論文の内容の要旨

Quantum entanglement and holography
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BACKGROUND

A key concept in modern quantum gravity theory is holography or holographic principle, which opened the door to a non-perturbative definition of quantum gravity as an equivalent non-gravity quantum theory in one lower dimensions. The equivalent non-gravity theory usually can be regarded as a theory located on the boundary of the gravity theory at infinity, in some sense. This equivalence between a $(d+1)$-dimensional gravitational theory (usually called "bulk theory") and a d-dimensional non-gravitational theory (usually called "boundary theory") is sometimes called holographic duality. This duality allows us to interpret a gravitational world as a holographic image constructed from the data on its boundary at infinity, because all the information of the world is already encoded somehow on its boundary. This is why this principle is named "holography"; the world emerges from its boundary.

In quantum gravity theory, the notion of spacetime becomes vague because general states are quantum superpositions of possible spacetime geometries, but in the classical limit, classical on-shell spacetime geometries dominate and we can say more surely that there is a spacetime in the world, which satisfies classical equations of motions like Einstein equations. Since we already know classical gravity theories, this classical limit in the bulk theory is one of the fundamental tools to study holographic duality. Typically, the classical limit in the bulk side corresponds to the large degrees of freedom limit in the boundary side, such as the large N limit in gauge theories or the large central charge limit in two dimensional conformal field theories. The spacetime thus emerges as a macroscopic or collective notion, similarly to the emergence of thermodynamics or fluid dynamics in coarse-graining microscopic details. We can also obtain a different geometry in this classical limit by taking a different quantum state from the start; a thermal state gives a black hole geometry and a time-dependent state gives a time-dependent geometry.

The most successful example of the holography is so-called AdS/CFT correspondence, which says that a quantum gravity theory on $(d+1)$-dimensional Anti de-Sitter space, AdS_{d+1}, is equivalent to a d-dimensional conformal field theory, CFT_d. This holographic duality is originally derived from considerations of D branes, which are non-perturbative extended objects on which strings can end in string theory, a successful theory of quantum gravity particulary suitable for perturbative calculations. We can also get a different bulk by modifying the theory itself, and so there are a variety of directions to extend AdS/CFT correspondence, whose examples include a capped off geometry given by the vacuum state of a gapped theory. Now holography is widely studied in more general situations than AdS/CFT correspondence, no matter whether they have stringy constructions with branes or not, such as many kinds of holographic models of condensed matters, generalizations to non-relativistic theories or to higher spin theories, and dS/CFT correspondence or flat space holography.

Mainly based on the study of AdS/CFT correspondence, a considerable number of dictionaries have been composed to translate physical quantities or concepts in one theory to the other via the holographic duality, such as global symmetries, correlation functions, responses to external sources, partition functions, thermodynamics, phase transitions, and spectrum of operators. The holographic duality remains mysterious, though, especially on how the information or structure of the bulk spacetime is encoded in the boundary quantum theory in one lower dimensions. This mystery would be very fundamental to holography in the sense that quantum gravity theory is a quantum theory for spacetimes.

There have been a huge amount of attempts to probe the bulk structure via holography, of which one of the most important breakthroughs is the holographic formula of entanglement entropy assigning a surface area in the bulk side to quantum entanglement in the boundary side, at the rate of one (qu-)bit per four Planck areas. This relation is so important because area is one of the most elemental quantities for spacetime geometries, and in fact, the formula is a realization of the original idea of the holographic principle that asserts in quantum gravity theory, the degrees of freedom live not in volumes but in areas. This relation is also a generalization of the famous Bekentein-Hawking for black hole entropy, extending the scope of the application from just the thermal entropy, namely, the entanglement entropy of the whole spacial region for a thermal state, to more general quantum information or entanglements, or from just black hole horizons to more general bulk surfaces, and can be a lot of help to study how the information or structure of the bulk spacetime is encoded in the boundary quantum theory in one lower dimensions. In fact, to bulk spacetimes, this formula is so fundamental that we can derive linearized Einstein equations from this formula with the aid of mathematical properties of entanglements, while it is not known whether we can derive full non-linear Einstein equations only from this formula.

MY RESEARCH

In this thesis, aiming to understand better how quantum gravity theory encodes spacetime structure as physical degrees of freedom or equivalently quantum information, I have studied various aspects of quantum entanglement from the viewpoint of holographic principle, which I believe is the most fundamental principle of quantum gravity theory. My research on this topic is divided into four parts.

Chapter 3

In the first part, we propose a new universal behaviour of quantum entanglement with respect to a mass gap in field theories based on a study on mutual information, a measure of quantum entanglement shared by separate regions. This proposal is checked for some explicit examples, a free field theory and the holographic dual of a strongly coupled field theory, for an annular region whose shape is complicated enough to construct meaningful mutual information. This observation thus gives not only a consistency check of the holographic principle, but also gives us a new insight into quantum entanglement in field theories.

In particular, we have examined entanglement entropies of an annulus or equivalently mutual informations across the annulus in three dimensions, resorting to numerical calculations. Massless and massive free scalar theories are studied by lattice discretization method, and a CFT_3 and a gapped theory holographically described by AdS_4 and CGLP background respectively, are studied by shooting method. As a consequence, we observed that the mutual information across the annulus exponentially decays with the mass gap times the annulus width, both in massive free scalar theory and in CGLP background. This term depends on the annulus width and thus is non-local with respect to the entangling region. From this observation, we conjectured that this exponentially decaying nonlocal behaviour is universal for mutual information in gapped theories, and because mutual information is composed of entanglement entropies, this conjecture directly means that in gapped theories, entanglement entropies also should have such exponentially decaying non-local terms, which have been neglected in expanding entropies with respect to the inverse of the mass gap.

Chapter 4

In the second part, with a concrete holographic setup of a thermalizing state represented by a growing black hole geometry, we demonstrate that entanglement entropy can grow linearly with time even without growth of time slice or wormhole in gravity side, while a famous literature attributes the linear growth of a holographic entanglement entropy of a thermalizing state to the growth of the volume of time slice or wormhole through a black hole. We speculate that the true origin of the linear growth of entropies is not the growth of some time slice but the situation that the corresponding surfaces enter into the event horizon of the black holes. This study thus improves our understanding of how the time dependence of entanglement entropies is represented in gravity side of holography, especially for thermalization processes. As the holographic dual of a thermalizng state, we took the time-dependent Janus black hole geometry in three dimensions, whose holographic dual CFT_2 state is proposed.

Specifically, we have investigated holographic entanglement entropies in three dimensional time-dependent Janus black hole with covariant holographic entanglement entropy formula, a covariant generalization of the original holographic entanglement entropy formula. We studied entropies of one interval, whose corresponding extremal surface is luckily obtained analytically, and consequently we see a time dependence typical of thermalizing states, that is, the entropies grow linearly with time at first and become saturated at some time proportional to the interval size in the end. Some limits that simplify the results are also studied.

Chapter 5

In the third part, we examine how we can extend the notion of renormalized entanglement entropy (REE) from a flat space to curved spaces, where REE is known to be a measure of degrees of freedom in the sense that it monotonically decreases along renormalization group (RG) flows. There are two ways to extend it according to whether we interpret the derivative with respect to the size of the spacial region in the definition of the renormalized entanglement entropy as enlarging the spacial region or as scaling the space, and our example shows that the former successfully decreases along RG flow while the latter does not. This result serves a concretized realization of our intuition that physical degrees of freedom are encoded in quantum entanglement of spacial regions.

As a concrete setup, we have defined two types of REE, $\mathcal{F}_{\rm C}$ and $\mathcal{F}_{\rm LM}$, out of cap entropies on a cylindrical spacetime $\mathbb{R} \times \mathbb{S}^2$, and calculated cap entropies and thus these REEs by applying a mass perturbation $m^2\phi^2$ to a conformally coupled scalar ϕ, analytically in the ultraviolet ($mR \ll 1$) and the infrared ($mR \gg 1$) regions, and numerically in the whole region of the scale mR. The results show that $\mathcal{F}_{\rm C}$ monotonically decreases all the way along the RG flow in our setup, while $\mathcal{F}_{\rm LM}$ does not.

Chapter 6

In the fourth part, we show that the recent proposal for the holographic formula of Rényi entropy, which contains more detailed information about quantum entanglement than entanglement entropy, expectedly satisfies inequalities that Rényi entropies should obey, given that bulk geometries are stable. This study thus gives a nontrivial consistency check of the formula and of the holographic principle itself. Moreover, we reformulate quantities representing quantum entanglement in analogy with statistical mechanics, which provides us a concise interpretation of the Rényi entropic inequalities as the positivities of entropy, energy and heat capacity, and makes clear a thermodynamic structure in the derivation of the holographic formula for quantum entanglements. This analogy is given by identifying the Rényi parameter n with the inverse temperature β and the modular Hamiltonian $-\log\rho$ with the usual Hamiltonian H, namely, applying statistical mechanics to the escort density matrix $\rho_n = \rho^n/\mathrm{Tr}[\rho^n]$. In light of this analogy, the modular entropy is identified with the thermal entropy, and so-called capacity of entanglement is identified with the heat capacity.

Particularly, our research has revealed that capacity of entanglement is holographically related to the Hessian matrix of the action with respect to metric perturbations, and that its non-negativity, the most nontrivial inequality, is holographically related to the stability of the bulk spacetime, which is usually implicitly assumed in holographic studies. Since the inequalities are roughly just the consequences of the unitarity of quantum theories, this work suggests some connection between the unitarity and the bulk stability.