数理科学実践研究レター 2018–4 July 06, 2018

A modified model on a traffic network and signal optimization

by

Xinchi Huang, Kazune Takahashi, Yasuhisa Kishi,

Masahiko Kanai, Takafumi Mase

UNIVERSITY OF TOKYO

GRADUATE SCHOOL OF MATHEMATICAL SCIENCES

KOMABA, TOKYO, JAPAN

A modified model on a traffic network and signal optimization

Xinchi Huang[1] (Graduate School of Mathematical Sciences, the University of Tokyo)
Kazune Takahashi[2] (Graduate School of Mathematical Sciences, the University of Tokyo)
Yasuhisa Kishi[3] (Technology Planning Department, NISSAN Motor Co., Ltd.)
Masahiko Kanai[4] (Graduate School of Mathematical Sciences, the University of Tokyo)
Takafumi Mase[5] (Graduate School of Mathematical Sciences, the University of Tokyo)

Abstract

In this article, we consider a modified model for a traffic network and an optimal control problem of determining the optimal parameters at each signal. First we establish a nonlinear model for an $N \times N$ traffic network. Then Split Cycle Offset Optimisation Technique (SCOOT) as well as an objective function (average waiting time) is introduced. Finally, we give the numerical results of the optimal parameters.

1 Introduction

As we know, there are heavy traffic jams in the center parts of the cities all over the world. It seems to be an important issue to give a reasonable way to control the traffic signals, for example, determining the signal parameters. A macroscopic model (a system of ODEs) introduced by J. Imura snd M. Kamal [1] is used to formulate the optimal control of the traffic signals based on the "on-off" strategy. Later on, K. Aihara, K. Ito, J. Nakagawa and T. Takeuchi [2] apply this linear ODE model and use a novel binary optimization method to develop a real-time optimal control law.

In this article, rather than give a real-time optimal control, we are interested in suggesting a reasonable and efficient choice of the three important parameters of the traffic signals, i.e., Split, Offset and Cycle under the knowledge of the average turning rates at each signal. Compared with [2], our settings are more practical to the real traffic problems and a sophisticated model is established. However, heavy calculations are required which prevent us from reaching a real-time optimal control.

The article is organized as follows: In Section 2, notations and a traffic flow model is introduced. In Section 3, we discuss the three signal parameters and the objective function. In Section 4, provided that the average turning rates at each signal are known, we apply a numerical test and give reasonable choice of signal parameters which is one local solution to our optimal control problem.

2 A modified macroscopic traffic flow model

We consider the square grids (i, j), $i, j = 1, ..., N$ for the traffic network (Fig.1). Fix a minimum time interval $\Delta t > 0$ (s). During each time interval $[k\Delta t, (k+1)\Delta t)$ (s), $k = 0, ..., K$ and at each junction (i, j), $i, j = 1, ..., N$ we assign a traffic signal which is described by $u^k_{i,j}, v^k_{i,j}$ taking value 0 or 1. Here $u^k_{i,j}$ indicates the traffic flows in the east-west direction while $v^k_{i,j}$ is related to the north-south direction. As a practical condition, we have

$$u^k_{i,j} v^k_{i,j} = 0, \qquad i, j = 1, ..., N, \ k = 0, ..., K-1.$$

In particular, $u^k_{i,j} = v^k_{i,j} = 0$ means that the traffic flows in both directions are not allowed at the junction (i, j) during the time interval $[k\Delta t, (k+1)\Delta t)$ (s). Let $E^k_{i,j}, W^k_{i,j}, N^k_{i,j}$ and $S^k_{i,j}$ denote the traffic volumes near the junction (i, j) in the direction of east, west, north and south respectively (see Fig.2) at time $k\Delta t$ (s). In addition, we assume that the turning rates in each direction at the junction (i, j) are given by $b^X_{i,j} \geq 0$ (left turn), $c^X_{i,j} \geq 0$ (right turn) and $a^X_{i,j} := 1 - b^X_{i,j} - c^X_{i,j} \geq 0$ (go straight) where $X \in \{E, W, N, S\}$ stands for the direction (Fig.3). Furthermore, we assume the time delay between the adjacent junctions is $n\Delta t$ seconds, $n \in \mathbb{N}$. That is, it takes $n\Delta t$ seconds

[1] huangxc@ms.u-tokyo.ac.jp
[2] kazunetakahashi@g.ecc.u-tokyo.ac.jp
[3] yasu-kishi@mail.nissan.co.jp
[4] mkanai@ms.u-tokyo.ac.jp
[5] mase@ms.u-tokyo.ac.jp

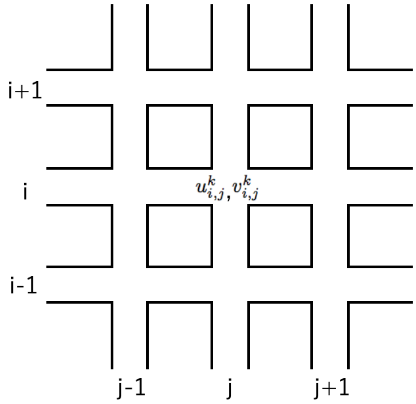


Figure 1: Square grids for the traffic network.

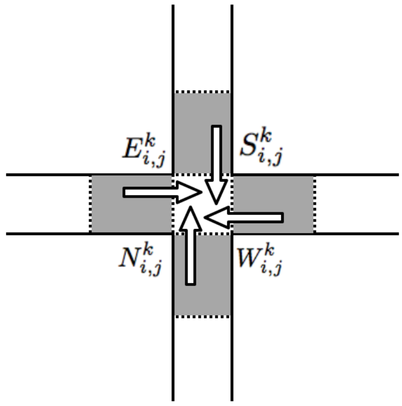


Figure 2: Traffic volumes.

to go from one junction to the next one. Equipped with the above notations, we can easily obtain the following equation (see also Fig.4):

$$\begin{aligned}
E^{k+1}_{i,j} &= E^k_{i,j} - u^k_{i,j}\lambda(E^k_{i,j})E^k_{i,j}, \qquad i=1,...,N,\ j=2,...,N,\ k=0,...,n-1\\
E^{k+1}_{i,j} &= E^k_{i,j} - u^k_{i,j}\lambda(E^k_{i,j})E^k_{i,j} + u^{k-n}_{i,j-1}a^E_{i,j-1}\lambda(E^{k-n}_{i,j-1})E^{k-n}_{i,j-1} + v^{k-n}_{i,j-1}b^S_{i,j-1}\lambda(S^{k-n}_{i,j-1})S^{k-n}_{i,j-1}\\
&\quad + v^{k-n}_{i,j-1}c^N_{i,j-1}\lambda(N^{k-n}_{i,j-1})N^{k-n}_{i,j-1}, \qquad i=1,...,N,\ j=2,...,N,\ k=n,...,K-1
\end{aligned}$$

where the passing ratio function λ is defined as

$$\lambda(Y) := \begin{cases} 1 & Y \le Y_0, \\ Y_0/Y & Y > Y_0 \end{cases}$$

with a positive constant Y_0. Similarly, we have equations in the other three directions:

$$\begin{aligned}
W^{k+1}_{i,j} &= W^k_{i,j} - u^k_{i,j}\lambda(W^k_{i,j})W^k_{i,j}, \qquad i=1,...,N,\ j=1,...,N-1,\ k=0,...,n-1\\
W^{k+1}_{i,j} &= W^k_{i,j} - u^k_{i,j}\lambda(W^k_{i,j})W^k_{i,j} + u^{k-n}_{i,j+1}a^W_{i,j+1}\lambda(W^{k-n}_{i,j+1})W^{k-n}_{i,j+1} + v^{k-n}_{i,j+1}b^N_{i,j+1}\lambda(N^{k-n}_{i,j+1})N^{k-n}_{i,j+1}\\
&\quad + v^{k-n}_{i,j-1}c^S_{i,j-1}\lambda(S^{k-n}_{i,j-1})S^{k-n}_{i,j-1}, \qquad i=1,...,N,\ j=1,...,N-1,\ k=n,...,K-1,
\end{aligned}$$

$$\begin{aligned}
N^{k+1}_{i,j} &= N^k_{i,j} - v^k_{i,j}\lambda(N^k_{i,j})N^k_{i,j}, \qquad i=2,...,N,\ j=1,...,N,\ k=0,...,n-1\\
N^{k+1}_{i,j} &= N^k_{i,j} - v^k_{i,j}\lambda(N^k_{i,j})N^k_{i,j} + v^{k-n}_{i-1,j}a^N_{i-1,j}\lambda(N^{k-n}_{i-1,j})N^{k-n}_{i-1,j} + u^{k-n}_{i-1,j}b^E_{i-1,j}\lambda(E^{k-n}_{i-1,j})E^{k-n}_{i-1,j}\\
&\quad + u^{k-n}_{i-1,j}c^W_{i-1,j}\lambda(W^{k-n}_{i-1,j})W^{k-n}_{i-1,j}, \qquad i=2,...,N,\ j=1,...,N,\ k=n,...,K-1
\end{aligned}$$

and

$$\begin{aligned}
S^{k+1}_{i,j} &= S^k_{i,j} - v^k_{i,j}\lambda(S^k_{i,j})S^k_{i,j}, \qquad i=1,...,N-1,\ j=1,...,N,\ k=0,...,n-1\\
S^{k+1}_{i,j} &= S^k_{i,j} - v^k_{i,j}\lambda(S^k_{i,j})S^k_{i,j} + v^{k-n}_{i+1,j}a^S_{i+1,j}\lambda(S^{k-n}_{i+1,j})S^{k-n}_{i+1,j} + u^{k-n}_{i+1,j}b^W_{i+1,j}\lambda(W^{k-n}_{i+1,j})W^{k-n}_{i+1,j}\\
&\quad + u^{k-n}_{i+1,j}c^E_{i+1,j}\lambda(E^{k-n}_{i+1,j})E^{k-n}_{i+1,j}, \qquad i=1,...,N-1,\ j=1,...,N,\ k=n,...,K-1.
\end{aligned}$$

Considering the boundary input, we have

$$\begin{aligned}
E^{k+1}_{i,j} &= E^k_{i,j} - u^k_{i,j}\lambda(E^k_{i,j})E^k_{i,j} + F^k_{i,0}, \qquad i=1,...,N,\ j=1,\ k=0,...,K-1\\
W^{k+1}_{i,j} &= W^k_{i,j} - u^k_{i,j}\lambda(W^k_{i,j})W^k_{i,j} + F^k_{i,N+1}, \qquad i=1,...,N,\ j=N,\ k=0,...,K-1\\
N^{k+1}_{i,j} &= N^k_{i,j} - v^k_{i,j}\lambda(N^k_{i,j})N^k_{i,j} + F^k_{0,j}, \qquad i=1,\ j=1,...,N,\ k=0,...,K-1\\
S^{k+1}_{i,j} &= S^k_{i,j} - v^k_{i,j}\lambda(S^k_{i,j})S^k_{i,j} + F^k_{N+1,j}, \qquad i=N,\ j=1,...,N,\ k=0,...,K-1.
\end{aligned}$$

Here $F^k_{i,j}$ $(i=0,N+1,\ j=0,N+1,\ k=0,...,K-1)$ represents the traffic inflows at the boundary junctions.

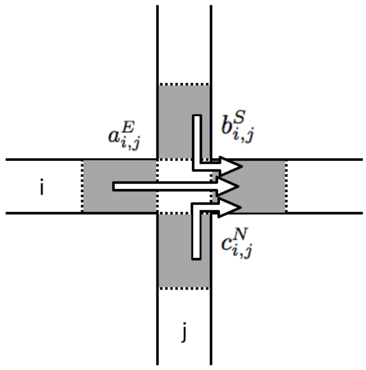


Figure 3: Turning rates.

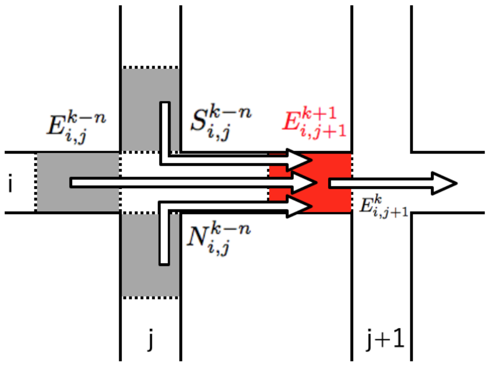


Figure 4: Image of the model.

Denote the $4N^2$ column vector for the traffic volumes $(E^k_{i,j}, W^k_{i,j}, N^k_{i,j}, S^k_{i,j})$ at time $k\Delta t$ (s) by D^k, $D^k := (E^k_{1,1}, E^k_{2,1}, ..., E^k_{N,N}, W^k_{1,1}, ..., N^k_{1,1}, ..., S^k_{N,N})^T$ $(k = 0, ..., K)$. Then we can rewrite the above equations in a matrix form:

$$D^{k+1} = A(D^k; U^k)D^k + B(D^{k-n}; U^{k-n})D^{k-n} + F^k, \qquad k = 0, ..., K-1 \tag{1}$$

where U^k is the $4N^2$ signal control vector defined by

$$U^k := (u^k_{1,1}, u^k_{2,1}, ..., u^k_{N,N}, u^k_{1,1}, ..., u^k_{N,N}, v^k_{1,1}, ..., v^k_{N,N}, v^k_{1,1}, ..., v^k_{N,N}), \qquad k = 0, ..., K-1 \tag{2}$$

and $D^k := 0$ (zero vector) if $k < 0$, F^k is the boundary inflows.
Remark. If we take $Y_0 = \infty$ and $n = 0$, then (1) leads to a simple linear model which is the discretization of the macroscopic model introduced in [1] and [2].

3 SCOOT and objective function

Split Cycle Offset Optimisation Technique (SCOOT) is an adaptive traffic control system for the coordination and control of traffic signals across an urban road network. SCOOT is originally developed by the Transport Research Laboratory for the Department of Transport in 1980. It is used extensively in many countries all over the world. In this article, we adapt the system of SCOOT (Fig.5) and consider the problem of determining the three signal parameters.

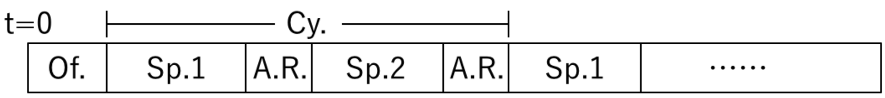


Figure 5: System of SCOOT.

Next we employ matrices $Cy, Sp1, Sp2, Of \in \mathbb{N}^{N\times N}$ to describe the signals at all the junctions which means at junction (i, j), one cycle of the signal takes $(Cy)_{i,j}\Delta t$ (s), the green light in the east-west direction (resp.north-south direction) lasts $(Sp1)_{i,j}\Delta t$ (resp.$(Sp2)_{i,j}\Delta t$) (s) and one cycle begins at $(Of)_{i,j}\Delta t$ (s). Moreover, we define all red by $AR := (Cy - Sp1 - Sp2)/2$. In terms of these parameters, we can uniquely determine the signal controls $u^k_{i,j}, v^k_{i,j}$ as follows:

$$u^k_{i,j} = \begin{cases} 1, & 0 \le \left[\left(k - (Of)_{i,j}\right) \bmod (Cy)_{i,j}\right] \le (Sp1)_{i,j}, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

$$v^k_{i,j} = \begin{cases} 1, & 0 \le \left[\left(k - (Of)_{i,j} - (Sp1)_{i,j} - (AR)_{i,j}\right) \bmod (Cy)_{i,j}\right] \le (Sp2)_{i,j}, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

Now we introduce an objective function that represents the average waiting time. That is

$$WT(\{U^k\}^K_{k=0}) := \frac{\Delta t}{G} \sum_{k=0}^{K-1} (1 - U^k)D^k. \tag{5}$$

Here D^k, U^k are the same notations as (1)-(2) and constant G denotes the total traffic volume going through this network in $K\Delta t$ seconds.
Let us now assume all red AR is given, which implies $Sp2$ is given by

$$Sp2 = Cy - Sp1 - 2AR.$$

Then in terms of (5), we express WT as a function of $Cy, Sp1$ and Of:

$$WT(Cy, Sp1, Of) = \frac{\Delta t}{G} \sum_{k=0}^{K-1} \big(1 - U^k(Cy, Sp1, Of)\big) D^k(Cy, Sp1, Of). \tag{6}$$

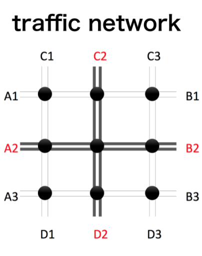
If there is no confusion, we use the same notation WT as well as U^k and D^k here and henceforth.
Remark. The choice of objective function is different from that in [2]. Actually, it is clear that here we take into account the traffic volume waiting in the network due to the red signals. Thus, WT is the average waiting time for a vehicle going through the traffic network in time $K\Delta t$ seconds.

4 Numerical test

In this section, we carry out a numerical test for the optimization problem (OP):

$$(Cy^\star, Sp1^\star, Of^\star) = \\ \operatorname{argmin}\{WT(Cy, Sp1, Of) |\ Cy_{min} \le Cy \le Cy_{max}, 0 \le Sp1 \le Cy - 2AR, 0 \le Of \le Cy\}$$

with the domain of function WT being in $\mathbb{N}^{N\times N}$.
First we fix some parameters: $\Delta t = 2$, $K = 1800$, $N = 3$, $AR = 1$, $Cy_{min} = 10$, $Cy_{max} = 60$. In addition, according to a sampling data (Fig.6), the turning rates $a^X_{i,j}, b^X_{i,j}, c^X_{i,j}$, $i, j = 1, ..., N, X \in \{E, W, N, S\}$, boundary inflows F^k, $k = 0, ..., K-1$ as well as the total traffic volume G can be calculated. Furthermore we consider a simple case that the time delay number is a constant $n = 10$ and constant $Y_0 = 5/6$. Finally we apply the idea of Alternating Direction Method of Multipliers (ADMM) and have numerical calculations. The results are given in Fig.7 and Fig.8-9.



OD(Origin-Destination) matrix

		OUT												
		A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3	SUM
IN	A1		0	0	25	50	25	25	50	25	25	50	25	300
	A2	0		0	25	400	25	25	50	25	25	50	25	650
	A3	0	0		25	50	50	25	50	25	25	50	25	325
	B1	50	50	25		0	0	25	50	25	25	50	25	325
	B2	25	300	25	0		0	25	50	25	25	50	25	550
	B3	25	50	50	0	0		25	50	25	25	50	25	325
	C1	25	50	25	25	50	25		0	0	50	50	25	325
	C2	25	50	25	25	50	25	0		0	25	300	25	550
	C3	25	50	25	25	50	25	0	0		25	50	50	325
	D1	25	50	25	25	50	25	50	50	25		0	0	325
	D2	25	50	25	25	50	25	25	400	25	0		0	650
	D3	25	50	25	25	50	25	25	50	50	0	0		325
	SUM	250	700	250	225	800	250	250	800	250	250	700	250	

Figure 6: Sampling data of OD matrix.

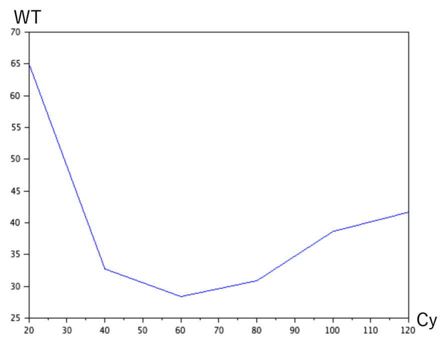


Figure 7: $WT - Cy$ in the time interval $[10, 60]$.

Fig.7 shows the relation between Cy and WT in $[10, 60]$ Δt(s) with intervals of $10\Delta t$ seconds. This indicates that the optimal cycle is $C^\star_y \Delta t \approx 60$ seconds under the above settings.

22	26	42
40	28	40
40	30	38

Figure 8: An optimal matrix of Split (sec).

22	0	46
22	34	20
48	58	56

Figure 9: An optimal matrix of Offset (sec).

We also obtain one pair of optimal choice of the Split and Offset in Fig.8 and Fig.9. Due to the high nonlinearity of our model, this pair of solutions is not necessarily a global optimizer to our problem (OP). In fact, it is almost impossible to derive a global optimizer numerically since the dimension of the control is too large. However, comparing with a simple half-half control, i.e. each component of $Sp1$ is $28/\Delta t = 14$ and Of is 0, we successfully reduce the object WT from about 42.342 seconds to only 27.984 seconds. This means our control is quite reasonable and works efficiently.

Acknowledgement

The authors thank all the members in the discussion group on this topics, Yoshida, Kataoka, Matsuzawa, Watanabe and Shoda, for their kind listening and comments. This work was supported by the Program for Leading Graduate Schools, MEXT, Japan.

References

[1] J. Imura, M. Kamal, Discussion on Traffic, Tokyo Institute of Technology, 2012.

[2] K. Aihara, K. Ito, J. Nakagawa, T. Takeuchi, Optimal control laws for traffic flow, Applied Mathematics Letters 26, 2013: 617-623.