論 文 の 内 容 の 要 旨

Hamiltonian Formalism of Generalized Magnetohydrodynamics—
Structures Created on Casimir Leaves
（一般化磁気流体力学のハミルトニアン形式——
カシミール葉層に形成される構造）

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One of the most important aspects of plasma physics is the existence of a scale hierarchy. The scale hierarchy encapsulates the complexity of plasmas, which is reflected in the richness of plasma behavior in laboratory as well as in astrophysical and space plasmas. Such plasma phenomena require a nonlinear interaction between distinct scales. A typical example of a plasma fluid model endowed with scale hierarchy is extended magnetohydrodynamics (MHD). Extended MHD is the generalization of ideal MHD, and it is endowed with two primary two-fluid effects, i.e., Hall drift and electron inertia effects. These effects constitute the scale hierarchy in the extended MHD model—three disparate scales; one of them is the large scale defining the macroscopic structures, while the other two arise because of the ion and electron skin depths. By invoking the framework of Hamiltonian mechanics, the role of small scales in the creation of nonlinear structures in plasma can be thoroughly delineated.

Hence, in this dissertation, we present a rigorous and complete mathematical formulation of the noncanonical Hamiltonian structure of the extended MHD model *for the first time*. The underlying Poisson structure of the basic dynamical equations is obtained using a novel Lie algebra (generating bracket). This generating bracket satisfies an extended permutation law, which gives a unified framework for proving the important Jacobi's identity for hydrodynamical and magnetohydrodynamical models. The formulated Poisson algebra is shown to possess a nontrivial center, i.e., the Hamiltonian system is noncanonical in nature. Hence, this property gives rise to the Casimir invariants (generalized helicities). These Casimir invariants for extended MHD and the subsumed models are calculated, i.e., for Hall, inertial, and ideal MHD. Moreover, the necessary boundary conditions for extended MHD are investigated.

The extended MHD model is applied to derive nonlinear Alfvén, helicon, and TG waves, as well as for studying turbulence in the solar wind. The Casimir invariants of the system, which are features of the noncanonical nature of the Hamiltonian of the system, are the key to studying such plasma processes. Since the dynamics of plasma are restricted to stay on the surfaces of constant Casimir, thus all of these nonlinear phenomena appear as structures embodied on Casimir leaves.

Firstly, using the Casimir invariants in determining the equilibrium of the system by extremizing the energy-Casimir functional, the exact nonlinear Alfvén wave solutions of the fully nonlinear extended MHD system are derived for the first time. The solutions consist of two Beltrami eigenfunctions, which incorporate different length scales. A remarkable feature of the inclusion of these "small-scale effects" is that the wave patterns are no longer arbitrary; the large-scale component of the wave cannot be independent of the small-scale component, and the coexistence of them forbids the large-scale component to have a free wave form. This is in marked contrast to the ideal MHD picture where the Alfvén wave propagating on a uniform ambient magnetic field keeps its arbitrary shape constant.

Second, we originate a rigorous nonlinear theory for helicon and Trivelpiece-Gould (TG) waves that delineates the multi-scale structure of electromagnetic waves in extended MHD. The derived analytical solutions, which satisfy the set of nonlinear equations of extended MHD, manifest the intrinsic coupling of the large scale and the electron skin depth small scale; the former is realized as a helicon mode and the latter as a TG mode. In the regime of relatively low frequency or high density, however, the combination is shown to be comprised of the TG mode and an ion cyclotron wave (slow wave). The energy partition between these modes is determined by the helicities carried by the wave fields.

Finally, we use the nonlinear Alfvénic wave solutions to derive the kinetic and magnetic spectra by resorting to a Kolmogorov-like hypothesis based on the assumption of constant cascading rates of the energy and generalized helicities of extended MHD. The magnetic and kinetic spectra are derived in the ideal, Hall, and electron inertia regimes. The resultant spectra are compared against the observational evidence and shown to be in good agreement.

Thus, by tackling these set of problems, the utility and elegance of the Hamiltonian formalism in understanding the scale hierarchy of plasma fluid models has been demonstrated.