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KELVIN-HELMHOLTZ INSTABILITY FOR SUPERSONIC SHEAR FLOW AT THE MAGNETOSPHERIC BOUNDARY

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Abstract. It is demonstrated by means of a MHD simulation that a finite thick velocity shear layer with super-Alfvénic velocity jump is unstable to the Kelvin-Helmholtz (KH) instability no matter how large the sonic Mach number, a result suggesting that the tail flank boundary is unstable to the KH instability. For supersonic shear flow the unstable mode becomes damped-oscillatory in the magnetosheath. For both subsonic and supersonic shear flows, the energy flux density into the magnetosphere by the KH instability is large enough to replenish the plasma in the low latitude boundary layer with the tailward flow kinetic energy of observed intensity. A significant fraction of the energy flux density can reach deeper into the magnetosphere and its intensity is comparable to an energy flux density required for excitation of a ULF wave in the magnetosphere.

Introduction

It has long been suggested that the magnetopause boundary is unstable to the KH instability [*Dungey*, 1955]. The importance of the KH instability in the viscous interaction has recently been emphasized [*Miura*, 1984, 1987]. This instability has also been suggested as an important mechanism in exciting a ULF wave (Pc 5 pulsation of small azimuthal mode number ($m < 10$)) in the magnetosphere (see recent reviews by *Southwood and Hughes* [1983]; *Allan and Poulter* [1984]).

A measure of the energy flux required for the viscous interaction is the kinetic energy flux associated with the tailward flow in the low latitude boundary layer (LLBL). According to *Eastman* [1984], this energy flux is about 7 to 33 GW, which is about 0.06 % to 0.3 % of Φ ; here, Φ is the total kinetic energy flux of the solar wind flow incident on the magnetospheric cross section, which is nearly equal to 1.2×10^4 GW. The ionospheric Joule dissipation associated with the Pc 5 toroidal mode resonance is about 6 GW [*Greenwald and Walker*, 1980], which is about 0.05 % of Φ.

A substantial portion of the LLBL is on the closed field lines [e.g., *Mitchell et al.*, 1987]; therefore, if there is no replenishment of the tailward flow momentum across the magnetopause, the tailward flow in the LLBL should slow down rapidly, with increasing distance, by the ionospheric Joule dissipation [*Lemaire*, 1977; *Sonnerup*, 1980; *Nishida*, 1989]. Observation by *Hones et al.* [1972], however, shows that a LLBL-like tailward flow is present even inside the tail flank boundaries, a fact suggesting that there is a continuous replenishment of the tailward flow momentum across the tail flank boundary.

Since the transport of momentum and energy by the KH instability is ultimately determined by its nonlinear process, the nonlinear treatment of the instability [*Miura*, 1984, 1987; *Wu*, 1986; *LaBelle-Hamer et al.*, 1988; *Belmont and Chanteur*, 1989] is essential in evaluating the nonlinear transport. The purpose of this letter is to extend the previous simulation study of the KH instability [*Miura*, 1987] to an unexplored parameter regime, i.e., the tail flank boundary, where the magnetosheath flow is supersonic [*Spreiter et al.*, 1966], and to evaluate quantitatively the importance of the KH instability in the viscous interaction and in the ULF wave generation.

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Model and Eigenmode Structure

Figure 1 shows a MHD model of the finite thick tangential discontinuity representing the magnetospheric boundary on the equatorial plane. Both flow velocity and magnetic field are sheared across the boundary. Thicknesses of the velocity shear layer and the magnetopause are equal and represented by 2a, and the sound speed C_S is uniform across the boundary. The magnetosheath flow is characterized by $M_S = V_0/C_S$ and $M_A = V_0/V_{Ash}$, where V_0 is the total jump of the flow velocity across the boundary and V_{Ash} is the Alfvén speed in the magnetosheath. M_A is fixed to 10. The plasma β in the magnetosheath β_{sh} is given by $(6/5)(M_A/M_S)^2$ and that in the magnetosphere β_{sp} is taken equal to 0.2. A periodic boundary condition is imposed at $y = 0$ and $y = L_y$. We imposed a boundary condition, such as there being no mass flow ($v_x = 0$) across boundaries at $x = \pm 10a$. It then follows that B_x and derivatives, with respect to x of the remaining MHD quantities (ρ, v_y, v_z, B_y, B_z, p), must vanish at the boundaries ($x = \pm 10a$). Time is normalized by $2a/V_0$.

For the finite thick velocity shear layer, the growth rate of the KH instability is peaked at a wavelength comparable to $2\pi \times 2a$ [*Ong and Roderick*, 1972; *Walker*, 1981; *Miura and Pritchett*, 1982; *Mishin and Morozov*, 1983]. For a realistic set of 2a and V_0 at the magnetopause the fastest growing mode has a wave period in the Pc 4 to Pc 5 range (45 – 600 sec). This is the basis for considering the KH instability for the finite thick velocity shear layer as an excitation mechanism of the Pc 5 toroidal mode resonance. We should note that the magnetosheath flow changes from subsonic in the subsolar region to supersonic at the flank [*Spreiter et al.*, 1966]. The discontinuous vortex sheet becomes unstable to the KH instability only when V_0 lies between two critical velocities. The upper critical velocity for the two dimensional wave propagating in the direction of the shear flow is of the order of the sound speed [*Landau*, 1944] for the hydrodynamic case, and the fast magnetosonic speed for the MHD case. Whereas, for the finite thick velocity shear layer, there is no such upper critical velocity and the shear layer becomes unstable no matter how large the sonic Mach number [*Blumen et al.*, 1975].

The asymptotic form of the linear eigenmode equation for the total pressure perturbation δp^* at $x < -a$ in the magnetosheath becomes [*Miura and Pritchett*, 1982],

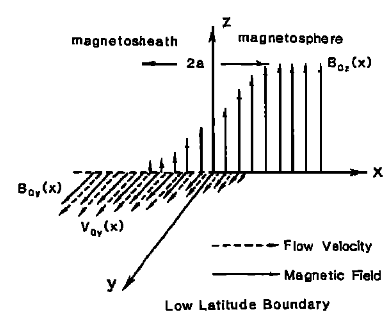

Fig. 1. A model of the finite thick tangential discontinuity representing the low latitude magnetospheric boundary.

$$d^2\delta p^*/dx^2 - \kappa_{sh}^2\, \delta p^* = 0 \qquad (1)$$

where

$$\kappa_{sh}^2 = k_y^2 - \Omega^2/[(1 - k_y^2 V_{Ash}^2/\Omega^2)C_S^2 + V_{Ash}^2]$$
$$= -(\Omega^2 - k_y^2 C_S^2)(\Omega^2 - k_y^2 V_{Ash}^2)$$
$$\times[(\Omega^2 - k_y^2 V_{Ash}^2)\, C_S^2 + \Omega^2 V_{Ash}^2]^{-1} \qquad (2)$$

and $\Omega = \omega - k_y V_0 = \omega_r + i\gamma - k_y V_0$, k_y being the wavenumber in the y direction. For the medium wavelength mode satisfying $\omega_r >> |\gamma|$, we can neglect the imaginary part of κ_{sh}^2. If $\kappa_{sh}^2 > 0$, δp^* becomes evanescent in the magnetosheath; for $\kappa_{sh}^2 < 0$, δp^* becomes oscillatory. Since $\Omega^2 - k_y^2 V_{Ash}^2$ is positive for $M_A = 10$, whether the eigenmode is evanescent or oscillatory depends on the sign of $\Omega^2 - k_y^2 C_S^2$. If we define the magnetosheath convective Mach number M_{Csh}, which characterizes the nature of a disturbance as subsonic or supersonic [*Papamoshou and Roshko*, 1988], by $M_{Csh} = -\Omega_r/(k_y C_S) = (V_0 - \omega_r/k_y)/C_S$, $M_{Csh} < 1$ gives an evanescent eigenmode and $M_{Csh} > 1$ gives an oscillatory mode. If the non-zero growth rate is taken into account in equation (2), the asymptotic behaviour of the eigenmode is damped-oscillatory for the supersonic case.

The eigenmode equation at x > a in the magnetosphere becomes [*Miura and Pritchett*, 1982]

$$d^2\delta p^*/dx^2 - \kappa_{sp}^2\, \delta p^* = 0 \qquad (3)$$

where $\kappa_{sp}^2 = k_y^2(1 - M_{Csp}^2)$. Here, M_{Csp} is the magnetospheric convective Mach number defined by $M_{Csp} = \omega_r/(k_y C_{Fsp})$, where C_{Fsp} is the fast magnetosonic speed in the magnetosphere defined by $C_{Fsp} = (C_S^2 + V_{Asp}^2)^{1/2}$, V_{Asp} being the Alfvén speed in the magnetosphere.

Simulation Results

A MHD simulation is initiated by adding a small seed of unstable perturbation, which has a wavelength equal to L_y, to the flowing equilibrium shown in Figure 1; the KH instability is, therefore, treated as an absolute instability. For all simulation runs, we used the same wavenumber k_y, which satisfies $2k_y a = 0.8$; this wavenumber is nearly equal to that of the fastest growing mode.

Figure 2 shows temporal evolution of the peak of $|v_x|$ normalized by V_0 for $M_S = 0.5 \sim 5.0$. For each sonic Mach number the peak of $|v_x|$ grows linearly in an early phase and tends to saturate in a later phase. We have also performed simulation for $M_S > M_A = 10$ ($\beta_{sh} < 1$) and have found that the initial perturbation grows and saturates. This result is consistent with the linear analysis of *Blumen et al.* [1975]. Saturation levels for the supersonic cases are smaller than those for the subsonic ones. This is due to the fact that, together with the vortex formation, standing compressional waves are also excited in the magnetosheath for the supersonic cases (see Figure 5).

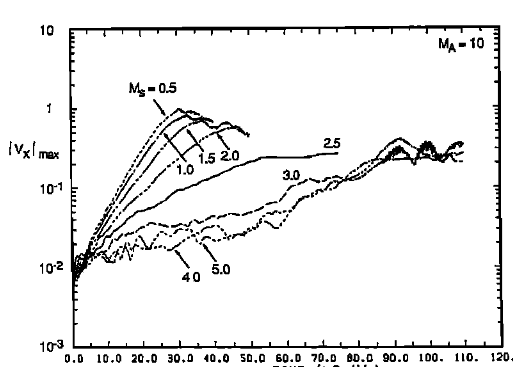

Fig. 2. Temporal evolution of the peak of $|v_x|$ normalized by V_0 for different sonic Mach numbers M_S and a fixed Alfvén Mach number ($M_A = 10$).

Figure 3 shows M_{Csh} and M_{Csp} calculated from the tailward phase velocity V_{ph} ($=\omega_r/k_y$) as a function of M_S. Both convective Mach numbers increase with increasing M_S. M_{Csh} is less than unity for $M_S < 2.3$, but it exceeds unity for $M_S > 2.3$. Therefore, it is expected that the KH eigenmode in the magnetosheath becomes oscillatory for $M_S > 2.3$. On the other hand, M_{Csp} remains less than unity for $M_S = 1.0$~5.0. Therefore, the KH eigenmode in the magnetosphere should be an evanescent form. Since M_{Csp} increases and approaches to unity with increasing M_S, we expect that the e-folding distance of the evanescent eigenmode increases as M_S increases.

We show, in Figure 4, profiles along x of the perturbations of the plasma pressure, the magnetic pressure, and the flow kinetic energy, which are normalized by the unperturbed plasma pressure in the magnetosheath and are averaged in the y direction over one wavelength for subsonic ($M_{Csh} < 1$)

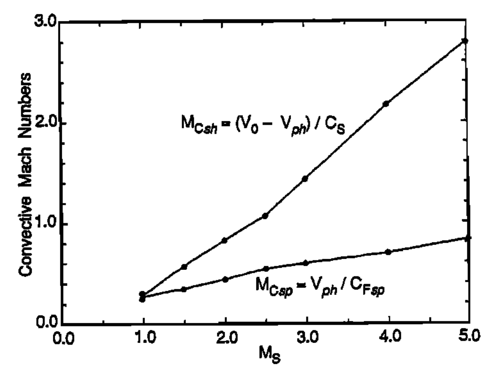

Fig. 3. Convective Mach numbers M_{Csh} and M_{Csp} as a function of the sonic Mach number M_S.

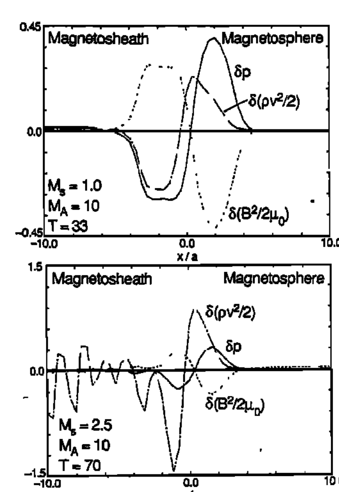

Fig. 4. Profiles along x of the perturbations of the pressure (solid line), the magnetic pressure (dotted line), and the flow kinetic energy (dot-dash line), which are normalized by the unperturbed plasma pressure in the magnetosheath and are averaged in the y direction over one wavelength, for $M_{Csh} < 1$ (upper panel) and $M_{Csh} > 1$ (lower panel) at their saturation stages.

(upper panel) and supersonic ($M_{Csh} > 1$) (lower panel) cases at their saturation stages; here, the perturbation $\delta F(t)$ is defined by $\delta F(t) = F(t) - F(t = 0)$. The eigenmode is damped-oscillatory in the magnetosheath for $M_{Csh} > 1$ (lower panel), although it is an evanescent form in the magnetosheath for $M_{Csh} < 1$ (upper panel). The eigenmode in the magnetosphere is evanescent for both supersonic and subsonic cases, as is expected from $M_{Csp} < 1$ (Figure 3). For both cases the pressure perturbation (solid line) near the magnetopause is out of phase with the magnetic pressure perturbation (dotted line) and, hence, the perturbation is a slow-mode type near the magnetopause. In the magnetosphere, however, both perturbations are in phase and the perturbation is a fast-mode type. The flow kinetic energy increased inside the magnetopause and decreased outside of it in both cases. This is due to the fact that the tailward flow momentum is transported from the magnetosheath into the magnetosphere by the instability and a velocity boundary layer (VBL), with a tailward flow, is formed inside the magnetopause.

Shown in Figure 5 are three dimensional views of the top surfaces of the magnetic pressure distributions for $M_{Csh} < 1$ (upper panel) and $M_{Csh} > 1$ (lower panel) at their saturation stages. It can be clearly seen that the standing, compressional oscillations are excited in the magnetosheath for the supersonic case (lower panel).

As we might well expect, we have found from a simulation run with $M_S = 2.5$ ($M_{Csh} > 1$) and $M_A = 10$ that a leading edge of the magnetosheath compressional wave (propagating in the positive y direction), generated by the KH instability at the magnetopause, steepens nonlinearly and finally develops into a shock discontinuity, whose shock front is nearly parallel to the x axis.

By taking a spatial average over one wavelength of the energy conservation equation $\partial \varepsilon/\partial t = -\nabla\cdot\mathbf{Q}$ [*Miura*, 1984], where ε is the energy density and $\mathbf{Q}$ is the energy flux density, we obtain a spatially averaged energy conservation equation $\partial\langle\varepsilon\rangle/\partial t = -\partial\langle Q_x\rangle/\partial x$, where the brackets represent the spatial average over one wavelength and $\langle Q_x\rangle$ is expressed by

$$\langle Q_x\rangle = \langle(\rho v^2/2 + \Gamma p/(\Gamma - 1))v_x + (\mathbf{E}\times\mathbf{B})_x/\mu_0\rangle \qquad (4)$$

Γ being the ratio of the specific heats. Shown in Figure 6 are profiles of each energy flux density averaged over one

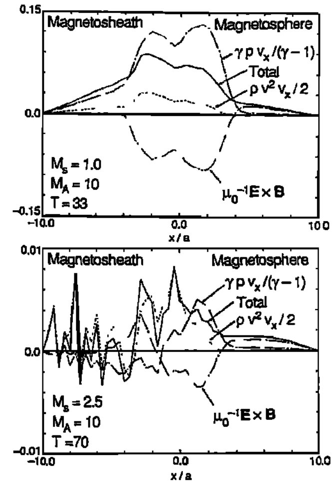

Fig. 6. Profiles along x of the kinetic energy flux density (dotted line), the enthalpy flux density (dot-dash line), the Poynting flux density (double dots-dash line), and the total energy flux density (solid line), which are averaged over one wavelength and normalized by the magnetosheath kinetic energy flux density $\rho_0 V_0^3/2$, for $M_{Csh} < 1$ (upper panel) and $M_{Csh} > 1$ (lower panel).

wavelength for $M_{Csh} < 1$ (upper panel) and $M_{Csh} > 1$ (lower panel). Each energy flux density is calculated in the rest frame of the magnetosphere and normalized by the unperturbed magnetosheath kinetic energy flux density $\rho_0 V_0^3/2$. The energy flux density is defined to be positive, if it is directed from the magnetosheath into the magnetosphere. For both cases, the Poynting flux, shown by the double dots-dash curve, is directed to the magnetosheath near the magnetopause, but it is cancelled by a part of the enthalpy flux, which is shown by the dot-dash curve and directed into the magnetosphere. Therefore, the total energy flux density $\langle Q_x\rangle$, which is shown by the solid curve, is directed from the magnetosheath into the magnetosphere, for both cases, near the magnetopause. As is expected from $\kappa_{sp}{}^2 = k_y{}^2(1 - M_{Csp}{}^2)$ the total energy flux density decays more slowly to zero with increasing x in the magnetosphere for M_S=2.5 than M_S=1.0.

The peak of the total energy flux density, which occurs near the magnetopause, is a measure of the net energy transported from the magnetosheath into the magnetosphere by the KH instability. We have found for $M_A = 10$ that the peak of the energy flux density normalized by $\rho_0 V_0^3/2$ decreases with increasing M_S and approaches a constant value 0.4 % of $\rho_0 V_0^3/2$ for large M_S (> 4), which occurs at the tail flank.

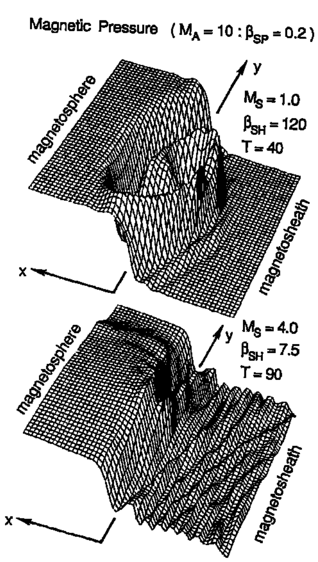

Fig. 5. Three-dimensional views of top surfaces of the magnetic pressure distributions for $M_{Csh} < 1$ (upper panel) and $M_{Csh} > 1$ (lower panel) at their saturation stages.

Discussion

In order to explain the plasma entry from the magnetosheath onto the closed field lines in the LLBL, several mechanisms [*Eviatar and Wolf*, 1968; *Lemaire*, 1977; *Heikkila*, 1982; *Tsurutani and Thorne*, 1982; *Nishida*, 1989], which cause or require breaking down of the ideal MHD, have been proposed. Whatever the mechanism of the plasma penetration is, a penetrated plasma with a tailward flow momentum is quickly slowed down with increasing distance. At the tail flank the energy flux density by the KH instability into the magnetosphere is larger than 0.4 % of $\rho_0 V_0^3/2$. This means that 0.4 % of Φ is transported into the magnetosphere, as the solar wind flow incident on the magnetospheric cross section steps aside from the magnetosphere and becomes a flow tangent to the flank boundary in the magnetosheath. Therefore, the energy flux density into the magnetosphere by the KH instability is large enough to replenish the plasma in the LLBL with the tailward flow kinetic energy of observed intensity (see Introduction). We expect, therefore, that a LLBL-like tailward flow exists even inside the tail flank, this being consistent with the observation by *Hones et al.* [1972].

We have seen in Figure 4 that the energy transported into the magnetosphere by the KH instability at the magnetopause is deposited in the magnetosphere in two different ways. First, the transported energy contributes to an increase in the flow kinetic energy in VBL produced inside the magnetopause. By this viscous interaction the tailward flow energy in the LLBL would be replenished when the flow kinetic energy in the LLBL is dissipated as Joule heat in the ionosphere. Second, a significant fraction of the energy flux density can reach deeper into the magnetosphere than VBL, and contributes to compression of magnetic field lines (dotted lines at x > 4a in Figure 4) and plasma in the form of a fast magnetosonic wave. This fast mode energy would be responsible for an excitation of a Pc 5 pulsation in the magnetosphere by the mechanism of field line resonance [*Southwood*, 1974; *Chen and Hasegawa*, 1974]. The fast mode energy can reach deeper into the magnetosphere as M_S increases, since κ_{sp} decreases with increasing M_{Csp}. Since the present simulation is performed for an energy-conserving system without ionospheric coupling, above speculations on the energetics of the KH instability should be tested more realistically in future by taking account of the fact that the magnetic field lines in the magnetosphere are tied to the ionosphere.

The lower panel of Figure 6 shows that the total energy flux density directed into the magnetosphere at x~6a, which is located deeper in the magnetosphere than VBL, is dominated by the Poynting flux and is about 1/5 of the peak energy flux density at x~0. Therefore, the energy flux density by the KH instability which can reach deep into the magnetosphere seems to be large enough to provide energy for the Pc 5 toroidal mode resonance (see Introduction). The simulation results further suggest that the KH instability at the tail flank, where the magnetosheath flow is supersonic, is an important source of compressional, oscillatory disturbances including shocks, in the magnetosheath.

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