論文の内容の要旨

Geometrical approaches to defect operators in the class S theories

(クラスS理論における欠陥演算子への幾何学的アプローチ)

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The field theories we would like to analyze are interacting superconformal field theories called "class S theory" which are obtained as twisted compactifications of the 6d $\mathcal{N}=(2,0)$ SCFTs on Riemann surfaces C with punctures. Interestingly, even though almost all of the SCFTs have no definition based on the Lagrangians, some of their BPS observables have been evaluated assuming the dualities following their geometrical constructions. Indeed, in addition to the partition functions, these 4D/2D dualities also offer new geometrical descriptions of supersymmetric defects in such SCFTs, which are main subjects of this thesis.

Let us focus on the 4D gauge theory at first. As basic objects, there are supersymmetric Wilson-'t Hooft line operators They are one-dimensional objects, *loops* in 4D and the natural extensions of Wilson loop operators by replacing the electrically charged particles (quarks) by both electrically and magnetically charged heavy probes (dyons). As another generalization of loop operators, there are half-BPS surface operators. They are two-dimensional objects in 4D. They correspond to the insertions of heavy string-like objects or some non-dynamical vortex. Here we quote a fact that 6D $\mathcal{N}=(2,0)$ SCFTs have codimension-two defects and codimension-four defects. Then, before the dimensional reduction via C (a punctured Riemann surface), both loops and surface operators are expected to be some codimension four defects. Equivalently speaking, both defects come from codimension-four defects appearing in 6D $\mathcal{N}=(2,0)$ SCFTs and both have the same origin in 6D. Notice that 4D loop defects and 4D surface defects looks like codimension-one defects and local defects on the 2D side, respectively. Their appearances in those 2D theories on C look totally different.

Let us focus on the 2D theory. The 4D loop operators correspond to Verlinde network operators/Wilson network operators in the Liouville-Toda CFTs/q-deformed Yang-Mills theories. On the other hand, the 4D surface operators are mapped into (fully degenerate) vertex operators/difference operators in the CFTs/Yang-Mills theories. Geometrically, they can be represented as special punctures on the Riemann surface in both the set-ups. These works seems to give the answers to the above questions. However, they are far from the complete answer. One of the reasons is that they discuss only $SU(2)$ gauge theories and some special cases of $SU(3)$ type theories at most. Our first work provided the extension to general $SU(N)$-type theory.

The key concept in this work is the class S skein relation. To explain that, we regard the above 4D/2D duality relation as follows. This duality relates the charge of 4D loops and the network geometry in 2D and, naïvely, it is expected that this correspondence is one-to-one. However, this is not true actually and the map is many-to-one, that is to say, there are infinitely many networks to give the same 4D loop operators. In this thesis, we view the skein relations as the equivalence relations. Among the skein relations, there is a special class of skein relations called crossing resolutions. This relation connects the OPEs of loop operators in 4D and the resolutions of all crossings between networks in 2D. We found that these relations naturally coincide with those already well-known in mathematics. We do many consistency checks that this conjecture is true in the thesis.

At this stage, there appear no surface defects. Let us consider a situation in which both line operators and surface operators coexist. A key observation to describe their skein relations on the geometrical side as follows. On both the Liouville/Toda CFT and the q-deformed Yang-Mills theory, the concept of crossings of networks exists and, in fact, they correspond to the ordering of corresponding half-BPS line operators in one space direction determined by the unbroken supersymmetry in 4D gauge theory. Then, the existence of crossings among several networks suggests that there is a hidden direction which is exactly identified with one of physical directions on the 4D gauge theory side. When we recall that the expectation values of BPS loops are independent of the positions on that direction, it is natural to speculate that the networks are still topological in the new geometry. In this new three dimensional geometry, codimension four defects are expressed as knot with junctions and both surface defects and line defects are on the same ground. We refer the corresponding defects in the q-deformed Yang-Mills theory to as "punctured networks".

Once we have understood the geometrical relations on the 2D side, we go to the second question before : How to compute the correlators for the given general network defects in the 2D q-deformed Yang-Mills theories ? Naively speaking, it seems to be enough to replace ordinary Lie groups by "quantum group" as gauge groups at mathematical level. Indeed, the rigorous definition of Wilson loops without junctions in that case was already given based on quantum groups, but its extension to any networks is not obvious yet for several reasons. Furthermore, even if it can be well-defined, it is not useful for the actual computations because it needs the general invariant tensors in the quantum group sense. Instead of giving rigorous definitions, we will propose the direct procedure to obtain the conjectural expressions.

Finally, we make a few comments on the applications of the skein relations. The first concerns the dyonic loop operators of $\mathcal{N}=4$ $SU(N)$ Yang-Mills theory. When the electric charge and the magnetic charge of a dyonic loop are parallel in the weight system, there is an obvious realization of such dyonic loop in the class S language as a loop wrapping the torus. When they are not parallel, it was expected that they are represented by networks on the torus. We will give a complete description in the case of $SU(3)$. Later, we discuss the extension to general $SU(N)$ cases. The other is proposals of new kinds of skein relations.

In the recent developments of 4D supersymmetric gauge theories, many $\mathcal{N}=2$ SCFTs are constructed from the 6D $\mathcal{N}=2$ SCFTs with several codimension two defects. They are called class S theories. Although their field theoretical treatments are far from clear yet because of the lack of Lagrangians, we can describe BPS sectors of them geometrically once we admit the 4D/2D duality relations. Additions of several defects are interesting problems and, in fact, their developments have revealed the properties of (BPS) defects themselves and supported the 4D/2D duality relations. However, as for the correspondence between 4D BPS loop/line operators and 2D some topological network defects, there are only a few works before our developments.

We believe that our works shed new light on these subjects, at least, the geometrical descriptions and the computation of class S Schur indices with any closed 6D codimension four defects. The main discussions in this thesis are summarized as follows.

- Developments of class S skein relations and the charge/network correspondence in the 4D/2D duality relations
- Discussions of composite surface-line defects in the context of the Schur indices and their geometrical counterparts, what we call, Wilson punctured network defects in the 2D q-deformed Yang-Mills theory
- Proposal of conjectural formula for the Wilson punctured network defects and its non-trivial consistency checks
- Discovery of new kinds of skein relations

The first work discusses the relation between the "charge" of 4D BPS loop operators and their network realizations on the punctured Riemann surfaces with skein relations. For 4D $\mathcal{N}$=4 super Yang-Mills where some Lagrangian exists, we establish the correspondence dictionary for leading charges of 4D Wilson-'t Hooft loop operators. The second work is new in the context of the 4D/2D duality relations and unify both loop ans surface defects in term of the 6D SCFTs Such networks are geometrically knots with junctions in the three dimensional space which includes C and one of 4D directions and expected to be the counterparts of composite surface-line systems in 4D. The third work is the proposal of totally new formula although there is no derivation and many evidences are given in the final work. This allows us to compute the Schur indices in the presence of 4D BPS loop operators specified by networks on C. It seems to be strange because there is no definition of charges for loop operators but instead they are defined by the geometry on 2D side. However, even in the absence of defects, we cannot define the 4D theory in their own framework and we believe that these developments will sometimes help us to understand the 4D SCFTs themselves.