Coimbra [6]: Assumed that E(r|x) of the residuals are zero and that it and its Var (r|x) are normal. Differs from [1] by calculating standard deviation was through ANN regression.	95%	Generally achieved (no specific value reported), with large deviations clustered in the same day.	NdFD meteorological data, intra-day and day-ahead forecasts of GHI.	The prediction intervals were tested in 3 days of hourly data. (intra-day and day-ahead forecasts)
Hirata el al [11]: Multi-horizon forecasts wiht times series approach and clustering of past data based on an extension of Kwasniok and Smith method.	92% and 96%	99.7% and 98.4% respectively (but with different setups)	Solar irradiation data provided by JMA for Tokyo.	Data from 2002 to 2006, with 10m resolution resampled to 1h units (0h to 36h ahead forecasts)
Ohtake et al [12]: Method using statistical evaluations based on measured and NWP forecasts of GHI.	80%	93%	Measured and NWP data provided by JMA for Kanto in Japan. Evaluations of intervals for one day in regional and local scales.	5 years of data from 2008 to 2012 with hourly resolution (day-ahead forecasts).
Proposed Method	85%, 90%, 95%, 97.5%	86.9%, 91.3%, 95.3%, 97.7% (median values)	Day-ahead PV power forecasts for 432 PV systems in Japan.	1 year of hourly data from April/2010 to Mar/2011.

prepared in case large deviations occur and avoid frequency instabilities and imbalances between power supply and demand. For power markets participants, coverage of high errors is important because of the cost of selling an amount of PV power that will not be delivered due to a forecast error, for example. Thus, most researchers in this field are not interested in the full distribution (distribution here meaning the full range of confidence levels) of the forecast error at all hours, all weather conditions and all seasons. Although methods that can give such information can be developed (specially in the presence of large amounts of past data), priority is given to consistent prediction intervals at high confidence levels. Finally, the results in Table 2 show that with our approach good consistency between expected confidence levels and effective error coverage were achieved.

6. Conclusions

The objective of this study was to validate a method to yield prediction intervals with high confidence levels for day-ahead forecasts of PV power generation, and to evaluate the suitability of 4 distribution assumptions in the calculation of the intervals. From the results obtained with a set of 432 PV systems and 1 year of hourly data, we conclude that, within the 4 distributions tested, the non-parametric assumption was the best one, followed by the Laplacian distribution assumption.

The latter, had the disadvantage of being symmetric, which does not properly characterize properly the PV power forecast error in all hours, seasons and weather conditions. In spite of that, the Laplacian distribution assumption yielded prediction intervals which were still better than the ones obtained with the Hyperbolic distribution, which is asymmetric, and with the Gaussian one.

Regarding the fitness of the non-parametric distribution, it is expected to be a good approach when there are extensive data containing all important seasonal, weather, and hour variations of the PV power forecast error. The lack of such a large database for all PV systems, in a given area for example, is what justified the study of known distributions with the maximum likelihood estimation in the first place.

Nevertheless, the results showed that, for high confidence level values, it is possible to select a proper subsets of data to characterize the forecast error with a non-parametric distribution assumption from just two months of past forecasts. This fact, the simplicity of application of the proposed method, associated with its low requirements regarding computational costs and availability of past forecasts, make the method an interesting and valid approach to obtain prediction intervals for day-ahead forecasts of PV power. The applicability of the method is particularly useful for recently installed PV systems, when there is not extensive past information about its power generation.

An important limitation of the proposed method is regarding its application. With it, it is unlikely that the full forecast error distribution will be obtained without using more past data. To a complete description of the forecast error distribution in lower quantiles (confidence levels lower than 85%), more than 2 months of forecasts similar to the target one may be necessary. Thus, the proposed method application works well and it is recommended in cases when high confidence levels are required.

Finally, the difference noted between the confidence levels and the forecast error coverage achieved, particularly for a confidence level of 85%, show that there is still margin for improvements. Better selection of past data may yield better results. This item, and the applicability of the proposed method on regional scale, and with different time horizons will be addressed in further studies.

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