論文の内容の要旨

論文題目

Formulation of unified models for beam and shell based on meta-modeling theory
(メタモデリング理論に基づいた梁・シェルの統一モデル定式化)

氏名　ヘーワー　ボールッパゲ　イレシカ　カルナーラーシナ

This thesis presents formulation of unified models for beam and shell in curvilinear coordinate system based on meta-modeling theory. Main objectives of this work are to derive governing equations for beam and shell of arbitrary geometry, formulation of unified elements for FEM (Finite Element Method) implementation and extension from Lagrangian to Hamiltonian for algorithm specific formulation.

Modern civil engineering structures involve components of complex geometries to achieve competitive aesthetic demand and efficient load transferring. As a result, many structures incorporate beams and shells of complex geometries. Being slender in lateral dimensions, it is important to accurately model these structural components, taking exact geometry into account. Though beam and shell theories are centuries old, still there are difficulties in analyzing a structural member of complex shape both analytically and numerically.

Currently governing equations are available for limited simple geometries and restricted to simple coordinate systems, like Cartesian or Cylindrical, restricting the geometrically exact formulation of structural elements. As a result there are difficulties in analytically studying complex geometries. This limitation can be eliminated by formulating beam and shell theories using curvilinear coordinate system with tensor algebra. Governing equations for any specific geometry can be easily obtained simply by substituting the metric tensor of the coordinate system for the problem. This could lead to further advancements in structural mechanics.

However, the traditional formulations of governing equation of structural elements are mainly based on the force and moment equilibrium of free-body diagrams. It is too tedious and error-prone process. Continuum mechanics is not the starting point of deriving governing equations for structural elements, though any part of a structure is a subset of continuum mechanics. This obscures; the mathematical approximations involved, how to implement higher accurate models of structural elements, proper

treatment of non-linear behaviors, etc. The natural starting point for formulating beam and shell theories in curvilinear coordinate system is continuum mechanics. Structural elements can be considered as dimension reduced 3D continuum mechanics problems as being slender in one (i.e. shells) or two (i.e. beams) dimensions. Therefore, continuum mechanics is not only the natural starting point in formulating structural elements, but also less error-prone. Though some of research work uses continuum mechanics to formulate structural elements, those methods involve non-physical assumptions on Poisson's ratio. Meta-modeling proposed by Hori *et. al.* eliminates the need of any non-physical assumptions of Poisson's ratio and rigorously establishes the connection between continuum mechanics and structural mechanics by using Hellinger-Reissner functional and proper approximations for the field variables involved. As a demonstration, Hori *et. al.* have derived the existing beam and plate theories based on meta-modeling.

When it comes to numerical analysis, structural elements of complex geometries are modeled as a large collection of simple FEM elements. As an example, a simple problem like a twisted curved beam cannot be analytically studied using existing beam formulations, and a large number of tiny elements have to be used with FEM. The needs for higher accuracy, modeling exact geometry and reduction of computational cost have motivated development of various structural elements with curved geometries. Though there are number of FEM models, those are restricted to simple geometries easily representable in Cylindrical and Spherical coordinate systems, mainly.

In addition, current dynamic analysis is mainly based on numerical method such as Wilson- theta, Newmark beta etc., which do not respect the properties of the physical problem, like conservation of energy and momentum.

As is seen, there is a room to advance structural mechanics overcoming limitations by improvements as explained in above. Motivated by that, this study aims to formulate beam and shell theories in curvilinear coordinate system so as to achieve consistency with continuum mechanics and capturing geometry accurately. The main methodology used is meta-modeling theory proposed by Hori *et. al.* with tensor analysis in curvilinear coordinates. Meta-modeling theory rigorously establishes relation between continuum mechanics and structural mechanics. The use of tensor algebra in curvilinear coordinates allows formulating governing equations and FEM models for beams and shells of arbitrary geometry, taking exact geometry into account.

Beam and shell models are recognized as first order approximations to involved field variables, and approximations in curvilinear coordinates are presented. Proper coordinate systems are selected to define dimension reduced members by considering their kinematic and dynamic characteristics. Basically, this formulation addresses all the above mentioned limitations of current structural mechanics. Obtaining governing equations or FEM models for each specific geometry is straight forward; setup a suitable coordinate system to describe the geometry and substitute corresponding metric tensor to the equations given in this thesis.

The resulting models will have both analytical and numerical advantages. Some analytical advantages are: availability of governing equations for arbitrary geometries; possible rigorous treatment of material non-linearity in structural mechanics; stability analysis; etc.

Two models are derived for each of beam and shell; one is based on the traditional rotation vector interpretation, and the other based on purely first order approximation of field variables with Taylor expansion. A first model is considered because of classical beam and shell theories are involved with rotation based interpretation. However it provides complex governing equations and involves with mechanical interpretation. Second model provides simple equations and there are convenient to use in FEM formulation.

Consistent models for beam are derived by considering Euler- Bernoulli hypothesis and shear deformation effect (related to Timoshenko beam theory) separately. Effect of axial extension, bi axial bending and torsion are considered in beam formulation. We can clearly see that coupling of displacement and rotations components unlike straight beam. By using derived set of equations, we can easily obtain for relevant equations related to in-plane and out-of-plane deformation by appropriately simplifying based on specific deformation modes. Similarly consistent models for shell are derived. Consistent model for shell can be appropriately used to obtain membrane action of shell and obtain governing equations for plate.

Derived models are verified with literature and it can be clearly see that they are well matched with literature except some additional terms in derived equations. Additional terms in this formulation are due to considering effect of rotary inertia and change of base vectors. Since these formulations are based on continuum mechanics, the consistency of the derived beam and shell models with continuum mechanics is guaranteed.

Moreover, this research aims to formulate unified FEM elements. Unified FEM element for shell is derived in curvilinear coordinate system and similar to geometric approximation, field variables are approximated. This process is similar with initial steps in isogeometric analysis. This FEM formulation includes covariant derivative in well manner so as to improve accuracy of FEM formulation of shell. Some numerical advantages of this formulation are: reduction of per-node number of degrees of freedom (DOF) (e.g. a shell element only require 5 DOFs, while that of traditional element is 6); faster convergence of iterative solvers due to the elimination of one DOF; reduction of number of elements required due to the availability of accurate FEM elements for curved and twisted geometries; increase in accuracy; etc.

Further, by deriving Hamilton canonical forms for structural elements, it advances dynamic analysis. This algorithm specific formulation makes it possible to use advanced Symplectic time integrators used in physics, which preserves properties of the physical system like energy and momentum. Rigorous derivation of Lagrangian to Hamiltonian for structural mechanics is done based on curvilinear coordinate system. This work is extended work of proposed continuum Hamiltonian by Hori *et. al.* Hamilton canonical forms for structural mechanics are derived in detail by considering shell model.

This research provides consistent models for shell and beams by introducing straightway process to obtain governing equations for any geometries. Moreover it is entirely based on mathematical approximations and there is room to extend this work by considering stress distribution over cross section accurately, warping effect, and capturing material non-linearity etc. Further unified FEM elements are derived by identifying possible numerical advantages of this formulation. Finally Canonical forms for structural elements are derived to advance dynamic analysis in structural mechanics.