論 文 の 内 容 の 要 旨

論文題目　Stable Matchings on Matroidal Structures
（マトロイド的構造における安定マッチング）

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The stable matching model, introduced by Gale and Shapley (1962), is a mathematical formulation of two-sided markets, where each agent has a preference on the opposite agent set. Gale and Shapley provided an algorithm to find a stable matching, in which no agent has an incentive to change the current assignment. The algorithm has various applications such as the admission market between colleges and students and the labor market between doctors and hospitals. As the growth of application, however, there arise various factors which cannot be handled by existing models. Then, the development of the stable matching theory shows no sign of slowing down even half a century after its appearance.

In this thesis, we cope with some complicated settings of two-sided market models by extending the matroidal framework proposed by Fleiner (2001). We consider models with certain types of lower quotas, integer- or real- variables, and some complicated preferences. We capture these settings through matroids and extensions. Also, as a novel application of stable matchings, we solve a list coloring problem with supermodular constraints.

We first investigate a stable matching model in which agents have lower quotas in addition to upper quotas. This setting is well captured using generalized matroids. We provide a polynomial-time algorithm which finds a stable matching or reports the nonexistence. We also show properties of stable matchings such as the distributive lattice structure.

Next, we consider stable allocations, real-variable versions of stable matchings. We design the first polynomial-time algorithm to find a stable allocation in polymatroid intersection. The algorithm combines the policy of the Gale–Shapley algorithm with the augmenting path technique, which is common in the matroid literature.

Thirdly, we introduce a new notion "matroidal choice functions" to represent preferences under matroid constraints, which cannot be derived from modular value functions. We find a strong relationship between the greedy algorithm on matroids and the substitutability of choice functions, an essential property for the existence of a stable matching.

In the last part, utilizing a certain generalization of a stable matching, we prove the list supermodular coloring theorem, which generalizes Galvin's list edge coloring theorem for bipartite graphs. The existence of a stable matching plays a key role in the proof.

Our research deals with different types of generalizations of the stable matching model. In all our models, matroidal structures work effectively when we design algorithms and analyze inherent properties. We can realize that many desirable results, such as the existence of polynomial-time algorithms and the lattice structure of stable matchings, are supported by simple axioms of matroidal structures.

Since there are various kinds of two-sided markets in practice, it is important to establish comprehensive guidelines to handle current and forthcoming problems. Our results suggest that matroidal structures can be useful tools for that purpose.