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Stabilization of the Kelvin-Helmholtz instability by the transverse magnetic field in the magnetosphere-ionosphere coupling system

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Abstract. A sheared flow equilibrium in the magnetosphere-ionosphere (M-I) coupling system and its stability against the Kelvin-Helmholtz (K-H) instability are investigated within the ideal MHD by using a box-shaped magnetospheric model. Without forcing, the unperturbed transverse (convection) electric field responsible for the $\mathbf{E} \times \mathbf{B}$ drift declines exponentially with time due to the ionospheric Joule dissipation and the decay (e-folding) time is larger than one-half of the Alfvén bounce period. The restoring force due to the line bending associated with the transverse magnetic field is responsible for the existence of a critical height-integrated Pedersen conductivity $\Sigma_{Pc} \sim (\mu_0 V_A)^{-1}$, where V_A is the average Alfvén velocity along the field line, above which the K-H instability in the magnetosphere is suppressed completely.

Introduction

Previous electrostatic models studying the stability of the velocity shear layer in the M-I coupling system included the ionospheric coupling, but they are not fully 3-D, because the magnetosphere is either height-integrated [*Keskinen et al.*, 1988] or a current-voltage relationship has been used [*Lotko and Shen*, 1991; *Wei and Lee*, 1993].

MHD modes with non-zero $\mathbf{k} \cdot \mathbf{B}_0$, where $\mathbf{k}$ is the wave vector and $\mathbf{B}_0$ is the background magnetic field, distort the field line and the restoring force of field line bending contributes a stabilizing influence. The stabilizing, line-tying effect [*Miura and Kan*, 1992; hereafter referred to as MK92], which is due to non-zero $k_z B_{0z}$, where k_z is the vertical wave number and B_{0z} is the vertical unperturbed field strength, cannot be neglected when the Alfvén transit time between the two ionospheres becomes comparable to or smaller than the growth (e-folding) time of the K-H instability. When there is a transverse electric field causing shear of the electric field drift, the other non-zero term, $k_y B_{0y}$, in the line bending term $\mathbf{k} \cdot \mathbf{B}_0 = k_y B_{0y} + k_z B_{0z}$, where B_{0y} is the transverse field in the magnetosphere and k_y is a wave number in the direction of the unperturbed flow, has also a stabilizing influence. The purpose of the present paper is to describe the decaying nature of the sheared flow equilibrium in the 3-D M-I coupling system and to show the existence of a critical Pedersen conductivity Σ_{Pc}, above which the K-H instability is suppressed completely owing to the non-zero $k_y B_{0y}$.

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Sheared Flow Equilibrium in the M-I Coupling System

The upper panel of Figure 1 shows shear of the $\mathbf{E} \times \mathbf{B}$ drift velocity ($\mathbf{V}_E$) in the magnetosphere. This M-I coupling system can be modeled by a simple box-shaped M-I coupling system in the lower panel. The vertical component of the static background magnetic field is represented by $B_0 \hat{z}$, where $\hat{z}$ is the unit vector in the z direction and $B_0 > 0$. Each ionosphere has thickness h. Therefore, the ionospheres are located from $z = l$ to $z = l + h$ and from $z = -l$ to $z = -l - h$. We consider a limit $h \rightarrow 0$, so that we do not need to take into account the unperturbed magnetic field B_{0y} in the ionosphere. We take the unperturbed electric field $\mathbf{E}_0$ parallel to the x axis and assume that it is uniform for $-l \leq z \leq l$. As $\mathbf{E}_0$ decays with time due to the ionospheric Joule dissipation, $\mathbf{E}_0$ should actually be a function of x, z, and t.

Figure 2 is a cross section of the magnetosphere at a fixed z ($0 < z < l$). The field $\mathbf{E}_0$ has only the x component $E_{0x}(x, z, t)$, which is constant at $x \neq 0$, but is discontinuous at $x = 0$, and

$$E_{0x}(x, z, t) = \begin{cases} \mathcal{E}_{0x}(t) \; (x < 0, -l \leq z \leq l) \\ -\mathcal{E}_{0x}(t) \; (x > 0, -l \leq z \leq l) \end{cases} \tag{1}$$

where $\mathcal{E}_{0x}(t) > 0$. This electric field drives Pedersen and Hall currents in the ionosphere. The unperturbed ionospheric Pedersen current is connected to the unperturbed field-aligned current J_{0z} at $x = 0$. In the limit of $h \rightarrow 0$, these unperturbed currents produce an unperturbed y component of the field

$$B_{0y}(x, z, t) = \begin{cases} \mu_0 \Sigma_P E_{0x}(x, z, t) \; (0 < z < l) \\ -\mu_0 \Sigma_P E_{0x}(x, z, t) \; (-l < z < 0) \end{cases} \tag{2}$$

in the magnetosphere, which is shown by the horizontal dashed arrows in Figure 2. The unperturbed field-aligned current at $x = 0$ and the unperturbed ionospheric Pedersen current produce no magnetic field under the ionosphere (i.e., in the atmosphere) [*Fukushima*, 1976]. That is,

$$|B_{0y}(x, z, t)| = \mathcal{B}_{0y}(t) > 0 \; (\text{at } x \neq 0, 0 < z < l, \text{ and } -l < z < 0) \tag{3}$$

where

$$\mathcal{B}_{0y}(t) = \mu_0 \Sigma_P \mathcal{E}_{0x}(t) \tag{4}$$

The Hall current produces an x component of the unperturbed magnetic field

$$B_{0x}(x, z, t) = \begin{cases} \mu_0 \Sigma_H E_{0x}(x, z = l, t) \; (z > l + h) \\ -\mu_0 \Sigma_H E_{0x}(x, z = -l, t) \; (z < -l - h) \end{cases} \tag{5}$$

in the atmosphere. The frozen-in law in the magnetosphere is

$$\mathbf{E}_0 + \mathbf{V}_0 \times \mathbf{B}_0 = 0 \tag{6}$$

where the subscript 0 denotes the unperturbed quantities, $\mathbf{V}_0(x, z, t) = (0, V_{0y}(x, z, t) \equiv V_0(x, z, t), V_{0z}(x, z, t))$, and $\mathbf{B}_0(x, z, t) = (0, B_{0y}(x, z, t), B_0)$. Notice that $B_0 = B_{0z}$ and

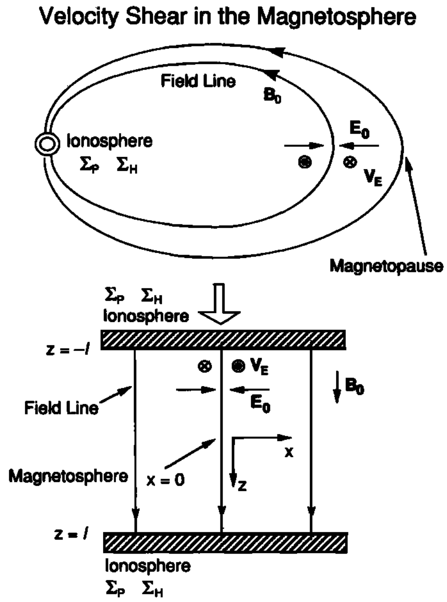

Figure 1. A shear of the $\mathbf{E} \times \mathbf{B}$ drift velocity in the magnetosphere (upper panel) and a simplified box-shaped M-I coupling system (lower panel). $\mathbf{B}_0$ is the static background field. The horizontal component of $\mathbf{B}_0$ is not shown explicitly. Ionospheres are represented by hatched regions.

$$V_0(x, z, t) \equiv V_{0y}(x, z, t) = \begin{Bmatrix} -V_0(t)\ (x < 0, -l < z < l) \\ V_0(t)\ (x > 0, -l < z < l) \end{Bmatrix} \quad (7)$$

where $V_0(t) > 0$ and

$$|V_{0z}(x, z, t)| = V_{0z}(t)\ (\text{at } x \neq 0, -l < z < 0 \text{ and } 0 < z < l) \quad (8)$$

with $V_{0z}(t) = E_{0x}B_{0y}/(B_0^2 + B_{0y}^2)$. The equation (6) has the only x component, which can be written as

$$E_{0x} = -(V_0 B_0 - B_{0y} V_{0z}) \quad (9)$$

Figure 3 shows a 3-D view of the magnetic field configuration in the magnetosphere. The unperturbed magnetic field line is straight and the magnetic field points downward only at x = 0 and it has a transverse component B_{0y} (x, z, t) at

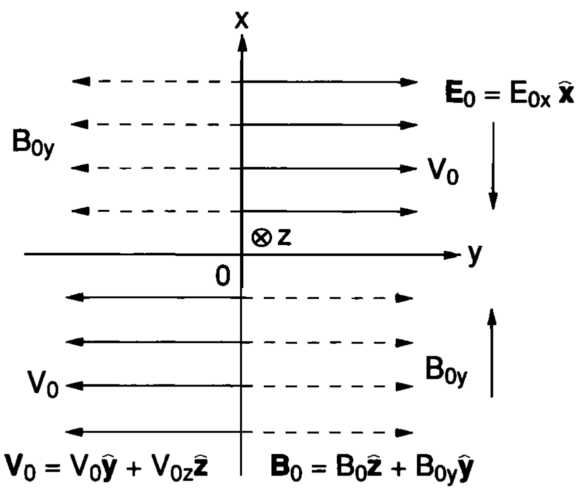

Figure 2. A cross section of the magnetosphere at a fixed z ($0 < z < l$). The unperturbed field B_{0y} is shown by the dashed arrows and the unperturbed y component of the electric drift V_{0y} = V_0 is shown by the solid arrows.

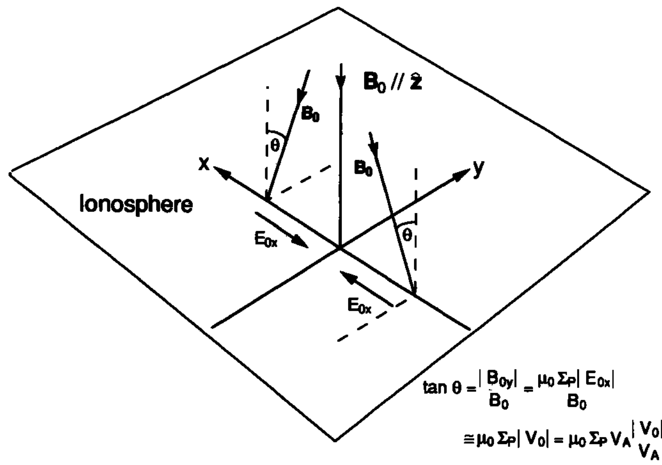

Figure 3. A 3-D view of the magnetic field configuration. The rectangular surface represents the ionosphere. Three magnetic field lines at x > 0, x = 0, and x < 0 are shown by solid lines. The unperturbed magnetic field points downward only at x = 0 and it has a component B_{0y} at $x \neq 0$.

$x \neq 0$, which is uniform at $x \neq 0$ and $z \neq 0$, and is discontinuous at x = 0 and z = 0. The angle θ (> 0) between the magnetic field line at $x \neq 0$ and the z-axis is given by

$$\tan\theta = B_{0y}/B_0 = \mu_0\Sigma_P E_{0x}/B_0 \cong \mu_0\Sigma_P V_0 \quad (10)$$

where we assumed $B_{0y} << B_0$.

Figure 4 shows a cross section of the M-I coupling system at a fixed x for a small V_0 (left panel) and for a large V_0 (right panel). The solid line in the magnetosphere represents the magnetic field line at x > 0 and the dashed lines represent magnetic field lines at $x \leq 0$. The field line is kinked at z = 0, because the ionospheric foot of the field line cannot move freely because of the finite Pedersen conductivity. Therefore, the magnetic field line is stretched in the y direction by the magnetospheric plasma flow.

Figure 5 shows unperturbed quantities in the M-I coupling system at a fixed y. The Pedersen current in the ionosphere is connected to a field-aligned current J_{0z} at x = 0 (note that B_{0y} = 0 at x = 0).

Decaying Nature of the Equilibrium

When the "unperturbed" $\mathbf{E}_0$ is set up in the magnetosphere, $\mathbf{E}_0$ must decay with time, because of the ionospheric Joule dissipation. In other words, the unperturbed configuration

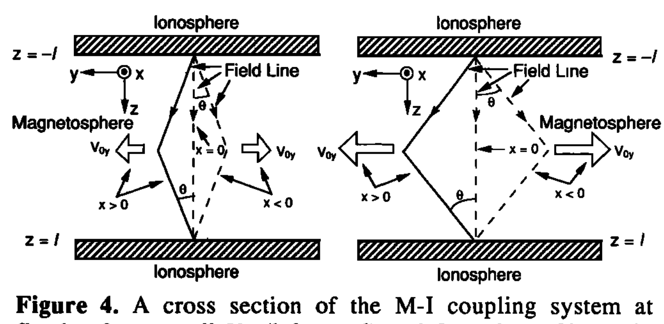

Figure 4. A cross section of the M-I coupling system at fixed x for a small V_0 (left panel) and for a large V_0 (right panel). The solid line in the magnetosphere represents the field line at x > 0 and the dashed lines represent field lines at x $\leq$ 0. Large white arrows represent electric drift by $\mathbf{E}_0$.

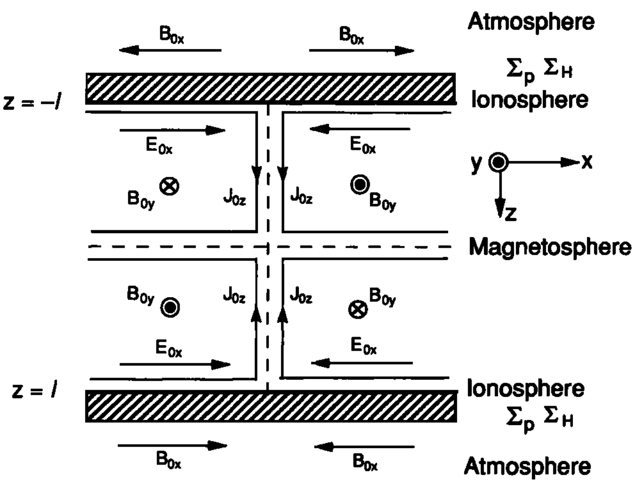

Figure 5. A cross section of the unperturbed electric field, magnetic field, and current in the magnetosphere at a fixed y. The solid vertical arrows represent the direction of the current.

shown in Figures 4 and 5 is not in strict dynamical equilibrium as *Galinsky and Sonnerup* [1994] noted, because the y component of $\mathbf{J} \times \mathbf{B}$ force is not balanced by any other force.

Here, we calculate the decay time of $\mathbf{E}_0$. Since we consider the uniform $\mathbf{E}_0$ at $x \neq 0$, the advection term $(\mathbf{V}_0 \cdot \nabla)\mathbf{V}_0$ vanishes at $x \neq 0$. Therefore, incompressible MHD equations yield

$$\partial/\partial t\, [\, \rho\, \mathbf{V}^2/2 + \mathbf{B}^2/(2\mu_0)] = -\mu_0^{-1}\, \nabla \cdot (\, \mathbf{E} \times \mathbf{B}\,) - \nabla \cdot (\, p\, \mathbf{V}\,) \quad (11)$$

where ρ is constant with time. We take a volume integral of (11) in the magnetosphere at $x \neq 0$. For the volume of the integral let us take a volume characterized by $0 \le x \le l_x$, $0 \le y \le l_y$, and $-l \le z \le l$. We obtain from (11)

$$\frac{\partial}{\partial t}\int_0^{l_x} dx \int_0^{l_y} dy \int_{-l}^{l} dz \left(\frac{1}{2}\rho_0\, \mathbf{V}_0^2 + \frac{\mathbf{B}_0^2}{2\mu_0}\right)$$
$$= -\frac{1}{\mu_0}\left[\int_{z=l-\varepsilon} (\, \mathbf{E}_0 \times \mathbf{B}_0\,)\cdot d\mathbf{S} + \int_{z=-l+\varepsilon} (\, \mathbf{E}_0 \times \mathbf{B}_0\,)\cdot d\mathbf{S}\,\right] \quad (12)$$

where the subscript 0 denotes unperturbed quantities and we let ε (>0) $\to$ 0. The contribution from the last term of the R.H.S. of (11) vanishes because of the uniform p_0 and the incompressibility. The assumption of the uniform p_0 is justified, because only the region inside the magnetosphere is considered. Contributions to the R.H.S. of (12) from surface integrals at $y = 0$ and $y = l_y$ canceled out, because $\mathbf{E}_0$ and $\mathbf{B}_0$ are uniform at $x \neq 0$ and $z \neq 0$.

From the continuity of the tangential electric field we have

$$\mathbf{E}_0\, (\, z = l - \varepsilon\,) = \mathbf{E}_0^{I.S.} \quad (13)$$

where $\mathbf{E}_0^{I.S.}$ is the ionospheric electric field. The unperturbed field component $\mathbf{B}_{0\perp}$ (perpendicular to the z-axis) is given by

$$\mathbf{B}_{0\perp}\, (\, z = l - \varepsilon\,) = -\mu_0\, \Sigma_P\, \mathbf{E}_0^{I.S.} \times \hat{\mathbf{z}} \quad (14)$$

Using (13) and (14) we obtain

$$\int_{z=l-\varepsilon} (\, \mathbf{E}_0 \times \mathbf{B}_0\,)\cdot d\mathbf{S} = l_x\, l_y\, \mu_0\, \Sigma_P E_{0x}^2 \quad (15)$$

The same equation also holds at $z = -l + \varepsilon$. Substitution of (15) and the same equation at $z = -l + \varepsilon$ into (12) yields

$$l\; \partial/\partial t\, [\, \rho_0 (\, V_0^2 + V_{0z}^2\,)/2 + B_{0y}^2/(2\mu_0)\,] = -\Sigma_P\, E_{0x}^2 \quad (16)$$

where B_{0y} is given by (4) and

$$V_{0z}/V_0 = B_{0y}/B_0 = \tan\theta \cong \mu_0\, \Sigma_P\, V_0 = \mu_0\, \Sigma_P\, V_A \cdot M_A \quad (17)$$

where $V_A = B_0/(\mu_0\rho_0)^{1/2}$ and $M_A = V_0/V_A$. Therefore, we obtain

$$\frac{\rho_0}{2}(V_0^2 + V_{0z}^2) + \frac{B_{0y}^2}{2\mu_0} \cong \frac{\rho_0}{2}\frac{E_{0x}^2}{B_0^2}\left[1 + \mu_0^2\, \Sigma_P^2 V_A^2 (1 + \mu_0^2\, \Sigma_P^2 V_A^2 \cdot M_A^2)\right] \quad (18)$$

where we assumed $B_{0y} \ll B_0$. Therefore, if we assume reasonably $(\mu_0 \Sigma_P V_A M_A)^2 \ll 1$ (note that $M_A \ll 1$ for the magnetospheric flow), substitution of (18) into (16) shows that $\mathbf{E}_0$ declines exponentially with an exponential decay rate given by

$$\tau_d^{-1} = \frac{V_A}{l}\frac{\mu_0\, \Sigma_P\, V_A}{1 + (\mu_0\, \Sigma_P\, V_A)^2} \quad (19)$$

The term $(\mu_0 \Sigma_P V_A)^2$ in the R.H.S arises from the magnetic energy associated with B_{0y}. Since the R.H.S. becomes maximum at $\mu_0 \Sigma_P V_A = 1$, we obtain $\tau_d \ge 2l/V_A$. This is an important relationship, because if the decay time of the unperturbed state is smaller than $2l/V_A$, such an "unperturbed" state will decay during one Alfvén transit time and will never be realized. In such a case the concept of "convection" of an entire flux tube would be useless and a stability analysis of the M-I coupling system would not be well posed.

Stabilizing Effect of the Transverse Magnetic Field B_{0y}

We assume that $\mathbf{B}_0$, $\mathbf{V}_0$, and $\mathbf{E}_0$ are constant with time. Such an assumption is valid only when we know posteriorly that the growth rate γ obtained from the perturbation analysis satisfies $\gamma\, \tau_d \gg 1$. That is, the present analysis is invalid when the instability development time becomes comparable with the decay time of the unperturbed state (or roughly Alfvén transit time for the limiting case). In MK92 the effect of B_{0y} and V_{0z} were included only in the ionospheric boundary condition and not in the magnetospheric normal-mode equation. Now the effect of B_{0y} is included in the magnetosphere. According to *Miura and Pritchett* [1982] the K-H instability is suppressed in an incompressible plasma by the magnetic tension force associated with the field line bending when

$$C = \int_{-\infty}^{\infty} [\rho_0 k_y^2 v_{0y}^2 - \mu_0^{-1} f^2]\, |\phi|^2 dx \le 0 \quad (20)$$

where $v_{0y}(x)$ is the y component of the unperturbed flow velocity, $\partial/\partial z$ is simply approximated by ik_z, and

$$f \equiv \mathbf{k}\cdot\mathbf{B}_0 = k_y\, B_{0y} + k_z\, B_0 \quad (21)$$

$$|\phi|^2 \equiv \left[\left|\frac{d}{dx}\left(\frac{\delta B_x}{f}\right)\right|^2 + (k_y^2 + k_z^2)\left|\frac{\delta B_x}{f}\right|^2\right] \quad (22)$$

where δB_x is the x component of the field perturbation and $B_0 = B_{0z}$. From (20) a sufficient condition for the stability is

$$[\rho_0 k_y^2 v_{0y}^2 - \mu_0^{-1} f^2]_{max} \le 0 \quad (23)$$

where the subscript max means the maximum value at $-\infty < x < \infty$. By taking the most stringent condition, one can reduce (23) further to

$$[\rho_0 k_y^2 v_{0y}^2]_{max} - [\mu_0^{-1} f^2]_{min} \le 0 \quad (24)$$

where the subscript min means the minimum value at $-\infty < x < \infty$. Since the addition of a constant to the unperturbed velocity profile does not affect the stability property of the flow, it is sufficient to consider the case of an antisymmetric $v_{0y}(x)$ as shown in Figure 2. If we denote the total velocity jump by $\Delta V = 2V_0 \cong 2E_{0x}/B_0$, (24) can be written as

$$\rho_0 k_y^2 (\Delta V/2)^2 - \mu_0^{-1} f_{min}^2 \le 0 \quad (25)$$

where

$$|f|_{min} = \big|\, |k_z\, B_0| - |k_y\, B_{0y}|\, \big| \quad (26)$$

From (25) and (26) a sufficient condition for the stability is

$$|\Delta V / V_{A0y}| \le 2\, \big|\, 1 - |k_z\, B_0/(k_y\, B_{0y})|\, \big| \quad (27)$$

where $V_{A0y} \equiv B_{0y}/(\mu_0\rho_0)^{1/2}$. Since

$$\frac{|\Delta V|}{|V_{A0y}|} = \frac{B_0}{B_{0y}} 2\, M_A = \frac{B_0}{\mu_0 \Sigma_P E_{0x}} 2\, M_A \cong \frac{2\, M_A}{\mu_0 \Sigma_P V_0} = \frac{2}{\mu_0 \Sigma_P V_A} \quad (28)$$

we obtain from (27) with the use of (10)

$$\frac{1}{\mu_0 \Sigma_P V_A} \leq \left| 1 - \left|\frac{k_z}{k_y}\right| \frac{1}{\mu_0 \Sigma_P V_0} \right| \qquad (29)$$

for the stability. Let us consider a case, where the following condition is satisfied:

$$\mu_0 \Sigma_P V_A \geq M_A^{-1} |k_z / k_y| \qquad (30)$$

In this case (29) can be reduced to

$$\mu_0 \Sigma_P V_A \geq 1 + M_A^{-1} |k_z / k_y| \qquad (31)$$

This is a sufficient condition for the absolute stability. It follows that the necessary condition for the instability is

$$\mu_0 \Sigma_P V_A < 1 + M_A^{-1} |k_z / k_y| \qquad (32)$$

From (31) it is obvious that when $\Sigma_P = \infty$, the K-H instability is suppressed completely. The first term in the R.H.S. of (31) (equal to unity) originates from the stabilizing, line bending term associated with non-zero $k_y B_{0y}$. Only the second small term (practically much smaller than unity) in the R.H.S. of (31) originates from the line-tying effect due to non-zero $k_z B_{0z}$. Note that in MK92 the local stabilizing effect by the field line bending associated with B_{0y} was not included in the magnetosphere and hence the stabilizing effect by B_{0y} found in the present study is different from the line-tying effect due to non-zero $k_z B_{0z}$, which was clarified in MK92.

Discussion

The stabilizing influence of the line bending on the K-H instability was first clarified by *Chandrasekhar* [1961]. While the *Chandrasekhar's* stability criterion that when the magnetic field is parallel to the flow, the total jump of the velocity must exceed twice the Alfvén velocity for the instability, is valid only for a uniform magnetic field, the present result (32) is valid for a self-consistent 3-D M-I couling system with inhomogenious magnetic field. *Seyler* [1988] performed 3-D two-fluid MHD simulations of auroral structures by including a dispersive effect of the finite electron inertia and reported that when the ionosphere is represented as an open circuit boundary (zero conductivity), the shear flow instability dominates as is consistent with (32). Furthermore, *Seyler* [1990] argues that for an oblique Alfvén wave model satisfying $E_{0x}/B_{0y} \equiv V_A$ [*Haerendel*, 1983], a 3-D instability found in his simulation is not the K-H instability but a collisionless tearing mode, because the total jump of the $\mathbf{E} \times \mathbf{B}$ flow velocity in his simulation is $\Delta V = 2V_{A0y}$, which satisfies the *Chandrasekhar's* criterion. But it should be noted that when the unperturbed magnetic field reverses its sign at x = 0, as in Figure 2, and the perturbation is 3-D, the stability condition is given by (27), which is slightly different from the *Chandrasekhar's* criterion. *Rankin et al.* [1993] showed a development of the K-H instability driven by field line resonances for $\Sigma_P \to \infty$, which should give the absolute stability according to the present study. However, since B_{0y} vanishes at the equator in their simulation, the growth of the K-H instability is possible near the equator even for the infinite Σ_P as has been observed in their simulation. A 3-D simulation of *Galinsky and Sonnerup* [1994] showing that the sufficiently large Σ_P can slow down the development of the K-H instability or suppress it completely, seems to be consistent with the existence of a critical Σ_P corresponding to $\mu_0 \Sigma_{Pc} V_A$ of order 1.0 (see their Figure 1), which is found in the present study.

A small-scale auroral vortex street (curl) often seen in association with discrete auroral arcs [*Hallinan and Davis*, 1970; *Oguti*, 1974] is visible evidence of the K-H instability driven by the shear of the $\mathbf{E} \times \mathbf{B}$ drift velocity. Figure 5 in MK92 shows that when $\mu_0 \Sigma_P V_A = 0.1 < \mu_0 \Sigma_{Pc} V_A \sim 1$ and $V_A/V_0 = 100$, the growth rate satisfies $\gamma \tau_d > \gamma 2l/V_A >> 1$ for $k_y l V_0 / V_A >> 0.5$. Since $k_y l V_0 / V_A >> 0.5$ is well satisfied for small-scale curls (~10km) and $l = 20R_e$, this demonstrates that small-scale curls satisfy the condition for the instability, i.e., $\Sigma_P < \Sigma_{Pc}$ and $\gamma \tau_d >> 1$. The existence of the critical Σ_P (see (31)) may explain the sporadic appearance of curls and rays (auroral vortex streets seen from the side) in the nightside auroras, because Σ_P inside discrete arcs is enhanced, and it may also explain relative scarcity of rays in the dayside cusp auroras (*Hallinan*, private communication, 1987), because the dayside Σ_P may be larger than Σ_{Pc} owing to a slight presence of sunlight.

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