研　究　速　報

Interaction Surface of Plastic Resistances for Exposed-Type Steel Column Base under Bi-axial Bending

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1. INTRODUCTION

Considerable damage was observed in some of steel structures during the 1995 Hyogo-ken Nanbu Earthquake. Among them, many exposed-type column bases failed in various patterns, such as brittle base plate fracture, excessive anchor bolt elongation, unexpected early anchor bolt failure due to inferior construction work, etc. An exposed type column base receives axial force and bi-axial bending, when receiving an arbitrary multi-directional earthquake motion. For this reason, it is more desirable to consider the effect of bi-axial bending at the ultimate stage of inelastic response in the design of steel column bases.

In this research, the inelastic behavior and the ultimate resistance of exposed-type steel column bases subjected to bi-axial bending are examined experimentally, and to clarify the interaction between the x-axis and y-axis plastic resistances in a column base, the limit analysis on the column base is performed. No axial load is applied to the column.

2. OUTLINE OF EXPERIMENTAL PROGRAM

These studies are conducted on exposed-type steel column bases composed of a square hollow steel column, two types of base plates of thickness, 9mm and 19mm, anchor bolts all with screws, and a base block. The system composed of these elements is set up as a cantilever column, and each test specimen is subjected to a different lateral displacement history. The test specimens are fabricated of cold-formed square hollow steel column of size: B = 150mm, D = 150mm, t = 6mm, effective height h = 1500mm. The test setup and the dimensions of the test specimen are shown in Fig. 1 and Fig. 2, respectively. Table 1 gives the notation of the test codes used herein.

Table 1 Tests code

Loading type / t	Uniaxial-Y (UY)	Biaxial-Linear (BL)	Biaxial-Circular (BC)
19mm	ECB19SCUY	ECB19SCBL	ECB19SCBC
9mm	ECB9SCUY	ECB9SCBL	ECB9SCBC

Note: ECB stands for 'Exposed Column Base' and SC stands for ' Static Cyclic'. t : Thickness of base plate

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2.1 Loading programs

During the cyclic loading tests under biaxial bending, the column is subjected to lateral force on each loading direction. The experiment is conducted in 6 series of tests, first uni-axial static cyclic loading in y-direction (SCUY), then biaxial linear static cyclic loading (SCBL), finally biaxial circular static cyclic loading (SCBC) for 9mm and 19mm thick column bases. In order to ensure an accurate loading path in the x-y plane, only the displacement control is adopted. At the beginning of the tests, the

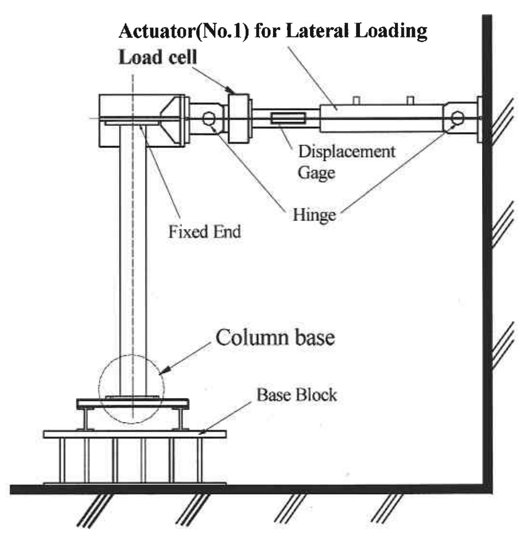
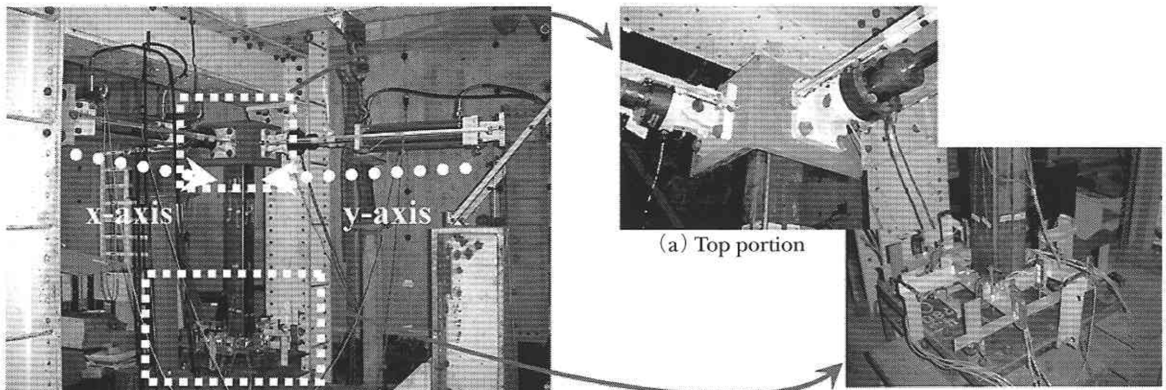


Fig. 1 Test setup

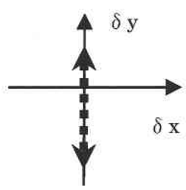
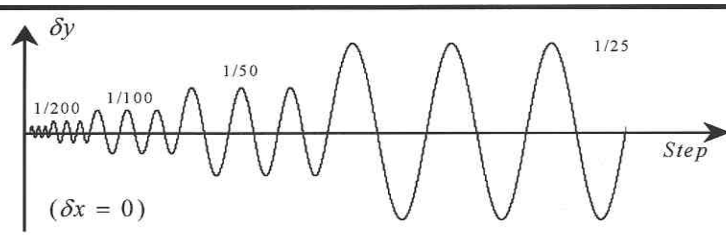
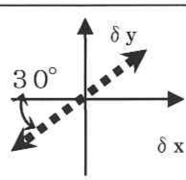
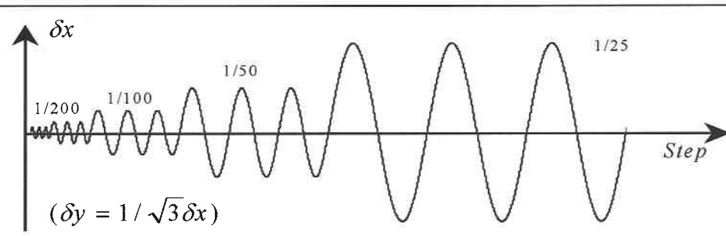
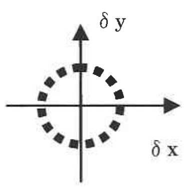
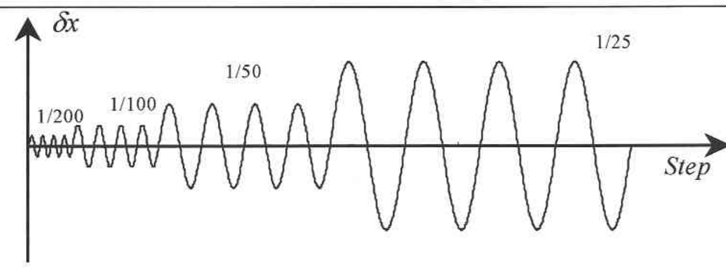
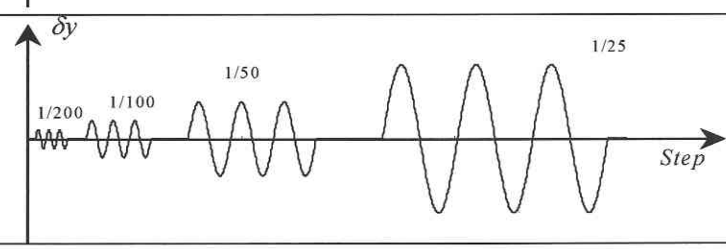


(a) Top portion

(b) Device of measurement

Photo 1 General views of test set-up

Table 2 Loading Program

Loading Type	Displacement path	Control program for top displacement
SC-UY	 Uniaxial-Y	
SC-BL	 Biaxial-Linear	
SC-BC	 Biaxial-Circular	
		

*δ: displacement of the top of the column

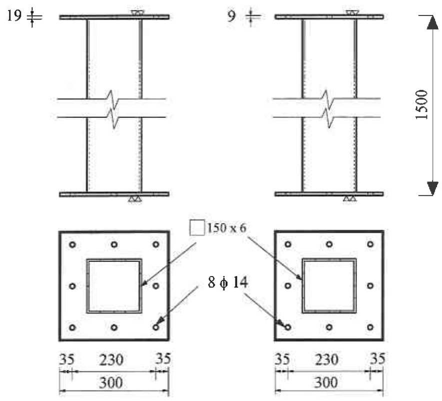


Fig. 2 Detail of specimen

Table 3 Mechanical properties of materials (in MPa)

	σ_y	σ_u
Base plate 9mm (JIS SS400)	270	450
Base plate 19mm (JIS SS400)	250	430
SHS 150x150x6 (JIS STKR 400)	780	900
		(in kN)
	T_y	T_u
M12 A. bolt (JIS SS400) (with All screw)	53	57

target amplitude of δ/L ratio, which is the drift angle, is taken 1/200, where L denotes the effective length of column and δ denotes the displacement at the top of the column. After the completion of every 3 loading cycles, the displacement amplitude is increased to 1/100, 1/50, 1/25, one after another. (in case of SCBC every 4 loading cycles). Every test loading is performed quasi-statically, slower than 0.001 m/sec. In uni-axial static cyclic loading in y-direction (SCUY), the load is applied uni-axially only in y-direction; there is no loading in x-direction. In biaxial linear static cyclic loading (SCBL), the ratio of δ_x / δ_y is controlled to be $1/\sqrt{3}$, which means that the loading direction makes an angle of 30 degrees with the y-axis. In biaxial circular static cyclic loading (SCBC), the column rotates in a circular manner around the axis passing through the center of the base plate along its length. In biaxial circular static cyclic loading (SCBC), there is a uni-axial loading inserted between each three cycles.

2.2 Material properties

The material properties for the steel plates were determined from tensile tests on strip cuts taken from different parts of the plates forming the steel square tube. Furthermore, in case of anchor bolts, ten pieces were chosen randomly and tested. The yield stress σ_y and the tensile strength σ_u of base plates with 9 mm, 19 mm thickness, square hollow steel column and the tension load capacity of each type of anchor bolt are summarized in Table 3.

3. ANALYTICAL MODEL FOR LIMIT ANALYSIS

To consider the interaction between the x-axis and y-axis bending resistances in a column base, we propose two different multi-spring models: one is base plate yielding type and the other is anchor bolt yielding type.

3.1 The model of anchor bolt yielding Type

The column base is modeled with an analytical model, where the anchor bolts yield but the base plate does not yield, as shown in Fig. 3. It is assumed that the base plate rotates around the compression springs beneath the column flange and tension spring is set at the anchor bolt positions. The equilibrium equation is derived by considering only the anchor bolt spring and compression spring forces.

3.1.1 Compact procedure

In the limit analysis based on the linear programming (Compact Procedure, Ref.[2]), the following problem is solved;

Maximize λ .... (1)

Subject to: Equilibrium equation

$$\{P\} = [\,Con\,] \cdot \{m\} \quad \cdots\cdots\cdots\cdots (2)$$

Plastic condition $|m_i| \leq m_{pi}$ .... (3)

Where, λ is load factor, $\{P\}$ is load pattern vector, $[Con]$ is connectivity matrix, $\{m\}$ is element force vector, and m_{pi} is element plastic capacity.

The equilibrium equation of this model shown in Fig. 3 and 4, which gives a relationship between the load and element force, is the following:

$$\begin{Bmatrix} M_X \\ M_Y \\ N \end{Bmatrix} = \begin{bmatrix} a & a & a & 0 & -a & -a & -a & 0 & -\frac{D}{2} & 0 & \frac{D}{2} & 0 \\ -a & 0 & a & a & a & 0 & -a & -a & 0 & -\frac{D}{2} & 0 & \frac{D}{2} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} \quad \cdots\cdots\cdots (4)$$

The plastic condition of this model is given by:

Anchor bolt: $0 \leq B_i \leq B_{ui}$,

Compression spring: $-C_{ui} \leq C_i \leq 0$ .... (5)

$C_{ui} = \infty$: that is, foundation is not failed.

3.1.2 Convex set theory (Ref.[3])

The fundamental of convex set theory (Ref. [3]) is as follows:

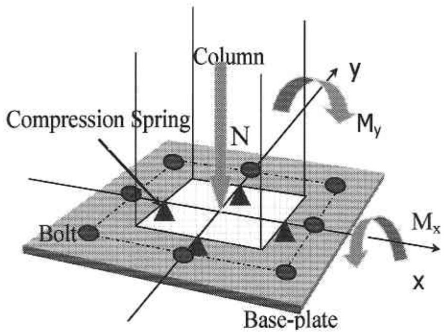


Fig. 3 Analytical model in case of anchor bolt yielding type

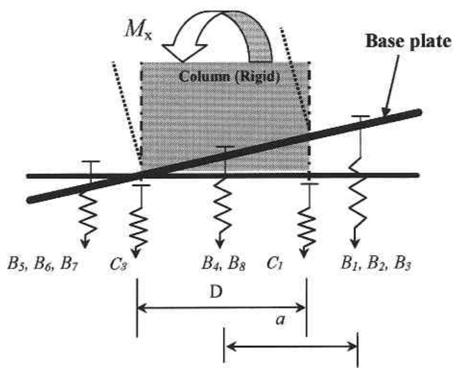


Fig. 4 Equilibrium between axial-spring and generalized stress in y-axis

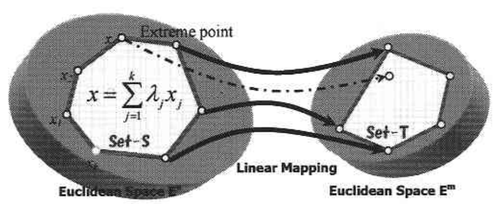


Fig. 5 Convex set theory[3]

1) Each point in a convex set is expressed by a convex combination of extreme points.

2) A set-T in an Euclidean space E^m formed by linear-mapping from convex set-S in E^n space becomes a convex set.

3) The extreme points of convex set-T exist in the images of the extreme points in convex set-S.

3.2 The model of base plate yielding type

The column base is modeled as shown in Fig. 8, where the anchor bolts don't yield but the base plate yields. The base plate is approximated as a set of beam element having a certain effective width as shown in Fig. A (see Appendix).

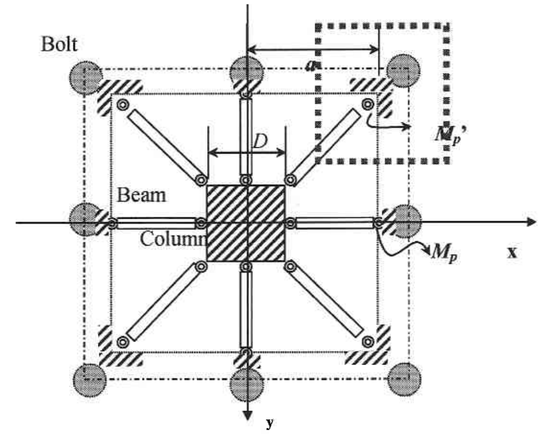


Fig. 6 Analytical model for base plate yielding type

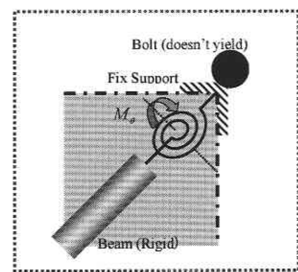


Fig. 7 Detail of rotation spring for base plate yielding type

3.2.1 Uni-axial bending moment

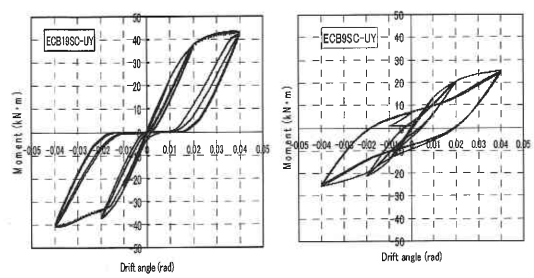
When bending moment is applied uni-axially, the ultimate strength is given by the following equation based on the virtual-work principle.

$$M_{Xp} = M_{Yp} = Mp' \cdot \sqrt{2}\frac{2+2\gamma}{1-\gamma} + Mp\frac{1+7\gamma}{1-\gamma} = \frac{\sqrt{2}\zeta(2+2\gamma)+1+7\gamma}{1-\gamma}Mp \qquad (6)$$

Where, ζ is the ratio of the effective width of base plate near the column corner to that of the base plate near the column side (Appendix), $\zeta = \frac{b_{eff}'}{b_{eff}}$, γ is the ratio of the lengths D/2 to a.

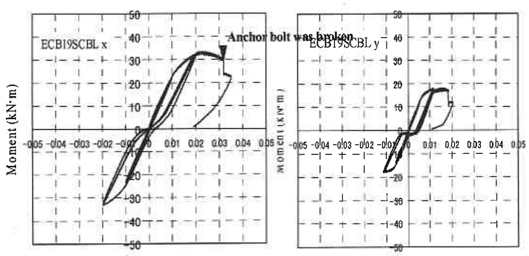
3.2.2 Bi-axial bending moment (45 degree)

When bi-axial bending is applied, the ultimate strength in the direction making 45 degree of angle with x and y axis is the largest. The ultimate strength in the direction making 45 degrees of angle with both of the axis is given by the following.

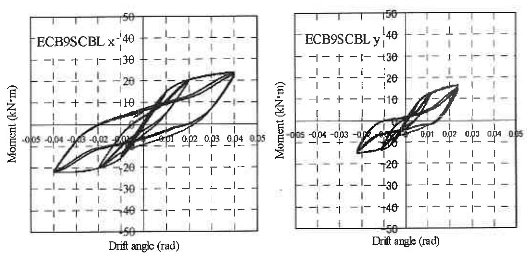


(a) ECB19SCUY　　(b) ECB9SCUY

Fig. 8 Hysteresis loops for SCUY

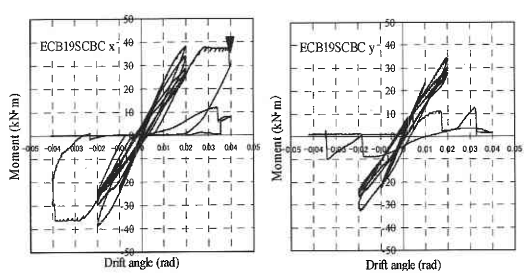


(a) ECB19SCBL-x　　(b) ECB19SCBL-y

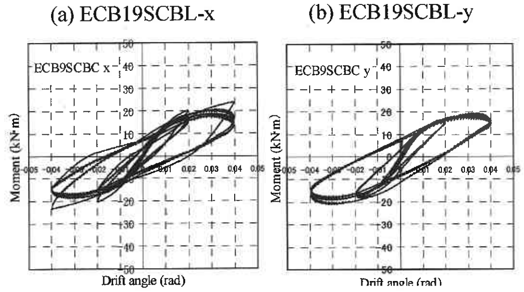


(c) ECB9SCBC-x　　(d) ECB9SCBC-y

Fig. 9 Hysteresis loops for SCBL



(a) ECB19SCBL-x　　(b) ECB19SCBL-y



(c) ECB9SCBC-x　　(d) ECB9SCBC-y

Fig. 10 Hysteresis loops for SCBC

$$M_{45p} = 2\frac{1+3\gamma}{1-\gamma}Mp' + 2\frac{3\sqrt{2}\gamma}{1-\gamma}Mp$$

$$= 2\frac{\zeta(1+3\gamma)+(1+3\sqrt{2})\gamma}{1-\gamma}Mp \quad \cdots\cdots (7)$$

4. TEST RESULTS AND DISCUSSION

4.1 Hysteretic responses

During the tests, the rectangular column remained elastic. In all directions, the case with anchor bolt yielding type showed a typical pinched hysteretic behavior. Although any failure did not occur for uniaxial loading, in case of biaxial cyclic loading for ECB19SCBL and ECB19SCBC, a bolt failure occurred when the drift angle on x-component has just passed the 0.03 rad. On the other hand, the case with base plate yielding type showed less pinched but slight degrading in hysteresis loops. In the first cycle of each 3-cycle group we have bilinear characteristics, but at the 2nd and 3rd cycle of each cycle group, pinched loops are observed. Additionally, it is observed that the hysteresis for the biaxial-circular path becomes more rounded around the unloading point.

4.2 Moment locus and drift angle Locus of bi-axial bending

The shape of bi-axial interaction surface is different for anchor bolt yielding type and base plate yielding type in the biaxial circular loading. Anchor bolt yielding type keeps an almost circular shape until the bolt failure. On the other hand, base plate yielding type shows a circular shape in the elastic range and changes to a square shape with rounded corners in the inelastic range, which leads to the approximation that strengths about two axes may be assumed independent under such a square-shaped interaction surface.

4.3 Resistance interaction surface of bi-axial bending

4.3.1 Interaction surface of Anchor bolt yielding type

The interaction surface of bi-axial bending resistances formed by linear mapping, based on the convex set theory, is shown in Fig. 11. Analytical parameters are taken as follows: Ultimate strength of bolt: $B_u = 56$ kN (based on the tensile strength of material σ_u), Ratio of length: $\gamma = 0.65$ （$a = 115$mm）

Although we observe that the resistance force of the loading path gradually degrading, it is confirmed from Fig. 11 that interaction surface of bi-axial bending based on the anchor bolt yielding type model agrees with the test results approximately. The shape of analyzed interaction surface based on the anchor bolt yielding type model becomes a circle (or diamond) and similar to the test results (SCBC and SCBL). Also, the uniaxis strength derived from the anchor bolt yielding type model is consistent with

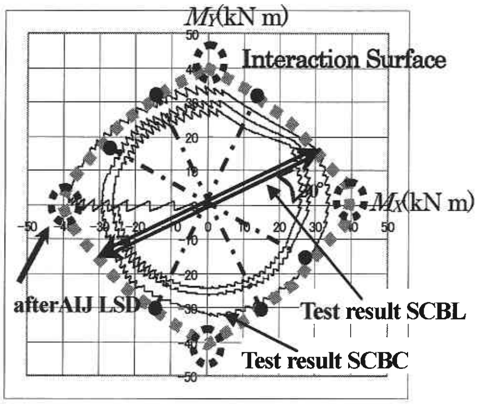


Fig. 11 Comparison of results in case of 19 mm base plate

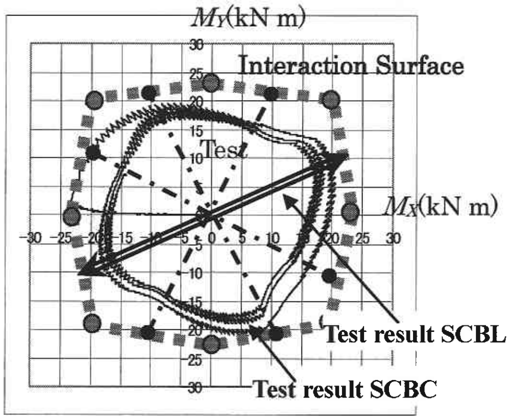


Fig. 12 Comparison of results in case of 9 mm base plate

the design formula of AIJ/LSD recommendation (Ref. [1]).

4.3.2 Interaction surface of base plate yielding Type

A safety domain under bi-axial bending of base plate yielding type is obtained by calculating the resistance forces in both uni-axial direction and in the direction deviating 45 degree from x or y-axis. To estimate the safety domain, the following theorem can be used:

1) The safety domain is a convex set.
2) A convex combination of arbitrary two points in a convex set, that is line-segment between two points, is a part of the convex set.

Here, the effective width to calculate the moment capacity of the base plate is estimated after Ref. [4] (Appendix).

Moment capacity of base plate: $M_p = 0.87$ kN m

$(b_{eff} = 92$ mm, $\sigma_u = 450MPa)$

Ratio of lengths: $\gamma = 0.65$

Ratio of effective width: $\varsigma = 0.8$

At the latter of first cycle, that is the initiation of circular movement, the analyzed strength matches with the test resistance. However, in the following reversals, the test resistance diminished as seen in Fig. 12. The reason is maybe that some small weld crack appeared during loading path between the column corner and the base plate, and some deterioration of resistance is observed. However, the shape of analyzed interaction surface of base plate yielding type model is a square and similar to the test results.

5. CONCLUSIONS

A series of experiment on the exposed type steel column bases under static cyclic loading consisted of bi-axial bending is carried out. Also, the conformity with experimental results of resistance interaction which is calculated by the limit analysis is performed. The conclusion drawn from this study is summarized as follows:

1) In all the directions, the case with anchor bolt yielding type showed a typical pinched hysteretic behavior. On the other hand, the case with base plate yielding type showed a less pinched but a slightly degraded hysteresis loop.
2) Different shapes of bi-axial interaction surface are observed for the anchor bolt yielding type and the base plate yielding type in the biaxial circular loading. (a circular or diamond shape for the anchor bolt yielding type and a square shape for the base plate yielding type)
3) It is shown that bi-axial interaction surface of limit analysis for the anchor bolt yielding type and the base plate yielding type agree with the test results approximately. The uniaxial strength derived from the anchor bolt yielding type is consistent with the design formula of AIJ/LSD recommendation.
4) The difference of interaction surface based on analytical models between the anchor bolt yielding type and the base plate yielding type can well explain the test obserbation.

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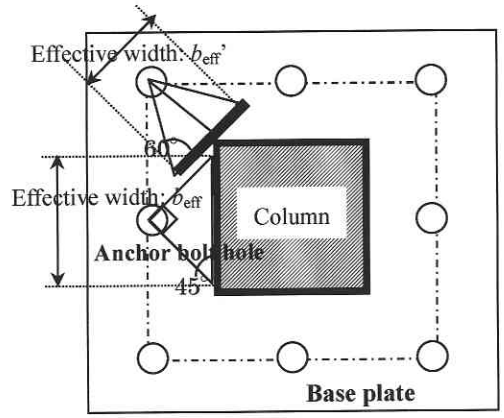

Fig. A Effective width at the corner of base plate

Appendix

Moment resisting capacity of base plate [4]

Effective width of base plate resisting to the bending moment is estimated after Ref. [4] as shown in Fig. A.