論文内容の要旨

Correlations in quantum many-body systems and quantum information theoretic measures

（量子多体系における相関と量子情報的測度）

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Correlations between constituents are sources of various phenomena in matters. Generic properties of multipartite correlations in quantum many-body systems are far from fully understood in contrast to bipartite correlations. In this thesis, we study generic properties of multipartite correlations in quantum many-body systems by using theoretical tools developed in quantum information theory.

The first subject of this thesis is multipartite correlations in gapped ground states on a two-dimensional (2D) spin lattice. These systems can exhibit topologically ordered phases which cannot be described by conventional theory of phases based on symmetry-breaking. It has been discovered that ground states and excitations in topologically ordered phases are useful for constructing fault-tolerant quantum computer. Ground states in topologically ordered phases contain characteristic multipartite correlations at large-scale, while all bipartite correlations vanish at a long distance. These characteristic multipartite correlations are known to be a necessary resource for protecting stored quantum information from perturbations, and therefore a key feature of topologically ordered phases. However, a concrete classification or characterization (even rigorous definition) of the characteristic multipartite correlations has not been established.

A quantity that is considered as an indicator of the characteristic correlations is the topological entanglement entropy (TEE). However, it is difficult to accept that the TEE provides a proper classification or quantification of multipartite correlations, since the definition does not satisfy some of the criterions considered to be necessary for a good measure of multipartite correlations in quantum information theory. Moreover, the definition of the TEE is only valid for ground states, and cannot be straightforwardly generalized to mixed states. This disadvantage is problematic since investigating topologically ordered phases in finite temperature is highly desirable in condensed matter physics and quantum information theory/experiment.

In Chapter 4, we provide two new quantitative characterizations of the characteristic correlations based on quantum information theory, after presenting preliminaries on quantum information (Chapter 2) and topologically ordered phases (Chapter 3). In quantum information theory, correlations are often quantified by a geometrical or operational function. We show that both characterizations are possible for the characteristic multipartite correlations in topologically ordered phases. In addition, these characterizations can be applied for general quantum states and thus they are applicable for states at finite-temperature as well. The geometrical characterization is given by a function called the irreducible correlation. The irreducible correlation provides a decomposition of the amount of the multipartite correlation into different "orders" based on a geometrical structure of quantum states. We show that the multipartite correlations can be quantified by the highest-order irreducible correlation for regions depicted in Fig.1, and it coincides to the TEE under the condition of zero correlation length. States with a zero correlation length appear as fixed-points of the renormalization procedure, and therefore our result may represent a universal property of phases in the thermodynamic limit.

In information theory, a function is said to have an operational meaning when we can interpret the value of the function as a rate or efficiency of some information theoretical task. The operational characterization is given via a connection to a particular type of information processing protocols called secret sharing. We show that we can perform a secret sharing protocol by using a reduced state in a topologically ordered phase, and show that the TEE represents the optimal rate of the protocol. In this interpretation, the characteristic multipartite correlations in topologically ordered phases are necessary resources to perform the secret sharing protocol. This result is also valid for states with a zero correlation length.

A key concept behind our results is a condition on tripartite correlations called the Markov property. States or probability distributions satisfying the Markov property are called (spatial) Markov chains (or Markov networks). Properties of Markov chains are well-established both for classical and quantum systems. When a ground state has a zero correlation length, the reduced state associated to a certain region forms a Markov chain. We utilize properties of Markov chains to overcome difficulties in studying multipartite correlations.

In more realistic situations, ground states have nonzero correlation length and the Markov property only holds approximately. While in classical systems states approximately satisfying the Markov property can be well-approximated by Markov chains, this is not the case in quantum systems. Characterizing quantum states approximately satisfying the Markov property has been a long-standing problem in quantum information theory. Fawzi and Renner have provided a partial answer to this problem. We use their result to extend some of our results beyond the case of zero correlation length at the end of Chapter 4.

In classical physics, Gibb states of a short-range Hamiltonians are equivalent to Markov chains. This statement is known as the Hammersley-Clifford theorem providing a characterization of multipartite correlations in Gibbs states. The Hammersley-Clifford theorem has been generalized to quantum systems as well. For 1D systems, the generalized theorem shows that any quantum Gibbs state of a short-range commuting Hamiltonian is equivalent to a Markov chain. Since general quantum Hamiltonians contain non-commuting terms, it is natural to ask whether this relation still holds for general 1D Gibbs states.

The second subject of this thesis is the Markov property of 1D Gibbs states of short-range Hamiltonians. In Chapter 5, we investigate this question and provide an affirmative answer. Namely, we show that any 1D Gibbs state of a short-range Hamiltonian approximately satisfies the Markov property, and conversely, any state approximately satisfying the Markov property can be well-approximated by a 1D Gibbs state of a short-range Hamiltonian. Our results establish a generic property of 1D Gibbs state and also provide a characterization of states approximately satisfying the Markov property in terms of short-range Hamiltonians. We then provide several applications of our results for the area law of Gibbs states, a dimension independent bound on an entanglement measure and preparation algorithm of 1D Gibbs states on a quantum computer.

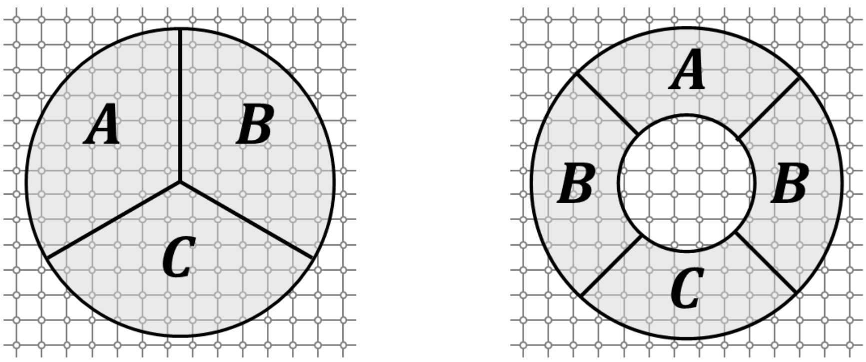

Fig.1: Examples of regions used to calculate the TEE.