論文の内容の要旨

Renormalization-Group Analysis of Classical Spins on Networks
(ネットワーク上の古典スピンの繰り込み群解析)

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In this thesis, we present both analytical and numerical treatment of classical spins on networks. This thesis consists of renormalization-group studies of two systems. The first one is the self-avoiding walk on a complex network and the second one is the Ising model on a square lattice. At first glance, the first system is not a spin model and the second system is not on a network but on a lattice, so that the title of this thesis may not be appropriate. We, however, explain that both problems can be actually treated as spin models on networks. Although renormalization-group study of classical systems have a long history and have been studied by many people, we will delve into the problem from a new viewpoint.

In the study of the first model, we focus on an analytical aspect. We treat complex networks, which have been a hot topic for the last fifteen years. The network has a repeated fractal structure like a matrioshka and is especially suitable to study by renormalization-group technique. We study the self-avoiding walk on the complex fractal networks called the (u,v)-flower by mapping it to the N-vector model in a generating-function formalism and carrying out the renormalization-group calculation of the generating function. First, we analytically calculate the critical exponent ν and the connective constant by a renormalization-group analysis in arbitrary fractal dimensions. We find that the exponent ν is equal to the displacement exponent, which describes the speed of diffusion in terms of the shortest distance. Second, by obtaining an exact solution for the (u,u)-flower, we provide an example which supports the conjecture that the universality class of the self-avoiding walk on graphs is *not* determined only by the fractal dimension.

In the study of the second model, we focus on a numerical aspect. We discuss dynamics of the model by using a new data structure, called tensor network. We propose a tensor-network algorithm for discrete-time stochastic dynamics of a homogeneous system in the thermodynamic limit. We map a d-dimensional nonequilibrium Markov process to a (d+1)-dimensional infinite tensor network by using a higher-order singular-value decomposition. As an application of the algorithm, we compute the nonequilibrium relaxation from a fully magnetized state to equilibrium of the one- and two-dimensional Ising models with the periodic boundary condition. Utilizing the translational invariance of the systems, we analyze the behavior in the thermodynamic limit directly. We estimated the dynamical critical exponent $z=2.16(5)$ for the two-dimensional Ising model. Our approach fits well with the framework of the nonequilibrium-relaxation method. On one hand, our algorithm can compute the time evolution of the magnetization of a large system precisely for a relatively short period. In the nonequilibrium-relaxation method, on the other, one needs to simulate dynamics of a large system for a short time. The combination of the two provides a new approach to the study of critical phenomena.