博士論文（要約）

論文題目　Characteristic class and the ε-factor of an étale sheaf

（エタール層の特性類と ε 因子）

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CHARACTERISTIC CLASS AND THE ε-FACTOR OF AN ÉTALE SHEAF

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ABSTRACT. Recently, the singular support and the characteristic cycle of a constructible sheaf on a smooth variety over an arbitrary perfect field are constructed by Beilinson and Saito, respectively. Saito also defines the characteristic class of a constructible sheaf as the intersection of the characteristic cycle and the zero section of the cotangent bundle.

In this paper, based on their theory, we prove a twist formula for the ε-factor of a constructible sheaf on a projective smooth variety over a finite field in terms of characteristic class of the sheaf. This formula was conjectured by Kato and Saito in [3, Conjecture 4.3.11].

As a corollary of our formula, we prove that the characteristic classes of constructible étale sheaves on projective smooth varieties over a finite field are compatible with proper push-forward.

1. INTRODUCTION

Let k be a finite field of characteristic p and let $f\colon X \to \mathrm{Spec} k$ be a smooth projective variety. Let Λ be a finite field of characteristic $\ell \neq p$ or $\Lambda = \overline{\mathbb{Q}}_\ell$. For a constructible complex $\mathcal{F}$ of Λ-modules on X, let $D(\mathcal{F})$ be the dual $R\mathcal{H}om(\mathcal{F}, \mathcal{K}_X)$ of $\mathcal{F}$ where $\mathcal{K}_X = Rf^!\Lambda$ is the dualizing complex. Let $\chi(X_{\bar{k}}, \mathcal{F})$ be the Euler characteristic of $\mathcal{F}$. The L-function $L(X, \mathcal{F}, t)$ satisfies the following functional equation

$$L(X, \mathcal{F}, t) = \varepsilon(X, \mathcal{F}) \cdot t^{-\chi(X_{\bar{k}}, \mathcal{F})} \cdot L(X, D(\mathcal{F}), t^{-1}), \tag{1.0.1}$$

where

$$\varepsilon(X, \mathcal{F}) = \det(-\mathrm{Frob}; R\Gamma(X_{\bar{k}}, \mathcal{F}))^{-1} \tag{1.0.2}$$

is the epsilon factor (the constant term of the functional equation (1.0.1)). Let $cc_X\mathcal{F} \in CH_0(X)$ be the characteristic class of $\mathcal{F}$ defined in [7, Definition 5.7] using the characteristic cycle $CC\mathcal{F}$ of $\mathcal{F}$. The reciprocity map $CH_0(X) \to \pi_1(X)^{\mathrm{ab}}$ is defined by sending the class $[x]$ of a closed point $x \in X$ to the geometric Frobenius Frob_x. Let $\bar{x}$ be a geometric point of X and let ρ be a continuous representation of $\pi_1(X, \bar{x})$ over Λ of finite dimension. We also denote by $\det\rho\colon CH_0(X) \to \Lambda^\times$ the composition of $\det\rho$ and the reciprocity map $CH_0(X) \to \pi_1(X)^{\mathrm{ab}}$.

In this paper, we prove the following Theorem 1.1, which is conjectured by Kato and T. Saito in [3, Conjecture 4.3.11]. We note that their formula is written in terms of the Swan class of $\mathcal{F}$.

Theorem 1.1. *We have*

$$\det\rho(-cc_X\mathcal{F}) = \frac{\varepsilon(X, \mathcal{F} \otimes \rho)}{\varepsilon(X, \mathcal{F})^{\dim\rho}}. \tag{1.1.1}$$

When $\mathcal{F}$ is the constant sheaf Λ, this is proved by S. Saito [5]. If $\mathcal{F}$ is a smooth sheaf on an open dense subscheme U of X such that $\mathcal{F}$ is tamely ramified along $D = X \setminus U$, then Theorem 1.1 is a consequence of [6, Theorem 1]. If $\dim X = 1$, the formula (1.1.1) follows from the product formula of Deligne and Laumon [1, 7.11] and [4, 3.2.1.1]. In [9, 10], I. Vidal proved a similar result on a proper smooth surface over a finite field of characteristic $p > 2$ under one of the following two assumptions:

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(1) the sheaf $\mathcal{F}$ is of rank 1 and the corresponding character is of order np with $(n,p)=1$;
(2) the wild ramification of $\mathcal{F}$ is totally non-fierce, c.f., [10, Théorème 2.2].

As a corollary of Theorem 1.1, we prove the following compatibility of characteristic class with proper push-forward by using the injectivity of the reciprocity map [2, Theorem 1].

Corollary 1.2. *Let $f: X \to Y$ be a proper map between smooth projective varieties over k and let $\mathcal{F}$ be a constructible complex of Λ-modules on X. Then we have an equality in $CH_0(Y)$:*

$$f_*(cc_X\mathcal{F}) = cc_Y f_*\mathcal{F}. \tag{1.2.1}$$

In general, T. Saito conjectures that the characteristic cycle (resp. characteristic class) should be compatible with proper push-forward, cf., [7, 7.2] and [8, Conjecture 1].

Here is a rough idea of the proof of Theorem 1.1. We follow a similar strategy as S. Saito's proof in [5]. By taking a good pencil, we prove Theorem 1.1 by induction on the dimension of X. By using the product formula of Deligne and Laumon, it is reduced to computing local contributions on both sides. In order to calculate local contributions, we use the Milnor formula for characteristic cycle.

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