博士論文（要約）

# Essays in prices versus quantities in mixed oligopolies

# (混合寡占における価格数量の内生化に関する研究)

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# Essays in prices versus quantities in mixed oligopolies

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- Chapter 2 is based on the paper “Price versus Quantity in a Mixed Duopoly with Foreign Penetration” by Junichi Haraguchi and Toshihiro Matsumura.

-Chapter 3 is based on the paper “Cournot-Bertrand Comparison in a Mixed Oligopoly” by Junichi Haraguchi and Toshihiro Matsumura.

-Chapter 4 is based on the paper “Price versus Quantity in a Mixed Duopoly under Uncertainty” by Junichi Haraguchi.

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# Contents

# Chapter 1

# Introduction

## 1.1 The background of this dissertation

The Cournot and Bertrand models are the two classic and popular models that describe competition among firms in oligopolistic markets, and these models clearly depict firms' strategic interactions that are frequently used in oligopoly theory. In the Cournot model, firms compete on quantities. In the Bertrand model, prices are the strategic variable. The literature contains numerous comparative studies of the price and quantity models.[1] It is known that price competition is tougher than quantity competition in oligopolies with private firms. Thus, consumer surplus and social surplus under price competition are higher than under quantity competition. On the other hand, profit under price competition is lower than under quantity competition. Thus, firms prefer to strategically vary quantities rather than prices.

The endogenous competition structure (in price and quantity) has first been examined by Singh and Vives (1984). They consider a standard differentiated duopoly model formulated by Dixit (1979) and assume that each firm can choose one of two different types of strategy variables: prices and quantities. Firms can commit to one of these variables. They consider the following two-stage game. The firms simultaneously choose a strategy variable in the first stage. In the second stage, after observing their opponent's first-stage choice, the firms compete using their committed strategies. In other words, firms first choose which strategy variable (price and quantity) they adopt, and they set the magnitude of the strategy variable afterward. Singh and Vives (1984) suppose that firms can commit to a strategic variable and that they must compete in the committed strategy. They focus on the sub-game perfect equilibrium, and thus they solve this problem by backward induction. They show that committing to compete on quantity (res. price) is the dominant strategy for each firm if the goods are substitutes (complements) and thus that Cournot (Bertrand) competition is the endogenous competition structure. Since Singh and Vives (1984), the endogenous competition structure (in prices and quantities) has been extensively discussed.[2]

The supply function equilibrium(SFE) was introduced by Klemperer and Meyer (1989) and is an alternative to the Cournot and Bertrand models. The SFE is an equilibrium in a game where firms choose their own supply functions. Firms offer their own supply schedules simultaneously. Then, the market is cleared such that the total supply matches the demand at a certain price. In the model with demand uncertainty, the market is cleared after the realization of the uncertainty. The

[1] See Vives (1985), Okuguchi (1986), Dastidar (1996), and Häckner (2000).

[2] See, Cheng (1985), Tanaka (2001a,b), and Tasnádi (2006).

SFE is defined as the Nash equilibria in this game. The SFE model has potential for expansion, and it has theoretical applications.[3]

Oligopoly markets with public firms are called mixed oligopoly markets. Merrill and Schneider (1966) were the first to examine a mixed market. Since then mixed oligopoly has been extensively discussed in the literature.[4] Mixed oligopolies occur in various industries, such as the airline, steel, automobile, railway, natural gas, electricity, postal service, education, hospital, home loan, and banking industries. In addition, we have repeatedly observed the nationalization of private enterprises facing financial problems, such as General Motors, Japan Airlines, and Tokyo Electric Power Corporation. Studies on mixed oligopolies involving both state-owned public enterprises and private enterprises have recently attracted heightened attention and have become increasingly popular.
In mixed oligopoly markets, the effects of the public firms are not straightforward because of strategic interactions. For instance, De Fraja and Delbono (1989) show that welfare may be higher when a public firm is a profit-maximizer rather than a welfare-maximizer.

Even if firms are fully symmetric but are concerned about social welfare, the economic implications could be completely different from standard models with profit-maximizing firms.[5] For instance, Ghosh and Mitra (2014) examine an oligopoly market where every firm maximizes a weighted average of its own profit and social welfare. Competition among firms concerned about social welfare may represent transitioning and developing economies where the extent of private ownership is restricted, or competition among firms considering corporate social responsibility (CSR). Ghosh and Mitra (2014) compare Cournot and Bertrand competition in a symmetrically differentiated market and show that Bertrand competition yields higher profit and lower social welfare than Cournot competition when the weight on profit is sufficiently low. Therefore, we must be careful when we consider the existence of (partially) public firms.

The purpose of this dissertation is to extend the previous models of mixed oligopoly and to examine the properties of the existence of (partially) public firms. Since Ghosh and Mitra (2010), the comparison between price and quantity competition in a mixed market has been discussed in the literature. Ghosh and Mitra (2010) analyze these models in a mixed duopoly market. In their model, a welfare-maximizing public firm competes against a profit-maximizing private firm. They show that the social welfare and the private firm's profit are higher under price competition than under quantity competition. Matsumura and Ogawa (2012) endogenize the competition structure (in prices and quantities) in a mixed duopoly market. They shows that price competition is the endogenous competition structure. These results are sharp contrasts to the case of private duopoly. We extend these models and investigate the implications in chapters 2, 3, and 4.
In chapter 2, we consider a foreign private firm. In the mixed oligopoly literature, the public firm's objective is domestic welfare then the public firm cares about domestic firm's profit but not foreign firm's. The existence of a foreign investor plays a key role in mixed market. Chapter 3 discusses the comparison between price and quantity competition and the endogenous competition structure in a mixed oligopoly market. We consider one public firm competing against private firms. We analyze the effect of the number of private firms in a mixed oligopoly. We also discuss the case

[3]See Holmberg and Newbery (2010), Vives (2011), and Holmberg et al. (2013)

[4]See De Fraja and Delbono (1989), Fjell and Pal (1996), Matsumura and Kanda (2005), and Ghosh and Mitra (2010).

[5]See, Matsumura and Ogawa (2014).

of multiple public firms. In chapter 4, we consider the effect of demand shocks on the endogenous competition structure in a mixed duopoly market.
In chapter 5, we analyze the effect of the existence of a (partially) public firm in the SFE, and we introduce non-profit-maximizing firms into the SFE. Chapter 6 discusses government-leading collusion in price and quantity competition.

## 1.2 The outline of this dissertation

The remainder of this dissertation is organized as follows.

Chapter 2 discusses the endogenous competition structure (in prices or quantities) in a mixed duopoly market with foreign investors and foreign consumers. In this market, a welfare-maximizing public firm competes against a profit-maximizing private firm. As Matsumura and Ogawa (2012) discuss, price competition appears in the equilibrium in a mixed duopoly market. This chapter mainly focuses on the effect of the existence of foreign investors and foreign consumers. No studies that discuss the endogenous competition structure consider foreign penetration or integrated market. In other words, these studies assume a domestically-owned private firm and domestic consumers, and they ignore any aspect of international competition. In a mixed oligopoly market, however, the nationality of the private firm affects the result. Additionally, the integrated market is discussed in international trade. For these reasons, we considers the existence of foreign investors and foreign consumers explicitly.
In this chapter, we show that the existence of foreign investors do not change the competition structure and that the existence of foreign consumers can change the competition structure. In other words, price competition is the dominant strategy for both a private and a public firm whether a foreign investor exists or not, but quantity competition can be the equilibrium if the market is integrated. This implies that the results of Ghosh and Mitra (2010) and Matsumura and Ogawa (2012) is robust if there are foreign investors in a mixed market.

In Chapter 3, we compare price and quantity competition in a mixed oligopoly market. In this chapter, we assume that the market includes one public firm and multiple private firms. In a private oligopoly market, the number of private firms does not affect the ranking of social welfare and profit for the private firms.In other words, the private firms yield higher profit in quantity competition than in price competition, and price competition yields higher welfare than quantity competition. We discuss the impact of the number of private firms on the social welfare and the profit for private firms in a mixed market. We also endogenize the competition structure (in prices or quantities) in a mixed oligopoly market.
We show that regardless of the number of private firms, price competition yields higher welfare than quantity competition, but the profit ranking depends on the number of private firms. We find that quantity competition can yield higher profit for the private firms if the number of private firms is greater than or equal to five. Thus, the number of private firms is important in a mixed oligopoly market, in contrast to a private oligopoly market. We also show that Bertrand competition can fail to be an equilibrium if there exists only one private firm. We find that choosing to compete on prices in the first stage is not a dominant strategy for the private firms.

Chapter 4 characterizes the endogenous competition structure in a mixed duopoly market with demand shocks. Reisinger and Ressner (2009) introduce demand uncertainty into the Singh and Vives (1984) model. They show that price competition can be the equilibrium if there exists a

demand shock that affects the slope of the demand curve. This implies that the demand shock affects the firm's strategy choice. Thus, we consider the demand shock in a differentiated mixed duopoly market, and we discuss the effect of demand uncertainty.
We show that demand uncertainty which affects slope of demand curve does not affect the firm's first-stage choice. Thus, price competition appears in equilibrium if the slope of demand curve is uncertain. Then, a demand shock to the slope is unworkable in the mixed market. This is in sharp contrast to the Reisinger and Ressner (2009). Additionally, we try to introduce two-dimensional uncertainty, which affects the slope and the intercept of the demand curve, into the mixed market. We find that quantity competition can be the equilibrium if the covariance of the demand shocks is sufficiently negative. Thus, the firm's choice of strategy variable is affected by the demand shock in a mixed market as well.

Chapter 5 also considers demand shocks in the duopoly market, but in this chapter we study the supply function equilibrium (SFE). The SFE is an equilibrium in a game in which firms flexibly choose their own supply functions. Firms offer their own supply schedule simultaneously, and then, the market is cleared such that total supply matches the demand at a certain price. In the model with demand uncertainty, the market is cleared after the realization of uncertainty. The SFE is defined as the (pure strategy) Nash equilibria in this game. In this chapter, we assume that each firm considers not only its own profit but also the social surplus. The extreme case is a mixed duopoly market.
We generally characterize the supply function equilibria in partially privatized markets. We show the necessity of a positive slope for an SFE with symmetric objectives. We specify the demand and cost functions and show that not only partially public firms but also private firms offer more flat supply functions when the publicity of the public firm is enhanced. Thus, in supply function competitions, the existence of a public firm improves the social welfare. We also confirm that the supply function equilibrium converges to the (inverse) marginal cost function when the publicity of firms or the extent of social concern is increased symmetrically.

Chapter 6 discusses government-leading welfare-improving collusion in a mixed duopoly model. We formulate an infinitely repeated game in which a public firm and a private firm coexist. We suppose that the government proposes welfare-improving collusion and this is sustainable if both firms have incentives to collude. We compare the Cournot and Bertrand models in this long-run context.
We find that the deviation incentive is stronger under Cournot competition than under Bertrand competition. This leads the government to establish welfare-improving collusion more easily under Bertrand competition, and thus, Bertrand competition can yield greater welfare. On the other hand, in a mixed duopoly, competition is more severe, and thus, the punishment for deviation is stricter under Cournot competition. This leads the government to establish collusion more easily under Cournot competition, and thus, Cournot competition can yield greater welfare. Thus, Cournot competition is better for social welfare when firms are sufficiently patient.

# Chapter 2

# Price versus Quantity in a Mixed Duopoly with Foreign Penetration

**Abstract**

We characterize the endogenous competition structure (in prices or quantities) in a differentiated duopoly between a public firm that maximizes domestic welfare and a private firm that can be owned by domestic or foreign investors. The market for which they compete can be domestic or integrated: in the first case Bertrand competition emerges endogenously and in the second case Cournot competition can emerge if the fraction of domestic consumers in the integrated market is low enough. We also determine the optimal degree of foreign penetration showing the optimality of a partial foreign ownership. Finally, we extend the model to increasing marginal cost confirming the robustness of the results.

**JEL classification numbers:** H42, L13

**Keywords:** Cournot, Bertrand, Mixed Markets, International Competition, Trade

**Based article:** Haraguchi J, Matsumura T (2014) Price versus quantity in a mixed duopoly with foreign penetration. Research in Economics 68(4), 338–353.
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# Chapter 3

# Cournot-Bertrand Comparison in a Mixed Oligopoly

**Abstract**

We revisit the classic discussion comparing price and quantity competition, but in a mixed oligopoly in which one state-owned public firm competes against private firms. It has been shown that in a mixed duopoly, price competition yields a larger profit for the private firm. This implies that firms face weaker competition under price competition, which contrasts sharply with the case of a private oligopoly. Here, we adopt a standard differentiated oligopoly with a linear demand. We find that regardless of the number of firms, price competition yields higher welfare. However, the profit ranking depends on the number of private firms. We find that if the number of private firms is greater than or equal to five, it is possible that quantity competition yields a larger profit for each private firm. We also endogenize the price-quantity choice. Here, we find that Bertrand competition can fail to be an equilibrium, unless there is only one private firm.

**JEL classification numbers:** H42, L13

**Key words:** Cournot, Bertrand, Mixed Markets, Differentiated Products, Oligopoly

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# Chapter 4

# Price versus Quantity in a Mixed Duopoly under Uncertainty


**Abstract**

In this study, I characterize an endogenous competition structure (price and quantity) in a differentiated mixed duopoly under demand uncertainty. The results reveal that price competition yields higher welfare and private firm profit under one-dimensional uncertainty, which affects the slope of the demand curves. In addition, I endogenize the price-quantity choice and find that Bertrand competition appears in equilibrium under one-dimensional uncertainty. However, the ranking of welfare and profit for private firm can be reversed in the case of two-dimensional uncertainty, which affects the slope and intercept of the demand curves. I also show that Cournot competition can be an endogenous competition structure under two-dimensional uncertainty.



**JEL classification numbers:** H42, L13

**Key words:** Cournot, Bertrand, Mixed Markets, Differentiated Products, Demand Uncertainty

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## 4.1 Introduction

The literature contains numerous comparative studies of price and quantity competition. In oligopolies with private firms, it is well known that price competition is tougher, yielding lower profits than in the case of quantity competition.[1] Singh and Vives (1984) endogenized a competition structure (in terms of price and quantity) and concluded that firms often choose between adopting a price or quantity contract. Assuming a private duopoly, where both firms maximize profits, and linear demand and product differentiation, Singh and Vives (1984) showed that a quantity contract is the dominant strategy for each firm in the case of substitutable goods. However, when goods are complements, a price contract is the dominant strategy. Cheng (1985), Tanaka (2001a,b), and Tasnádi (2006) extended this analysis to asymmetric oligopolies, more generic demand and cost conditions, and vertical product differentiation, confirming the robustness of the results. However, these results depend on the assumption that all firms are private and profit-maximizers. Therefore, they may not apply to the increasingly important and popular mixed oligopolies, in which state-owned public firms compete against private ones.

Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly and showed that, contrary to the case of private duopoly, quantity competition is tougher than price competition, resulting in a smaller profit for the private firm.[2] Matsumura and Ogawa (2012) examined an endogenous competition structure in a mixed duopoly, where one of the two firms is public, and found that a price contract is the dominant strategy for both the private and public firm, regardless of whether goods are substitutes or complements.[3] However, these studies assume that demand is certain; in other words, they neglect the effect of a demand shock.

Weitzman (1974) analyze the choice between setting price and setting quantity in the market with uncertainty. Using expected social welfare as the objective function, he showed that the choice between two regulatory instruments depends on slopes of the marginal cost and demand functions. If the demand function is steep and marginal cost function is flat then quantity regulation is more desirable than price regulation, while in the reverse, price regulation is preferred over quantity regulation.

Reisinger and Ressner (2009), which is closely related to the present study, endogenized a competition structure in a private duopoly market under demand uncertainty. They showed that

[1] See Shubik and Levitan (1980) and Vives (1985).

[2] See also Nakamura (2013), Scrimitore (2014), and Haraguchi and Matsumura(2016)

[3] Haraguchi and Matsumura (2014) showed that this result holds, regardless of the private firm's nationality. Chirco *et al.* (2014) showed that both firms choose a price contract when the organizational structure is endogenized. However, Scrimitore (2013) showed that both firms can choose a quantity contract if a production subsidy is introduced.

one-dimensional uncertainty, which affects the demand function slope, can change the equilibrium competition structure. Their results imply that one-dimensional demand shock affects the firm's choice of strategy under a private duopoly. They also studied two-dimensional uncertainty, which affects the slope and intercept of the demand curve, and checked the robustness of their result. However, they did not consider the existence of the public firm. In this study, I investigate the effect of demand shock in a mixed duopoly market.

First, I endogenize the competition structure (i.e., price and quantity) using Singh and Vives' (1984) model. I show that Bertrand competition appears in equilibrium despite a slope-affecting demand shock.

Next, I revisit this price-quantity comparison in a mixed duopoly with an exogenous demand shock. I adopt a standard differentiated oligopoly with linear demand (Dixit, 1979) and show that, despite the existence of a demand shock, the Bertrand model yields higher welfare and private firm profit.

Finally, I consider two-dimensional demand shock, which affects the slope and intercept. I show that Cournot competition can be an endogenous competition structure and the Cournot model yields higher welfare and private firm profit.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 endogenizes the competition structure (i.e., price and quantity contract) in the case of a shock affecting the demand curve slope. Section 4 presents the main result. Section 5 considers two-dimensional uncertainty. Section 6 concludes.

## 4.2 Model

I adopt a standard differentiated oligopoly with linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is:

$$U(q_0, q_1) = \alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2\theta} + y,$$

where $q_i$ is the consumption of good $i$ produced by firm $i$ ($i = 0, 1$), and $y$ is the consumption of an outside good that is competitively provided (with a unit price). Parameters $\alpha$ and $\beta$ are positive constants, and $\delta \in (0, 1)$[4] represents the degree of product differentiation: a smaller $\delta$ indicates a larger degree of product differentiation. I assume that $\theta$ is a random variable with support $[\underline{\theta}, \overline{\theta}]$, where $\overline{\theta} > \underline{\theta} \geq 0$. The distribution of $\theta$ is characterized by the cumulative density function $F(\theta)$ and it has a mean of $E[\theta] = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta = 1$, where $f(\theta)$ denotes the density function of $\theta$. I denote

[4]If $\delta >$ ($<$) 0, the products are substitutes (complements). Although we restrict our attention to the case of substitute products only, we can show that our main propositions hold when $\delta \in (-1, 0)$ as well.

$Var(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta^2 f(\theta)d\theta - 1 = \sigma_\theta^2$ and $E[\frac{1}{\theta}] = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\theta} f(\theta)d\theta = z$. By Jensen's inequality, $z > 1$ and it increases in $\sigma_\theta^2$.

Firm 0 and firm 1 produce differentiated commodities, for which the inverse demand function is given by

$$p_i = \alpha - \frac{\beta}{\theta} q_i - \frac{\beta}{\theta} \delta q_j \ (i = 0, 1, i \neq j), \tag{4.1}$$

where $p_i$ and $q_i$ are firm $i$'s price and quantity. The marginal production costs are constant. Let $c_i$ denote firm $i$'s marginal cost. I assume $\alpha > c_i$.

Firm 0 is a state-owned public firm, and its payoff is the social surplus, given by

$$SW = (p_0 - c_0)q_0 + (p_1 - c_1)q_1 + \left[ \alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2\theta} - p_0 q_0 - p_1 q_1 \right].$$

Firm 1 is a private firm, and its payoff is its own profit: $\pi_1 = (p_1 - c_1)q_1$.
The game runs as follows. In the first stage, each firm chooses whether to adopt a price or quantity contract. In the second stage, after observing the rival's choice in the first stage, each firm simultaneously chooses its own strategy, on the basis of the decision taken in the first stage. Thereafter, the shock is realized, the market clears, and welfare and profit are accrued.

## 4.3 Second-stage games

First, I discuss four possible subgames: both firms choose a quantity contract (q-q game), both firms choose a price contract (p-p game), only firm 0 chooses the quantity contract (q-p game), or only firm 0 chooses the price contract (p-q game). I assume that the solutions in all the games are interior, that is, equilibrium prices and quantities for both firms are strictly positive. I define $a_i \equiv \alpha - c_i$. This assumption is hold if and only if $a_1 - \delta a_0 > 0$ and $a_0 - \delta a_1 > 0$. I adopt superscript $'ij'$ to denote the equilibrium outcome when firm 0 chooses $i \in \{p, q\}$ and firm 1 chooses $j \in \{p, q\}$.

### 4.3.1 Cournot model (q-q game)

First, I discuss the Cournot model (q-q game), in which both firms choose quantities. Substituting (4.1) into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned} SW &= (\alpha - c_0)q_0 + (\alpha - c_1)q_1 - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2\theta}, \\ \pi_1 &= (\alpha - \frac{\beta}{\theta} q_1 - \frac{\beta}{\theta} \delta q_0 - c_1)q_1. \end{aligned}$$

Thus, I have the following maximization problems for the public firm and private firm:

$$\begin{aligned}\max_{q_0}\int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \max_{q_0}\int_{\underline{\theta}}^{\overline{\theta}}\left[(\alpha-c_0)q_0+(\alpha-c_1)q_1-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)}{2\theta}\right]f(\theta)d\theta\\ &= \max_{q_0}\,(\alpha-c_0)q_0+(\alpha-c_1)q_1-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)}{2}\int_{\underline{\theta}}^{\overline{\theta}}\frac{1}{\theta}f(\theta)d\theta\\ &= \max_{q_0}\,(\alpha-c_0)q_0+(\alpha-c_1)q_1-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)z}{2},\end{aligned}$$

$$\begin{aligned}\max_{q_1}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= \max_{q_1}\int_{\underline{\theta}}^{\overline{\theta}}\left[(\alpha-\frac{\beta}{\theta}q_1-\frac{\beta}{\theta}\delta q_0-c_1)q_1\right]f(\theta)d\theta\\ &= \max_{q_1}\,(\alpha-\beta q_1\int_{\underline{\theta}}^{\overline{\theta}}\frac{1}{\theta}f(\theta)d\theta-\beta\delta q_0\int_{\underline{\theta}}^{\overline{\theta}}\frac{1}{\theta}f(\theta)d\theta-c_1)q_1\\ &= \max_{q_1}\,(\alpha-c_1-z\beta q_1-z\beta\delta q_0)q_1.\end{aligned}$$

The first-order conditions for the public firm and private firm are

$$\begin{aligned}\frac{\partial}{\partial q_0}\int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= a_0-\beta q_0 z-\beta\delta q_1 z=0,\\ \frac{\partial}{\partial q_1}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= a_1-2\beta q_1 z-\beta\delta q_0 z=0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for firm 0 and firm 1:

$$\begin{aligned}R_0^{qq}(q_1) &= \frac{a_0-\beta\delta q_1 z}{\beta z},\\ R_1^{qq}(q_0) &= \frac{a_1-\beta\delta q_0 z}{2\beta z}.\end{aligned}$$

These functions lead to the following expression for the equilibrium quantities:

$$\begin{aligned}q_0^{qq} &= \frac{2a_0-\delta a_1}{\beta(2-\delta^2)z},\\ q_1^{qq} &= \frac{a_1-\delta a_0}{\beta(2-\delta^2)z}.\end{aligned}$$

Substituting these equilibrium quantities into the demand and payoff functions, I have the following expected welfare and expected profit for firm 1:

$$E[SW^{qq}] = \frac{(4-\delta^2)a_0^2-2\delta(3-\delta^2)a_0a_1+(3-\delta^2)a_1^2}{2\beta(2-\delta^2)^2 z}, \tag{4.2}$$

$$E[\pi_1^{qq}] = \frac{(a_1-\delta a_0)^2}{\beta(2-\delta^2)^2 z}. \tag{4.3}$$

### 4.3.2 Bertrand model (p-p game)

I now characterize the Bertrand model (p-p game), in which both firms choose prices. Based on (4.1) the direct demand function is given by

$$q_i = \frac{\theta(\alpha - \alpha\delta - p_i + \delta p_j)}{\beta(1-\delta^2)}, \quad (i = 0, 1, i \neq j).$$

Substituting these direct demand functions into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned}
SW &= (\alpha - c_0)\Big\{\frac{\theta(\alpha - \alpha\delta - p_0 + \delta p_1)}{\beta(1-\delta^2)}\Big\} + (\alpha - c_1)\Big\{\frac{\theta(\alpha - \alpha\delta - p_1 + \delta p_0)}{\beta(1-\delta^2)}\Big\} \\
&- \frac{\beta\theta}{2}\Bigg[\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}^2 + 2\delta\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \\
&+ \Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\}^2\Bigg], \\
\pi_1 &= (p_1 - c_1)\Big\{\frac{\theta(\alpha - \alpha\delta - p_1 + \delta p_0)}{\beta(1-\delta^2)}\Big\}.
\end{aligned}$$

Thus, I have following maximization problems for the public firm and private firm:

$$\begin{aligned}
\max_{p_0} \int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \max_{p_0} \int_{\underline{\theta}}^{\overline{\theta}} \Bigg[(\alpha - c_0)\Big\{\frac{\theta(\alpha - \alpha\delta - p_0 + \delta p_1)}{\beta(1-\delta^2)}\Big\} + (\alpha - c_1)\Big\{\frac{\theta(\alpha - \alpha\delta - p_1 + \delta p_0)}{\beta(1-\delta^2)}\Big\} \\
&- \frac{\beta\theta}{2}\Bigg[\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}^2 + 2\delta\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \\
&+ \Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\}^2\Bigg]\Bigg] f(\theta)d\theta \\
&= \max_{p_0} (\alpha - c_0)\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta \\
&+ (\alpha - c_1)\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta \\
&- \frac{\beta}{2}\Bigg[\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}^2 + 2\delta\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \\
&+ \Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\}^2\Bigg] \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta \\
&= \max_{p_0} (\alpha - c_0)\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\} + (\alpha - c_1)\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \\
&- \frac{\beta}{2}\Bigg[\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}^2 + 2\delta\Big\{\frac{\alpha - \alpha\delta - p_0 + \delta p_1}{\beta(1-\delta^2)}\Big\}\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\} \\
&+ \Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\}^2\Bigg],
\end{aligned}$$

$$\begin{aligned}
\max_{p_1} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= \max_{q_1} \int_{\underline{\theta}}^{\overline{\theta}} \Bigg[(p_1 - c_1)\Big\{\frac{\theta(\alpha - \alpha\delta - p_1 + \delta p_0)}{\beta(1-\delta^2)}\Big\}\Bigg] f(\theta)d\theta \\
&= \max_{p_1} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta (p_1 - c_1)\Big\{\frac{\theta(\alpha - \alpha\delta - p_1 + \delta p_0)}{\beta(1-\delta^2)}\Big\} \\
&= \max_{p_1} (p_1 - c_1)\Big\{\frac{\alpha - \alpha\delta - p_1 + \delta p_0}{\beta(1-\delta^2)}\Big\}.
\end{aligned}$$

The first-order conditions for the public and private firms are

$$\begin{aligned}
\frac{\partial}{\partial p_0} \int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \frac{c_0 - p_0 - \delta c_1 + \delta p_1}{\beta(1-\delta^2)} = 0, \\
\frac{\partial}{\partial p_1} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= \frac{c_1 - 2p_1 + \alpha + \delta p_0 - \delta\alpha}{\beta(1-\delta^2)} = 0.
\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$\begin{aligned} R_0^{pp}(p_1) &= c_0 + \delta(p_1 - c_1), \\ R_1^{pp}(p_0) &= \frac{c_1 + \alpha + p_0\delta - \alpha\delta}{2}. \end{aligned}$$

These functions lead to the following expression for the equilibrium prices:

$$\begin{aligned} p_0^{pp} &= \frac{\alpha\delta - \alpha\delta^2 + 2c_0 - \delta c_1}{2 - \delta^2}, \\ p_1^{pp} &= \frac{\alpha - \alpha\delta + c_1 + \delta c_0 - \delta^2 c_1}{2 - \delta^2}. \end{aligned}$$

Substituting these equilibrium prices into the payoff functions, I have the following resulting expected welfare and expected profit for firm 1:

$$E[SW^{pp}] = \frac{(4 - 5\delta^2 + 2\delta^4)a_0^2 + (3 - 3\delta^2 + \delta^4)a_1^2 - 2\delta(3 - 3\delta^2 + \delta^4)a_0 a_1}{2\beta(1-\delta^2)(2-\delta^2)^2}, \tag{4.4}$$

$$E[\pi_1^{pp}] = \frac{(a_1 - \delta a_0)^2}{\beta(1-\delta^2)(2-\delta^2)^2}. \tag{4.5}$$

### 4.3.3 p-q game

I discuss the situation in which firm 0 chooses the price contract and firm 1 chooses the quantity contract. Based on (4.1), the demand functions in this sub-game are given by

$$\begin{aligned} q_0 &= \frac{\theta(\alpha - p_0)}{\beta} - \delta q_1, \\ p_1 &= \delta p_0 - \alpha\delta + \alpha - \frac{\beta(1-\delta^2)q_1}{\theta}. \end{aligned}$$

Substituting these demand systems into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned} SW &= (\alpha - c_0)\Big\{\frac{\theta(\alpha - p_0)}{\beta} - \delta q_1\Big\} + (\alpha - c_1)q_1 - \frac{\beta}{2}\Big\{\frac{\theta(\alpha - p_0)^2}{\beta^2} + \frac{(1-\delta^2)q_1^2}{\theta}\Big\}, \\ \pi_1 &= \Big\{\delta p_0 - \alpha\delta + \alpha - \frac{\beta(1-\delta^2)q_1}{\theta} - c_1\Big\}q_1. \end{aligned}$$

Thus, I have the following maximization problems for the public firm and private firm:

$$
\begin{aligned}
\max_{p_0} \int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \max_{p_0} \int_{\underline{\theta}}^{\overline{\theta}} \Big[(\alpha - c_0)\Big\{\frac{\theta(\alpha-p_0)}{\beta} - \delta q_1\Big\} + (\alpha - c_1)q_1 \\
&- \frac{\beta}{2}\Big\{\frac{\theta(\alpha-p_0)^2}{\beta^2} + \frac{(1-\delta^2)q_1^2}{\theta}\Big\}\Big] f(\theta)d\theta \\
&= \max_{p_0}\ (\alpha - c_0)\Big\{\frac{(\alpha-p_0)}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta - \delta q_1\Big\} + (\alpha - c_1)q_1 \\
&- \frac{\beta}{2}\Big\{\frac{(\alpha-p_0)^2}{\beta^2}\int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta + (1-\delta^2)q_1^2 \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\theta} f(\theta)d\theta\Big\} \\
&= \max_{p_0}\ (\alpha - c_0)(\frac{\alpha - p_0}{\beta} - \delta q_1) + (\alpha - c_1)q_1 - \frac{(\alpha-p_0)^2}{2\beta} - \frac{\beta(1-\delta^2)q_1^2 z}{2},
\end{aligned}
$$

$$
\begin{aligned}
\max_{q_1} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= \max_{q_1} \int_{\underline{\theta}}^{\overline{\theta}} \Big[\Big\{\delta p_0 - \alpha\delta + \alpha - \frac{\beta(1-\delta^2)q_1}{\theta} - c_1\Big\} q_1\Big] f(\theta)d\theta \\
&= \max_{q_1}\ \Big\{\delta p_0 - \alpha\delta + \alpha - \beta(1-\delta^2)q_1 \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\theta} f(\theta)d\theta - c_1\Big\} q_1 \\
&= \max_{q_1}\ (\delta p_0 - \alpha\delta + \alpha + (\beta\delta^2 - \beta)q_1 z - c_1)q_1.
\end{aligned}
$$

The first-order conditions for firms 0 and 1 are

$$
\begin{aligned}
\frac{\partial}{\partial p_0}\int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \frac{c_0 - p_0}{\beta} = 0, \\
\frac{\partial}{\partial q_1}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta &= \alpha - \delta\alpha - c_1 + \delta p_0 - 2\beta(1-\delta^2)q_1 z = 0.
\end{aligned}
$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$
\begin{aligned}
R_0^{pq}(q_1) &= c_0, \\
R_1^{pq}(p_0) &= \frac{\alpha - \delta\alpha - c_1 + \delta p_0}{2\beta(1-\delta^2)z}.
\end{aligned}
$$

These functions lead to the following expression for the equilibrium price and quantity:

$$
\begin{aligned}
p_0^{pq} &= c_0, \\
q_1^{pq} &= \frac{a_1 - \delta a_0}{2\beta(1-\delta^2)z}.
\end{aligned}
$$

Substituting these equilibrium price and quantity into the payoff functions, I have the following resulting expected welfare and expected profit for firm 1:

$$E[SW^{pq}] = \frac{4(1-\delta^2)a_0^2 z + 3(a_1 - \delta a_0)^2}{8\beta(1-\delta^2)z}, \tag{4.6}$$

$$E[\pi_1^{pq}] = \frac{(a_1 - \delta a_0)^2}{4\beta(1-\delta^2)z}. \tag{4.7}$$

### 4.3.4 q-p game

I now discuss the situation in which firm 0 chooses the quantity contract and firm 1 chooses the price contract.

$$\begin{aligned} p_0 &= \delta p_1 - \alpha\delta + \alpha - \frac{\beta(1-\delta^2)q_0}{\theta}, \\ q_1 &= \frac{\theta(\alpha - p_1)}{\beta} - \delta q_0. \end{aligned}$$

Substituting these demand systems into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned} SW &= (\alpha - c_0)q_0 + (\alpha - c_1)\Big\{\frac{\theta(\alpha - p_1)}{\beta} - \delta q_0\Big\} - \frac{\beta}{2}\Big\{\frac{\theta(\alpha - p_1)^2}{\beta^2} + \frac{(1-\delta^2)q_0^2}{\theta}\Big\}, \\ \pi_1 &= (p_1 - c_1)\Big\{\frac{\theta(\alpha - p_1)}{\beta} - \delta q_0\Big\}. \end{aligned}$$

Thus, I have the following maximization problems for the public firm and private firm:

$$\begin{aligned} \max_{q_0} \int_{\underline{\theta}}^{\overline{\theta}} SWf(\theta)d\theta &= \max_{q_0} \int_{\underline{\theta}}^{\overline{\theta}} \Big[(\alpha - c_0)q_0 + (\alpha - c_1)\Big\{\frac{\theta(\alpha - p_1)}{\beta} - \delta q_0\Big\} \\ &- \frac{\beta}{2}\Big\{\frac{\theta(\alpha - p_1)^2}{\beta^2} + \frac{(1-\delta^2)q_0^2}{\theta}\Big\}\Big] f(\theta)d\theta \\ &= \max_{q_0} (\alpha - c_0)q_0 + (\alpha - c_1)\Big\{\frac{(\alpha - p_1)}{\beta} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta - \delta q_0\Big\} \\ &- \frac{\beta}{2}\Big\{\frac{(\alpha - p_1)^2}{\beta^2} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta + (1-\delta^2)q_0^2 \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\theta}\theta f(\theta)d\theta\Big\} \\ &= \max_{q_0} (\alpha - c_0)q_0 + (\alpha - c_1)(\frac{\alpha - p_1}{\beta} - \delta q_0) - \frac{(\alpha - p_1)^2}{2\beta} - \frac{\beta(1-\delta^2)q_0^2 z}{2}, \end{aligned}$$

$$\max_{p_1} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta = \max_{p_1} \int_{\underline{\theta}}^{\overline{\theta}} \Big[(p_1 - c_1)\Big\{\frac{\theta(\alpha - p_1)}{\beta} - \delta q_0\Big\}\Big] f(\theta)d\theta$$
$$= \max_{p_1} (p_1 - c_1)\Big\{\frac{(\alpha - p_1)}{\beta} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)d\theta - \delta q_0\Big\}$$
$$= \max_{p_1} (p_1 - c_1)\Big\{\frac{(\alpha - p_1)}{\beta} - \delta q_0\Big\}.$$

The first-order conditions for firms 0 and 1 are

$$\frac{\partial}{\partial q_0} \int_{\underline{\theta}}^{\overline{\theta}} SW f(\theta)d\theta = a_0 - \delta a_1 - \beta(1 - \delta^2)q_0 z = 0,$$
$$\frac{\partial}{\partial p_1} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f(\theta)d\theta = \frac{c_1 - 2p_1 + \alpha - \beta\delta q_0}{\beta} = 0.$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$R_0^{qp}(p_1) = \frac{a_0 - \delta a_1}{\beta(1 - \delta^2)z},$$
$$R_1^{qp}(q_0) = \frac{\alpha + c_1 - \beta\delta q_0}{2}.$$

These functions lead to the following expression for the equilibrium quantity and price:

$$q_0^{qp} = \frac{a_0 - \delta a_1}{\beta(1 - \delta^2)z},$$
$$p_1^{qp} = \frac{\alpha + c_1}{2} - \frac{\delta(a_0 - \delta a_1)}{2(1 - \delta^2)z}.$$

Substituting these equilibrium quantity and price into the payoff functions, I have the following resulting expected welfare and expected profit for firm 1:

$$E[SW^{qp}] = \frac{3(1 - \delta^2)^2 a_1^2 z^2 + 2(1 - \delta^2)(a_0 - \delta a_1)(a_0 - 2\delta a_1)z - \delta^2(a_0 - \delta a_1)}{8\beta(1 - \delta^2)^2 z^2}, \quad (4.8)$$
$$E[\pi_1^{qp}] = \frac{((1 - \delta^2)a_1 z - \delta(a_0 - \delta a_1))^2}{4\beta(1 - \delta^2)^2 z^2}. \quad (4.9)$$

## 4.4 Result

I now discuss the choice in the first stage.

**Lemma 1** *(i)* $E[SW^{pq}] > E[SW^{qq}]$, *(ii)*$E[SW^{pp}] > E[SW^{qp}]$, *(iii)* $E[\pi_1^{pp}] > E[\pi_1^{pq}]$, *and (iv)*

$E[\pi_1^{qp}] > E[\pi_1^{qq}]$

**Proof** (i)From (4.6) and (4.2), I have

$$E[SW^{pq}] - E[SW^{qq}] = \frac{H_1}{8\beta(1-\delta^2)(2-\delta^2)^2 z},$$

where $H_1 \equiv 4(1-\delta^2)\Big\{2(z-1)(2-\delta^2)+\delta^2(2+z\delta^2)\Big\}a_0^2+\delta^2\Big\{3\delta^4 a_0^2+2\delta^3 a_0 a_1+(2-\delta^2)a_1^2+2(a_1-\delta a_0)a_1\Big\}$. This is positive for $z > 1$.

(ii) From (4.4) and (4.8), I have

$$E[SW^{pp}] - E[SW^{qp}] = \frac{H_2 z^2 + H_3 z + H_4}{8\beta(1-\delta^2)^2(2-\delta^2)^2 z^2},$$

where $H_2 \equiv (1-\delta^2)(3a_1^2\delta^6 - 8a_0a_1\delta^5 + (8a_0^2 - 11a_1^2)\delta^4 + 24a_0a_1\delta^3 - 4(5a_0^2 - 3a_1^2)\delta^2 - 24a_0a_1\delta + 16a_0^2)$, $H_3 \equiv 2(1-\delta^2)(2-\delta^2)^2(2a_0-\delta a_1)(a_0-\delta a_1)$, and $H_4 \equiv \delta^2(2-\delta^2)(a_0-\delta a_1)^2$.

Substituting z=1 into this equation, I have

$$E[SW^{pp}] - E[SW^{qp}]|_{z=1} = \frac{\delta^2(4-2\delta^3)(a_1-\delta a_0)^2}{8\beta(1-\delta^2)^2(2-\delta^2)^2 z} > 0.$$

Differentiating $E[SW^{qp}]$ with respect to z, I have

$$\frac{\partial E[SW^{qp}]}{\partial z} = \frac{(a_0-\delta a_1)\Big\{\delta^2(a_0-\delta a_1)-(1-\delta^2)(2a_0-\delta a_1)z\Big\}}{4\beta(1-\delta^2)^2 z^3}.$$

This is decreasing in z. Substituting $z = 1$ into this, I have

$$\frac{\partial E[SW^{qp}]}{\partial z}|_{z=1} = -\frac{(a_0-\delta a_1)\Big\{2(1-\delta^2)(a_0-\delta a_1)+\delta(a_1-\delta a_0)\Big\}}{4\beta(1-\delta^2)^2} < 0.$$

Thus, $E[SW^{qp}]$ is decreasing in $z$ for $z > 1$ and $E[SW^{pp}]$ is not affected by z. Therefore, $E[SW^{pp}] - E[SW^{qp}]$ is positive for $z > 1$.

(iii) From (4.5) and (4.7), I have

$$E[\pi_1^{pp}] - E[\pi_1^{pq}] = \frac{(a_1-\delta a_0)^2(4(z-1)+\delta^2(4-\delta^2))}{4\beta(1-\delta^2)(2-\delta^2)^2 z}.$$

This is positive for $z > 1$.

(iv) From (4.9) and (4.3),

$$E[\pi_1^{qp}] - E[\pi_1^{qq}] = \frac{H_5 z^2 + H_6 z + H_7}{4\beta(1-\delta^2)^2(2-\delta^2)^2 z},$$

where $H_5 \equiv a_1^2(1-\delta^2)^2(2-\delta^2)^2$, $H_6 \equiv 2(1-\delta^2)(a_1^2\delta^6 - a_0a_1\delta^5 - 2(2a_1^2 - a_0^2)\delta^4 + 2(3a_1^2 - a_0^2)\delta^2 - 2a_1^2$, and $H_7 \equiv \delta^2(2-\delta^2)^2(a_0 - \delta a_1)^2$.
Substituting $z = 1$ into this, I have

$$E[\pi_1^{qp}] - E[\pi_1^{qq}]|_{z=1} = \frac{\delta^2(4-3\delta^2)(a_1 - \delta a_0)^2}{4\beta(1-\delta^2)^2(2-\delta^2)^2} > 0.$$

Differentiating $E[\pi_1^{qp}]$ with respect to $z$, I have

$$\frac{\partial E[\pi_1^{qp}]}{\partial z} = \frac{\delta(a_0 - \delta a_1)\Big\{(1-\delta^2)(z-1)a_1 + (a_1 - \delta a_0)\Big\}}{2\beta(1-\delta^2)^2 z^3}.$$

This is positive for $z > 1$ and $E[\pi_1^{qp}]$ is increasing in $z$ for $z > 1$. On the other hand, $E[\pi_1^{qq}]$ is decreasing in $z$ for $z > 1$. Thus, $E[\pi_1^{qp}] - E[\pi_1^{qq}]$ is positive for $z > 1$. I now present the main result:

**Proposition 1** *Bertrand competition is the endogenous competition structure for any degree of demand shock.*

**Proof** Lemma 3(i) and Lemma 3(ii) imply that choosing p is the dominant strategy for firm 0. Lemma 3(iii) and Lemma 3(iv) imply that choosing p is the dominant strategy for firm 1. Q.E.D.

I explain why one-dimensional demand uncertainty dose not change the competition structure. I can straightforwardly apply the explanation of strategic advantage of price setting discussed in Matsumura and Ogawa (2012). First, I check the private firm's incentive. Suppose a public firm chooses the price contract. $R_0^{pq} = c_0$ indicates that firm 0 engages in marginal cost pricing, regardless of the private firm's output. Since a private firm's quantity is given, marginal cost pricing is the best for welfare. From $R_0^{pp}(p_1) = c_0 + \delta(p_1 - c_1)$, the public firm chooses a price higher than its marginal cost, responding to the private firm's pricing, when a private firm chooses a price contract. If the private firm chooses a price contract, its output depends on the public firm's price and a lower public firm pricing reduces the private firm's output and this reduces social welfare. Thus, the public firm chooses a price higher than its marginal cost to reduce welfare loss. This higher price is beneficial for the private firm. In addition to this strategic advantage of price setting, there exists a uncertainty based advantage of price setting. As discussed in Reisinger and Ressner (2009), shock to the slope does not affect the ex-post optimal price if private firm commit to the price contract. On the other hand ex-post optimal output is affected by the shock to the slope. Since $\frac{\partial R_1^{pq}(q_0)}{\partial z} = -\frac{\alpha - \delta\alpha - c_1 + \delta p_0}{2\beta(1-\delta^2)z^2} < 0$, an increase in $z$ decreases the private firm's output. This reduces the private firm's profit in the p-q game and the price contract is more attractive for the private firm.

Suppose that the public firm chooses the quantity contract and the private firm chooses the quantity contract. Substituting $R_0^{qq}(q_1) = \frac{a_0 - \beta\delta q_1}{\beta}$ into the expected inverse demand function of the public firm results in the public firm choosing price, such that $p_0(R_0^{qq}(q_1), q_1) = c_0$. Suppose the private firm chooses the price contract. Substituting $R_0^{qp}(p_1) = \frac{a_0 - \delta a_1}{\beta(1-\delta^2)z}$ into the expected demand function of the public firm causes the public firm to choose price, such that $p_0(R_0^{qp}(p_1), p_1) = c_0 + \delta(p_1 - c_1) > c_0$. If the private firm chooses the price contract, its output depends on the public firm's output. A lager public firm output reduces private firm quantity and as mentioned, a smaller private firm output reduces welfare. Therefore, the public firm chooses quantity less than that of the private firm when it chooses a quantity contract and this smaller quantity is beneficial for private firm. In addition, since $\frac{\partial R_1^{qq}(q_0)}{\partial z} = -\frac{a_1}{2\beta z^2} < 0$, an increasing $z$ decreases the private firm output in the q-q game and profit. Thus, the private firm prefers the price contract over the quantity contract.

Second, I examine the public firm's incentive. As Singh and Vives(1984) discussed, the demand elasticity of the private firm is higher when the public firm chooses a price contract rather than a quantity contract. Thus, the private firm becomes more aggressive when the public firm chooses a price contract rather than a quantity contract, thus improving welfare. In addition, as discussed above, the existence of uncertainty decreases the private firm's quantity in the q-q and p-q games. This reduction in the private firm's output in turn reduces social welfare. Thus, the public firm has a strict incentive to choose a price contract under uncertainty.

I show that Matsumura and Ogawa's (2012) result is robust under one-dimensional demand uncertainty. This result is in sharp contrast to Reisinger and Ressner's (2009). They show that a firm's choice of strategy depends on one-dimensional uncertainty in a private duopoly and thus, it is possible that a different competition structure can be the equilibrium. On the other hand, I demonstrate that one-dimensional uncertainty cannot change the competition structure in a mixed duopoly.

I discuss the Cournot–Bertrand comparison in a mixed duopoly.

**Proposition 2** *The Bertrand model yields higher welfare and private firm profit than the Cournot model, regardless of the degree of demand uncertainty.*

**Proof** From (4.4) and (4.2), I have

$$E[SW^{pp}] - E[SW^{qq}] = \frac{H_8}{2\beta(1-\delta^2)(2-\delta^2)^2 z}, \tag{4.10}$$

where $H_8 \equiv \Big\{ -2\delta^5 a_0 a_1 + \delta^4(2a_0^2 + a_1^2)\Big\}z + 2\delta^3 a_0 a_1(3z-1) + \delta^2\Big\{(1-5z)a_0^2 + (1-3z)a_1^2\Big\} + 6\delta a_0 a_1(1-z) + (4a_0^2 + 3a_1^2)(z-1)$. Substituting z=1 into (4.10), I have

$$E[SW^{pp}] - E[SW^{qq}]|_{z=1} = \frac{\delta^2(a_1 - \delta a_0)^2}{2\beta(1-\delta^2)(2-\delta^2)^2} > 0.$$

$E[SW^{pp}]$ is not affected by z and $E[SW^{qq}]$ is decreasing in $z$ for $z > 1$. Therefore, (4.10) is always positive for $z > 1$. From (4.5) and (4.3), I have

$$E[\pi_1^{pp}] - E[\pi_1^{qq}] = \frac{(a_1 - \delta a_0)^2(z - (1 - \delta^2))}{\beta(1 - \delta^2)(2 - \delta^2)^2 z}$$

This is positive for $z > 1$ Q.E.D.

As shown in Ghosh and Mitra (2010), in mixed duopolies, the Cournot model yields tougher competition among firms than the Bertrand model. Since $\frac{\partial q_0^{qq}}{\partial z} < 0$ and $\frac{\partial q_1^{qq}}{\partial z} < 0$, an increasing $z$ decreases the firm's output in Cournot competition. This decreases firm 1's profit and social welfare in Cournot competition. In addition, as discussed in Reisinger and Ressner (2009), a demand shock that affects the slope dose not change the equilibrium outcome in the Bertrand model. Therefore, an increasing $z$ makes the Bertrand model more attractive than the Cournot model for both public and private firms. I show that Ghosh and Mitra's (2010) result is robust under one-dimensional demand uncertainty.

## 4.5 Two dimensional demand uncertainty

This section extends the model. I consider two-dimensional demand uncertainty. Thus far, I have only assumed a demand shock that affects the slope. In this section, however, I also consider a shock to the intercept. I adopt a standard differentiated oligopoly with linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is

$$U(q_0, q_1) = (\alpha + \epsilon)(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2\theta} + y.$$

The parameter $\theta$ is a random variable with support $[\underline{\theta}, \overline{\theta}]$, where $\overline{\theta} > \underline{\theta} > 0$. The parameter $\epsilon$ is a random variable with support $[\underline{\epsilon}, \overline{\epsilon}]$, where $\overline{\epsilon} > \underline{\epsilon} > -\alpha$. The distribution of $\theta$ and $\epsilon$ is characterized by the joint cumulative density function $F_{\theta,\epsilon}(\theta, \epsilon)$. I assume the density function for $\theta$ and $\epsilon$ is $f_{\theta,\epsilon}(\theta, \epsilon)$. As discussed in section 2, parameter $\theta$ has mean $E[\theta] = \int_{\underline{\theta}}^{\overline{\theta}} \theta f_\theta(\theta) d\theta = 1$, where $f_\theta(\theta) = \int_{\underline{\epsilon}}^{\overline{\epsilon}} f_{\theta,\epsilon}(\theta, \epsilon) d\epsilon$ denotes the marginal density function of $\theta$. I denote $Var(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta^2 f(\theta) d\theta - 1 = \sigma_\theta^2$ and $E[\frac{1}{\theta}] = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{\theta} f_\theta(\theta) d\theta = z$. By Jensen's inequality, $z > 1$ and it increases in $\sigma_\theta^2$. Without loss of generality, I assume $E[\epsilon] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \epsilon f_\epsilon(\epsilon) d\epsilon = 0$, where $f_\epsilon(\epsilon) = \int_{\underline{\theta}}^{\overline{\theta}} f_{\theta,\epsilon}(\theta, \epsilon) d\theta$ is the marginal density function of $\epsilon$. I denote $Var(\epsilon) = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \epsilon^2 f_\epsilon(\epsilon) d\epsilon = \sigma_\epsilon^2 > 0$. Next, I denote $E[\theta, \epsilon] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\epsilon f_{\theta,\epsilon}(\theta, \epsilon) d\theta d\epsilon = \sigma_{\theta\epsilon}$ and $E[\theta\epsilon^2] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\epsilon^2 f_{\theta,\epsilon}(\theta, \epsilon) d\theta d\epsilon = \sigma_{\theta\epsilon^2}$.

The inverse demand is given by

$$p_i = \alpha + \epsilon - \frac{\beta}{\theta} q_i - \frac{\beta}{\theta} \delta q_j \ (i = 0, 1, i \neq j). \tag{4.11}$$

In this section the social surplus is denoted by

$$SW=(p_0-c_0)q_0+(p_1-c_1)q_1+\left[(\alpha+\epsilon)(q_0+q_1)-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)}{2\theta}-p_0q_0-p_1q_1\right].$$

Firm 1 is a private firm, and its payoff is its own profit: $\pi_1=(p_1-c_1)q_1$.

### 4.5.1 Cournot model (q-q game)

First, I discuss the Cournot model (q-q game), in which both firms choose quantities. Substituting (4.11) into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned} SW &= (\alpha+\epsilon-c_0)q_0+(\alpha+\epsilon-c_1)q_1-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)}{2\theta}, \\ \pi_1 &= (\alpha+\epsilon-\frac{\beta}{\theta}q_1-\frac{\beta}{\theta}\delta q_0-c_1)q_1. \end{aligned}$$

Thus, I have the following maximization problems for the public firm and private firm:

$$\begin{aligned} \max_{q_0}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \max_{q_0}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}}\Bigg[(\alpha+\epsilon-c_0)q_0+(\alpha+\epsilon-c_1)q_1 \\ &- \frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)}{2\theta}\Bigg]f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon \\ &= \max_{q_0}\ (\alpha-c_0)q_0+(\alpha-c_1)q_1-\frac{\beta(q_0^2+2\delta q_0q_1+q_1^2)z}{2}, \end{aligned}$$

$$\begin{aligned} \max_{q_1}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \max_{q_1}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}}\left[(\alpha+\epsilon-\frac{\beta}{\theta}q_1-\frac{\beta}{\theta}\delta q_0-c_1)q_1\right]f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon \\ &= \max_{q_1}\ (\alpha-c_1-z\beta q_1-z\beta\delta q_0)q_1. \end{aligned}$$

The first-order conditions for the public firm and private firm are

$$\begin{aligned} \frac{\partial}{\partial q_0}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= a_0-\beta q_0z-\beta\delta q_1z=0, \\ \frac{\partial}{\partial q_i}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= a_1-2\beta q_1z-\beta\delta q_0z=0. \end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for firm 0 and firm 1:

$$\begin{aligned} R_0^{qq}(q_1) &= \frac{a_0-\beta\delta q_1z}{\beta z}, \\ R_1^{qq}(q_0) &= \frac{a_1-\beta\delta q_0z}{2\beta z}. \end{aligned}$$

These functions lead to the following expression for the equilibrium quantities:

$$
\begin{aligned}
q_0^{qq} &= \frac{2a_0 - \delta a_1}{\beta(2-\delta^2)z}, \\
q_1^{qq} &= \frac{a_1 - \delta a_0}{\beta(2-\delta^2)z} \quad .
\end{aligned}
$$

Substituting these equilibrium quantities into the demand and payoff functions, I have the following expected welfare and expected profit for firm 1:

$$
E[SW^{qq}] = \frac{(4-\delta^2)a_0^2 - 2\delta(3-\delta^2)a_0a_1 + (3-\delta^2)a_1^2}{2\beta(2-\delta^2)^2 z}, \tag{4.12}
$$

$$
E[\pi_1^{qq}] = \frac{(a_1-\delta a_0)^2}{\beta(2-\delta^2)^2 z}. \tag{4.13}
$$

### 4.5.2 Bertrand model (p-p game)

I now characterize the Bertrand model (p-p game), in which both firms choose prices. Based on (4.11), the direct demand function is given by

$$
q_i = \frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_j - p_i)}{\beta(1-\delta^2)}, \quad (i = 0, 1, i \neq j).
$$

Substituting these direct demand functions into the payoff functions, I have the following welfare and profit for firm 1:

$$
\begin{aligned}
SW &= (\alpha+\epsilon-c_0)\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\right\} + (\alpha+\epsilon-c_1)\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\right\} \\
&- \frac{\beta}{2\theta}\left[\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\right\}^2 + 2\delta\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\right\}\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\right\}\right. \\
&+ \left.\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\right\}^2\right], \\
\pi_1 &= (p_1 - c_1)\left\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\right\}.
\end{aligned}
$$

Thus, I have the following maximization problems for the public firm and private firm:

$$\begin{aligned}
\max_{p_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \max_{p_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \Bigg[ (\alpha+\epsilon-c_0)\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\Big\} \\
&+ (\alpha+\epsilon-c_1)\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\Big\} - \frac{\beta}{2\theta}\Bigg[\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\Big\}^2 \\
&+ 2\delta\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_1 - p_0)}{\beta(1-\delta^2)}\Big\}\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\Big\} \\
&+ \Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\Big\}^2\Bigg]\Bigg] f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon \\
&= \max_{p_0} \frac{H_9}{2\beta(1-\delta^2)}
\end{aligned}$$

$$\begin{aligned}
\max_{p_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \max_{q_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \Bigg[(p_1 - c_1)\Big\{\frac{\theta(\alpha+\epsilon)}{\beta(1+\delta)} + \frac{\theta(\delta p_0 - p_1)}{\beta(1-\delta^2)}\Big\}\Bigg] f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon \\
&= \max_{q_1} (p_1 - c_1)\Big\{\frac{\alpha + \sigma_{\theta\epsilon}}{\beta(1+\delta)} + \frac{\delta p_0 - p_1}{\beta(1-\delta^2)}\Big\},
\end{aligned}$$

where $H_9 \equiv -p_1^2 + (2\delta p_0 - 2\delta c_0 + 2c_1)p_1 - p_0^2 - (2c_1\delta - 2c_0)p_0 + 2(1-\delta)\sigma_{\theta\epsilon^2} + 2(1-\delta)(a_0 + a_1)\sigma_{\theta\epsilon} + 2\alpha(1-\delta)(\alpha - c_0 - c_1)$.

The first-order conditions for the public and private firms are

$$\begin{aligned}
\frac{\partial}{\partial p_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \frac{c_0 - p_0 - \delta c_1 + \delta p_1}{\beta(1-\delta^2)} = 0, \\
\frac{\partial}{\partial p_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \frac{(\alpha + \sigma_{\theta\epsilon})(1-\delta) + c_1 - 2p_1 + \delta p_0}{\beta(1-\delta^2)} = 0.
\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$\begin{aligned}
R_0^{pp}(p_1) &= c_0 + \delta(p_1 - c_1), \\
R_1^{pp}(p_0) &= \frac{c_1 + \alpha + p_0\delta - \alpha\delta + \sigma_{\theta\epsilon}(1-\delta)}{2}.
\end{aligned}$$

The equilibrium price of the public firm can be derived as

$$p_0^{pp} = \frac{(\delta - \delta^2)\sigma_{\theta\epsilon} - \alpha\delta^2 + (\alpha - c_1)\delta + 2c_0}{2 - \delta^2}$$

and that of the private firm is

$$p_1^{pp} = \frac{(1-\delta)\sigma_{\theta\epsilon} - c_1\delta^2 - (\alpha - c_0)\delta + c_1 + \alpha}{2 - \delta^2}.$$

Substituting these equilibrium prices in to the demand and payoff functions I have the following expected welfare and firm 1's expected profit:

$$E[SW^{pp}] = \frac{H_{10}}{2\beta(1-\delta^2)(2-\delta^2)^2}, \tag{4.14}$$

$$E[\pi_1^{pp}] = \frac{(a_1 - \delta a_0 + \sigma_{\theta\epsilon}(1-\delta))^2}{\beta(1-\delta^2)(2-\delta^2)^2}, \tag{4.15}$$

where $H_{10} \equiv 2(1-\delta)(2-\delta^2)^2\sigma_{\theta\epsilon^2} - (1-\delta)^3(1+\delta)\sigma_{\theta\epsilon}^2 + 2(1-\delta)(\delta^4(a_0+a_1) + \delta(1-\delta^2)a_0 + (1-\delta^2)(4a_0+3a_1))\sigma_{\theta\epsilon} + (\delta^4 - 3\delta^2 + 3)a_1 + (2\delta^4 - 5\delta^2 + 4)a_0^2 - 2\delta(\delta^4 - 3\delta^2 + 3)$.

### 4.5.3 p-q game

I discuss the case in which firm 0 chooses a price contract and firm 1 chooses a quantity contract. Based on (4.11), the demand functions in this sub-game are given by

$$q_0 = \frac{\theta(\alpha+\epsilon-p_0)}{\beta} - \delta q_1,$$

$$p_1 = \delta p_0 + (1-\delta)(\alpha+\epsilon) - \frac{\beta(1-\delta^2)q_1}{\theta}.$$

Substituting these demand systems into the payoff functions, I have the following welfare and profit for firm 1:

$$SW = (\alpha+\epsilon-c_0)\Big\{\frac{\theta(\alpha+\epsilon-p_0)}{\beta} - \delta q_1\Big\} + (\alpha+\epsilon-c_1)q_1 - \frac{\beta}{2}\Big\{\frac{\theta(\alpha+\epsilon-p_0)^2}{\beta^2} + \frac{(1-\delta^2)q_1^2}{\theta}\Big\},$$

$$\pi_1 = \Big\{\delta p_0 + (1-\delta)(\alpha+\epsilon) - \frac{\beta(1-\delta^2)q_1}{\theta} - c_1\Big\}q_1.$$

Thus, I have following maximization problems for the public firm and private firm:

$$\begin{aligned}\max_{p_0}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\theta}}^{\bar{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \max_{p_0}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\theta}}^{\bar{\theta}}\Big[(\alpha+\epsilon-c_0)\Big\{\frac{\theta(\alpha+\epsilon-p_0)}{\beta} - \delta q_1\Big\} + (\alpha+\epsilon-c_1)q_1 \\ &- \frac{\beta}{2}\Big\{\frac{\theta(\alpha+\epsilon-p_0)^2}{\beta^2} + \frac{(1-\delta^2)q_1^2}{\theta}\Big\}\Big] f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon \\ &= \max_{p_0} \frac{-p_0^2 + 2c_0p_0 + \sigma_{\theta\epsilon^2} + 2\sigma_{\theta\epsilon}(\alpha-c_0) - 2\alpha c_0 + \alpha^2}{2\beta} - \frac{(1-\delta^2)z\beta q_1^2}{2} \\ &+ \big\{(\alpha-c_1) - \delta(\alpha-c_0)\big\}q_1\end{aligned}$$

$$\begin{aligned}\max_{q_1}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\theta}}^{\bar{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \max_{q_1}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\theta}}^{\bar{\theta}}\Big[\Big\{\delta p_0 + (1-\delta)(\alpha+\epsilon) - \frac{\beta(1-\delta^2)q_1}{\theta} - c_1\Big\}q_1\Big] f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon \\ &= \max_{q_1} \big\{\delta p_0 - \alpha\delta + \alpha + (\beta\delta^2 - \beta)q_1 z - c_1\big\}q_1.\end{aligned}$$

The first-order conditions for firms 0 and 1 are

$$\begin{aligned}
\frac{\partial}{\partial p_0}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \frac{c_0-p_0}{\beta}=0,\\
\frac{\partial}{\partial q_1}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon)d\theta d\epsilon &= \alpha-\delta\alpha-c_1+\delta p_0-2\beta(1-\delta^2)q_1 z=0.
\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$\begin{aligned}
R_0^{pq}(q_1) &= c_0,\\
R_1^{pq}(p_0) &= \frac{\alpha-\delta\alpha-c_1+\delta p_0}{2\beta(1-\delta^2)z}.
\end{aligned}$$

These functions lead to the following expression for the equilibrium price and quantity:

$$\begin{aligned}
p_0^{pq} &= c_0,\\
q_1^{pq} &= \frac{a_1-\delta a_0}{2\beta(1-\delta^2)z}.
\end{aligned}$$

Substituting the equilibrium price and quantity into the payoff functions, I have the following resulting expected welfare and expected profit for firm 1:

$$E[SW^{pq}] = \frac{8(1-\delta^2)a_0\sigma_{\theta\epsilon}z+4(1-\delta^2)(a_0^2+\sigma_{\theta\epsilon^2})z+3(a_1-\delta a_0)^2}{8\beta(1-\delta^2)z}, \tag{4.16}$$

$$E[\pi_1^{pq}] = \frac{(a_1-\delta a_0)^2}{4\beta(1-\delta^2)z}. \tag{4.17}$$

### 4.5.4 q-p game

Here, I discuss the case in which firm 0 chooses a quantity contract and firm 1 chooses a price contract. Based on (4.11), the demand functions in this sub-game are given by

$$\begin{aligned}
p_0 &= \delta p_1+(1-\delta)(\alpha+\epsilon)-\frac{\beta(1-\delta^2)q_0}{\theta},\\
q_1 &= \frac{\theta(\alpha+\epsilon-p_1)}{\beta}-\delta q_0.
\end{aligned}$$

Substituting these demand systems into the payoff functions, I have the following welfare and profit for firm 1:

$$\begin{aligned}
SW &= (\alpha+\epsilon-c_0)q_0+(\alpha+\epsilon-c_1)\Big\{\frac{\theta(\alpha+\epsilon-p_1)}{\beta}-\delta q_0\Big\}-\frac{\beta}{2}\Big\{\frac{\theta(\alpha+\epsilon-p_1)^2}{\beta^2}+\frac{(1-\delta^2)q_0^2}{\theta}\Big\},\\
\pi_1 &= (p_1-c_1)\Big\{\frac{\theta(\alpha+\epsilon-p_1)}{\beta}-\delta q_0\Big\}.
\end{aligned}$$

Thus, I have the following maximization problems for the public firm and private firm:

$$
\begin{aligned}
\max_{q_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \max_{q_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[ (\alpha+\epsilon-c_0)q_0 + (\alpha+\epsilon-c_1)\Big\{ \frac{\theta(\alpha+\epsilon-p_1)}{\beta} - \delta q_0 \Big\} \\
&- \frac{\beta}{2}\Big\{ \frac{\theta(\alpha+\epsilon-p_1)^2}{\beta^2} + \frac{(1-\delta^2)q_0^2}{\theta} \Big\} \Big] f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon \\
&= \max_{q_0} \frac{-p_1^2 + 2c_1 p_1 + \sigma_{\theta\epsilon^2} + 2\sigma_{\theta\epsilon}(\alpha - c_1) - 2\alpha c_1 + \alpha^2}{2\beta} - \frac{(1-\delta^2) z\beta q_0^2}{2} \\
&+ \big\{ (\alpha - c_0) - \delta(\alpha - c_1) \big\} q_0
\end{aligned}
$$

$$
\begin{aligned}
\max_{p_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \max_{p_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[ (p_1 - c_1)\Big\{ \frac{\theta(\alpha+\epsilon-p_1)}{\beta} - \delta q_0 \Big\} \Big] f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon \\
&= \max_{p_1} (p_1 - c_1)\Big\{ \frac{\alpha + \sigma_{\theta\epsilon} - p_1}{\beta} - \delta q_0 \Big\}.
\end{aligned}
$$

The first-order conditions for firms 0 and 1 are

$$
\begin{aligned}
\frac{\partial}{\partial q_0} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} SW f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= -(1-\delta^2) z\beta q_0 + a_0 - \delta a_1 = 0, \\
\frac{\partial}{\partial p_1} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \int_{\underline{\theta}}^{\overline{\theta}} \pi_1 f_{\theta,\epsilon}(\theta,\epsilon) d\theta d\epsilon &= \frac{\alpha + \sigma_{\theta\epsilon} - 2p_1 + c_1}{\beta} - \delta q_0 = 0.
\end{aligned}
$$

The second-order conditions are satisfied. From the first-order conditions, I obtain the following reaction functions for the public and private firms:

$$
\begin{aligned}
R_0^{qp}(p_1) &= \frac{a_0 - \delta a_1}{\beta(1-\delta^2)z}, \\
R_1^{qp}(q_0) &= \frac{\alpha + c_1 - \beta\delta q_0 + \sigma_{\theta\epsilon}}{2}.
\end{aligned}
$$

The equilibrium quantity of the public firm can be derived as

$$
q_0^{qp} = \frac{a_0 - \delta a_1}{\beta(1-\delta^2)z}
$$

and the equilibrium price of the private firm can be derived as

$$
p_1^{qp} = \frac{\alpha + \sigma_{\theta\epsilon} + c_1}{2} - \frac{\delta(a_0 - \delta a_1)}{2(1-\delta^2)z}.
$$

Substituting the equilibrium quantity and price in to the demand and payoff functions, I have the following expected welfare and firm 1's expected profit:

$$
E[SW^{qp}] = \frac{H_{11}}{8\beta(1-\delta^2)^2}, \tag{4.18}
$$

$$
E[\pi_1^{qp}] = \frac{(a_1 + \sigma_{\theta\epsilon})^2 z}{4\beta} - \frac{\delta(a_0 - \delta a_1)}{\beta(1-\delta^2)} - \frac{\delta^2(a_0 - \delta a_1)^2}{4\beta(1-\delta^2)^2 z}, \tag{4.19}
$$

where $H_{11} \equiv (1-\delta^2)^2 z^2(-\sigma_{\theta\epsilon}^2 + 6a_1\sigma_{\theta\epsilon} + 3a_1^2 + 4\sigma_{\theta\epsilon^2}) + 2(1-\delta^2)(a_0 - \delta a_1)(\delta\sigma_{\theta\epsilon} + 2a_0 - \delta a_1)z - \delta^2(a_0 - \delta a_1)^2$.

I now present my main result:

**Proposition 3** *(i) Cournot competition can be an endogenous competition structure if there exists a two-dimensional demand shock.*
*(ii) Cournot competition can be an endogenous competition structure only if the covariance between the shock is negative.*
*(iii) Bertrand competition fail to be an equilibrium if*

$$\sigma_{\theta,\epsilon} \in \Big( -\frac{\sqrt{z}\{2(\sqrt{z}+1)-\delta^2\}(a_1-\delta a_0)}{2(1-\delta)z}, -\frac{\sqrt{z}\{2(\sqrt{z}-1)+\delta^2\}(a_1-\delta a_0)}{2(1-\delta)z} \Big).$$

**Proof** (i) From (4.16) and (4.12), I have

$$E[SW^{pq}] - E[SW^{qq}] = \frac{H_{12}}{8\beta(1-\delta^2)(2-\delta^2)^2 z}, \tag{4.20}$$

where $H_{12} \equiv 4(1-\delta^2)(2-\delta^2)^2(2a_0\sigma_{\theta\epsilon}a_0^2 + \sigma_{\theta\epsilon^2})z + (3\delta^6 - 16(1-\delta^2)^2)a_0^2 + \delta^2(4-\delta^2)(a_1 - 2\delta a_0)a_1$.
From (4.14) and (4.18), I have

$$E[SW^{pp}] - E[SW^{qp}] = \frac{H_{13}}{8\beta(1-\delta^2)^2(2-\delta^2)^2 z^2}, \tag{4.21}$$

where $H_{13} \equiv \delta(2-\delta)(1-\delta^2)^2(4-2\delta-\delta^2)\sigma_{\theta\epsilon}^2 z^2 + 2(1-\delta)^2(1+\delta)\Big\{-3\delta^5 a_1 + \delta^4(4a_0+a_1) - 4\delta^3(a_0 - 3a_1) - 16\delta^2 a_0 + 4\delta(a_0 - 3a_1) + 16a_0\Big\}\sigma_{\theta\epsilon}z^2 + (1-\delta^2)\Big\{\delta^6(3a_1^2 + 4\sigma_{\theta\epsilon^2}) - 8\delta^5(a_0a_1 + \sigma_{\theta\epsilon^2}) - \delta^4(8a_0^2 + 11a_1^2) + 8\delta^3(3a_0a_1 + \sigma_{\theta\epsilon^2}) - 4\delta^2(5a_0^2 - 3a_1^2) - 8\delta(3a_0a_1 + 4\sigma_{\theta\epsilon^2}) + 16(a_0^2 + \sigma_{\theta\epsilon^2})\Big\}z + \delta^2(2-\delta^2)^2(a_0 - \delta a_1)^2$.
From (4.15) and (4.17), I have

$$E[\pi_1^{pp}] - E[\pi_1^{pq}] = \frac{4\Big\{(1-\delta)\sigma_{\theta\epsilon} + (a_1 - \delta a_0)\Big\}^2 z - (2-\delta^2)^2(a_1-\delta a_0)^2}{4\beta(1-\delta^2)(2-\delta^2)^2 z}. \tag{4.22}$$

From (4.19) and (4.13), I have

$$E[\pi_1^{qp}] - E[\pi_1^{qq}] = \frac{H_{14}}{4\beta(1-\delta^2)^2(2-\delta^2)^2 z}, \tag{4.23}$$

where $H_{14} \equiv (1-\delta^2)^2(a_1 - \sigma_{\theta\epsilon})^2 - 4(1-\delta^2)\Big\{(1-\delta^2)(a_1-\delta a_0)^2 - \delta^3(4-\delta^2)a_0 - \delta^2(2-\delta^2)^2 a_1\Big\}z + \delta^2(2-\delta^2)^2(a_0-\delta a_1)^2$.
Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.1, z = 2.3, \sigma_{\theta\epsilon} = 0.375$, and $\sigma_{\theta\epsilon^2} = 0.25$

into (4.20), (4.21), (4.22), and (4.23), I have $E[SW^{pq}] - E[SW^{qq}] \approx 0.78, E[SW^{pp}] - E[SW^{qp}] \approx 0.61, E[\pi_1^{pp}] - E[\pi_1^{pq}] \approx 0.88$, and $E[\pi_1^{qp}] - E[\pi_1^{qq}] \approx 0.97$. Then, Bertrand competition can be the endogenous competition structure. Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.2, z = 2.3, \sigma_{\theta\epsilon} = -0.75$, and $\sigma_{\theta\epsilon^2} = 0.25$ into (4.20), (4.21), (4.22), and (4.23), I have $E[SW^{pq}] - E[SW^{qq}] \approx -0.34, E[SW^{pp}] - E[SW^{qp}] \approx -0.26, E[\pi_1^{pp}] - E[\pi_1^{pq}] = -0.01$, and $E[\pi_1^{qp}] - E[\pi_1^{qq}] \approx -0.04$. In this case, a quantity contract is the dominant strategy for firms and thus, the Cournot model can appear in equilibrium.
(ii) Since $E[\pi_1^{pp}]$ is increasing in $\sigma_{\theta\epsilon}$ and $E[\pi_1^{pp}]|_{\sigma_{\theta\epsilon}=0} > 0$ and $E[\pi_1^{pq}]$ is not affected by $\sigma_{\theta\epsilon}$, the private firm deviates from the p-p game only if $\sigma_{\theta\epsilon}$ is negative. Since $E[\pi_1^{qq}]$ is not affected by $\sigma_{\theta\epsilon}$ and $E[\pi_1^{qp}]$ is increasing in $\sigma_{\theta\epsilon}$ and $E[\pi_1^{qp}]|_{\sigma_{\theta\epsilon}=0} > 0$, the quantity contract is the dominant strategy for the private firm only if $\sigma_{\theta\epsilon}$ is negative. In addition, $E[SW^{pq}]$ is increasing in $\sigma_{\theta\epsilon}$ and $E[SW^{qp}]|_{\sigma_{\theta\epsilon}=0} > 0$ and $SW^{qq}$ is not affected by $\sigma_{\theta\epsilon}$. Thus, given the private firm's price contract, the public firm has an incentive to commit to the price contract. For these reasons, Cournot competition can be an endogenous competition structure only if $\sigma_{\theta\epsilon}$ is negative.
(iii) From (4.22) private firm has an incentive to deviate from Bertrand competition if

$$\frac{4\Big\{(1-\delta)\sigma_{\theta\epsilon} + (a_1 - \delta a_0)\Big\}^2 z - (2-\delta^2)^2(a_1 - \delta a_0)^2}{4\beta(1-\delta^2)(2-\delta^2)^2 z} < 0.$$

Solving this inequality with respect to $\sigma_{\theta,\epsilon}$ and I have

$$-\frac{\sqrt{z}\{2(\sqrt{z}+1) - \delta^2\}(a_1 - \delta a_0)}{2(1-\delta)z} < \sigma_{\theta,\epsilon} < -\frac{\sqrt{z}\{2(\sqrt{z}-1) + \delta^2\}(a_1 - \delta a_0)}{2(1-\delta)z}.$$

Q.E.D.

Next, I explain how two-dimensional demand uncertainty can change the competition structure. First, I explain why the private firm has an incentive to deviate from the p-p game to p-q game. In p-p game, decreasing $\sigma_{\theta\epsilon}$ induces both the public and private firm's lower pricing. In other words, a lower $\sigma_{\theta\epsilon}$ makes competition severe in the p-p game. This reduces private firm profit. On the other hand, $\sigma_{\theta\epsilon}$ dose not affect the public firm's pricing in the p-q game. Then, the private firm can earn larger profits in the p-q game than in the p-p game for some $\sigma_{\theta\epsilon}$. Suppose the public firm chooses the quantity contract and the private firm chooses the quantity contract. Then, in the q-q game, $\sigma_{\theta\epsilon}$ does not affect the equilibrium outcomes. Suppose the private firm chooses a price contract. Then, in the q-p game, a decreasing $\sigma_{\theta\epsilon}$ induces lower pricing by the private firm and this is harmful for the private firm's profit. Therefore, $\sigma_{\theta\epsilon}$ is negative and the private firm prefers the quantity contract to the price contract.

Second, I explain the public firm's incentive to deviate from the p-p game to q-p game. A lower $\sigma_{\theta\epsilon}$ induces aggressive pricing in the p-p game for both firms. This improves welfare in the p-p game. However, the impact of decreasing $\sigma_{\theta\epsilon}$ is different and the private firm chooses more aggressive pricing than the public firm. This increases the difference in output between the public and private firm, leading to welfare loss. The former's welfare-improving effect is dominated by the latter's and a lower $\sigma_{\theta\epsilon}$ can decrease welfare in the p-p game. On the other hand, public firm output is free from $\sigma_{\theta\epsilon}$ and a lower $\sigma_{\theta\epsilon}$ induces aggressive pricing by the private firm. This aggressive pricing increases the private firm's output and this improves welfare. Therefore, the public firm has the incentive to deviate from the p-p game to the q-p game if $\sigma_{\theta\epsilon}$ is sufficiently negative. I examine whether the public firm has an incentive to deviate from the p-q game if $\sigma_{\theta\epsilon}$ is negative. As argued above, $\sigma_{\theta\epsilon}$ dose not affect the equilibrium outcomes in the q-q game. Substituting $R_0^{pq}(q_1) = c_0$ into the public firm's demand function in the p-q game, I have $q_0^{pq}(R_0^{pq}(q_1), q_1) = \frac{a_1+\sigma_{\theta\epsilon}}{\beta} - \delta q_1$. Thus, a lower $\sigma_{\theta\epsilon}$ decreases quantity for the public firm, causing a welfare loss. Thus, decreasing $\sigma_{\theta\epsilon}$ can achieve larger welfare in the q-q game than in the p-q game.

Finally, I explain why each firm has a incentive to deviate from the price contract when the covariance between shock is negative. As discussed in Reisinger and Ressner (2009), the negative correlation means that a positive shock on the intercept goes along with a steeper expected slope of the inverse residual demand, and this implies that the variation of ex-post optimal prices relative to the variation of ex-post optimal quantities increases in the covariance. Thus, there exists uncertainty based quantity setting if $\sigma_{\theta,\epsilon}$ is negative.

**Proposition 4** *If there is two-dimensional uncertainty, Cournot competition can yield higher welfare than Bertrand competition*
**Proof** From (4.14) and (4.12), I have

$$E[SW^{pp}] - E[SW^{qq}] = \frac{H_{15}}{2\beta(1-\delta^2)(2-\delta^2)^2 z}, \tag{4.24}$$

where $H_{15} \equiv -(1-\delta)^3(1+\delta)\sigma_{\theta\epsilon}^2 z + 2(1-\delta)\Big\{(\delta^4 - \delta^3 - 4\delta^2 + \delta + 4)a_0 + (\delta^4 - 3\delta^2 + 3)a_1^2 + 2(\delta^5 - \delta^4 - 4\delta^3 + 4\delta^2 + 4\delta - 4)\sigma_{\theta\epsilon^2}\Big\}z - (1-\delta^2)\Big\{2\delta(3-\delta)a_0a_1 + (4-\delta^2)a_0^2 + (3-\delta^2)a_1^2\Big\}$.
Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.2, z = 2.3, \sigma_{\theta\epsilon} = 0.375, and \sigma_{\theta\epsilon^2} = 0.25$ into (4.24), I have $E[SW^{pp}] - E[SW^{qq}] \approx 2.13$. Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.1, z = 2.3, \sigma_{\theta\epsilon} = -0.75$, and $\sigma_{\theta\epsilon^2} = 0.25$ into (4.24), I have $E[SW^{pp}] - E[SW^{qq}] \approx -0.49$. Q.E.D.

Under two-dimensional uncertainty, as discussed above, a lower $\sigma_{\theta\epsilon}$ leads to welfare loss in the Bertrand model and $\sigma_{\theta\epsilon}$ dose not affect the equilibrium outcome in the Cournot model. The lower pricing improves social welfare, however, the private firm tends to have a more aggressive pricing than the public firm and this causes an output gap between the public and private firm, which is harmful for welfare. Then, the Cournot model can yield welfare higher than the Bertrand model for some $\sigma_{\theta\epsilon}$ .

**Proposition 5** *Cournot competition yields higher profit for private firm if*

$$\sigma_{\theta,\epsilon} \in \Big( -\frac{\sqrt{z}(\sqrt{z}+\sqrt{1-\delta^2})(a_1-\delta a_0)}{(1-\delta)z}, -\frac{\sqrt{z}(\sqrt{z}-\sqrt{1-\delta^2})(a_1-\delta a_0)}{(1-\delta)z} \Big)$$

**Proof** From (4.15) and (4.13), I have

$$E[\pi_1^{pp}] - E[\pi_1^{qq}] = \frac{\Big\{(1-\delta)\sigma_{\theta\epsilon} + (a_1-\delta a_0)\Big\}^2 z - (1-\delta^2)(a_1-\delta a_0)^2}{\beta(1-\delta^2)(2-\delta^2)^2 z}. \tag{4.25}$$

This is negative if

$$\frac{\Big\{(1-\delta)\sigma_{\theta\epsilon} + (a_1-\delta a_0)\Big\}^2 z - (1-\delta^2)(a_1-\delta a_0)^2}{\beta(1-\delta^2)(2-\delta^2)^2 z} < 0. \tag{4.26}$$

Solving this inequality with respect to $\sigma_{\theta,\epsilon}$, I have

$$-\frac{\sqrt{z}(\sqrt{z}+\sqrt{1-\delta^2})(a_1-\delta a_0)}{(1-\delta)z} < \sigma_{\theta,\epsilon} < -\frac{\sqrt{z}(\sqrt{z}-\sqrt{1-\delta^2})(a_1-\delta a_0)}{(1-\delta)z}.$$

Q.E.D.

Under two-dimensional uncertainty, a negative $\sigma_{\theta\epsilon}$ induces aggressive pricing by both the public and private firm in the Bertrand model. This aggressive pricing decreases the private firm's profit in the Bertrand model. The existence of $\sigma_{\theta\epsilon}$ is not important in the Cournot model. Therefore, the Cournot model can yield higher profits for the private firm if $\sigma_{\theta\epsilon}$ takes a negative value.

## 4.6 Conclusion

In this study, I revisit the classic discussion of the comparison between price and quantity competition, but in a mixed duopoly under demand uncertainty. Ghosh and Mitra (2010) considered certain demand and showed that in a mixed duopoly, price competition yields higher welfare and a larger profit for private firm. I show that regardless of the degree of demand uncertainty, price competition yields higher welfare and a larger profit for the private firm under one-dimensional demand shock. I also endogenize the choice between a price and quantity contract. Matsumura and Ogawa (2012) considered certain demand and showed that choosing a price contract is the dominant strategy for both firms. I find that both firms choose price contracts in unique equilibrium, regardless of the degree of shock to the slope. This suggests that one-dimensional uncertainty has no effect in a mixed duopoly market. As Reisinger and Ressner (2009) discussed, a firm's choice of strategy is affected by one-dimensional uncertainty in the private duopoly market, which is in sharp contrast to the result of the present study. I also show that the Cournot model can yield higher welfare and profit for the private firm and quantity competition appears in equilibrium if there is two-dimensional demand uncertainty. This indicates that Ghosh and Mitra(2010) and Matsumura and Ogawa's (2012) result is not robust under demand uncertainty.

The present study is subject to the following limitations. First, it assumes that a linear demand system. As discussed in Weitzman (1974), my result can be applied to the general demand system and it may be possible to check the robustness of my results. However, I was unable to extend this previous work given the strategic interaction in my study. Thus, these generalizations remains an area for future research.

Second, this study assumes a constant marginal cost for simplicity. However, as pointed by Weitzman (1974) the slope of marginal cost function affects a player's price-quantity choice in a market with uncertainty. Furthermore, the shape of the cost function plays an important role in a mixed market. As Matsumura and Okamura (2015) showed, the results of a model with constant marginal costs may be contrasting to those of a model with increasing marginal costs. An extension of the model with more general costs remains for future research.

In reality, many mixed markets are oligopoly markets and the number of firms is important in mixed markets; moreover, there is more than one public firm in certain mixed oligopoly markets. For instance, the banking sector in Japan, Germany, and India; the energy market in the European Union; and many sectors in China and Russia suggest the existence of mixed oligopoly markets with multiple public firms. In the literature, Matsumura and Shimizu (2010), Matsumura and Okumura (2013), and Haraguchi and Matsumura (2016) consider mixed oligopoly markets with more than one public firm. In the mixed oligopoly literatures, endogenizing the number of firms by considering free-entry market is also important and popular, because free-entry markets often yield different implications. [5] This paper can be extend in these direction in future research.

[5] For discussion on free-entry markets in the mixed oligopolies, see Matsumura and Kanda(2005), Fujiwara(2007), and Ino and Matsumura(2010). For recent developments in this field, see Cato and Matsumura(2012,2013) and Ghosh et al. (2015).

# Chapter 5

# Supply Function Equilibria and Nonprofit-Maximizing Objectives

**Abstract**

We examine the *supply function equilibrium* (SFE), which is often used in the analysis of multi-unit auctions such as wholesale electricity markets, among (partially) public firms. In a general model, we characterize the SFE by such firms and examine the properties of symmetric SFE. In a duopoly model with linear demand and quadratic cost functions, we analyze asymmetric SFE and show that not only a partially public firm but also a profit-maximizing firm offers flatter supply functions as equilibrium strategies when the publicity of the public firm is enhanced. We also confirm that in the linear-quadratic model, the SFE converges to the (inverse) marginal cost function when the firms' social concern is improved symmetrically in the industry.

**Key words:** supply function equilibrium, electricity markets, partial privatization, corporate social responsibility, mixed oligopoly

**JEL code:** H42, L13, L33

**Based article:** Haraguchi J, Yasui Y (2015) Supply Function Equilibria and Nonprofit-Maximizing Objectives. Mimeo.

## 5.1 Introduction

Since Green and Newbery (1992) have applied the *supply function equilibrium* (henceforth, SFE) to the analysis of wholesale electricity markets, a bunch of applications to electricity or treasury bills market have appeared in the literature.[1] In this paper, we assume that firms are concerned with social welfare, which can be interpreted as (partially) public firms or as firms concerned with cooperate social responsibility (CSR). At an equilibrium in the model with linear demand and quadratic cost functions, not only a partially public firm but also a private firm offers supply functions closer to their marginal cost functions when the publicity of the public firm is enhanced. Thus, in contrast to Matsumura (1998), the society benefits from the existence of perfectly public firm. We also confirmed that in the linear-quadratic model, the supply function equilibrium converges to the (inverse) marginal cost function when the publicity of firms or the extent of social concern is improved symmetrically in the industry, while it is not guaranteed in the general model.

### 5.1.1 SFE and its applications

SFE, introduced by Klemperer and Meyer (1989), is an equilibrium in a game where firms choose their own supply function flexibly. Firms offer their own supply schedule simultaneously, and then, the market is cleared such that the total supply matches the demand at a certain price. In the model with demand uncertainty, the market is cleared after the realization of uncertainty. The SFE is defined as the (pure strategy) Nash equilibria in this game. A remarkable feature of the SFE is that it is characterized as a locus of *ex post* optimal price–quantity pairs given the others' supply functions. The logic is as follows. Each firm guesses the others' (fixed) supply schedules. After the realization of demand uncertainty, combined with the others' supply functions, a residual demand function is determined. Suppose that the firm chooses its ex post optimal price–quantity pair along with the residual demand. Since the ex post optimal points vary according to realizations of uncertainty even though she assumes the others' supply functions fixed, the locus of ex post optimal points becomes a function from price to quantity. Since she can obtain ex post optimized profit through this supply function, she has no incentive to take other supply functions in the first stage, given others ' supply functions. Thus, the locus is a best response to the others ' supply functions. By considering such best responses for each firm, we obtain the Nash equilibrium (NE)—or in other words, the SFE—in this game.

The most famous application of the SFE would be wholesale electricity markets. In deregulated wholesale electricity markets, generating companies offer their own supply schedules and retailers bid to supply their own customers such as consumers or other companies. If there are many retailers, the setting of supply function competition fits well with the structure in those markets

[1] See Holmberg and Newbery (2010) for recent applications of SFE on electricity markets.

because the retail demand is subject to exogenous shocks such as whether, holidays, and major sporting events. Since Green and Newbery (1992) have applied the SFE to the analysis of wholesale electricity markets, it is extensively examined theoretically and empirically as a tool to analyze the electricity markets.[2]

Another important, but less focused, interpretation would be the equilibria in the *conjectural variation* model with appropriately specified strategies. In general, the conjectural variation first introduced by Bowley (1924) implies that each firm chooses its quantity while believing that other firms change their quantities by $r_j dq_i$ in response to change in firm $i$'s quantity $dq_i$. It has been used in empirical literature to capture various market structures in a reduced form.[3] A major critique on this model is that it contradicts to the equilibrium concept in the field of economics, that is, firms are *not* assumed to choose quantity *given others' quantity*.[4] However, in supply function competitions, strategies are (usually positive sloped) supply functions. Therefore, other firms actually change their quantity in response to change in a firm ' s quantity. In other words, at the SFE, the conjectural parameter $r_j$ is determined by the equilibrium strategy of firm $j$. Of course, there might be countless equilibria if there is no uncertainty in the market, because the shape of each supply function only affects the off-equilibrium outcomes and works as an empty threat. However, in the market with demand uncertainty, the SFE restricts the set of possible outcomes much more narrowly and helps us to understand the market.

### 5.1.2 Mixed Oligopoly, Partial Privatization, and CSR

In this paper, we introduce (partially) public firms into the SFE.[5] Oligopoly markets with public firms, called mixed oligopoly, is first examined by Merrill and Schneider (1966) and extensively discussed in the literature.[6] Matsumura(1998) generalizes it to the model of partial privatization by introducing a partially public firm that maximizes a weighted average of its own profit and the social welfare. In mixed oligopoly markets, effects of the public firms are not straightforward because of

[2] Vives (2011) introduces uncertainty in cost functions rather than demand function and treats it as asymmetric information. Holmberg et al. (2013) shows that the SFE in step functions like actual offers in the electricity markets converge to continuous SFE as steps becomes finer. Since it is difficult to obtain the analytical solution of the SFE, computation methods to calculate it are also developed in the literature of operations research (see Holmberg (2009).

[3] Iwata (1974) and Gollop and Roberts (1979) propose empirical methods to estimate or test the conjectural variation. They also apply it to Japanese plate glass industry and US coffee industry. Brander and Zhang (1993) estimate the conjectual variation in the US airline industry with dynamic settings. See also Bresnahan (1989) for review and discussion on the conjectual variation.

[4] Farrell and Shapiro (1990) explain that we can consider it as "the equilibrium of an (unmodeled) dynamic oligopolistic game", and Cabral (1995) provides an explicit example of the repeated game of which the CV solution is an exact reduced form of the equilibrium.

[5] Mixed oligopolies occur in various industries, such as the airline, steel, automobile, railway, natural gas, electricity, postal service, education, hospital, home loan, and banking industries.

[6] See, De Fraja and Delbono (1989), Fjell and Pal (1996), Matsumura and Kanda (2005), and Ghosh and Mitra (2010).

strategic interactions.[7] For instance, De Fraja and Delbono (1989) show that welfare may be higher when a public firm is a profit-maximizer rather than a welfare-maximizer, and Matsumura (1998) finds the optimal level of partial privatization is neither full privatization nor full nationalization. Matsumura and Sunada (2013) examine a mixed oligopoly with misleading advertising competition and find that a public firm engage in rather than canceling out the misleading advertisement. Even if firms are fully symmetric but concerning about social welfare, economic implication could be completely different from standard models with profit-maximizing firms. For instance, Ghosh and Mitra (2014) examine an oligopoly market where every firm maximizes a weighted average of its own profit and social welfare. The competition among firms concerned with social welfare can be interpreted as that in transition and developing economies where the extent of private ownership is restricted, or competition among firms that consider CSR. They compare Cournot and Bertrand competition in a symmetrically differentiated market and show that Bertrand competition yields higher profit and lower social welfare than Cournot competition when the weight on profit is sufficiently low. Therefore, we must be careful when we consider the existence of (partially) public firms. Fortunately, in supply function competitions, it turns out that a public firm encourages private firms to take socially better action through strategic complementarities in terms of the slopes of each supply function.

The remainder of the paper is organized as follows. Section 2 presents the general model. Section 3 analyzes the linear-quadratic model. Section 4 concludes the paper. The proofs of propositions can be found in Appendix.

## 5.2 The General Model

With symmetric and private firms, Klemperer and Meyer (1989) characterize SFE under demand uncertainty as solutions of a differential equation and examined some general properties of that. In this subsection, we characterize SFE with partially public firms analogously and examine the general properties. In the next subsection, we specify a model with a linear demand and quadratic cost functions in order to derive more clear implication on the effects of the public firm.

Let denote demand function as $Q = D(p) + \epsilon$, where $p$ is a price of the product and $\epsilon$ is a scalar random variable with strictly positive density everywhere on the support $[\underline{\epsilon}, \bar{\epsilon}]$. We assume $-\infty < D'(p) < 0$, and $D''(p) \leq 0$. The firms have identical cost functions $C$ s.t. $C'(q) \geq 0$ and $0 < C''(q) < \infty$ $\forall q \in [0, \infty)$. Without loss of generality, let $C'(0) = 0$.[8] A strategy for firm $i\,(i = 1, 2)$ is defined as a function mapping from price into quantity: $S_i : [0, p) \to (-\infty, \infty)$.

[7] If we can assume that public firms can commit to a certain strategy, a general analysis on effects by "exogenous competition" by Weyl and Fabinger (2013) can be applied. However, if we cannot assume that the public firm cannot make a commitment and respond to others' strategies, an analysis of mixed oligopoly is required.

[8] If $C'(0) \neq 0$, we can normalize it by considering $\tilde{p} = p - C'(0)$ as in Klemperer and Meyer (1989).

Here, we focus on pure strategy Nash equilibria, in which $S_i$ maximizes $i$'s payoff given that $j$ chooses $S_j$ $(i,\, j = 1,\, 2,\, j \neq i)$.

Given $S_j$, firm $i$'s ex post objective function is as follows:

$$\begin{aligned} v_i(p) &= (1-\theta)\left[p\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)-C\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)\right] \\ &+ \theta\left[\int_p^{\hat{p}} D\left(\dot{p},\,\epsilon\right)d\dot{p}+pD\left(p,\,\epsilon\right)-C\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)-C\left(S_j\left(p\right)\right)\right]. \end{aligned} \tag{5.1}$$

Therefore, the first-order condition is derived as follows:

$$\begin{aligned} \frac{\partial v_i}{\partial p} &= (1-\theta)\left[\left(D\left(p\right)+\epsilon-S_j\left(p\right)\right)+\left(p-C'\left(D\left(p\right)+\epsilon-S_j\left(p\right)\right)\right)\left(D'\left(p\right)-S_j'\left(p\right)\right)\right] \\ &+ \theta\left[\left(p-C'\left(D\left(p\right)+\epsilon-S_j\left(p\right)\right)\right)D'\left(p\right)+\left(C'\left(D\left(p\right)+\epsilon-S_j\left(p\right)\right)-C'\left(S_j\left(p\right)\right)\right)S_j'\left(p\right)\right]=0. \end{aligned} \tag{5.2}$$

Since $S_i$ is determined by the locus of the optimal price $p^*\left(\epsilon\right)$ and the corresponding quantity $D\left(p^*\left(\epsilon\right)\right)+\epsilon-S_j\left(p^*\left(\epsilon\right)\right)$, we can replace $D\left(p\right)+\epsilon-S_j\left(p\right)$ by $S_i$. Then, the above equation becomes a differential equation:

$$S_j'\left(p\right)=\frac{(1-\theta_i)\left[S_i+\left(p-C'\left(S_i\right)\right)D'\left(p\right)\right]+\theta_i\left[\left(p-C'\left(S_i\right)\right)D'\left(p\right)\right]}{(1-\theta_i)\left(p-C'\left(S_i\right)\right)+\theta_i\left[\left(p-C'\left(S_i\right)\right)-\left(p-C'\left(S_j\right)\right)\right]}\equiv f_i\left(p,\,S_i,S_j\right). \tag{5.3}$$

Since a pair of functions $(S_1,\, S_2)$ that solves a system of differential equations $S_j'\left(p\right)=f_i\left(p,\,S_i,S_j\right)$ for $i=1,2,\ j\neq i$ satisfy the first-order condition given the other strategy, it is SFE if the payoff function given others ' strategies satisfy the second-order conditions. In a model with only private firms, it is known that the symmetric SFE strategy $S=S_i$ $(i=1,\,2)$ must satisfy $0<S'\left(p\right)<\infty$ for all $p$ if $\epsilon$ has a full support $(\underline{\epsilon}=-D(0),\,\bar{\epsilon}=\infty)$. In the competition among partially public firms, this property still holds, and the set of SFE is bounded by a function that shifts upwards (or becomes flatter if we take quantity in the horizontal line and price in the vertical line) as $\theta$ increases.

**Proposition 1 (Necessity of positive slope)** *If $\epsilon$ has full support $(\underline{\epsilon}=-D(0),\,\bar{\epsilon}=\infty)$ and $S$ is symmetric SFE tracing through ex post optimal points, then $\forall p\geq 0$, $S$ satisfies (5.3) and $0<S'\left(p\right)<\infty$. Furthermore, $S$ is bounded from below by a function $S^0\left(p,\,\theta\right)$ with its derivative*

$$S^{0\prime}\left(p\right)=-\frac{D_p\left(p\right)+D_{pp}\left(p\right)\left(p-C'\left(S^0\left(p\right)\right)\right)}{(1-\theta)-D_p\left(p\right)C''\left(S^0\left(p\right)\right)}. \tag{5.4}$$

**Proof.** See the Appendix D.

As can be seen from eq. (5.4), $S^{0\prime}(p,\,\theta)$ is increasing in $\theta$ for all $p$, and thus, the lower bound $S^0(p,\,\theta)$ gets closer to the (inverse) marginal cost function when $\theta$ increases. It is also worth noting that even if $\theta$ goes to 1, $S^0(p,\,\theta)$ does not converge to the (inverse) marginal cost function, $C'^{-1}(p)$, in general. Thus, we cannot guarantee that firms take (almost) socially optimal behavior even if they are concerned with social welfare and hardly care about their profits.

For analysing the asymmetric SFE and the role of a public firm, we specify the demand and the cost functions in the following section.

## 5.3 Linear Demand and Quadratic Cost Function

We specify the cost functions and a demand function to have an analytical solution. The identical cost functions are defined as $C(S) = \frac{c}{2}S^2$ and the total demand function is defined as $Q = D(p,\epsilon) = \alpha + \epsilon - mp$. That is, $U(Q) = \left(\frac{\alpha+\epsilon}{m}\right)Q - \frac{1}{m}Q^2/2$, where $U$ is a surplus function. Suppose player $j$'s strategy to be $q_j = S_j(p) = a + bp$. Then, the residual demand is written as

$$q_i(p,\,\epsilon) = D(p,\epsilon) - s_j(p) \quad = \quad \alpha + \epsilon - mp - a - bp. \tag{5.5}$$

Therefore, the *ex post* profit maximization problem for firm i given others ' strategies is written as follows:

$$\begin{aligned} \max_p \quad & (1-\theta_i)\left[pq_i(p,\,\epsilon) - C(q_i(p,\,\epsilon))\right] \\ + \quad & \theta_i\left[U(q_i(p,\,\epsilon) + S_j(p)) - C(q_i(p,\,\epsilon)) - C(S_j(p))\right]. \end{aligned} \tag{5.6}$$

We can consider the ex post optimal price for each $\epsilon$ by taking the first-order condition with respect to $p$. Since the corresponding $q_i$ for each $\epsilon$ is determined along the residual demand function $q_i(p,\,\epsilon)$, we can obtain the following locus of optimal points by canceling out $\epsilon$ in the FOC and (5.5):

$$q_i = \frac{(m+b) + \theta_i(bc-1)\,b}{(1-\theta_i) + c(m+b)}p + \frac{\theta_i abc}{(1-\theta_i) + c(m+b)}.$$

Then, the following supply functions construct the SFE:

$$S_i(p) \quad = \quad a_i^* + b_i^* p \ \text{ for all } i = 1,\,2,$$

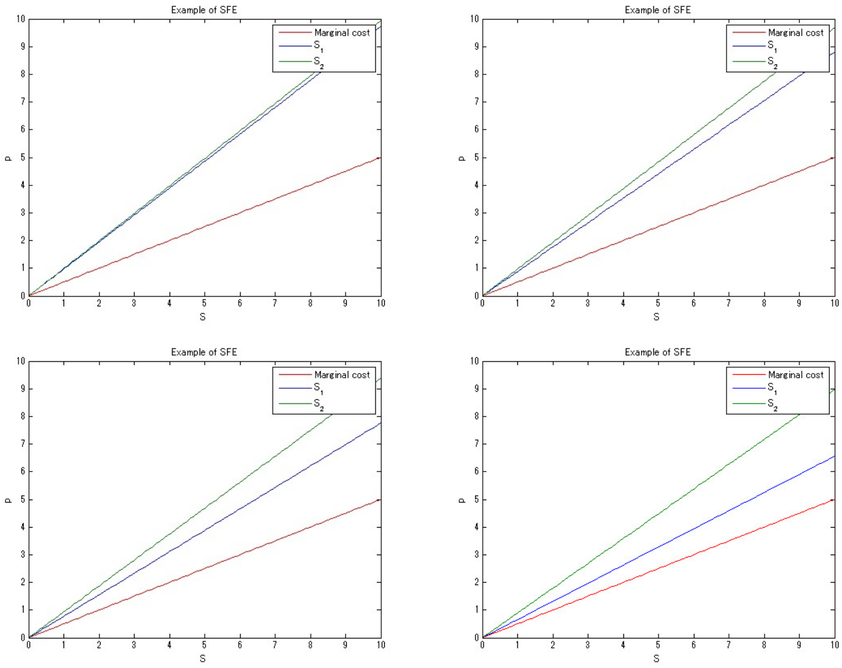


Figure 5.1: Numerical examples of SFE where $m = 1$, $c = 0.5$ and $\theta_2 = 0$. $\theta_1 = 0.1$: upper left, $\theta_1 = 0.4$: upper right, $\theta_1 = 0.7$: lower left, and $\theta_1 = 0.999$: lower right

where

$$a_i^* = \frac{\theta_i a_j^* b_j^* c}{(1-\theta_i) + c\left(m + b_j^*\right)}, \text{ and} \tag{5.7}$$

$$b_i^* = \frac{\left(m + b_j^*\right) + \theta_i \left(b_j^* c - 1\right) b_j^*}{(1-\theta_i) + c\left(m + b_j^*\right)} \; (i = 1,\, 2,\, j \neq i). \tag{5.8}$$

Suppose $b_j^* \neq 0$. Then, since $a_1^* = a_2^* = 0$ must hold to satisfy eq. (5.7), we can obtain the SFE by solving eq. (5.8).

Even though it is difficult to solve eq. (5.8) analytically, we can obtain some properties of the equilibrium without solving it explicitly. If we take $b_i^*$ as a function of $b_j^*$, $b_i^*$ shifts upward when $\theta_i$ increases. On the other hand, $b_i^*$ is increasing in $b_j^*$ if firm $i$ is a private firm ($\theta_i = 0$). Here, suppose that firm 1 is partially privatized and firm 2 is completely private. If we increase $\theta_1$, $b_1^*$ is shifted upward. Therefore, not only $b_1^*$ but also $b_2^*$ would increase since $b_2^*$ is increasing in $b_1^*$. (Numerical examples are illustrated in Fig. 5.1)

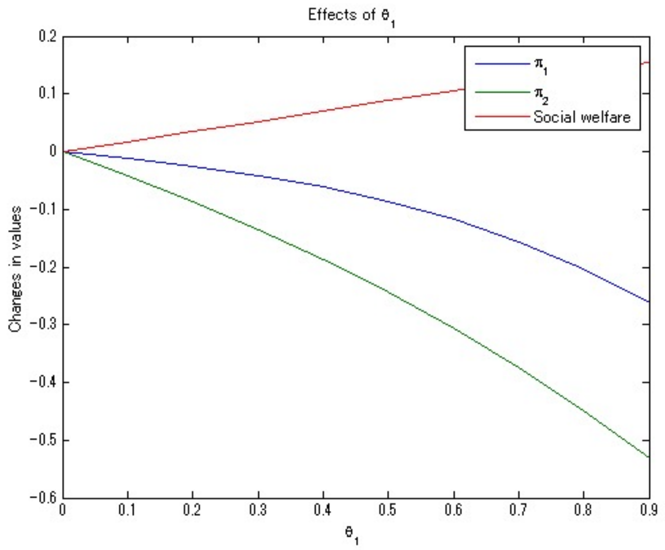


Figure 5.2: Effects on social welfare and profits. ($\alpha = 5$, $m = 1$, $c = 0.5$, $E[\epsilon] = 0$, $E[\epsilon^2] = 2$, and $\theta_2 = 0$.)

**Proposition 2: (Effect of partial privatization)** *If we consider a linear demand function and quadratic cost functions, a SFE is characterized as $S_i(p) = b_i^* p \ \forall i = 1, 2$, where $b_i^* = \left[\left(m + b_j^*\right) + \theta_i \left(b_j^* c - 1\right) b_j^*\right] / \left[(1 - \theta_i) + c\left(m + b_j^*\right)\right]$ $(j \neq i)$. Furthermore, if firm 2 is a private firm ($\theta_2 = 0$), then, both $S_1'(p)$ and $S_2'(p)$ increase when $\theta_1$ increases.*

**Proof.** See the Appendix D.

Since both firms' supply functions become closer to the (inverse) marginal cost function for larger $\theta_1$, the social welfare is also increased if we increase $\theta_1$. Then, we can state the following corollary.

**Corollary of Proposition 2: (Mixed oligopoly)** *Suppose firm 2 is a private firm ($\theta_2 = 0$). Then, $\theta_1 = 1$ (completely public) is the optimal level of partial privatization.*

Thus, in supply function competitions, existence of a public firm benefits the social welfare in contrast to the Cournot competition examined by Matsumura (1998). (Improvement of welfare is illustrated in fig. 5.2.)

Although we do not have any analytical solutions of eq. (5.8) for arbitrary $\theta_1$ and $\theta_2$, we can solve it for symmetric social concern: $\theta_1 = \theta_2 = \theta$. In addition, it turns out that the solution of eq. (5.8), which is derived from the assumption that the supply function is linear, coincides with the unique solution of the differential equation (5.3), which satisfies SOC of the payoff function for each firm. Therefore, uniqueness of the equilibrium is also proven for symmetric $\theta$. Since we have the analytical formula for SFE, we can confirm that the supply function converges to the (inverse) marginal cost function when $\theta$ approaches 1 under the symmetry assumption of $\theta$.

**Proposition 3: (Symmetric SFE in a linear-quadratic model)** *In a symmetric setting ($\theta_1 = \theta_2 = \theta$), we can obtain the unique symmetric SFE:*

$$S_i(p) = \frac{-m + \sqrt{m^2 + 4m(1-\theta)/c)}}{2(1-\theta)} p \ \ \forall i = 1, 2.$$

*Furthermore, it converges to the (inverse) marginal cost function as $\theta \to 1$.*

**Proof.** See the Appendix D.

We can interpret the symmetric $\theta$ as the extent of corporate social responsibility and increase of $\theta$ as the improvement of social concern in the industry. The model guarantees that the supply function shifts upward monotonically as social concern in the industry is improved and that it converges to the socially optimal level as $\theta \to 1$.

## 5.4 Conclusion

In this paper, we examine supply function equilibrium introduced by Klemperer and Meyer (1989) with (partially) public firms. First, we generally characterize the SFEs in partially privatized markets. We show the necessity of a positive slope for symmetric SFE. Second, we specify the demand and cost functions and show that not only a partially public firm but also a private firm offers flatter supply functions when the publicity of the public firm is enhanced. Thus, in supply function competitions, existence of a public firm benefits the social welfare, in contrast to the Cournot competition examined by Matsumura (1998). Finally, we confirmed that the supply function equilibrium converges to the (inverse) marginal cost function when the publicity of firms or the extent of social concern is improved symmetrically in the industry.

# Appendix D

# Proofs

We characterize the differential equation (5.3) by the following series of lemmas.

**Lemma 1** *The locus of points satisfying $f(p, S) = 0$ is a continuous, differentiable function $S = S^0(p)$, satisfying*

$$
\begin{array}{ll}
(i) & S^0(0) = 0, \\
(ii) & S^0(p) < (C')^{-1}(p), \forall p > 0, \\
(iii) & S^{0\prime}(p) \text{ is positive and increasing in } \theta, \forall p \geq 0, \text{ and} \\
(iv) & S^{0\prime}(0) < \frac{1}{C''(0)}.
\end{array}
$$

**Proof of Lemma 1:** Differentiation of (5.3) w.r.t. $S$ yields

$$f_S(p, S) = \frac{p - C'(S) + SC''(S)}{(p - C'(S))^2}$$

and thus, for all $(\bar{p}, \bar{S}) \neq (0, 0)$ such that $f(\bar{p}, \bar{S}) = 0$,

$$
\begin{aligned}
f_S(\bar{p}, \bar{S}) &= \frac{1}{\bar{p} - C'(\bar{S})} + \frac{\bar{S}}{\bar{p} - C'(\bar{S})} \frac{C''(\bar{S})}{\bar{p} - C'(\bar{S})} \\
&= \frac{1 - \frac{1}{(1-\theta)} D_p(\bar{p}) C''(\bar{S})}{\bar{p} - C'(\bar{S})} \neq 0.
\end{aligned}
$$

Therefore, by the implicit function theorem, $f(p, S) = 0$ implicitly defines, in the neighborhood of any such $(\bar{p}, \bar{S})$, a unique function $S = S^0(p)$, which is continuous and differentiable.

To prove $(i)$ and $(ii)$, observe that $\forall \theta \in [0, 1)$, $S^0(p)$ and $p - C'(S^0(p))$ are both positive or both zero since $-\infty < D_p(p) < 0$ and $f(p, S^0(p)) = 0$. Hence, $p > C'(S^0(p))$ whenever $S^0(p) > 0$. Furthermore, $S^0(0) = 0$ is the unique solution to $f(0, S) = 0$. For all $p > 0$, $S^0(p) > 0$ (otherwise, $S^0(p) = 0$ and $p - C'(S^0(p)) > 0$) and $p > C'(S^0(p))$. Since $C'' > 0$, we can take the inverse function of $C'$ and obtain $(C')^{-1}(p) > S^0(p)$ for all $p > 0$. Here, as $p \to 0$, the upper bound of $S^0(p)$ converges to zero and $S^0(p) > 0$ for all $p > 0$. Then, $S^0(p) \to 0$ as $p \to 0$. Thus, $S^0(p)$ is continuous at $p = 0$.

To prove $(iii)$ and $(iv)$, differentiate $f\left(p,\,S^0\left(p\right)\right)=0$ totally with respect to $p$ and substitute using this equation to get

$$S^{0\prime}\left(p\right)=-\frac{D_p\left(p\right)+D_{pp}\left(p\right)\left(p-C'\left(S^0\left(p\right)\right)\right)}{\left(1-\theta\right)-D_p\left(p\right)C''\left(S^0\left(p\right)\right)}.$$

Now, $\lim_{p\to 0}S^{0\prime}\left(p\right)$ exists and equals

$$-\frac{D_p\left(0\right)}{\left(1-\theta\right)-D_p\left(0\right)C''\left(0\right)}\equiv S^{0\prime}\left(p\right),$$

where $0<S^{0\prime}\left(p\right)<\frac{1}{C''(0)}$, and thus, $S^0\left(p\right)$ is continuous and differentiable at $p=0$. **Q.E.D.**

**Lemma 2** *The locus of points satisfying* $f\left(p,\,S\right)=\infty$ *is a continuous, differentiable function,* $S=S^{\infty}\left(p\right)\equiv\left(C'\right)^{-1}\left(p\right)$. *Hence,* $S^{\infty}\left(0\right)=0$ *and* $0<S^{\infty\prime}\left(p\right)<\infty\,\forall p\geq 0$.

**Proof of Lemma 2: (same as the proof of claim 2 in KM )** From (5.3), $S^{\infty}\left(p\right)$ solves $f\left(p,\,S^{\infty}\left(p\right)\right)=\infty$ implies that $S^{\infty}\left(p\right)$ solves $p-C'\left(S^{\infty}\left(p\right)\right)=0$, and hence, since $C''>0$, $S^{\infty}\left(p\right)=\left(C'\right)^{-1}\left(p\right)\,\forall p$. The stated properties of $S^{\infty}\left(p\right)$ follow from the assumptions on $C'\left(S\right)$. **Q.E.D.**

**Lemma 3** *For all points* $\left(p,\,S\right)$ *between the* $f=0$ *and* $f=\infty$ *loci,* $0<f\left(p,\,S\right)<\infty$. *For all points in the first quadrant above the* $f=0$ *locus or below the* $f=\infty$ *locus,* $0>f\left(p,\,S\right)>-\infty$.

**Proof of Lemma 3: (same as the proof of claim 3 in KM)** Since $\frac{S}{p-C'(S)}$ is finite and increasing in $S$ as long as $p>C'\left(S\right)$, for a given $\bar{p}$, $f\left(\bar{p},\,S\right)$ is finite and monotonically increasing in $S$ for $S\in\left[0,\,\left(C'\right)^{-1}\left(p\right)\right)$. Below the $f=\infty$ locus, $0>\frac{S}{p-C'(S)}>-\infty$, and hence, since $0>D_p>-\infty$, $0>f\left(p,\,S\right)>-\infty$. **Q.E.D.**

**Lemma 4** *If* $S\left(p\right)$ *solves (5.3) and the other firm takes* $S\left(p\right)$, *the second derivative of* $i$*'s payoff with respect to* $p$ *for a given* $\epsilon$ *evaluated at an intersection of* $S\left(p\right)$ *and residual demand function* $D\left(p\right)+\epsilon-S\left(p\right)$ *is written as*

$$\begin{aligned}\frac{\partial^2 v_i\left(p,\,\epsilon;\,S\left(p\right)\right)}{\partial p^2}|_{p=p^*} \;=\;& \left(D_p\left(p^*\right)-S'\left(p^*\right)\right)\left(\left(1-\theta\right)+C''\left(D\left(p^*\right)+\epsilon-S\left(p^*\right)\right)\right) \\ & -C''\left(D\left(p^*\right)+\epsilon-S\left(p^*\right)\right)\left(D_p\left(p^*\right)-S'\left(p^*\right)\right)^2-\left(1-\theta\right)S'\left(p^*\right),\end{aligned} \tag{5.9}$$

*where* $p^*$ *is a price that solves* $D\left(p^*\right)+\epsilon-2S\left(p^*\right)=0$.

**Proof of Lemma 4:** Given that $j$ chooses $S(p)$, the second-order derivative of $i$'s payoff with respect to $p$ for a given $\epsilon$ is

$$\begin{aligned}\frac{\partial^2 v_i(p,\,\epsilon;\,S(p))}{\partial p^2} &= (2-\theta)\left\{D_p(p)-S'(p)\right\}-C''(D(p)+\epsilon-S(p))\left(D_p(p)-S'(p)\right)^2\\ &\quad+\left(p-C'(D(p)+\epsilon-S(p))\right)\left(D_{pp}(p)-S''(p)\right)\\ &\quad+\theta S'(p)-\theta C''(S(p))\left(S'(p)\right)^2+\theta\left(p-C'(S(p))\right)S''(p). \qquad (5.10)\end{aligned}$$

If $S(p)$ solves (5.3), we can differentiate (5.3) totally with respect to $p$ to obtain an expression for $S''(p)$:

$$S''(p)=\frac{X_1}{\left((1-\theta)(p-C'(S(p)))\right)^2}, \qquad (5.11)$$

where

$$\begin{aligned}X_1 \;\equiv\; &\left[(1-\theta)S'(p)+\left(1-C''(S(p))S'(p)\right)D_p(p)+\left(p-C'(S(p))\right)D_{pp}(p)\right]\left[(1-\theta)\left(p-C'(S(p))\right)\right]\\ &-\left[(1-\theta)S(p)+\left(p-C'(S(p))\right)D_p(p)\right]\left[(1-\theta)\left(1-C''(S(p))S'(p)\right)\right].\end{aligned}$$

Using (5.3) to substitute for $S(p)$ in (5.11) gives

$$S''(p)=\frac{(1-\theta)S'(p)+(1-C''(S(p))S'(p))(D_p(p)-(1-\theta)S'(p))+(p-C'(S(p)))D_{pp}(p)}{(1-\theta)(p-C'(S(p)))}, \qquad (5.12)$$

and thus, when $S(p)$ solves (5.3), $S''(p)$ in (5.10) is replaced by (5.12). Moreover, if we evaluate at $p=p^*$ where $p^*$ solves $D(p^*)+\epsilon-2S(p)=0$, (5.10) becomes (5.9). **Q.E.D.**

By these lemmas, we can prove Proposition 1.

**Proof of Proposition 1** Satisfaction of (5.3) $\forall p\geq 0$ is a necessary condition for a supply function defined for all $p\geq 0$ to trace through ex post optimal points when the other firm commits to the same supply function. To show that $0<S'(p)<\infty\,\forall p\geq 0$ is also a necessary condition, we show that if, for some $p$, $S$ ever crosses either $f=0$ from below or $f=\infty$ from the left, then $S$ must eventually violate the global optimality[9].

Once trajectory $S$ crosses $f=0$ from below, $S'$ becomes and stays negative and, from (A2), $S''$ also becomes and stays negative. Therefore, the trajectory would eventually intersect the $S=0$ axis at a point $(p_0,\,0)$ with $p_0>C'(0)$, where $S'(p_0)=f(p_0,\,0)=\frac{1}{1-\theta}D_p(p_0)$. Therefore, for $\epsilon=e(0,\,p_0)$, $Q=D(p_0,\,\epsilon)=0$ by definition and then, residual demand $D(p_0,\,\epsilon)-S(p_0)=0$.

[9] Actually, such a part of $S$ represents one of the multiple intersections for a certain $\epsilon$, which results in a smaller profit than that of another intersection for the same $\epsilon$.

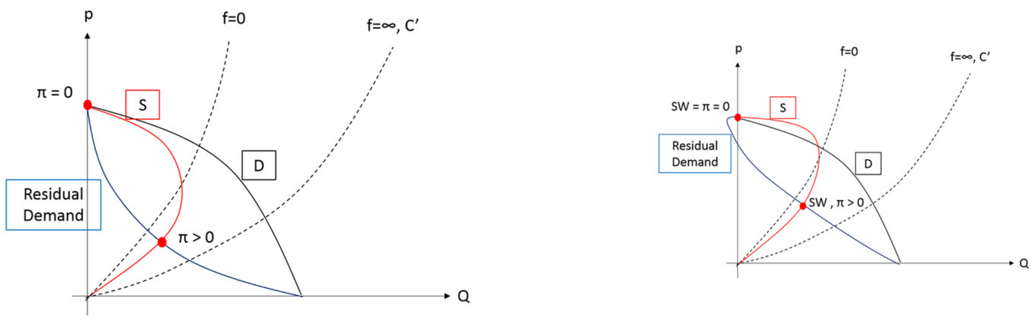


Figure 5.3: A supply function (satisfying FOC and symmetry) violating global optimality. (Left: $\theta = 0$, Right: $0 < \theta < 1$ )

Then, given firm $j$ takes $S$, $p_0$ satisfies the first-order condition but that results in $q_i = q_j = 0$ and $SW = \pi_i = v_i = 0$. On the other hand, since $S'(p_0) = \frac{1}{1-\theta} D_p(p_0)$, $S(p)$ and the residual demand $D(p, \epsilon) - S(p)$ for the same $\epsilon = e(0, p_0)$ cross each other at another point $(p_1, q_1)$ where $q_1 > 0$ and $p_1 > C'(q_1)$ (Fig.5.3). Since $SW$, $\pi_i$, $v_i > 0$ at $(p_1, q_1)$, firm $i$ has an incentive to adjust from $p_0$ to $p_1$. Thus, $S$ eventually violates the global optimality. **Q.E.D.**

**Proof of Proposition 2** For the proof of Proposition 2, it remains to be shown that

$$b_i^*(b_j, \theta_i) = \frac{(m + b_j) + \theta_i (b_j c - 1) b_j}{(1 - \theta_i) + c(m + b_j)} \; (j \neq i)$$

is increasing in $b_j$ when $\theta_i = 0$ and increasing in $\theta_i$ for any $b_j$.

$$\begin{aligned} \frac{\partial b_i^*(b_j, \theta_i)}{\partial b_j} &= \frac{\{1 + \theta_i (2b_j c - 1)\} \{(1 - \theta_i) + c(m + b_j)\} - c\{(m + b_j) + \theta_i (b_j c - 1) b_j\}}{\{(1 - \theta_i) + c(m + b_j)\}^2} \\ &= \frac{(1 - \theta_i) + \theta_i (2b_j c - 1) \{(1 - \theta_i) + c(m + b_j)\} - c\theta_i (b_j c - 1) b_j}{\{(1 - \theta_i) + c(m + b_j)\}^2}. \end{aligned}$$

Thus, if $\theta_i = 0$,

$$\frac{\partial b_i^*(b_j, \theta_i)}{\partial b_j} = \frac{1}{\{(1 - \theta_i) + c(m + b_j)\}^2} > 0.$$

On the other hand,

$$
\begin{aligned}
\frac{\partial b_i^*(b_j,\,\theta_i)}{\partial\theta_i} &= \frac{(1-\theta_i)\,(b_jc-1)\,b_j + c\,(m+b_j)\,(b_jc-1)\,b_j + (m+b_j) + \theta_i\,(b_jc-1)\,b_j}{\{(1-\theta_1)+c\,(m+b_2)\}^2} \\
&= \frac{b_j^3c^2 + \left(b_j^2c^2 - b_jc\right)m + m}{\{(1-\theta_1)+c\,(m+b_2)\}^2} \\
&= \frac{b_j^3c^2 + \left(b_jc-\frac{1}{2}\right)^2 m + \frac{3}{4}m}{\{(1-\theta_1)+c\,(m+b_2)\}^2} > 0
\end{aligned}
$$

Thus, if we suppose that $\theta_2 = 0$ and increase $\theta_1$, $b_2^*$ is increasing in $b_1^*$ and $b_1^*$ is shifted upward. Therefore, not only $b_1^*$ but also $b_2^*$ would increase since $b_2^*$ is increasing in $b_1^*$.**Q.E.D.**

**Proof of Proposition 3** By proposition 1, for $S$ to be a symmetric SFE, $S$ must satisfy (5.3) and $0 < S'(p) < \infty$. In the linear case, (5.3) is rewritten as follows:

$$
\begin{aligned}
S'(p) &= \frac{(1-\theta)\,S + \{p - C'(S)\}\cdot D_p(p)}{(1-\theta)\,\{p - C'(S)\}} \\
&= \frac{(1-\theta)\,S + \{p - cS\}\cdot(-m)}{(1-\theta)\,\{p - cS\}}.
\end{aligned}
$$

In the autonomous form,

$$
\begin{aligned}
\frac{dS}{dt} &= (1-\theta)\,S + \{p - cS\}\cdot(-m) \\
\frac{dp}{dt} &= (1-\theta)\,\{p - cS\}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\begin{bmatrix} \frac{dS}{dt} \\ \frac{dp}{dt} \end{bmatrix} &= \begin{bmatrix} (1-\theta+mc)\,S - mp \\ -c\,(1-\theta)\,S + (1-\theta)\,p \end{bmatrix} \\
&= \begin{bmatrix} (1-\theta+mc) & -m \\ -c\,(1-\theta) & (1-\theta) \end{bmatrix} \begin{bmatrix} S \\ p \end{bmatrix}.
\end{aligned}
$$

For any eigenvalue $r$, the following equation must be satisfied:

$$
\begin{aligned}
det\left(\begin{bmatrix} (1-\theta+mc) & -m \\ -c\,(1-\theta) & (1-\theta) \end{bmatrix} - rI\right) &= 0 \\
\Leftrightarrow r &= \frac{2\,(1-\theta) + mc \pm \sqrt{m^2c^2 + 4mc\,(1-\theta)}}{2}.
\end{aligned}
$$

We have different and unequal eigenvalues. For each eigenvalue $r_1$, $r_2$, the eigenvectors are defined as follows:

$$\left\{ \begin{bmatrix} (1-\theta+mc) & -m \\ -c(1-\theta) & (1-\theta) \end{bmatrix} - r_i I \right\} \begin{bmatrix} u_i \\ w_i \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{bmatrix} ((1-\theta+mc)-r_i)\,u_i - mw_i \\ -c(1-\theta)\,u_i + ((1-\theta)-r_i)\,w_i \end{bmatrix} = 0.$$

Then,

$$\begin{aligned} \frac{u_i}{w_i} &= \frac{(1-\theta)-r_i}{c(1-\theta)} \\ &= \frac{(1-\theta)-(1-\theta)-\frac{mb\pm\sqrt{m^2c^2+4mc(1-\theta)}}{2}}{c(1-\theta)} \\ &= \frac{-mb\mp\sqrt{m^2c^2+4mc(1-\theta)}}{2c(1-\theta)} \\ &= \frac{-m\mp\sqrt{m^2+\frac{4m(1-\theta)}{c}}}{2(1-\theta)}. \end{aligned}$$

Let the larger eigenvalue be $r_1$. Then, $\frac{u_1}{w_1} < 0$ and $\frac{u_2}{w_2} > 0$. Since the eigenvalues are real and unequal, the solution to the differential equation is written as

$$\begin{pmatrix} S \\ p \end{pmatrix} = A_1 e^{\lambda_1 t} \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} + A_2 e^{\lambda_2 t} \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} \tag{5.13}$$

where $A_1$ and $A_2$ are arbitrary constants. Here, if $A_1 \neq 0$,

$$\begin{aligned} \frac{S}{p} &= \frac{A_1 e^{\lambda_1 t} u_1 + A_2 e^{\lambda_2 t} u_2}{A_1 e^{\lambda_1 t} w_1 + A_2 e^{\lambda_2 t} w_2} \\ &= \frac{A_1 u_1 + A_2 \left(e^{(\lambda_2-\lambda_1)t}\right) u_2}{A_1 w_1 + A_2 \left(e^{(\lambda_2-\lambda_1)t}\right) w_2} \\ &\to \frac{u_1}{w_1} < 0 \text{ as } t \to \infty, \end{aligned}$$

and thus, all trajectories eventually leave the region between $f = 0$ and $f = \infty$ and their slope becomes negative. Therefore, the only remaining $S$ that satisfies the necessary conditions is (5.13) with $A_1 = 0$:

$$S(p) = \frac{-m+\sqrt{m^2+\frac{4m(1-\theta)}{c}}}{2(1-\theta)} p \equiv \frac{g(\theta)}{h(\theta)}. \tag{5.14}$$

Suppose that the other firm takes this linear supply function. Then, the local optimality for firm $i$'s payoff function is satisfied along $S(p)$ by lemma 5, and the residual demand is also linear since the demand function is defined as linear. Since given $\epsilon$, both residual demand and marginal cost are linear in $p$, firm $i$'s profit function $\pi_i$ is written as a function quadratic in $p$. On the other hand, since given $\epsilon$, the demand function and industrial marginal costs are linear in $p$, $SW$ is written as a function quadratic in $p$. Therefore, the payoff for firm $i$, which is the weighted average of $i$'s profit and $SW$, is written as a quadratic function. Therefore, the local optimal point given $\epsilon$ is actually a unique global maximizer given $\epsilon$. Thus, (5.14) is a symmetric SFE.

We check the effect of $\theta$. Since $g(1) = h(1) = 0$ by l'Hopital's rule, we have

$$\lim_{\theta \to 1} S(p) = \lim_{\theta \to 1} \frac{g(\theta)}{h(\theta)} = \lim_{\theta \to 1} \frac{g'(\theta)}{h'(\theta)} = \lim_{\theta \to 1} \frac{m}{c} \frac{p}{\sqrt{m^2 + \frac{4m(1-\theta)}{c}}} = \frac{p}{c}$$

Thus, $\theta$ converges to 1 and the supply function converges to marginal production cost. **Q.E.D.**

# Chapter 6

# Government-Leading Welfare-Improving Collusion


**Abstract**

We discuss government-leading welfare-improving collusion in a mixed duopoly. We formulate an infinitely repeated game in which a welfare-maximizing firm and a profit-maximizing firm coexist. The government proposes welfare-improving collusion and this is sustainable if both firms have incentives to follow it. We compare two competition structures—Cournot and Bertrand—in this long-run context. We find that Cournot competition yields greater welfare when the discount factor is sufficiently large, whereas Bertrand competition is better when the discount factor is small.



**JEL classification numbers:** L41, L13

**Key words:** repeated game, public collusion, Cournot-Bertrand welfare comparison

**Based article:** Haraguchi J, Matsumura T (2016) Government-Leading Welfare-Improving Collusion. Mimeo.

## 6.1 Introduction

Collusion among profit-maximizing firms raises prices, and thus, is harmful for consumer and economic welfare. However, if some firms are concerned with social welfare in the market, welfare-improving and consumer-benefiting collusion may be formed. In this study, we analyze an infinitely repeated game under complete information in a market in which a welfare-maximizing firm competes with a profit-maximizing firm.[1] The government proposes welfare-improving collusion and this is sustainable if incentive compatibility is satisfied for both firms.[2] We compare two competition structures—Cournot and Bertrand—in this long-run context. We find that Cournot competition (the quantity-setting model) yields greater welfare when the discount factor is sufficiently large, whereas Bertrand competition (the price-setting model) is better when the discount factor is small.

We show that the deviation incentive from welfare-improving collusion (one-shot gain of deviating from collusion) is greater under Cournot than Bertrand competition, in contrast to profit-maximizing private collusion. For this effect, it is more difficult for the government to form welfare-improving collusion under Cournot competition, and this is harmful for welfare. However, the punishment for the deviation is stricter under Cournot competition, again in contrast to a private duopoly. This punishment effect makes the collusion more stable. Therefore, it is easier to form welfare-improving collusion under Cournot competition, and this is beneficial for welfare. The former effect dominates when the discount factor is small, while the latter effect dominates when the discount factor is large. This leads to the above result.

In the literature on mixed oligopolies, Cournot–Bertrand comparisons are popular.[3] Ghosh and Mitra (2010), Matsumura and Ogawa (2012), and Haraguchi and Matsumura (2014) showed that Bertrand competition yields larger profit in the private firm, and Scrimitore (2014) and Haraguchi and Matsumura (2016) showed that profit ranking can be reversed.[4] However, these works showed that Bertrand competition yields greater welfare than Cournot competition under moderate conditions, whereas our study suggests that Cournot competition can be better for social welfare. More importantly, no study has discussed this problem in the context of long-run competition (an

[1]One natural interpretation of this market is that one firm is a state-owned public firm, which is adopted in the literature on mixed oligopolies. For the examples of mixed oligopolies and recent development of this field, see Ye (2016). Another interpretation is that one firm is concerned with corporate social responsibility (Ghosh and Mitra, 2014; Matsumura and Ogawa, 2014).

[2]For the reality of welfare-improving collusion in a mixed oligopoly, see Wen and Sasaki (2001). The government's intervention in collusion and competition occurs often in Japan and is discussed intensively in the context of industry policies. See Itoh et al. (1991).

[3]Another popular topic in the literature is private oligopolies. It is well known that under moderate conditions, price competition is stronger, yielding lower profits and greater welfare than in the case of quantity competition. See Shubik and Levitan (1980) and Vives (1985). However, it is not always true. See Chirco and Scrimitore (2013). Pal (2014, 2015).

[4]Nakamura (2015) investigated the bargaining between managers and owners in this context.

infinitely repeated game).

While Colombo (2016) discussed an infinitely repeated game in a mixed oligopoly, he discussed profit-maximizing partial collusion among private firms and investigated how the degree of privatization of the outsider (the public firm) affects the stability of private collusion. Thus, his analysis is completely different to ours.[5]

Wen and Sasaki (2001) is the most closely related to our study. They also discussed welfare-improving collusion and showed that the public firm's idle capacity stabilizes the collusion. However, they did not discuss a comparison between Bertrand and Cournot competition.[6]

## 6.2 The Model

We adopt a standard duopoly model with differentiated goods and linear demand (Dixit, 1979).[7] The quasi-linear utility function of the representative consumer is:

$$U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{\beta}{2}(q_0^2 + 2\gamma q_0 q_1 + q_1^2) + y, \tag{6.1}$$

where $q_0$ is the consumption of good 0 produced by the public firm, $q_1$ is the consumption of good 1 produced by the private firms, and $y$ is the consumption of an outside good that is competitively provided, with a unitary price. Parameters $\alpha$ and $\beta$ are positive constants and $\gamma \in (0, 1)$ represents the degree of product differentiation: a smaller $\gamma$ indicates a larger degree of product differentiation. The inverse demand functions for goods $i = 0, 1$ with $i \neq j$ are

$$p_i = \alpha - \beta q_i - \beta\gamma q_j, \tag{6.2}$$

where $p_i$ is the price of firm $i$.

The marginal cost of production is constant for both firms. Let us denote with $c_i$ the marginal cost of firm $i$, assuming $\alpha > c_i$. Firm 0 is a state-owned public firm whose payoff is the social surplus (welfare). This is given by:

$$SW = (p_0 - c_0)q_0 + (p_1 - c_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\gamma q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right]. \tag{6.3}$$

Firm 1 is a private firm and its payoff is its own profit:

$$\pi_1 = (p_1 - c_1)q_1. \tag{6.4}$$

[5]For the discussion on the stability collusion among non-profit-maximizers, see also Matsumura and Matsushima (2012).

[6]For long-run analysis not based on infinitely repeated game in mixed oligopolies, see Ishibashi and Matsumura (2006) and Nishimori and Ogawa (2002, 2005).

[7]This demand function is popular in the literature on mixed oligopolies. See Bárcena-Ruiz (2007), Ishida and Matsushima (2009), Matsumura and Shimizu (2010), and Haraguchi and Matsumura (2014,2016).

Firms engage in an infinitely repeated game. Let $\delta$ denote the discount factor between periods. Along the punishment path, the firms are assumed to use the grim trigger strategy of Friedman (1971).[8]

We consider government-leading welfare-improving collusion. The government proposes a pair of outputs $(q_0^C, q_1^C)$ in the quantity competition case and a pair of prices $(p_0^C, p_1^C)$ in the price competition case, where the superscript C denotes collusion. Both firms accept the proposal if it is sustainable in the infinitely repeated game under the grim trigger strategy.

## 6.3 Results

### 6.3.1 Bertrand case

First, we consider a competitive situation in which firms face a one-shot game. Let $a_i := \alpha - c_i$. We assume that the solution in the competition situation are interior, that is, equilibrium prices and quantities for both firms are strictly positive. The first-order conditions of firms 0 and 1 are

$$\frac{\partial SW}{\partial p_0} = \frac{c_0 - p_0 - \gamma c_1 + \gamma p_1}{\beta(1-\gamma^2)} = 0, \tag{6.5}$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{c_1 - 2p_1 + \alpha + \gamma p_0 - \alpha\gamma}{\beta(1-\gamma^2)} = 0, \tag{6.6}$$

respectively. The second-order conditions are satisfied. Let $R_i(p_j)$ $(i = 0, 1,\ i \neq j)$ be the reaction function of the one-shot game (stage game). From the above first-order conditions, we obtain

$$R_0(p_1) = c_0 + \gamma(p_1 - c_1), \tag{6.7}$$

$$R_1(p_0) = \frac{c_1 + \alpha + p_0\gamma - \alpha\gamma}{2}. \tag{6.8}$$

The equilibrium price, resulting profit of firm 1, and welfare are

$$p_0^N = \frac{\alpha\gamma - \alpha\gamma^2 + 2c_0 - c_1\gamma}{2-\gamma^2}, \tag{6.9}$$

$$p_1^N = \frac{\alpha - \alpha\gamma + c_1 + c_0\gamma - c_1\gamma^2}{2-\gamma^2}, \tag{6.10}$$

$$\pi_1(p_0^N, p_1^N) = \frac{(a_1 - \gamma a_0)^2}{\beta(1-\gamma^2)(2-\gamma^2)^2}, \tag{6.11}$$

$$SW(p_0^N, p_1^N) = \frac{(2\gamma^4 - 5\gamma^2 + 4)a_0^2 + (\gamma^4 - 3\gamma^2 + 3) - 2\gamma(\gamma^4 - 3\gamma^2 + 3)a_0 a_1}{2\beta(1-\gamma^2)(2-\gamma^2)^2}, \tag{6.12}$$

[8] This punishment strategy is not optimal (Abreu, 1988). We use the grim trigger strategy for simplicity and tractability. We believe that this is a very realistic punishment strategy because of its simplicity. Many works adopt this strategy when analyzing stability of agreements. See, among others, Deneckere (1983), Gibbons (1992), Maggi (1999), Gupta and Venkatu (2002), and Matsumura and Matsushima (2005).

where the superscript N denotes one-shot Nash equilibrium.

Next, we consider collusion in the infinitely repeated game. Both firms accept the government proposal $(p_0^C, p_1^C)$ if the following two inequalities are satisfied.

$$\frac{SW(p_0^C, p_1^C)}{1-\delta} \geq SW(R_0(p_1^C), p_1^C) + \frac{\delta SW(p_0^N, q_1^N)}{1-\delta}, \tag{6.13}$$

$$\frac{\pi_1(p_0^C, p_1^C)}{1-\delta} \geq \pi_1(p_0^C, R_1(p_0^C)) + \frac{\delta \pi_1(p_0^N, p_1^N)}{1-\delta}. \tag{6.14}$$

Sustainable pairs of prices must not yield smaller welfare than that of the one-shot Nash equilibrium because otherwise, the public firm never accepts them. Because the price of the private firm at one-shot Nash equilibrium is too high for social welfare and that of the public firm is optimal given $p_1^C$, $p_1^C \leq p_1^N$ must hold.

Sustainable pairs of prices must not yield smaller profit in the private firm than that of the one-shot Nash equilibrium because otherwise, the private firm never accepts them. Given $p_0$, $p_1^C$ $(< p_1^N)$ yields smaller profit in firm 1 than that of the one-shot Nash equilibrium. Thus, to compensate the private firm's profit, $p_0^C > p_0^N$ must hold when $p_1^C < p_1^N$. These lead to the following lemma (see Figure 6.1 for Lemma 1-ii).

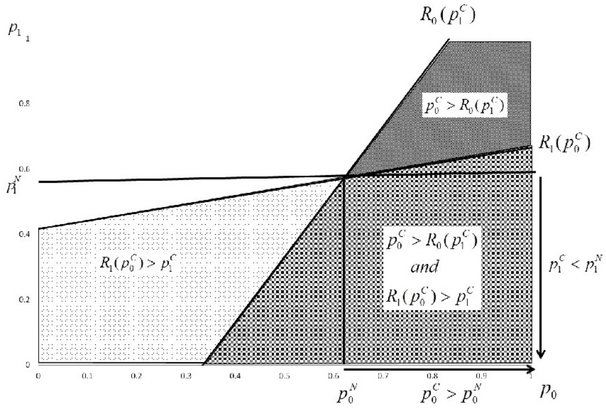


Figure 6.1: Lemma 1-ii

**Lemma 1** *(i)* $(p_0^C, p_1^C)$ *is sustainable only if* $p_0^C > p_0^N$ *and* $p_1^C < p_1^N$ *or* $(p_0^C, p_1^C) = (p_0^N, p_1^N)$. *(ii) If* $p_0^C > p_0^N$ *and* $p_1^C < p_1^N$, $p_0^C > R_0(p_1^C)$ *and* $p_1^C < R_1(p_0^C)$.

Lemma 1(i) presents a necessary (but not sufficient) condition for sustainable prices. Lemma 1(ii) states that firm 1 (res. firm 1) prefers a lower (res. higher) price than the collusive price given the rival's price.

### 6.3.2 Cournot case

First, we consider a competitive situation in which firms face a one-shot game. The first-order conditions of firms 0 and 1 are

$$\frac{\partial SW}{\partial q_0} = a_0 - \beta q_0 - \beta\gamma q_1 = 0, \tag{6.15}$$

$$\frac{\partial \pi_1}{\partial q_1} = a_1 - 2\beta q_1 - \beta\gamma q_0 = 0, \tag{6.16}$$

respectively. The second-order conditions are satisfied. Let $R_i(q_j)$ $(i = 0, 1,\ i \neq j)$ be the reaction function of the one-shot game (stage game). From the above first-order conditions, we obtain

$$R_0(q_1) = \frac{a_0 - \beta\gamma q_1}{\beta},$$

$$R_1(q_0) = \frac{a_1 - \beta\gamma q_0}{2\beta}.$$

The equilibrium output, resulting profit of firm 1, and welfare are

$$q_0^N = \frac{2a_0 - \gamma a_1}{\beta(2-\gamma^2)}, \tag{6.17}$$

$$q_1^N = \frac{a_1 - \gamma a_0}{\beta(2-\gamma^2)}, \tag{6.18}$$

$$\pi_1(q_0^N, q_1^N) = \frac{(a_1 - \gamma a_0)^2}{\beta(2-\gamma^2)^2}, \tag{6.19}$$

$$SW(q_0^N, q_1^N) = \frac{(4-\gamma^2)a_0^2 + (3-\gamma^2)a_1^2 - 2\gamma(3-\gamma^2)a_0 a_1}{2\beta(2-\gamma^2)^2}. \tag{6.20}$$

Next, we consider collusion in the infinitely repeated game. Both firms accept the government proposal $(q_0^C, q_1^C)$ if the following two inequalities are satisfied.

$$\frac{SW(q_0^C, q_1^C)}{1-\delta} \geq SW(R_0(q_1^C), q_1^C) + \frac{\delta SW(q_0^N, q_1^N)}{1-\delta}, \tag{6.21}$$

$$\frac{\pi_1(q_0^C, q_1^C)}{1-\delta} \geq \pi_1(q_0^C, R_1(q_0^C)) + \frac{\delta \pi_1(q_0^N, q_1^N)}{1-\delta}. \tag{6.22}$$

Sustainable pairs of quantities must not yield smaller welfare than that of the one-shot Nash equilibrium in the quantity competition because otherwise, the public firm never accept it. The quantity of the private firm at one-shot Nash equilibrium in the quantity competition is too low for social welfare and that of public firm is optimal given $q_1^C$, $q_1^C \geq q_1^N$ must hold.

Sustainable pairs of quantities must not yield smaller profit for the private firm than that of the one-shot Nash equilibrium in the quantity competition because otherwise, the private firm never accept it. Given $q_0$, $q_1^C(> q_1^N)$ yields smaller profit for the private firm than that of the one-shot Nash equilibrium in the quantity competition. Therefore, to compensate the private firm's profit, $q_0^C < q_0^N$ must hold when $q_1^C > q_1^N$. These lead to the following lemma.

**Lemma 2** $(q_0^C, q_1^C)$ *is sustainable only if* $q_0^C < q_0^N$ *and* $q_1^C > q_1^N$ *or* $(q_0^C, q_1^C) = (q_0^N, q_1^N)$.

Lemma 2 presents a necessary but not sufficient condition for sustainable outputs. The private (public) firm increases (decreases) its output expecting that the public (private) firm decreases (increases) its output.

### 6.3.3 Comparison

Before presenting the main results, we present a well-known result in the literature.[9]

**Result 1** $\pi_1(p_0^N, p_1^N) > \pi_1(q_0^N, q_1^N)$ *and* $SW(p_0^N, p_1^N) > SW(q_0^N, q_1^N)$.

In contrast to a private oligopoly, Bertrand competition yields larger profit in the private firm when the rival firm is a welfare maximizer.

We now present our main results. As mentioned in Subsection 3.1, the price of the private firm is too high for social welfare, and the government wants to decrease it. Thus, the government sets $p_1^C < p_1^N$. It sets $p_0^C > p_0^N$ because otherwise, firm 1 never accepts the collusion.

Although we cannot solve the optimal $p_i^C$ and $q_i^C$ explicitly, we derive a key property of the collusion. We show that the deviation incentive from the collusion is greater under the quantity case than under the price case, in contrast to the case of profit-maximizing collusion among profit-maximizing firms.

**Proposition 1** *Suppose that* $p_i^C = \alpha - \beta q_i^C - \beta\gamma q_j^C$. *Suppose that* $p_0^C > p_0^N$ *and* $p_1^C < p_1^N$. *Then* $SW(R_0(p_1^C), p_1^C) < SW(R_0(q_1^C), q_1^C)$ *and* $\pi_1(p_0^C, R_1(p_0^C)) < \pi_1(q_0^C, R_1(q_0^C))$.

**Proof** Let $p_1^D := R_1(p_0^C)$, and let $q_i^D$ be the resulting output of firm $i$ when $(p_0, p_1) = (p_0^C, p_1^D)$. Consider the Cournot case. Suppose that firm 1 deviates from the collusion and chooses $q_1 = q_1^D$ given $q_0 = q_0^C$. Its profit is $\pi_1(q_0^C, q_1^D)$. Because $q_1^D \neq R_1(q_0^C)$, $\pi_1(q_0^C, q_1^D) < \pi_1(q_0^C, R_1(q_0^C))$.

From Lemma 1(ii) we obtain $p_1^D > p_1^C$. We obtain $q_0^D > q_0^C$ because $q_0$ is increasing in $p_1$. Because $\pi_1(q_0, q_1)$ is decreasing in $q_0$, $\pi_1(q_0^C, q_1^D) > \pi_1(q_0^D, q_1^D) = \pi_1(p_0^D, p_1^C)$. These imply that $\pi_1(p_0^C, R_1(p_0^C)) < \pi_1(q_0^C, R_1(q_0^C))$.

A similar principle applies to the deviation incentive for firm 0. ■

We explain the intuition behind the result that the one-shot gain of the deviation is greater in the Cournot case than in the Bertrand case. If the private firm were to maximize current profit and not care about future profits, it would raise its price in the Bertrand case and reduce its output in

[9] See Ghosh and Mitra (2010).

the Cournot case. In the Bertrand case, the rival's price is given exogenously. Thus, the deviation increases the resulting output of the rival and is harmful for the private firm. By contrast, in the Cournot case, the rival's output is given exogenously, and thus, the abovementioned harmful effect dose not exist. Therefore, the private firm obtains a larger profit from the deviation in the Cournot case.

If the public firm were to maximize current welfare and not care about future welfare, it would reduce its price in the Bertrand case and increase its output in the Cournot case. In the Bertrand case, the rival's price is given exogenously. Thus, the deviation decreases the resulting output of the rival and is harmful for welfare. By contrast, in the Cournot case, the rival's output is given exogenously, and thus, the abovementioned harmful effect does not exist. Therefore, the public firm has a stronger incentive to deviate in the Cournot case, too.

Proposition 1 is in sharp contrast to the result in private oligopolies, in which one-shot gain of the deviation from a joint-profit-maximizing collusion is greater in the Bertrand case than in the Cournot case (Deneckere 1983, Gibbons, 1992).

Next, we investigate welfare implications. The following results state that Bertrand competition yields greater welfare than Cournot competition does when $\delta$ is sufficiently small (Proposition 2)[10], while the opposite result is obtained when $\delta$ is sufficiently large (Proposition 3).[11]

**Proposition 2** *If $\delta$ is close to 0, Bertrand competition yields greater welfare than Cournot competition.*

**Proof** Suppose that $\delta$ is sufficiently close to 0. Suppose that $(q_0^C, q_1^C)$ is sustainable and yields greater welfare than $SW(p_0^N, p_1^N)$. Because the deviation incentive is stronger under Cournot competition (Proposition 1), $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta\gamma q_1^C, \alpha - \beta q_1^C - \beta\gamma q_0^C)$ must be sustainable under Bertrand competition. Thus, Cournot competition never yields greater welfare than Bertrand competition.

Suppose that $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta\gamma q_1^C, \alpha - \beta q_1^C - \beta\gamma q_0^C)$ is sustainable and yields the greatest welfare among the sustainable outcomes. Then, either (6.13) or (6.14) is satisfied with equality because otherwise, a slight decrease in $p_1$ improves welfare, ensuring that (6.13) and (6.14) are satisfied. Under these conditions, $(q_0^C, q_1^C)$ must not be sustainable because the deviation incentive is stronger under Cournot competition for both firms and either (6.21) or (6.22) is not satisfied. Thus, Bertrand competition can yield strictly greater welfare than Cournot. ■

**Proposition 3** *If $\delta$ is close to 1, Cournot competition yields greater welfare than Bertrand competition.*

[10]This result does not depends on the assumption of grim trigger strategy because we use only Proposition 1 to derive this result.

[11]In the case of profit-maximizing collusion among private firms, both types of competition yield the same economic welfare when $\delta$ is sufficiently large because both yield the monopoly outcome.

**Proof** Suppose that $\delta$ is sufficiently close to 1. Suppose that $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta\gamma q_1^C, \alpha - \beta q_1^C - \beta\gamma q_0^C)$ is sustainable and yields greater welfare than $SW(p_0^N, p_1^N)$. Because the punishment for the deviation is more severe under Cournot competition (Result 1), $(q_0^C, q_1^C)$ must be sustainable under Cournot competition. Thus, Cournot competition never yields greater welfare than Bertrand competition.

Suppose that $(q_0^C, q_1^C)$ is sustainable and yields the greatest welfare among the sustainable outcomes. Then, either (6.21) or (6.22) is satisfied with equality because otherwise, a slight increase in $q_1$ improves welfare, ensuring that (6.21) and (6.22) are satisfied. Under these conditions, $(p_0^C, p_1^C) := (\alpha - \beta q_0^C - \beta\gamma q_1^C, \alpha - \beta q_1^C - \beta\gamma q_0^C)$ must not be sustainable because the punishment for the deviation is more severe under Cournot competition and either (6.13) or (6.14) is not satisfied. Thus, Cournot competition can yield strictly greater welfare than Bertrand competition. ■

On one hand, the deviation incentive is stronger under Cournot (Proposition 1) and this makes the collusion less stable. Therefore, it is more difficult for the government to form welfare-improving collusion under Cournot competition and this is harmful for welfare. On the other hand, the punishment effect is stricter under Cournot competition and this makes the collusion more stable. Therefore, it is easier for the government to form welfare-improving collusion under Cournot competition and this is beneficial for welfare. The former effect dominates when $\delta$ is small, while the latter effect dominates when $\delta$ is large. This leads to Propositions 2 and 3.

## 6.4 Concluding Remarks

In this study, we discuss welfare-improving collusion in mixed duopolies. We find that the deviation incentive is stronger under Cournot competition than under Bertrand competition. This leads the government to form welfare-improving collusion more easily under Bertrand competition, and thus, Bertrand competition can yield greater welfare. However, in a mixed duopoly, competition is more severe, and thus, the punishment for deviation is stricter under Cournot competition. This leads the government to form collusion more easily under Cournot competition, and thus, Cournot competition can yield greater welfare. The latter effect outweighs the former effect when the discount factor is large, and thus, Cournot competition is better for social welfare when firms are sufficiently patient.

In this study, we assume that a private firm is domestic. In the literature on mixed oligopolies, ownership of the private firm often matters[12] Our results, however, hold when the private firm is foreign. In this sense, our results are robust.

[12]See the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Fjell and Heywood (2002), Ogawa and Sanjo (2007), Heywood and Ye (2009), and Cato and Matsumura (2015).

Our results may be dependent on the assumption of duopoly. As discussed in chapter 3, Bertrand competition yields larger profit than Cournot competition as long as the number of private firms is equal to or smaller than four. However, they showed that Bertrand competition may yield smaller profit than Cournot competition if the number of private firms is equal to or larger than five, and always yields larger profit when the number of private firms is sufficiently large. Thus, if the number of private firms is large, the punishment effect becomes stricter under Bertrand competition for each private firm, whereas it remains weaker for the public firm, and therefore, the result becomes ambiguous. Moreover, if the number of private firms is sufficiently large, on one hand, it is more difficult to form collusion under both Bertrand and Cournot cases, and on the other hand, the welfare gain of collusion is small because competition yields an outcome close to the first-best outcome. Thus, in such a case, it might not be natural to discuss such welfare-improving collusion.[13]

[13]By contrast, in profit-maximizing collusion, the profit gain of collusion is greater when the number of firms is larger because more severe competition yields smaller profits.

## References


Abreu D (1988) On the theory of infinitely repeated games with discounting. Econometrica 56(2), 383–396.

Anderson, S P, de Palma A, Thisse, J-F (1997) Privatization and efficiency in a differentiated industry. European Economic Review 41(9), 1635–1654.

Bárcena-Ruiz J C (2007) Endogenous timing in a mixed duopoly: price competition. Journal of Economics 91(3), 263–272.

Bárcena-Ruiz J C, Garzón M B (2005) International trade and strategic privatization. Review of Development Economics 9(4), 502–513.

Bose A, Gupta B (2013) Mixed markets in bilateral monopoly. Journal of Economics, 110(2), 141–164.

Bowley A L (1924) The mathematical groundwork of Economics: an introductory treatise. Oxford University Press.

Brandão A, Castro S (2007) State-owned enterprises as indirect instruments of entry regulation. Journal of Economics 92(3), 263–274.

Brander J A, Spencer B J (1985) Export subsidies and international market share rivalry. Journal of International Economics 18(1-2), 83–100.

Brander J A, Zhang A (1993) Dynamic oligopoly behavior in the airline industry. International Journal of Industrial Organization 11(3), 407-435.

Bresnahan T F (1989) Empirical studies of industries with market power. Handbook of Industrial Organization II, 1012-1057.

Cabral L M B (1995) Conjectural variations as a reduced form. Economics Letters 49(4), 397-402.

Cato S, Matsumura T (2012) Long-run effects of foreign penetration on privatization policies. Journal of Institutional and Theoretical Economics 168(3), 444–454.

Cato S, Matsumura T (2013) Long-run effects of tax policies in a mixed market. FinanzArchiv 69(2), 215–240.

Cato S, Matsumura T (2015) Optimal privatization and trade policies with endogenous market structure. Economic Record 91(294), 309–323.

Cheng L (1985) Comparing Bertrand and Cournot equilibria: a geometric approach. RAND Journal of Economics 16(1), 146–152.

Chirco A, Colombo C, Scrimitore M (2014) Organizational structure and the choice of price versus quantity in a mixed duopoly. Japanese Economic Review 65(4), 521–542.

Chirco A, Scrimitore, M (2013) Choosing price or quantity? The role of delegation and network externalities. Economics Letters, 121(3), 482–486.

Colombo S (2016) Mixed oligopolies and collusion. Journal of Economics 118(2), 167-184.

Corneo G, Jeanne O (1994) Oligopole mixte dans un marche commun. Annales d'Economie et de Statistique 33, 73–90.

Dastidar K G (1997) Comparing Cournot and Bertrand in a homogeneous product market. Journal of Economic Theory 75(1), 205–212.

De Fraja G, Delbono F (1989) Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41(2), 302–311.

Deneckere, R (1983) Duopoly supergames with product differentiation. Economics Letters 11(1-2), 37–42.

Dixit A K (1979) A model of duopoly suggesting a theory of entry barriers. Bell Journal of Economics 10(1), 20–32.

Farrell J, Shapiro C (1990) Horizontal mergers: an equilibrium analysis. American Economic Review 80(1), 107-126.

Fjell K, Heywood J S (2002) Public Stackelberg leadership in a mixed oligopoly with foreign firms. Australian Economic Papers 41(3), 267–281.

Fjell K, Pal D (1996) A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29(3), 737–743.

Friedman J W (1971) A non-cooperative equilibrium for supergames. Review of Economic Studies 38(113), 1–12.

Fujiwara K (2007) Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92(1), 51–65.

Ghosh A, Mitra M (2010) Comparing Bertrand and Cournot in mixed markets. Economics Letters 109(2), 72–74.

Ghosh A, Mitra M (2014). Reversal of Bertrand-Cournot rankings in the presence of welfare concerns. Journal of Institutional and Theoretical Economics 170(3), 496–519.

Ghosh A, Mitra M, Saha B (2015) Privatization, underpricing and welfare in the presence of foreign competition. Journal of Public Economic Theory, 17(3), 433-460.

Ghosh A, Sen P (2012) Privatization in a small open economy with imperfect competition. Journal of Public Economic Theory 14(3), 441–471.

Gibbons R (1992) Game theory for applied economists, Cambridge, MA: Princeton University Press.

Gollop F M, Roberts M J (1979) Firm interdependence in oligopolistic markets. Journal of Econometrics 10(3), 313-331.

Green R J, Newbery D M (1992) Competition in the British electricity spot market. Journal of Political Economy 100(5), 929-953.

Gupta B, Venkatu G (2002) Tacit collusion in a spatial model with delivered pricing. Journal of Economics 76(1), 49–64.

Häckner J (2000) A note on price and quantity competition in differentiated oligopoly. Journal of Economic Theory 93(2), 233–239.

Hamilton J H, Slutsky S M (1990) Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. Games and Economic Behavior 2(1), 29–46.

Han L, Ogawa H (2008) Economic integration and strategic privatization in an international mixed oligopoly. FinanzArchiv 64(3), 352–363

Haraguchi J, Matsumura T (2014) Price versus quantity in a mixed duopoly with foreign penetration. Research in Economics 68(4), 338–353.

Haraguchi J, Matsumura T (2016) Cournot-Bertrand comparison in a mixed oligopoly. Journal of Economics 117(2), 117–136.

Heywood J S, Ye G (2009) Mixed oligopoly and spatial price discrimination with foreign firms. Regional Science and Urban Economics 39(5), 592–601.

Holmberg P (2009) Numerical calculation of an asymmetric supply function equilibrium with capacity constraints. European Journal of Operational Research 199(1), 285-295.

Holmberg P, Newbery D (2010) The supply function equilibrium and its policy implications for wholesale electricity auctions. Utilities Policy. 18(4), 209-226.

Holmberg P, Newbery D, Ralph D (2013) Supply function equilibria: step functions and continuous representations. Journal of Economic Theory 148(4), 1509-1551.

Ino H, Matsumura T (2010) What role should public enterprises play in free-entry markets? Journal of Economics 101(3), 213–230.

Ishibashi I, Matsumura T (2006) R&D competition between public and private sectors. European Economic Review 50(6), 1347–1366.

Ishida J, Matsushima N (2009) Should civil servants be restricted in wage bargaining? a mixed-duopoly approach. Journal of Public Economics 93(3–4), 634–646.

Itoh M, Kiyono K, Okuno-Fujiwara M, Suzumura K, Shell K (1991) Economic Analysis of Industrial Policy, Academic Press, New York.

Iwata G (1974) Measurement of conjectural variation in oligopoly. econometrica 42(5), 947-966.

Klemperer P, Meyer M (1989) Supply function equilibria in oligopoly under uncertainty. Econometrica 57(6), 1243-1277.

Lin M H, Matsumura T (2012) Presence of foreign investors in privatized firms and privatization policy. Journal of Economics 107(1), 71–80.

Maggi G (1999) The role of multilateral institutions in international trade cooperation. American Economic Review 89(1), 190–214.

Matsumura T (1998) Partial privatization in mixed duopoly. Journal of Public Economics 70(3), 473–483.

Matsumura T (2003) Stackelberg mixed duopoly with a foreign competitor. Bulletin of Economic Research 55(3), 275–287.

Matsumura T, Kanda O (2005) Mixed oligopoly at free entry markets. Journal of Economics 84(1), 27–48.

Matsumura T, Matsushima N (2005) Cartel stability in a delivered pricing oligopoly. *Journal of Economics* 86(3), 259–292.

Matsumura T, Matsushima N (2012) Airport privatization and international competition. Japanese Economic Review 63(4), 431–450.

Matsumura T, Matsushima N (2012) Competitiveness and stability of collusive behavior. Bulletin of Economic Research 64(s1), s22–s31.

Matsumura T, Matsushima N, Ishibashi I (2009) Privatization and entries of foreign enterprises in a differentiated industry. Journal of Economics 98(3), 203–219.

Matsumura, T, Ogawa, A (2010) On the robustness of private leadership in mixed duopoly. Australian Economic Papers 49(2), 149–160.

Matsumura T, Ogawa A (2012) Price versus quantity in a mixed duopoly. Economics Letters 116(2), 174–177.

Matsumura T, Ogawa A (2014) Corporate social responsibility or payoff asymmetry?: a study of an endogenous timing game. Southern Economic Journal 81(2), 457–473.

Matsumura T, Okamura M (2015) Competition and privatization policies revisited: the payoff interdependence approach. Journal of Economics 116(2), 137–150

Matsumura T, Okumura Y (2013) Privatization neutrality theorem revisited. Economics Letters 118(2), 324–326.

Matsumura T, Shimizu D (2010) Privatization waves. Manchester School 78(6), 609–625.

Matsumura T, Sunada T (2013) Advertising competition in a mixed oligopoly. Economics Letters 119(2), 183-185.

Matsumura T, Tomaru Y (2012) Market structure and privatization policy under international competition. Japanese Economic Review 63(2), 244–258.

Matsushima N, Matsumura T (2006) Mixed oligopoly, foreign firms, and location choice. Regional Science and Urban Economics 36(6), 753–772.

Merrill W C, Schneider N (1966) Government firms in oligopoly industries: a short-run analysis. Quarterly Journal of Economics 80(3), 400-412.

Mujumdar S, Pal D (1998) Effects of indirect taxation in a mixed oligopoly. Economics Letters 58(2), 199–204.

Mukherjee A, Suetrong K (2009) Privatization, strategic foreign direct investment and host-country welfare. European Economic Review 53(7), 775–785.

Nakamura Y (2013) Social welfare under quantity competition and price competition in a mixed duopoly with network effects: an analysis. Theoretical Economics Letters 3:211–215.

Nakamura Y (2015) Endogenous choice of strategic contracts in a mixed duopoly with bargaining over managerial delegation contracts. Australian Economic Papers 54(2), 121–134.

Nishimori A, Ogawa H (2002) Public monopoly, mixed oligopoly, and productive efficiency, Australian Economic Papers, 41(2), 185–190.

Nishimori A, Ogawa H (2005) Long-term and short-term contract in a mixed market. Australian Economic Papers, 44(3), 275–289.

Okuguchi K (1987) Equilibrium prices in the Bertrand and Cournot oligopolies. Journal of Economic Theory, 42(1), 128–139.

Ogawa H, Sanjo Y (2007) Location of public firm in the presence of multinational firm: a mixed duopoly approach. Australian Economic Papers 46(2), 191–203.

Pal D (1998) Endogenous timing in a mixed oligopoly. Economics Letters 61(2), 181–185.

Pal D, White M D (1998) Mixed oligopoly, privatization, and strategic trade policy. Southern Economic Journal 65(2), 264–281.

Pal R (2014) Price and quantity competition in network goods duopoly: a reversal result. Economics Bulletin, 34(2), 1019–1027.

Pal R (2015) Cournot vs. Bertrand under relative performance delegation: Implications of positive and negative network externalities. Mathematical Social Sciences, 75, 94-101.

Reisinger M, Ressner L (2009) The choice of price versus quantities under uncertainty. Journal of Economics and Management Strategy 18(4), 1155–1177.

Scrimitore M (2013) Price or quantity?: the strategic choice of subsidized firms in a mixed duopoly. Economics Letters 118(2), 337–341.

Scrimitore M (2014) Profitability under commitment in Cournot and Bertrand mixed markets. Journal of Institutional and Theoretical Economics 170(4), 684-703.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. RAND Journal of Economics 15(4), 546–554.

Shubik M, Levitan R (1980) Market structure and behavior (Harvard University Press, Cambridge, Massachussets, U.S.A.).

Tanaka Y (2001a) Profitability of price and quantity strategies in an oligopoly. Journal of Mathematical Economics 35(3), 409–418.

Tanaka Y (2001b) Profitability of price and quantity strategies in a duopoly with vertical product differentiation. Economic Theory 17(3), 693–700.

Tasnádi A (2006) Price vs quantity in oligopoly games. International Journal of Industrial Organization 24(3), 541–554.

Vives X (1985) On the efficiency of Bertrand and Cournot equilibria with product differentiation. Journal of Economic Theory 36, 166–175.

Vives X (2011) Strategic supply function competition with private information. Econometrica 79(6), 1919-1966

Wang LFS, Chen TL (2010) Do cost efficiency gap and foreign competitors matter concerning optimal privatization policy at the free entry market? Journal of Economics 100(1), 33–49.

Wang LFS, Lee JY (2013) Foreign penetration and undesirable competition. Economic Modelling 30(1), 729–732

Weitzman M (1974) Prices vs. quantities. Review of Economic Studies, 41, 477–491.

Wen M, Sasaki D (2001) Would excess capacity in public firms be socially optimal? Economic Record 77, 283–290.

Weyl E G, Fabinger M (2013) Pass-through as an economic tool: principles of incidence under imperfect competition. Journal of Political Economy 121(3), 528-583.

White M D (1996) Mixed oligopoly, privatization and subsidization. Economics Letters 53(2), 189–195.

Ye, G (2016) Leadership and privatisation in a mixed multi-product oligopoly: an endogenous timing model. Australian Economic Papers 55(2), 170–180.