

学 位 論 文

Dynamical Effects in Global and Local
Supersymmetric Theories

(大局的及び局所的な超対称性理論におけるダイナミカルな効果)

平成8年度 博士(理学)

東京大学大学院理学系研究科 物理学専攻

松 田 智 裕

Ph.D thesis

Dynamical Effects in Global and Local Supersymmetric Theories

Contents

1	Introduction	1
1.1	Overview	1
1.2	Organization of the thesis	2
2	Department of Physics, University of Tokyo, Bunkyo-ku, Japan	2
3	Tomohiro Matsuda	3
4	Summary of chapter 1	4
5	Summary of the manuscript	5
6	Summary of the introduction	6
7	Summary of chapter 2	7
8	Summary of chapter 3	8
9	Summary of chapter 4	9
10	Summary of chapter 5	10
11	Summary of chapter 6	11
12	Summary of chapter 7	12
13	Summary of chapter 8	13
14	Summary of chapter 9	14
15	Summary of chapter 10	15
16	Summary of chapter 11	16
17	Summary of chapter 12	17
18	Summary of chapter 13	18
19	Summary of chapter 14	19
20	Summary of chapter 15	20
21	Summary of chapter 16	21
22	Summary of chapter 17	22
23	Summary of chapter 18	23
24	Summary of chapter 19	24
25	Summary of chapter 20	25
26	Summary of chapter 21	26
27	Summary of chapter 22	27
28	Summary of chapter 23	28
29	Summary of chapter 24	29
30	Summary of chapter 25	30
31	Summary of chapter 26	31
32	Summary of chapter 27	32
33	Summary of chapter 28	33
34	Summary of chapter 29	34
35	Summary of chapter 30	35
36	Summary of chapter 31	36
37	Summary of chapter 32	37
38	Summary of chapter 33	38
39	Summary of chapter 34	39
40	Summary of chapter 35	40
41	Summary of chapter 36	41
42	Summary of chapter 37	42
43	Summary of chapter 38	43
44	Summary of chapter 39	44
45	Summary of chapter 40	45
46	Summary of chapter 41	46
47	Summary of chapter 42	47
48	Summary of chapter 43	48
49	Summary of chapter 44	49
50	Summary of chapter 45	50
51	Summary of chapter 46	51
52	Summary of chapter 47	52
53	Summary of chapter 48	53
54	Summary of chapter 49	54
55	Summary of chapter 50	55
56	Summary of chapter 51	56
57	Summary of chapter 52	57
58	Summary of chapter 53	58
59	Summary of chapter 54	59
60	Summary of chapter 55	60
61	Summary of chapter 56	61
62	Summary of chapter 57	62
63	Summary of chapter 58	63
64	Summary of chapter 59	64
65	Summary of chapter 60	65
66	Summary of chapter 61	66
67	Summary of chapter 62	67
68	Summary of chapter 63	68
69	Summary of chapter 64	69
70	Summary of chapter 65	70
71	Summary of chapter 66	71
72	Summary of chapter 67	72
73	Summary of chapter 68	73
74	Summary of chapter 69	74
75	Summary of chapter 70	75
76	Summary of chapter 71	76
77	Summary of chapter 72	77
78	Summary of chapter 73	78
79	Summary of chapter 74	79
80	Summary of chapter 75	80
81	Summary of chapter 76	81
82	Summary of chapter 77	82
83	Summary of chapter 78	83
84	Summary of chapter 79	84
85	Summary of chapter 80	85
86	Summary of chapter 81	86
87	Summary of chapter 82	87
88	Summary of chapter 83	88
89	Summary of chapter 84	89
90	Summary of chapter 85	90
91	Summary of chapter 86	91
92	Summary of chapter 87	92
93	Summary of chapter 88	93
94	Summary of chapter 89	94
95	Summary of chapter 90	95
96	Summary of chapter 91	96
97	Summary of chapter 92	97
98	Summary of chapter 93	98
99	Summary of chapter 94	99
100	Summary of chapter 95	100

Contents

1	Introduction	1
1.1	Overview	1
1.2	Organization of the paper	3
2	Review of $O(N)$ non-linear sigma model	5
2.1	Introduction	5
2.2	Review of the non-linear sigma model	6
2.3	Review of the four-fermion model	10
2.4	Phases in Supersymmetric Non-Linear Sigma Model	11
2.5	Some peculiar properties of the present model	15
2.6	Summary of chapter 2	19
3	Dynamical analysis in supersymmetric gauge theories	20
3.1	General Analysis	20
3.1.1	Supersymmetric pure Yang-Mills theory	20
3.1.2	Supersymmetric QCD	26
3.2	Large N expansion of global supersymmetric models	34
3.2.1	Gaugino condensation in supersymmetric QCD	34
3.2.2	Dilaton dependent coupling constant	39
3.3	Summary of chapter 3	48
4	Dynamical analysis in supergravity theories	49
4.1	Review of supergravity	50
4.1.1	Duality invariant Lagrangian	56
4.2	Review of the general analysis	59
4.3	Large N expansion	61
4.3.1	Introduction	61

4.3.2	Gaugino condensation in supergravity	62
4.4	Summary of chapter 4	66
5	Conclusions and discussions	69
A	Introduction to large N expansion	71
A.1	ϕ^4 theory	71
A.2	More on large N expansion	74
B	Review of the approach by G.G.Ross et al	77

2.1 Overview

When one considers the validity of the Standard Model (SM) theory in energy scales much higher than the electroweak scale, one has to take into account the possibility of a grand unified theory (GUT) or a more general theory. A typical example is the gauge hierarchy problem of the Standard Model of the strong and electroweak interactions, and its solution through the use of the renormalization group (RG) equations. From the Standard Model is extrapolated to the full energy scale Λ , the theory is no longer valid, and the theory of the strong interactions (QCD) has to be replaced by a more general theory. The most popular solution is the gauge hierarchy problem of the Standard Model is solved by a mechanism of global symmetry breaking, effectively breaking the gauge symmetry $U(1) \times SU(2) \times SU(3)$ to a general gauge g . These extensions of the Standard Model, however, the Minimal Supersymmetric Standard Model (MSSM) can be easily distinguished up to the scales very higher than the electroweak scale, such as the supersymmetric reflections scale 10^{16} GeV , the strong scale $M_s \sim 10^{17} \text{ GeV}$, or Planck scale $M_P \sim 1.2 \times 10^{19} \text{ GeV}$.

To go beyond the SM, one needs to consider a more fundamental theory, such as string theory, supersymmetric field theory. The possible candidates for such a theory is the d -dimensional supergravity, superstring theory, where the supersymmetry breaking of local supersymmetry is not necessary while with breaking supersymmetry. The theory, supersymmetry breaking can be

Chapter 1

Introduction

1.1 Overview

When one extends the validity of the low energy effective field theory to energy scales much higher than its characteristic mass scale, one faces to a scale hierarchy problem. A typical example is the gauge hierarchy problem of the Standard Model of the strong and electroweak interactions, seen as a low-energy effective theory. When the Standard Model is extrapolated to cut-off scales $\Lambda \gg 1\text{TeV}$, there is no symmetry protecting the mass of the elementary Higgs field from acquiring large value, and therefore the masses of the weak gauge bosons, receive large quantum corrections proportional to Λ . The most popular solution to the gauge hierarchy problem of the Standard Model is to extend it to a model with global $N=1$ supersymmetry, effectively broken at a scale $M_{\text{Soft}} \sim 1\text{TeV}$. (See ref.[1] for a general review.) These extensions of the Standard Model, for instance the Minimal Supersymmetric Standard Model(MSSM), can be safely extrapolated up to cut-off scales much higher than the electroweak scale, such as the supersymmetric unification scale $M_U \sim 10^{16}\text{Gev}$, the string scale $M_s \sim 10^{17}\text{Gev}$, or Planck scale $M_P = 2.4 \times 10^{18}\text{Gev}$.

To go beyond the MSSM, one must move to a more fundamental theory with spontaneous supersymmetry breaking. The possible candidate for such a theory is $N=1$ supergravity coupled to gauge and matter fields, where the spontaneous breaking of local supersymmetry is not incompatible with vanishing vacuum energy[2]. (Of course, supersymmetry breaking can be

transmitted by gauge interaction[3]. But here we concentrate on the supergravity mediated supersymmetry breaking models.) In $N=1$ supergravity, the spin 2 graviton has as superpartner, the spin $3/2$ gravitino. Here we consider the case that the supersymmetry breaking is spontaneous, via the super-Higgs mechanism[4]. One is then bound to interpret the MSSM as an effective low-energy theory derived from spontaneously broken supergravity. The scale of soft supersymmetry breaking in MSSM, M_{soft} , is related to the gravitino mass $m_{\frac{3}{2}}$, which sets the scale of the spontaneous breaking of local supersymmetry.

The idea of breaking supersymmetry in a dynamical way was first presented in ref.[10]. In those articles a general topological argument was developed in terms of the Witten index $Tr(-)^F$, showing that dynamical supersymmetry breaking cannot be achieved unless there is chiral matter or we include supergravity effects for which the index argument cannot apply. This is subsequently verified by explicitly studying gaugino condensation in pure supersymmetric Yang-Mills, a vector-like theory, for which gauginos condensate but do not break global supersymmetry. Breaking global supersymmetry with chiral matter was an open possibility in principle, but this approach ran into many problems when tried to be realized in practice[6].

The situation was improved very much with the coupling to supergravity. The reason was that the simple gaugino condensation was found to be sufficient to break supersymmetry once the coupling to gravity was included[5]. This works in the hidden sector mechanism where gravity is the messenger of supersymmetry breaking to the observable sector. However, this mechanism does not work when the gauge couplings are considered to be field dependent. Non-perturbative effects, like gaugino condensations, raises the moduli flat potentials, but it is very difficult to obtain a phenomenological vacuum state. The main difficulty lies in the runaway behavior of dilaton potential[7, 8]. Gaugino condensation with field dependent gauge couplings was anticipated and realized in a very natural way in string theory. As we will see in the review part of this paper, the gauge couplings are functions of dilaton and moduli fields. Furthermore, string theory provides a natural realization of the hidden sector models by having a hidden sector[9]. Thus it is important to consider the following questions: Does gaugino really condensate in supergravity theories? If so, is runaway potential stabilized? If runaway potential is not stabilized by gaugino condensation, what effect should be responsible for the moduli fixing? We know that an ordinary effective Lagrangian anal-

ysis cannot give us a satisfactory answer and we think it is very important to develop new ideas to analyze the dynamical properties of supergravity theories. (Assuming the scenario of two or more gaugino condensates, the effective Lagrangian with confined hidden sector stabilizes the dilaton potential and breaks supersymmetry with a more complicated dilaton superpotential generated by multiple gaugino condensations[8]. However, one solution for the stabilization of the dilaton in the effective Lagrangian requires a delicate cancellation between the contributions from different gaugino condensates, which is not very natural. Furthermore, it has a large and negative cosmological constant when supersymmetry is broken. The other solution generally requires the assistance of an additional source of supersymmetry breaking[11, 12].) In this paper, we have developed a new method for the analysis of the dynamical properties of supersymmetry theories. Of course, as is recently discussed by many authors[13], we may find a solution to this problem by introducing a new type of non-perturbative effects. One of the very promising candidates is the effects of strongly coupled strings, but we do not consider this possibility because at this moment we are not sure how to handle and apply this idea to phenomenological models.

In this paper, we analyze several types of supersymmetric models by using the large N expansion. In this limit, the relation between the effective Lagrangian approach with confined hidden sector and Nambu-Jona-Lasinio type approach is clarified. In the large N limit, these two correspond to different kinds of approximations of the exact solution.

1.2 Organization of the paper

In the present work, we discuss how the previously mentioned mechanism for dynamical supergravity breaking is realized in the framework of the large N expansion. In Chapter 2, in order to discuss the applicability of large N expansion in supersymmetric theories, we consider a simple toy model. This model, $O(N)$ sigma model, is very useful when one introduces large N expansion in supersymmetric theories. In Chapter 3, we use this method to analyze the dynamical properties of supersymmetric QCD. Finally in Chapter 4, we study the dynamical supersymmetry breaking in a local supersymmetric model. The driving force for gaugino condensation, which is not clear in other approaches, is now obvious. In general effective Lagrangian approach, the

form of the potential for gaugino condensation is not well defined near the origin ($\lambda \sim 0$) and it is difficult to understand how this condensation takes place. On the other hand, in Nambu-Jona-Lasinio type approach or in the large N expansion, the potential is given by the wine-bottle and we can easily imagine how fields rolls down to the condensating vacuum.

Chapter 2

Review of $O(N)$ non-linear sigma model

From a strictly technical point of view, the non-perturbative theories in supersymmetry should be of less interest for non-perturbative theories should be to see the realization of bosonic loops and fermionic ones. This is true for the supersymmetric extended cases considered, but not applicable to the $O(N)$ non-linear sigma model as a simple well known theory, we should emphasize that that the $O(N)$ non-linear sigma model is a theory with no supersymmetry, the generalization of it to supersymmetry is very difficult to achieve (see [1]).

On the other hand, it is very important to show that of all, that even the first approximation is enough to which we can exactly solve the supersymmetry by means of large N expansion. Here we consider the phase structure of supersymmetric $O(N)$ non-linear sigma model in two and three dimensions. The study follows with [2] and [3] in which we have analyzed this theory by using lattice method. Some shortcomings of the standard approach are here we consider will be given below.

2.1 Introduction

Many years ago, the study of the $O(N)$ non-linear sigma model was considered by Schwinger in connection with the study of the $O(N)$ non-linear sigma model. They obtain the expansion in powers of $1/N$ and the results are given in [4]. This leads to

Chapter 2

Review of $O(N)$ non-linear sigma model

People generally tend to think that, in supersymmetric theories, no gap equations should exist because the non-renormalization theorem seems to ensure the cancellation of bosonic loops and fermionic ones. This is true for the superpotential motivated mass terms[14], but not applicable to the D-term motivated ones, such as a gaugino soft mass. Here, we should remember that $N=1$ four-dimensional non-renormalization theorem says nothing about the renormalization of D-terms but only about the superpotential renormalization [15].

We believe it is very important to show, first of all, that one can find a supersymmetric model in which we can exactly solve the gap-equation by means of large N expansion. Here we examine the phase structures of supersymmetric $O(N)$ non-linear sigma model in two and three dimensions. We mainly follow refs.[16] and [17] in which we have re-analyzed this theory by using tadpole method. Some shortcomings of the previous papers on this topic are corrected and deeper insights are given.

2.1 Introduction

Many years ago, Gross and Neveu[18] have shown that dynamical symmetry breakdown is possible in asymptotically free field theories. They obtain an expansion in powers of $1/N$ that is non-perturbative in g^2 . This leads to

a massive fermion and to a $\bar{\psi}\psi$ bound state at threshold.

Polyakov[19] has pointed out that the $O(N)$ non-linear sigma model is asymptotically free and that the fundamental particle acquires a mass for $N > 2$.

Witten [20] has constructed a supersymmetric version of the two-dimensional $O(N)$ sigma model. This is a hybridization of the non-linear sigma model and Gross-Neveu model with Majorana fermions.

Then one question appears naturally. What is the difference between non-supersymmetric models and supersymmetric ones? If there is any difference, how is it realized? Many authors tried to answer this question[21, 22], but some questionable aspects of arguments are still left.

The purpose of this chapter is to clarify these ambiguities and present a systematic treatment of this model. To show explicitly what is going on, we are not going to eliminate the auxiliary fields at the first stage by using the equation of motion. If we eliminate all the auxiliary fields, it becomes difficult to find what relations we are dealing with.

2.2 Review of the non-linear sigma model

In this and the next sections we are going to review the well-known results of $O(N)$ non-linear sigma model and four-fermion model to fix the notations, and we show the strategy which is used throughout this paper.

The Lagrangian for $O(N)$ sigma model is defined by

$$L = -\frac{1}{2}n_j\partial^2n_j \quad (2.1)$$

with the local non-linear constraint

$$n_jn_j = \frac{N}{g^2}. \quad (2.2)$$

The sum over the flavor index j runs from 1 to N . This constraint can be implemented by introducing a Lagrange multiplier λ .

Let us consider the Euclidean functional integral in the form:

$$Z = \int D\vec{n} \delta\left((\vec{n})^2 - \frac{N}{g^2}\right) \exp\left(-\frac{1}{2} \int (\partial_\mu \vec{n})^2 d^Dx\right)$$

$$= \int D\lambda \int D\vec{n} \exp \left\{ -\frac{1}{2} \int \left[(\partial_\mu \vec{n})^2 + \lambda \left(\vec{n}^2 - \frac{N}{g^2} \right) \right] d^D x \right\} \quad (2.3)$$

The integral over n is Gaussian and can be performed in a standard fashion. We have:

$$Z = \int D\lambda \exp \left(\frac{N}{2g^2} \int \lambda d^D x - \frac{N}{2} \text{tr} \ln(-\partial^2 + \lambda) \right) \quad (2.4)$$

Let us first compute the variation of the action with respect to λ . We get the following equation[23]:

$$\begin{aligned} \frac{N}{2g^2} &= \frac{N}{2} \frac{\delta}{\delta \lambda(x)} \text{tr} \ln(-\partial^2 + \lambda(x)) \\ &= \frac{N}{2} G(x, x; \lambda) \end{aligned} \quad (2.5)$$

Here we have introduced the Green function:

$$G(x, x'; \lambda) = \langle x' | (-\partial^2 + \lambda)^{-1} | x \rangle \quad (2.6)$$

The meaning of the above equation becomes transparent if we notice that

$$\begin{aligned} \langle n_i(x) n_j(y) \rangle &= Z^{-1} \int D\lambda \int D\vec{n} \exp \left(-\frac{1}{2} \int \left\{ (\partial \vec{n})^2 + \lambda \left(\vec{n}^2 - \frac{N}{g^2} \right) \right\} d^D x \right) \\ &\quad \times n_i(x) n_j(y) \\ &= \delta_{ij} \frac{\int D\lambda e^{-W} G(x, y; \lambda)}{\int D\lambda e^{-W}} \\ W &= -\frac{N}{2g^2} \int \lambda d^D x + \frac{N}{2} \text{tr} \ln(-\partial^2 + \lambda). \end{aligned} \quad (2.7)$$

If λ integration is to be approximated by the saddle point λ_0 , we obtain

$$\langle n_i(x) n_j(y) \rangle = \delta_{ij} G(x, y; \lambda_0). \quad (2.8)$$

These equation shows that eq.(2.5) is nothing but the condition $\langle \vec{n}^2 \rangle = \frac{N}{g^2}$. In other words, the gap equation can be obtained directly from the constraint equation. Here we call this simple calculation a tadpole method after ref.[34].

Now let us solve eq.(2.5). Passing to the momentum representation, this "gap equation" is presented as:

$$\begin{aligned} G(x, x'; \lambda_0) &= \int \frac{d^D p}{(2\pi)^D} \frac{e^{ip(x-x')}}{p^2 + \lambda_0} \\ \frac{N}{g^2} &= NG(x, x; \lambda_0) \\ &= N \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \lambda_0}. \end{aligned} \quad (2.9)$$

This equation is applicable for any D dimensions. For $D=2$, we can obtain the precise form:

$$\begin{aligned} 1 &= \frac{g^2}{4\pi} \log \frac{\Lambda^2}{\lambda_0} \\ \lambda_0 &= \Lambda^2 \exp\left(-\frac{4\pi}{g^2}\right) \end{aligned} \quad (2.10)$$

For $D=3$, the situation differs slightly. We should include a critical coupling g_{cr}^2 defined by

$$1 = g_{cr}^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2}. \quad (2.11)$$

If the coupling is strong ($g^2 > g_{cr}^2$), the gap equation has a non-trivial solution at $\lambda_0 \neq 0$. (This critical coupling explicitly depends on the cut-off scale Λ , so in three dimensions, we regard this model as a low-energy effective theory of some high-energy physics. Of course one may find a good way to remove this cut-off dependence, but here we do not consider such a detailed analysis.) Using g_{cr} , we can rewrite (2.9) as:

$$\begin{aligned} 1 &= g^2 \int \frac{d^3 p}{(2\pi)^3 p^2} - g^2 \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{p^2} - \frac{1}{p^2 + \lambda_0} \right) \\ &= \frac{g^2}{g_{cr}^2} - g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{\lambda_0}{p^2(p^2 + \lambda_0)} \end{aligned} \quad (2.12)$$

The integral in (2.12) is convergent. Therefore, we have:

$$m^2 \equiv \lambda_0 = \text{const.} \left(\frac{1}{g_{cr}^2} - \frac{1}{g^2} \right)^2 \quad (2.13)$$

If we take $g^2 < g_{cr}^2$ something goes wrong with eq.(2.12). It does not have any solution, so the constraint $\langle \tilde{n}^2 \rangle = \frac{N}{g^2}$ cannot be satisfied in this way. To solve this puzzle, we should also consider the possibility of spontaneous breaking of $O(N)$ symmetry. In above discussions, we have implicitly assumed that the vacuum expectation value of \tilde{n} would vanish. Now we consider what will be changed if \tilde{n} itself gets a non-zero vacuum expectation value. Because of $O(N)$ symmetry, the vacuum expectation value of $\tilde{n} \equiv (n_1, n_2, \dots, n_N)$ may be written as

$$\langle \tilde{n} \rangle = (0, 0, \dots, \sqrt{N}v/g). \quad (2.14)$$

So that the constraint equation (2.9) is changed.

$$\begin{aligned} \langle (\tilde{n})^2 \rangle &= \langle \tilde{n} \rangle^2 + \langle 1-loop \rangle \\ &= N \left(\frac{v^2}{g^2} + \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + \lambda_0} \right) = \frac{N}{g^2} \end{aligned} \quad (2.15)$$

Of course, in two dimensions we cannot expect \tilde{n} to get any expectation value. Considering the spontaneous breaking of $O(N)$ symmetry, we introduce another important critical coupling constant g'_{cr} .

$$\frac{1-v^2}{g_{cr}^2} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} \quad (2.16)$$

If g is smaller than g_{cr} , then v grows such that the constraint equation is satisfied in the weak coupling region ($g'_{cr} \leq g \leq g_{cr}$) in a sense that not eq.(2.9) but eq.(2.15) is satisfied by some λ_0 . There appears, however, a peculiar "flat direction" in three dimensions. As we have seen in the above analysis (2.15), the constraint equation is satisfied for arbitrary values of λ_0 and v_0 once a certain relation is satisfied by these two variables. To satisfy the constraint equation, λ_0 and v should be related, but one cannot fix them at a unique point. This relation defines the "flat direction" of such a peculiar type. This is because once the constraint equation is satisfied, the potential term should vanish by definition.

As far as we deal with non-supersymmetric sigma model, we have no primary reason to believe that the vacuum expectation value of the field $v = \langle n_j \rangle$ would vanish in the strong coupling region.

2.3 Review of the four-fermion model

The four-fermion model is described by the Lagrangian

$$L = \frac{i}{2} \bar{\psi}_j \not{\partial} \psi_j + \frac{g^2}{8N} (\bar{\psi}_j \psi_j)^2 \quad (2.17)$$

where the sum of the flavor index j runs from 1 to N and we require that g^2 remain constant as N goes to infinity. By introducing a scalar auxiliary field σ ($\sigma = \frac{N}{2g^2} \bar{\psi}_j \psi_j$) we may rewrite (2.17) as

$$L = \frac{i}{2} \bar{\psi}_j \not{\partial} \psi_j + \frac{1}{2} \sigma \bar{\psi}_j \psi_j - \frac{N\sigma^2}{2g^2}. \quad (2.18)$$

Let us consider the functional integral in the form:

$$Z = \int D\psi_j D\sigma \exp \left[\int d^D x \left\{ \frac{1}{2} \bar{\psi}_j (i \not{\partial} + \sigma) \psi_j - \frac{N}{2g^2} \sigma^2 \right\} \right] \quad (2.19)$$

Integrating over the field ψ_j we get an effective action for the field σ :

$$Z = \int D\sigma \exp \left[-\frac{N}{2g^2} \int d^D x \sigma^2 + \frac{N}{2} \text{Tr} \ln (i \not{\partial} + \sigma) \right] \quad (2.20)$$

We impose the stationary condition which gives the gap equation.

$$\frac{N \langle \sigma \rangle}{g^2} - \frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \text{tr} \frac{1}{-\not{p} + \langle \sigma \rangle} = 0 \quad (2.21)$$

As is in non-linear sigma model discussed in the previous section, this gap equation represents the condition

$$\frac{N}{g^2} \langle \sigma \rangle = \frac{1}{2} \langle \bar{\psi}_j \psi_j \rangle |_{m_\psi = \langle \sigma \rangle}. \quad (2.22)$$

where non-zero σ corresponds to the condensation of $\bar{\psi}_j \psi_j$. For $D=2$, we obtain the equation:

$$\begin{aligned} \frac{1}{g^2} &= \int \frac{d^2 p}{(2\pi)^2} \frac{1}{p^2 + \langle \sigma \rangle^2} \\ \langle \sigma \rangle^2 &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right) \end{aligned} \quad (2.23)$$

For $D=3$, we have a critical coupling constant. The saddle point exists only within the branch

$$0 < \frac{1}{g^2} \leq \frac{1}{g_{cr}^2} \quad (2.24)$$

where

$$\frac{1}{g_{cr}^2} \equiv \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2}. \quad (2.25)$$

The crucial difference from the $O(N)$ non-linear sigma model is that eq.(2.22) always has a trivial solution at $\sigma = 0$. In the weak coupling region, non-trivial saddle point vanishes but the trivial solution always exists.

2.4 Phases in Supersymmetric Non-Linear Sigma Model

Supersymmetric non-linear sigma model is usually defined by the Lagrangian

$$L = \frac{1}{2} \int d^2 \theta (\bar{D} \Phi_j)(D \Phi_j) \quad (2.26)$$

with the non-linear constraint

$$\Phi_j \Phi_j = \frac{N}{g^2}. \quad (2.27)$$

where the sum of the flavor index j runs from 1 to N . The superfields Φ_j may be expanded in components

$$\Phi_j = n_j + \bar{\theta} \psi_j + \frac{1}{2} \bar{\theta} \theta F_j \quad (2.28)$$

and we define the super covariant derivative of the form:

$$D = \frac{\partial}{\partial \theta} - i \bar{\theta} \not{\partial} \quad (2.29)$$

In order to express the constraint (2.27) as a δ function, we introduce a Lagrange multiplier superfield Σ .

$$\Sigma = \sigma + \bar{\theta} \xi + \frac{1}{2} \bar{\theta} \theta \lambda \quad (2.30)$$

We thus arrive at the manifestly supersymmetric action for the supersymmetric sigma model.

$$S = \int d^D x d^2 \theta \left[\frac{1}{2} (\bar{D} \Phi_j) (D \Phi_j) + \frac{1}{2} \Sigma \left(\Phi_j \Phi_j - \frac{N}{g^2} \right) \right] \quad (2.31)$$

where $D=2,3$. In component form, the Lagrangian from (2.31) is explicitly written in the following form.

$$\begin{aligned} L = & -\frac{1}{2} n_j \partial^2 n_j + \frac{i}{2} \bar{\psi}_j \not{\partial} \psi_j + \frac{1}{2} F_j^2 - \sigma n_j F_j - \frac{1}{2} \lambda n_j^2 \\ & + \frac{1}{2} \sigma \bar{\psi}_j \psi_j + \bar{\xi} \psi_j n_j + \frac{N}{2g^2} \lambda \end{aligned} \quad (2.32)$$

We can see that λ, ξ , and σ are the respective Lagrange multipliers for the constraints:

$$\begin{aligned} n_j n_j &= \frac{N}{g^2} \\ n_j \psi_j &= 0 \\ n_j F_j &= \frac{1}{2} \bar{\psi}_j \psi_j \end{aligned} \quad (2.33)$$

The second and the third constraints are obtained by the supersymmetric transformations of the first one. We must not include kinetic terms for the field σ and ξ so as to keep these constraints manifest. We can examine these constraints in a manner we have used in the previous sections. As we have seen, we can directly solve the gap equation without calculating the explicit 1-loop potential. Below, we analyze each constraint and solve gap equations.

First we examine the two dimensional theory.

(1) Scalar part

$$\langle n_j n_j \rangle |_{m_n^2 = \langle \lambda \rangle + \langle \sigma^2 \rangle} = \frac{N}{g^2} \quad (2.34)$$

In two dimensions, as we have seen above, m_n is always non-zero.

$$\begin{aligned} m_n^2 &= \langle \lambda \rangle + \langle \sigma^2 \rangle \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right) \end{aligned} \quad (2.35)$$

This fixes m_n at a dynamical scale but does not fix $\langle \lambda \rangle$ and $\langle \sigma \rangle^2$ independently.

(2) Fermion part

$$\langle n_j F_j \rangle = \frac{1}{2} \langle \bar{\psi}_j \psi_j \rangle \quad (2.36)$$

One may think that the fermionic condensation should vanish to keep supersymmetry unbroken, but this notion is not always true. This relation includes auxiliary field F_j , to be eliminated by equation of motion. After substituting F_j by σn_j , we obtain at one-loop level:

$$\begin{aligned} \langle n_j F_j \rangle &= \langle \sigma n_j n_j \rangle \\ &= \langle \sigma \rangle \langle n_j n_j \rangle = \frac{1}{2} \langle \bar{\psi}_j \psi_j \rangle \end{aligned} \quad (2.37)$$

If we impose the $O(N)$ symmetric constraint $\langle n^2 \rangle = \frac{N}{g^2}$, we have

$$\begin{aligned} \frac{N}{g^2} \langle \sigma \rangle &= \frac{1}{2} \langle \bar{\psi} \psi \rangle |_{m_\psi = \langle \sigma \rangle} \\ \frac{N}{g^2} &= \frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \langle \sigma \rangle^2}. \end{aligned} \quad (2.38)$$

For $D=2$, the solution is

$$\langle \sigma \rangle^2 = \Lambda^2 \exp\left(-\frac{4\pi}{g^2}\right). \quad (2.39)$$

Substituting $\langle \sigma \rangle$ in the first constraint (2.34) with (2.39), we can find that $\langle \lambda \rangle$ must vanish (in this point ref.[22] was wrong). This means that the field ψ gains the same mass as n , and simultaneously supersymmetric order parameter $\langle \lambda \rangle$ vanishes. We can say that the supersymmetry is not broken in two dimensions as is predicted by Witten[10]. Moreover, we can examine the assumption of vanishing v by decomposing the constraint(2.36) as follows.

$$\langle \sigma \rangle \left(v^2 + \int \frac{d^2 p}{(2\pi)^2} \frac{1}{p^2 + \langle \sigma \rangle^2} \right) = \int \frac{d^2 p}{(2\pi)^2} \frac{\langle \sigma \rangle}{p^2 + \langle \sigma \rangle^2} \quad (2.40)$$

Bosonic and fermionic loops exactly cancel. Finally we obtain:

$$\langle \sigma \rangle v^2 = 0 \quad (2.41)$$

As $\langle \sigma \rangle$ is non-zero in two dimensions(2.39), we must set $v = 0$.

When $D=3$, we can find a solution for the eq.(2.34) only in the region $g > g'_{cr}$. The critical coupling g'_{cr} is defined by

$$\frac{1-v^2}{g'^2_{cr}} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} \quad (2.42)$$

while g_{cr} is defined as:

$$\frac{1}{g^2_{cr}} \equiv \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2}. \quad (2.43)$$

$O(N)$ symmetry is expected to be spontaneously broken in the region $g'_{cr} < g < g_{cr}$ by a non-zero value of v . And when $g = g'_{cr}$, m_n would vanish.

For the fermionic part(2.36), in $D=3$, we also have a critical coupling constant. As far as $g \geq g_{cr}$, we have nothing to worry about. In this strong coupling region, both supersymmetry and $O(N)$ symmetry are preserved in a fashion like two dimensions. In this region v cannot develop any non-zero value because eq.(2.41) is also true for the strongly coupled three dimensional theory. However, in the weak coupling region, something goes wrong. There is no non-trivial solution for fermionic constraint(2.36) and there is no fermionic condensation (This means that the only possible solution is $\langle \sigma \rangle = 0$). Thus we can see from eq.(2.41) that v can develop non-zero value in this weak coupling region. This is supported by the constraint (2.34), because this does not have any solution in the weak coupling region unless we allow v to develop non-vanishing value. Eq.(2.15) suggests:

$$v^2 = 1 - g^2 \int \frac{d^3p}{(2\pi)^2} \frac{1}{p^2} \quad (2.44)$$

Naive consideration also supports this analysis. In general, we can expect that quantum effects in correlation functions like $\langle n_j n_j \rangle$ or $\langle \bar{\psi} \psi \rangle$ would vanish in the weak coupling limit. But we have a $O(N)$ symmetric constraint. It is natural to think that the field n itself gains expectation value to complement quantum effects. This simply means that classical effects become more dominant in the weak coupling region, therefore the $O(N)$ symmetric constraint is satisfied classically. (i.e. in the weak coupling limit $g \rightarrow 0$ we obtain $v = 1$. This is a classical solution of the constraint.) As a result, in the weak coupling region $O(N)$ symmetry is spontaneously broken by non-zero value of v .

We should also note that, in the weak coupling region, there is also a possible solution of non-zero λ_0 if $g \neq g'_c$. It induces a supersymmetry breaking term to the Lagrangian of the form:

$$L_{break} = \lambda_0 \left((\vec{n})^2 - \frac{N}{g^2} \right) \quad (2.45)$$

On the constrained phase space $((\vec{n})^2 - \frac{N}{g^2} = 0)$, vacuum energy also seems to vanish for non-zero λ_0 as far as v valances to satisfy the constraints. (See section 2.2.) Can we think that there remains some unusual type of flat direction, with non-zero value of F-component? Of course this statement is wrong. After including effective kinetic terms, $\sim \lambda \lambda$ appears effectively (see ref.[24]). Then, we can find positive vacuum energy for supersymmetry breaking phase ($\lambda \neq 0$) as in the usual type of supersymmetric theories. So we can conclude:

(1) In two dimensions, both supersymmetry and $O(N)$ symmetry are not broken. This means that λ and v remain zero for any value of g .

(2) In three dimensions, both supersymmetry and $O(N)$ symmetry are not broken (i.e. λ and v remain zero) in the strong coupling region. $O(N)$ symmetry can be broken in the weak coupling region, but supersymmetry is kept unbroken in both phases.

2.5 Some peculiar properties of the present model

In this section we discuss the stability of the dynamically generated supersymmetric mass terms against the soft breaking term.

We examine the supersymmetric non-linear $O(N)$ sigma model with a soft breaking term. In two dimensions, we will find that the mass difference between supersymmetric partner fields vanishes accidentally. In three dimensions, the mass difference still remains in the strong coupling region, but $O(N)$ symmetry is always broken.

To see what happens, now let us extend the above analysis to include a supersymmetry breaking mass term. Here we consider a soft mass term of the form:

$$L_{break} = \frac{-1}{2} m_s^2 n_j^2 \quad (2.46)$$

Including this soft term, we can calculate the gap equation explicitly. For the scalar part it becomes:

$$\begin{aligned} n_j n_j |m_s^2 = \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} &= N \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} \\ &= \frac{N}{g^2} \end{aligned} \quad (2.47)$$

The fermionic part is unchanged by the soft breaking term. For $D = 2$ we can solve this equation explicitly.

$$\begin{aligned} 1 &= \frac{g^2}{4\pi} \log \frac{\Lambda^2}{\langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} \\ m_n^2 &= \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right) \end{aligned} \quad (2.48)$$

$\langle \sigma \rangle^2$ is determined by the fermionic part which is unchanged by the supersymmetry breaking term (2.46).

$$\begin{aligned} m_\psi^2 &= \langle \sigma \rangle^2 \\ &= \Lambda^2 \exp \left(-\frac{4\pi}{g^2} \right). \end{aligned} \quad (2.49)$$

These two equations suggest two consequences. One is that the supersymmetry breaking parameter λ gets non-zero value. The shift of λ that is induced by soft mass is:

$$\langle \lambda \rangle + m_s^2 = 0 \quad (2.50)$$

And the supersymmetry is broken. The second consequence is more peculiar. As we can see from explicit calculations, dynamically generated masses are unchanged and the mass degeneracy is not removed by the explicit soft mass term. This happens because the auxiliary field λ has absorbed m_s so that the two masses balance.

We conclude that, if we believe the validity of the large N expansion, the dynamical masses are unchanged while the supersymmetry breaking parameter develops non-zero value.

The crucial point of our observation lies in the fact that we can absorb the soft term by redefining a field. The simplest and trivial example is the

ordinary $O(N)$ non-linear sigma model with an explicit mass term. This is written as:

$$L = -\frac{1}{2}n_j\partial^2 n_j - \frac{1}{2}\lambda(n_j^2 - \frac{N}{g^2}) - m_s^2 n_j^2 \quad (2.51)$$

Does the explicit mass term changes the dynamical mass? The answer is no. This can easily be verified by redefining λ as $\lambda' = \lambda + m_s^2$. Lagrangian is now:

$$L = -\frac{1}{2}n_j\partial^2 n_j - \frac{1}{2}\lambda'(n_j^2 - \frac{N}{g^2}) - \frac{N}{2g^2}m_s^2 \quad (2.52)$$

We can find that the mass term is absorbed in λ' and only a constant is left. Of course, this constant does not change the gap equation.

In three dimensions, however, it is not so simple. Many fields and their equations form complex relations and determine their values each other. Let us see more details. In three dimensions, we should alter the above results. As is discussed above, this model has a weak coupling region where no dynamical mass is produced so no balancing effect between superpartner masses works in this region. Setting $\lambda = 0$, we find $m_n = m_s$ and $m_\psi = 0$ when g is small. This agrees with the naive expectation. What will happen if we go into the strong coupling region where the gap equations develop non-trivial solutions and the fermion becomes massive? If there were no soft term, $O(N)$ symmetry restoration occurs in this region. But because m_s is non-zero, v must develop non-zero value in order to compensate m_s and satisfy the constraint equation(2.15). In this case, we cannot set $\lambda = 0$ because λ and v should be determined by minimizing the full 1-loop potential.

To summarize, after adding a breaking term, some fields slide to compensate m_s but the mechanism is not trivial. Even in our simplest model, many complex relations determine their values. In two dimensions, we found that the mass difference between supersymmetric partner fields vanishes accidentally but the supersymmetry is broken. In three dimensions, the mass difference is always non-zero. In the weak coupling region it is m_s , but to determine the precise value of the mass difference in the strong coupling region, we should solve the following equation:

$$\begin{aligned} n_j n_j |_{m_s^2 = \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} &= v^2 + N \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2 + \langle \lambda \rangle + \langle \sigma \rangle^2 + m_s^2} \\ &= \frac{N}{g^2} \end{aligned} \quad (2.53)$$

where σ should be determined by the fermionic constraint. This equation (2.53) determines the relation between v and λ . To go further, we should minimize the full 1-loop potential for v and λ under the constraint (2.53). This calculation is too complicated to find exact solutions, but it is obvious that v and λ are simultaneously non-zero. So we can conclude that in the strong coupling region, mass difference is non-zero and $O(N)$ symmetry is also broken. In three dimensions $O(N)$ symmetry is always broken and mass difference is always non-zero. Its value (mass difference) coincides with m_s in the weak coupling region, but in the strong coupling region it is determined by very complicated relations.

What is new in refs. [16] and [17] is:

1) It was previously believed that the fermionic condensation is parameterized by λ which is the F-component of the Lagrange multiplier. In ref. [22], it is discussed that even if fermionic condensation occurs and λ becomes non-zero, supersymmetry is still maintained since the dynamical masses for fermion and scalar field are equal. By using tadpole method, we showed that λ should vanish in this theory.

2) If one calculates a naive 1-loop effective potential, one will find a fictitious negative energy solution at $\lambda \neq 0$. This problem can be evaded if effective $\lambda\lambda$ term is included.

3) We analyzed the effects of the soft supersymmetry breaking term on dynamical properties.

Recently, many groups have discussed the dynamical properties of softly broken supersymmetric theories [25], but some non-trivial assumptions were needed. (For example, they assumed that the non-perturbative superpotential W^{np} is not changed by the small soft mass. This is a very strong assumption which should be verified in another way. To be more specific, in $O(N)$ non-linear sigma model this assumption corresponds to: Neglect the m_s factor in the gap equations (2.48) and (2.53). Of course, this is obviously wrong.) Although our model is much simpler, we think it is important to consider the explicitly solvable models as a toy model for these complicated issues.

2.6 Summary of chapter 2

In this chapter we have reviewed ref.[16]. Supersymmetric $O(N)$ non-linear sigma model is already analyzed in refs.[22, 24] by using 1-loop effective potential, but some uncertainties were left. (See 1),2) and 3) in the last part of the previous section.) Using tadpole method, we have re-analyzed this theory and found that this method is very useful especially when we analyze large N expansions in supersymmetric theories. As we have stated in the previous section, many uncertainties are resolved. We have also considered the effect of the soft breaking term, which is not discussed in the previous papers.

Chapter 3

Dynamical analysis in supersymmetric gauge theories

3.1 General Analysis

3.1.1 Supersymmetric pure Yang-Mills theory

Let us first review the dynamical analysis of supersymmetric Yang-Mills theory. In this paper, the word pure supersymmetric Yang-Mills theory (SYM) means the supersymmetric gauge theory with $SU(N_c)$ gauge group and no matter field.

The Lagrangian is generally written as:

$$L = \frac{1}{g^2} \int d^2\theta W^{\alpha a} W_a^\alpha + h.c. \quad (3.1)$$

where the chiral representation of the vector superfield (W^α) contains gauge boson (A_μ) and its fermionic partner (λ^α).

$$W_\alpha^a = -i\lambda_\alpha^a + \left[\delta_\alpha^\beta D^a - \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu)_\alpha^\beta (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c) \right] \theta_\beta + \theta\theta (\sigma^\mu)_{\alpha\dot{\alpha}} \partial_\mu \bar{\lambda}^{a\dot{\alpha}} \quad (3.2)$$

It is known that Witten index is non-zero in this theory, so we can expect that supersymmetry should not be broken. On the other hand, it is also

known that gaugino should condensate and chiral symmetry is broken. This condensation can be checked by the instanton calculation in the weak coupling region or by using the effective (confined) Lagrangian analysis in the strong coupling region. These analyses present consistent results so we can believe that gaugino condensation occurs in supersymmetric pure Yang-Mills theory at any scale.

For later convenience, here we explicitly calculate the effective potential for the composite field $U = \frac{g^2}{32\pi^2} W^\alpha W_\alpha$. Other approaches, such as the instanton calculation, are entirely reviewed in ref.[6]. The underlying principle of this construction is t'Hooft's anomaly matching condition, which demands that the effective low energy Lagrangian, valid below some scale Λ , should reproduce the anomalies of the underlying constituent theory. In the case of the pure supersymmetric Yang-Mills model it is well known that R-symmetry and supersymmetry current as well as the energy momentum tensor lie in the same supersymmetry multiplet.

In terms of constituent fields the lowest component of U is proportional to the gaugino bilinear $\lambda\lambda$, so it makes sense to take U as the goldstone multiplet entering the low energy Lagrangian. The R-symmetry acts as

$$U(x, \theta) \rightarrow e^{3i\alpha} U(x, e^{-3i\alpha}\theta) \quad (3.3)$$

while the scale symmetry acts as

$$U(x, \theta) \rightarrow e^{3\gamma} U(xe^\gamma, \theta e^{\gamma/2}) \quad (3.4)$$

Assuming that the anomalies associated with the above classical invariances are reproduced by the superpotential W , one obtains the holomorphic equation for the superpotential (note that the imaginary part of U contains $\bar{F}F$)

- Anomaly from fundamental Lagrangian

$$\begin{aligned} \delta L &= 2N_c \alpha \frac{g^2}{32\pi^2} F \bar{F} \\ &= 2N_c \alpha i \int d^2\theta U + h.c. \end{aligned} \quad (3.5)$$

- Anomaly from composite Lagrangian

$$\int d^2\theta (-2i\alpha W + 2i\alpha U \frac{dW}{dU}) + h.c. \quad (3.6)$$

The resulting anomaly matching condition is:

$$U \frac{\partial W}{\partial U} - W = N_c U, \quad (3.7)$$

which has the general solution of the form

$$W = aU + N_c U \log \left(\frac{U}{\mu^3} \right) \quad (3.8)$$

where a is in general undetermined and corresponds to the rescaling of the condensation scale μ , and b is a function of the gauge coupling which is fixed by the anomaly constraints.

To examine this model explicitly, we need some information on the Kähler potential K . Usually one demands that the variation of the D-component of K is non-anomalous and it fixes its form to be:

$$K = \text{const.} (U\bar{U})^{1/3} \quad (3.9)$$

(If one takes $K = K(U\bar{U})$ then the variation vanishes identically for R-symmetry. However the variation under scale symmetry suggests that

$$\delta K|_D = \gamma \left[3 \frac{\partial K}{\partial U} U + 3 \frac{\partial K}{\partial \bar{U}} \bar{U} - 2K \right]_D \quad (3.10)$$

where only $K = \text{const.} (U\bar{U})^{1/3}$ is allowed.)

This leads to the effective potential of the form:

$$V = (K_{U\bar{U}})^{-1} |U|^2 \left| (a + N_c) + N_c \log \left(\frac{U}{\mu^3} \right) \right|^2 \quad (3.11)$$

This scalar potential has the minimum at $U = \mu^3 e^{-(a+N_c)/N_c} e^{2\pi i/N_c}$ which agrees with the analysis of the instanton calculations. This also agrees with the Witten index analysis, which suggests that this theory has supersymmetric vacuum with non-zero gaugino condensation. One may find an extra minimum at the origin ($U = 0$), but because it corresponds to a vanishing point of $K_{U\bar{U}}$ and it also contradicts to other approaches (Witten Index and the instanton calculation) we may think that the effective potential is not well defined near the origin and discard this possibility of finding a trivial minimum at $U = 0$. (See Fig.(3.1).)

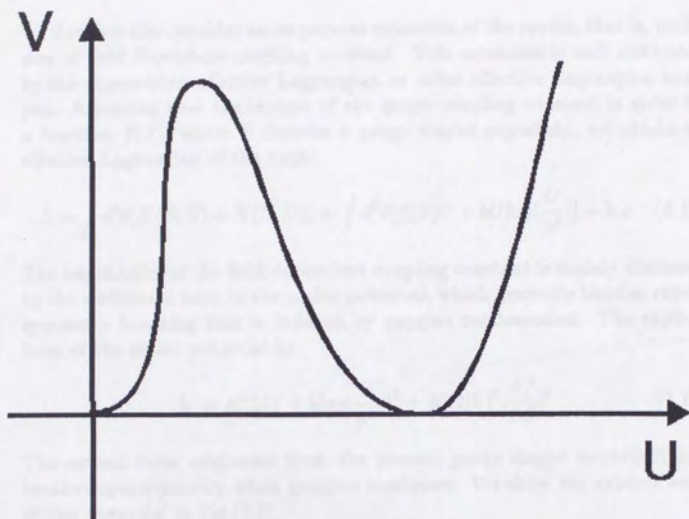


Figure 3.1: The scalar potential for gaugino condensate(U) for general supersymmetric Yang-Mills theory.

Here we also consider an important extension of the model, that is, inclusion of field dependent coupling constant. This extension is well motivated by the superstring effective Lagrangian or other effective Lagrangian analysis. Assuming that the inverse of the gauge coupling constant is given by a function $f(S)$, where S denotes a gauge singlet superfield, we obtain an effective Lagrangian of the form:

$$L = \int d^4\theta [K(S, \bar{S}) + K(U, \bar{U})] + \int d^2\theta [f(S)U + bU \log(\frac{U}{\mu^3})] + h.c. \quad (3.12)$$

The implication of the field dependent coupling constant is mainly discussed by the additional term in the scalar potential, which generally implies supersymmetry breaking that is induced by gaugino condensation. The explicit form of the scalar potential is:

$$V = K_{U\bar{U}}^{-1} |f + b \log(\frac{U}{\mu^3})|^2 + K_{S\bar{S}}^{-1} |U|^2 |\frac{\partial f}{\partial S}|^2 \quad (3.13)$$

The second term originates from the present gauge singlet superfield and breaks supersymmetry when gaugino condenses. We show the explicit form of this potential in Fig.(3.2)

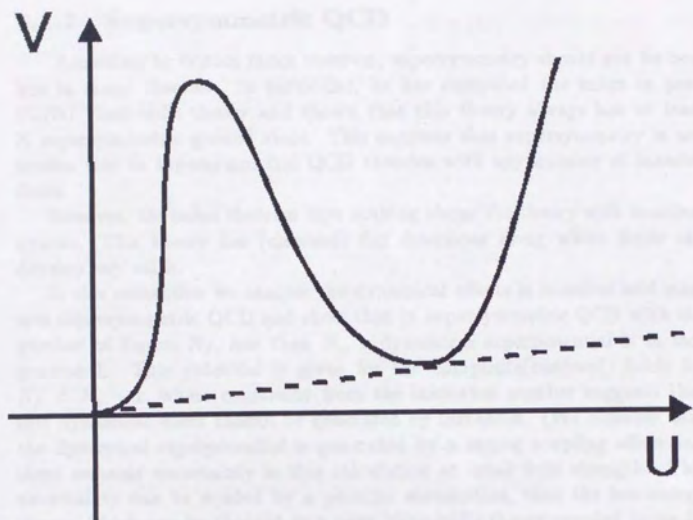


Figure 3.2: Eq.(3.13) is plotted. It is easy to find that gaugino condensation breaks supersymmetry once the gauge coupling constant becomes field dependent.

3.1.2 Supersymmetric QCD

According to Witten index theorem, supersymmetry should not be broken in many theories. In particular, he has computed the index in pure $SU(N)$ Yang-Mills theory and shown that this theory always has at least N supersymmetric ground state. This suggests that supersymmetry is not broken also in supersymmetric QCD theories with any number of massive fields.

However, the index theorem says nothing about the theory with massless quarks. This theory has (classical) flat directions along which fields can develop any value.

In this subsection we analyze the dynamical effects in massless and massive supersymmetric QCD and show that in supersymmetric QCD with the number of flavors N_f , less than N_c , a dynamical superpotential is in fact generated. This potential is given for the composite(confined) fields for $N_f < N_c - 1$, where constraint from the instanton number suggests that this dynamical effect cannot be generated by instanton. (We consider that the dynamical superpotential is generated by a strong coupling effect and there remains uncertainty in this calculation at small field strength. This uncertainty can be evaded by a peculiar assumption, that the low-energy theory, which can be thought as a pure Yang-Mills theory coupled to an R-Axion, has the same characteristics as the pure Yang-Mills theory. If this assumption is correct, we can expect that gaugino condensation is always non-zero even in the weak coupling limit.) However for $N_f = N_c - 1$, the instanton number constraint does not forbid the generation of the dynamical superpotential and we can show its generation by an explicit computation. (The F-term carries charge 2 under $U(1)_{R'}$ which cancels the charge -2 from the grassmannian integral. On the other hand, from

$$\partial_\mu j_R^\mu = \frac{1}{32\pi^2} 2(N_f - N_c) F^a \tilde{F}^a \quad (3.14)$$

we learn that instanton carries charge $2|N_c - N_f|$ under $U(1)_R$ symmetry. Thus we can see that the dynamical potential is allowed only for $N_f = N_c - 1$.)

Because we usually think that the instanton effects will dominate the dynamical effects in the weak coupling region, we think that the uncertainty does not exist in this case.

To obtain dynamical understanding of these theories, many authors attempted to construct effective Lagrangians to describe the low-energy dynamical

ics of these theories. These analysis made it clear that in these theories with massive quarks, the limit $m_q \rightarrow 0$ was likely to be peculiar. In particular, it was shown that if a dynamical superpotential is generated in supersymmetric QCD with massless quarks by non-perturbative effects, its form is uniquely determined by the symmetric constraints. Moreover, if a small mass term is added to this theory, we can obtain N vacuum states which agrees with the index arguments. (Of course tree level superpotential can alter the symmetries of the theory, which have constrained the dynamical superpotential in massless theories. Above we assumed that a small mass term will not change the qualitative character of the dynamical potentials.)

Here we mainly follow the paper ref.[28] which we regard as a "general" analysis.

By supersymmetric QCD we will mean a supersymmetric theory with gauge group $SU(N_c)$ and N_f flavors of quarks. The N_f quark flavors correspond to N_f chiral fields in the N_c representation and N_f chiral fields in the \overline{N}_c representation.

$$Q^{ir}(i = 1, \dots, N_c; r = 1, \dots, N_f), \quad \overline{Q}_{ir}(i = 1, \dots, N_c; r = 1, \dots, N_f) \quad (3.15)$$

These superfields can be written with component fields as:

$$\begin{cases} Q^{ir} = \phi^{ir} + \theta^\alpha \psi_\alpha^{ir} + \theta^2 F^{ir} \\ \overline{Q}_{ir} = \bar{\phi}_{ir} + \theta_\alpha \bar{\psi}^{\dot{\alpha}}_{ir} + \theta^2 \bar{F}_{ir} \end{cases} \quad (3.16)$$

The gauge fields $A_\mu^a (a = 1, \dots, N_c^2 - 1)$ are included in vector multiplets V^a accompanied by their super-partners, gauginos λ^a and auxiliary fields D^a . The total theory is given by

$$L = \frac{1}{4g^2} \int d^2\theta W^{aa} W_\alpha^a + h.c. + \int d^4\theta [Q^\dagger e^V Q + \overline{Q} e^{-V} \overline{Q}^\dagger] \quad (3.17)$$

Classically, this theory has a global $U(N_f)_{Left} \times U(N_f)_{Right} \times U(1)_R$ symmetry. The $U(N_f)_{Left} \times U(N_f)_{Right}$ symmetry is just like that of the ordinary QCD, corresponding to separate rotation of the Q and \overline{Q} fields. The symmetry $U(1)_R$ is an R-invariance, a symmetry under which the components of a given superfield transform differently. This corresponds to a rotation of the phases of the grassmannian variables θ^α ,

$$\begin{cases} \lambda \rightarrow e^{i\alpha} \lambda \\ \psi \rightarrow e^{i\alpha} \psi \\ \bar{\psi} \rightarrow e^{i\alpha} \bar{\psi} \end{cases} \quad (3.18)$$

with scalar and vector fields unrotated. This can be written as:

$$\begin{cases} W_\alpha(\theta) & \rightarrow e^{-i\alpha} W_\alpha(\theta e^{i\alpha}) \\ Q(\theta) & \rightarrow Q(\theta e^{i\alpha}) \\ \bar{Q}(\theta) & \rightarrow \bar{Q}(\theta e^{i\alpha}). \end{cases} \quad (3.19)$$

Just as in the ordinary QCD, some of these symmetries are explicitly broken by anomalies. A simple computation shows that the following symmetry, which is a combination of the ordinary chiral $U(1)_A$ and the $U(1)_R$ symmetry, is anomaly-free.

$$\begin{cases} W_\alpha(\theta) & \rightarrow e^{-i\alpha} W_\alpha(\theta e^{i\alpha}) \\ Q(\theta) & \rightarrow e^{i\alpha(N_c - N_f)/N_f} Q(\theta e^{i\alpha}) \\ \bar{Q}(\theta) & \rightarrow e^{i\alpha(N_c - N_f)/N_f} \bar{Q}(\theta e^{i\alpha}) \end{cases} \quad (3.20)$$

From now on, we call this non-anomalous global symmetry $U(1)_{R'}$.

It is important to stress that we are not looking for an explicit breaking of supersymmetry. Since it is believed that the supersymmetry current has no anomalies, the effective lagrangian should be supersymmetric and should be given by superfields. Its vacua, of course, need not to respect supersymmetry and other symmetries. The effective Lagrangian must respect all the (non-anomalous) symmetries whether the various symmetries of the theory is broken or not. Thus the dynamical superpotential which may be generated must be gauge and $G = SU(N_f)_{Left} \times SU(N_f)_{Right} \times U(1)_V \times U(1)_{R'}$ invariant. To be gauge and $SU(N_f)_{Left} \times SU(N_f)_{Right} \times U(1)_V$ invariant, such a F-term must be of the form:

$$F \sim \int d^2\theta f(\det_{rr'} \bar{Q}_{ir} Q^{ir'}) \quad (3.21)$$

Further requirement from R' -invariance determines the precise form as:

$$F \sim \int d^2\theta (\det_{rr'} \bar{Q}_{ir} Q^{ir'})^{-1/(N_c - N_f)} \quad (3.22)$$

This term is only meaningful for $N_f < N_c$. For $N_c = N_f$, it is meaningless. For $N_f > N_c$, it vanishes identically by a simple symmetry argument. The coefficient of this F-term should be dimensionful, and must be given by a power of the dynamically generated scale of the theory (Λ), which can be related to the scale of gaugino condensation.

$$F = \text{const.} \Lambda^{\frac{3N_c - N_f}{N_c - N_f}} \int d^2\theta (\det_{rr'} \bar{Q}_{ir} Q^{ir'})^{\frac{-1}{N_c - N_f}} \quad (3.23)$$

The question of whether the dynamics indeed generates this term or not is of course another problem and will be discussed in the following.

In $N_f = N_c - 1$, we can show that this term can be constructed by an explicit instanton calculation, so the dynamical origin is clear in this case.

Since this model has flat directions, it is reasonable to expect that Q and \bar{Q} may develop their vacuum expectation values along these directions. When $N_f < N - 1$, because instanton does not allow the generation of (3.23), we should explain its generation from another point of view. In this case, the gauge group is not completely broken and we can expect that the intermediate scale Lagrangian, which appears after gauge symmetry breaking, is pure Yang-Mills theory coupled to an axion superfield. At energies below the symmetry breaking scale, assuming that the supersymmetry is not explicitly broken by perturbative effects, we can obtain an effective Lagrangian:

$$L = \int d^2\theta \frac{1}{4g^2} \left[1 + \beta \ln \left(\frac{\phi}{\Lambda} \right) \right] W^\alpha W_\alpha + h.c.$$

$$\beta = \frac{g^2}{32\pi^2} N_f \quad (3.24)$$

where β is determined by R-anomaly. Eq.(3.24) contains a dimension five operator:

$$L_{AUX} = \frac{\beta g^2 \lambda \lambda}{v} F_\phi + h.c. + F_\phi^* F_\phi \quad (3.25)$$

where v is the symmetry breaking scale of $SU(N_c) \rightarrow SU(N_c - N_f)$, which can be identified with the cut-off scale of this effective Lagrangian. The field ϕ is considered as an anomaly compensating field for the non-anomalous R' symmetry of the original Lagrangian. When $N_c = 3$ and $N_f = 1$, this effective Lagrangian is explicitly calculated by integrating the heavy fields[33]. This extra term in L_{AUX} is very important when we think about the dynamical breaking of supersymmetry. Using the equation of motion for F_ϕ , we obtain the relation

$$\langle F_\phi^* \rangle \sim \langle \lambda \lambda \rangle / v \quad (3.26)$$

this means that once gaugino condensates in the effective Yang-Mills theory, supersymmetry should be broken dynamically. At the same time, the flat direction along ϕ is lifted, but the shape of this potential is rather problematic. Its minimum lies at infinity, so this potential is called "Run-away" potential.(See also Fig.(3.3).)

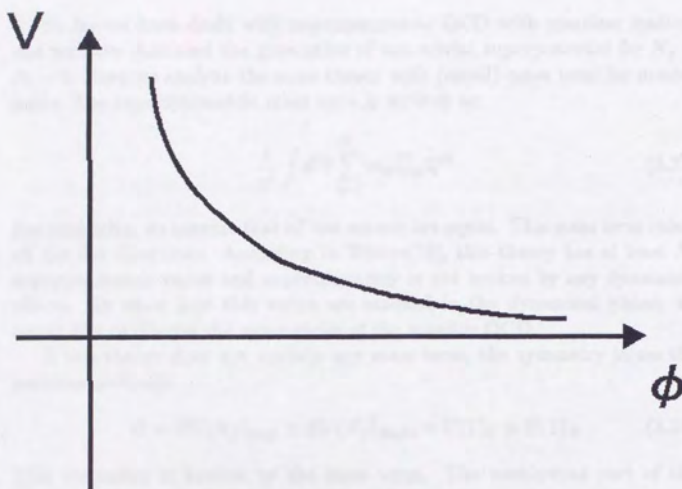


Figure 3.3: The flat direction is lifted but there remains a problematic behavior. This potential presents the so-called “Runaway potential”.

So far we have dealt with supersymmetric QCD with massless matters and we have discussed the generation of non-trivial superpotential for $N_f < N_c - 1$. Here we analyze the same theory with (small) mass term for matter fields. The supersymmetric mass term is written as:

$$\frac{1}{g^2} \int d^2\theta \sum_{k=1}^{N_f} m_k \bar{Q}_{ik} Q^{ik} \quad (3.27)$$

For simplicity, we assume that all the masses are equal. This mass term raises all the flat directions. According to Witten[10], this theory has at least N_c supersymmetric vacua and supersymmetry is not broken by any dynamical effects. To show how this vacua are realized in the dynamical phase, we would like to discuss the symmetries of the massive QCD.

If this theory does not contain any mass term, the symmetry is (see the previous section):

$$G = SU(N_f)_{Left} \times SU(N_f)_{Right} \times U(1)_V \times U(1)_R \quad (3.28)$$

This symmetry is broken by the mass term. The continuous part of the remaining symmetry is:

$$SU(N_f) \times U(1)_V \quad (3.29)$$

where $SU(N_f)$ is the vector subgroup of $SU(N_f)_{Left} \times SU(N_f)_{Right}$. Besides this continuous symmetry, this theory has discrete Z_{2N_c} symmetry:

$$\begin{aligned} W(\theta) &\rightarrow e^{-i\pi n/N_c} W(\theta e^{i\pi n/N_c}) \\ Q(\theta) &\rightarrow e^{-i\pi n/N_c} Q(\theta e^{i\pi n/N_c}) \\ \bar{Q}(\theta) &\rightarrow e^{-i\pi n/N_c} \bar{Q}(\theta e^{i\pi n/N_c}) \\ n &= 1, \dots, 2N_c \end{aligned} \quad (3.30)$$

Witten has suggested that the N_c vacua counted by index argument might be associated with the spontaneous breaking of this symmetry to a Z_2 subgroup. As far as the quark mass is small and can be considered as a small perturbation, one can show that the dynamical superpotential is still generated. The full superpotential is given by:

$$W_{dyn} = \text{const.} \Lambda^{\frac{3N_c - N_f}{N_c - N_f}} (\det \bar{Q} Q)^{\frac{-1}{N_c - N_f}} + m \bar{Q} Q \quad (3.31)$$

This superpotential has N supersymmetric vacua of the form:

$$\begin{aligned} \langle \overline{Q}Q \rangle &= \Lambda^2 \left(\frac{m(N_c - N_f)}{\text{const.} g^2 \Lambda} \right)^{\frac{N_c - N_f}{N_c}} e^{2\pi i n / N_c} \\ n &= 1, \dots, N_c \end{aligned} \quad (3.32)$$

This means that supersymmetric equation has exactly N_c different solution. The runaway potential is thus stabilized and supersymmetric vacuum is now well defined. (See Fig.(3.4))

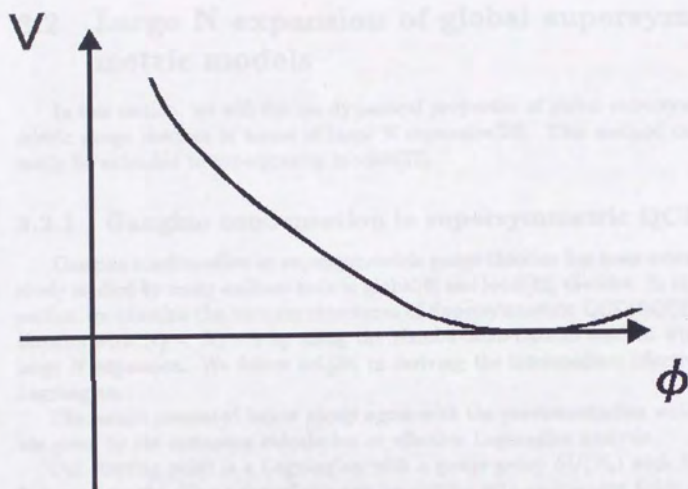


Figure 3.4: The scalar potential for SQCD with small mass term. The asymptotic behavior is changed. No runaway is observed and supersymmetry is kept unbroken.

3.2 Large N expansion of global supersymmetric models

In this section, we will discuss dynamical properties of global supersymmetric gauge theories in terms of large N expansion[26]. This method can easily be extended to supergravity models[27].

3.2.1 Gaugino condensation in supersymmetric QCD

Gaugino condensation in supersymmetric gauge theories has been extensively studied by many authors both in global[6] and local[32] theories. In this section we examine the vacuum structures of Supersymmetric QCD(SQCD) theories with $N_f < N_c - 1$ by using the Nambu-Jona-Lasinio method with large N expansion. We follow ref.[28] in deriving the intermediate effective Lagrangian.

The results presented below nicely agree with the previous studies which are given by the instanton calculation or effective Lagrangian analysis.

Our starting point is a Lagrangian with a gauge group $SU(N_c)$ with N_f flavors of quarks. These superfields can be written with component fields as (The following Lagrangian is the same as what was analyzed in the previous section.):

$$\begin{cases} Q^{ir} = \phi^{ir} + \theta^\alpha \psi_\alpha^{ir} + \theta^2 F^{ir} \\ \bar{Q}_{ir} = \bar{\phi}_{ir} + \theta_\alpha \bar{\psi}^{\dot{\alpha}}_{ir} + \theta^2 \bar{F}_{ir} \end{cases} \quad (3.33)$$

The gauge fields $A_\mu^a (a = 1, \dots, N_c^2 - 1)$ are included in vector multiplets V^a accompanied by their super-partners, gauginos λ^a and auxiliary fields D^a . The total theory is given by

$$L = \frac{1}{4g^2} \int d^2\theta W^{aa} W_a^a + h.c. + \int d^4\theta [Q^+ e^V Q + \bar{Q} e^{-V} \bar{Q}^+]. \quad (3.34)$$

The symmetries of this model are already discussed in the previous section.

Since this model has flat directions, it is reasonable to expect that Q and \bar{Q} may develop their vacuum expectation values along these directions. If $N_f < N - 1$, the gauge group is not completely broken. Moreover, we can see that instantons cannot generate a superpotential in this case, so we think that considering another type of non-perturbative effects in this model seems very important.

For simplicity, here we consider the case: $SU(N_c)$ gauge group is broken to $SU(N_c - N_f)$. The low-energy theory, where gauge interaction is still unconfined, consists of two parts: Kinetic terms for the unbroken pure $SU(N_c - N_f)$ gauge interaction and one for the massless chiral field. In addition to these terms, we should include higher dimensional operator. A dimension-five operator, in general, is generated at one-loop level[33]. As we have stated in the previous section, this can be obtained also from the renormalization of the effective coupling[6]:

$$L = \int d^2\theta \frac{1}{4g^2} \left[1 + \frac{g^2}{32\pi^2} N_f \ln \left(\frac{\phi}{\Lambda} \right) \right] W^\alpha W_\alpha + h.c. \quad (3.35)$$

where we can think that the renormalization of the effective gauge coupling constant is now field dependent:

$$g^{-2}(\phi) \equiv g^{-2} \left[1 + \frac{g^2}{32\pi^2} N_f \ln \left(\frac{\phi}{\Lambda} \right) \right] \quad (3.36)$$

Of course, this term itself is not five dimensional. Redefining the field as $\phi = \langle v \rangle + \phi'$, this term produces a dimension five operator, namely $\sim \frac{\phi'}{v} W^2$. Here v can be regarded as a symmetry breaking scale and can be treated as the cut-off scale of the low energy effective theory ($v \sim \Lambda$). In general, each Q can develop different values, but here, for simplicity, we assume every Q gains the same classical value. ϕ must be chosen to be invariant under all symmetries except for $U(1)_{R'}$. Detailed arguments on such a field dependence of coupling constant are given in ref.[6] and the references therein. The non-anomalous R' -symmetry of the original theory must be realized in the effective low-energy Lagrangian by the shift induced by ϕ . That determines the R' -charge of ϕ to be $(N_c - N_f)/N_f$.

For simplicity, we consider a generalized form

$$L = \int d^2\theta \frac{1}{4} f(\phi) W^\alpha W_\alpha + h.c. + \int d^4\theta \phi^* \phi \quad (3.37)$$

where $f(\phi)$ is the field dependent coupling constant.

$$f(\phi) = \frac{1}{g_0^2} + \beta \log \left(\frac{\phi}{\Lambda} \right) \quad (3.38)$$

Here β is a constant chosen to realize the anomaly free (mixed) R'-symmetry of the original Lagrangian. In our case, we take $\beta = \frac{N_f}{32\pi^2}$.

Generally, the kinetic term for the axionic superfield, $\phi\phi^*$ in eq.(3.37), is not calculable and one may expect it should have some other complicated form. However, as far as $K(\phi, \phi^*)$ is written by a function of the form $f(\phi\phi^*)$, the essential features are not changed. In that case, the auxiliary part of the kinetic term is changed to: $F_\phi F_\phi^* \rightarrow f'' F_\phi F_\phi^*$. Here, for convenience, we consider the simplest choice. The gauge group of the low energy theory is $SU(N_c - N_f)$. What we are concerned with is the auxiliary part of this Lagrangian:

$$L_{AUX} = \frac{\beta g^2 \lambda \lambda}{v} F_\phi + h.c. + F_\phi^* F_\phi \quad (3.39)$$

(This term can be derived by a direct integration of massive fields[33].) We can simply assume that the cut-off scale of this effective Lagrangian is v . The factor of g^2 appears because we have rescaled gaugino fields to have canonical kinetic terms. The equation of motion for F_ϕ is:

$$\begin{aligned} \frac{\partial L}{\partial F_\phi} &= \frac{\beta g^2}{v} \lambda \lambda + F_\phi^* \\ &= 0 \end{aligned} \quad (3.40)$$

This equation means that $\langle \lambda \lambda \rangle$ is proportional to F_ϕ . We can think that $\langle \lambda \lambda \rangle$ is the order parameter for the supersymmetry breaking. Using the tadpole method[34] we can derive a gap equation directly from (3.40).

$$\begin{aligned} F_\phi^* &\propto \left(1 - 4G^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0 \\ \begin{cases} G^2 = \frac{\beta^2 g^4}{v^2} n_g \\ m_\lambda^2 = \frac{|F_\phi|^2 g^2 \beta^2}{v^2} \end{cases} \end{aligned} \quad (3.41)$$

where β is proportional to N_f and n_g is the dimension of the low energy gauge group. (In this model n_g is defined as $n_g = (N_c - N_f)^2 - 1$)

Taking the limit $N_f \rightarrow \infty$, the above equation becomes a good approximation in a sense of large N expansion. (See fig.(3.5))

Of course one should be able to derive (3.41) by explicit calculation of 1-loop effective potential. But it will be very difficult because in calculating the explicit 1-loop effective potential we should include the superpartner of

gaugino condensation(may be a glueball), which makes this analysis much more complicated.

Let us examine the solution of this gap-equation. After integration we can rewrite it in a simple form.

$$\frac{4\pi^2}{G^2\Lambda^2} = 1 - \left(\frac{m_\lambda^2}{\Lambda^2}\right) \ln\left(1 + \frac{\Lambda^2}{m_\lambda^2}\right) \quad (3.42)$$

(See also Fig.(3.6).)

In the strong coupling region, this equation can have non-trivial solution. The explicit form of the scalar potential is shown in Fig.(3.7) and Fig.(3.8). (Here we ignore the trivial solution $F_\phi = 0$. We should note that such a solution does exist also in the effective Lagrangian analysis of pure Yang-Mills theory(see section 3.1), but it was neglected from several reasons.) Let us examine the behavior of this non-trivial solution. In pure supersymmetric Yang-Mills theories, gaugino condensation is observed even in the weak coupling region because of the instanton calculation and Witten index argument that suggests the invariance of Witten index under the deformation of coupling constants[10]. If we believe that the characteristics of the low energy Lagrangian of massless SQCD is also similar to pure SYM, the weak coupling region should be lifted by gaugino condensation effect. On the other hand, if we believe that non-compactness of the moduli space is crucial and believe that gaugino condensation should vanish in the weak coupling region, we can think that the potential represented in Fig.(3.8) is reliable and potential is flat in the weak coupling region. We cannot make definite answer to this question, but some suggestive arguments can be given by adding a small mass term for the field ϕ .

$$L_{mass}^{add} = \frac{1}{2}\epsilon\phi^2 \quad (3.43)$$

Existence of this term suggests that the moduli space is now compact. The resulting gap equation is drastically changed. We can naturally set F components to vanish, and the equation turns out to be a non-trivial equation for " ϕ ". Relevant terms are:

$$L_{AUX} = \left(\frac{\beta g^2}{v}\lambda\lambda + \epsilon\phi\right) F_\phi^* + h.c. + F_\phi^* F_\phi \quad (3.44)$$

The equation of motion for F_ϕ suggests that $\langle\lambda\lambda\rangle$ is now proportional to ϕ and no longer an order parameter for the supersymmetry breaking. The

gap equation is given by:

$$\epsilon\phi \times \left(1 - 4G^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 = \frac{\beta^2 g^4}{v^2} n_g \\ m_\lambda^2 = \frac{\epsilon^2 g^4 \beta^2 |\phi|^2}{v^2} \end{cases} \quad (3.45)$$

In general, this equation has a solution $m_\lambda = \text{const.}$ (see Fig.(3.9) and (3.10)) which does *not* break supersymmetry, and does not change Witten index for any (non-zero) value of ϵ and g_0 . In this case, the potential energy is always 0 for any value of g . Because the moduli space is compact and Witten index is well defined in this case, it is conceivable that there is no phase transition for gaugino condensation. (See Fig.(3.11).)

Here, we also comment on a peculiar properties of this model. As we have seen in the exactly calculable models, the dynamical mass is stable against the soft gaugino mass in the strong coupling region (i.e. when the gap-equation develops non-trivial solution.). This is precisely true in the large N limit, but we are not sure whether this property remains true also in the phenomenological models.

3.2.2 Dilaton dependent coupling constant

In this subsection we mainly focus on the supersymmetric Yang-Mills theory in which the gauge coupling constant is dependent only on the dilaton field S . The Lagrangian is now written as:

$$L = \frac{1}{4} \int d^2\theta f(S) W^\alpha W_\alpha + h.c. - \int d^4\theta \Lambda^2 \log(S + \bar{S}) \quad (3.46)$$

Here we assume $Re f(S) = Re S \equiv \frac{1}{g_0}$. Relevant part of the Lagrangian is:

$$L_{AUX} = g^2 F_S \lambda \lambda + h.c. + \frac{F_S^* F_S}{(S + \bar{S})^2} \Lambda^2 \quad (3.47)$$

Here g means the renormalized coupling constant. Using the tadpole method we can find the following gap equation,

$$F_S^* \times \left(\frac{\Lambda^2}{(S + \bar{S})^2} - 4G^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 = g^4 n_g \\ m_\lambda^2 = g^4 |F_S|^2 \\ n_g = N_c^2 - 1 \end{cases} \quad (3.48)$$

which can be rewritten as:

$$F_S^* \times \left(1 - 4G'^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 = \frac{4}{\Lambda^2} \left(\frac{g}{g_0} \right)^4 n_g \\ m_\lambda^2 = g^4 |F_S|^2 \end{cases} \quad (3.49)$$

This equation relies only on the parameter g/g_0 . This gap equation has the same characteristics of massless SQCD, so it is believed that $V(S)$ has runaway behavior. The potential is almost the same as Fig.(3.8).

Then what would happen if we add a small mass term, for example, $L_m = \epsilon \Lambda^2 S^2/2$? Now the auxiliary part of the Lagrangian is:

$$L_{AUX} = [g^2 F_S \lambda \lambda + \epsilon \Lambda^2 S F_S + h.c.] + \frac{F_S^* F_S}{(S + \bar{S})^2} \Lambda^2 \quad (3.50)$$

This means that the dilaton field S now becomes an order parameter for $\langle \lambda \lambda \rangle$. The gap equation is now given by:

$$\epsilon S \times \left(1 - 4G^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 &= \frac{g^4 (S + \bar{S})^2}{\Lambda^2} n_g \\ &= \frac{4}{\Lambda^2} \left(\frac{g}{g_0} \right)^4 N_c \\ m_\lambda^2 &= \epsilon^2 |S|^2 g^4 (S + \bar{S})^4 \\ &= 16 \epsilon^2 \left(\frac{g}{g_0} \right)^4 |S|^4 \end{cases} \quad (3.51)$$

This equation has a non-trivial solution at finite value as we have seen in the massive SQCD.

Finally we will comment on the relation to the phenomenological models. If a small mass term is induced by the small spacetime curvature or some effects of higher theories, and assuming that our analyses are correct in such theories, we can expect that the analysis above may be used to analyze the dynamical dilaton potential and stabilization of the runaway potential.

$$\left[\frac{\beta g^2}{v} \star \bigcirc^{\lambda \lambda} \right] + \left[\langle F_\phi \rangle \right]$$

$$O(g^2 N_f) \equiv O(g_0^2)$$

$$\left[\frac{\beta g^2}{v} \star \bigcirc^{\lambda \lambda} \right]$$

$$O(g^4 N_f) \equiv O(g_0^4 / N_f)$$

Figure 3.5: The gap equation (3.41) is written in a diagrammatic form. One of the typical correction is shown in the second diagram.

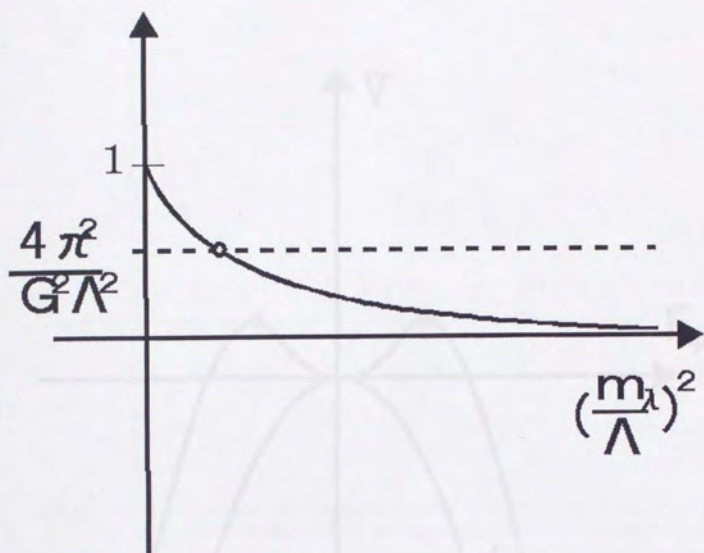


Figure 3.6: Left hand side of eq.(3.42)(dotted line) and right hand side(solid line) is plotted. If the four-fermi coupling is strong enough, non-trivial solution appears.

Figure 3.7: The relation between \tilde{S}_2 and the scalar potential is plotted for the weak and the strong coupling phases of equation (3.40) with $N_1 = N_2 = 1$. The explicit form of this potential is not calculated explicitly, but the stationary point of this potential is calculated in (3.61). The solution for the gap equation shows that at the stationary point of this potential because \tilde{S}_2 is an auxiliary field, the form of the potential is not important.

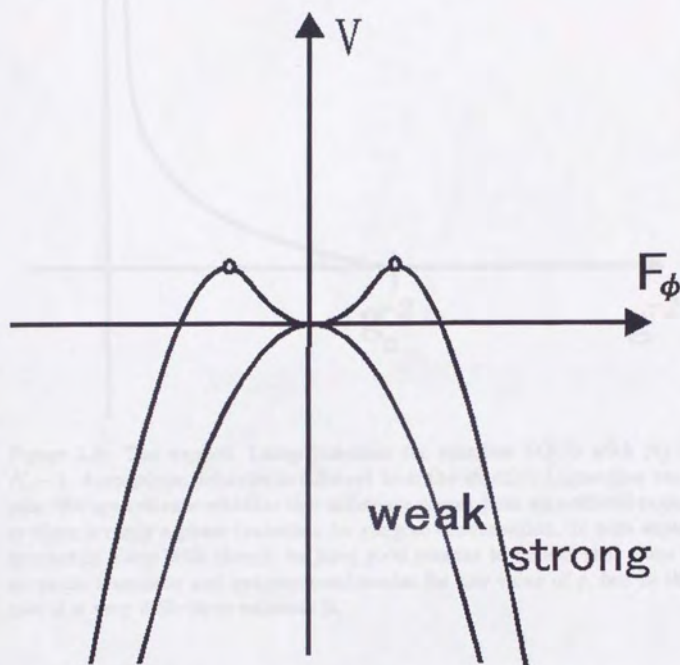


Figure 3.7: The relation between F_ϕ and the scalar potential is plotted for the weak and the strong coupling phases of massless SQCD with $N_f < N_c - 1$. The explicit form of this potential is not calculated explicitly, but the stationary point of this potential is already calculated in (3.41). The solution for the gap-equation always lies at the stationary point of this potential. Because F_ϕ is an auxiliary field, the form of the potential is not important.

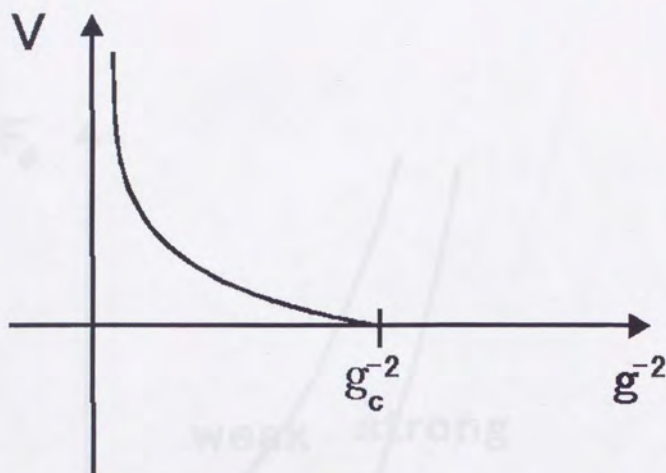


Figure 3.8: The explicit 1-loop potential for massless SQCD with $N_f < N_c - 1$. Asymptotic behavior is different from the effective Lagrangian analysis. We are not sure whether this difference comes from an artificial reason or there is really a phase transition for gaugino condensation. In pure supersymmetric Yang-Mills theory, we have good reasons to believe that there is no phase transition and gaugino condensates for any value of g , but in this case it is very difficult to examine it.

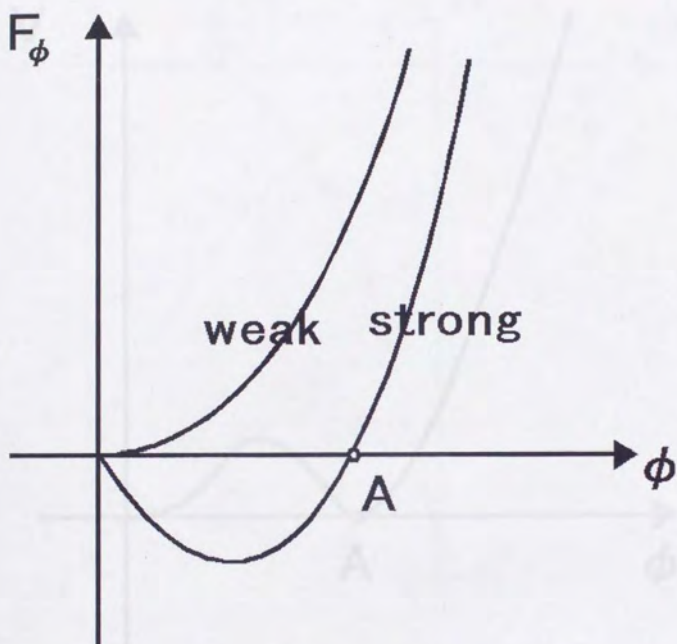


Figure 3.9: The relation between F_ϕ and ϕ for SQCD with small mass term is plotted. It is obvious that we can find a supersymmetric vacuum state with vanishing F_ϕ .

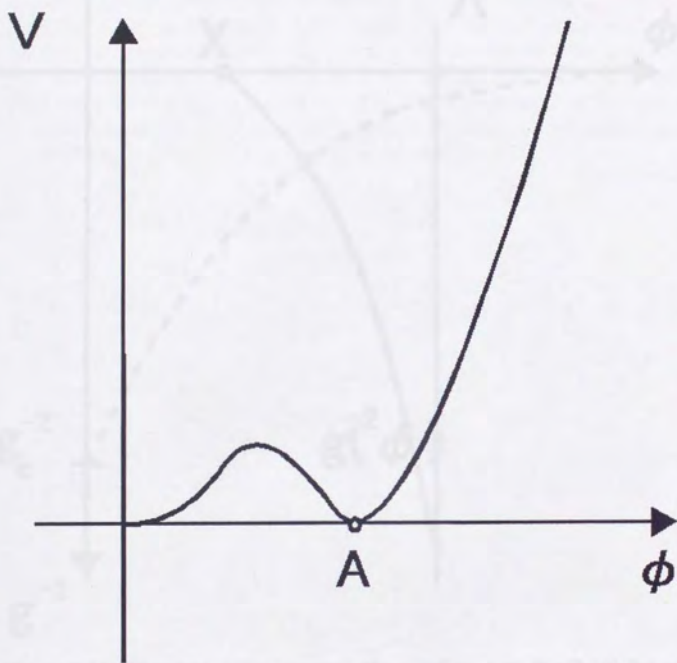


Figure 3.10: The explicit form of scalar potential is plotted for the weakly and the strongly coupled phases. We can find a “stable” supersymmetric vacuum state. Please compare this figure with Fig.(3.3), (3.4) and (3.8).

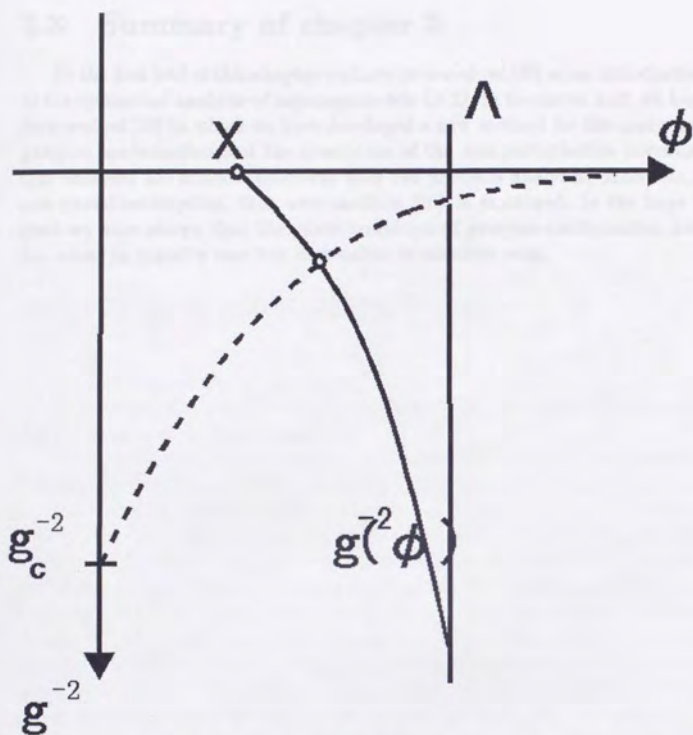


Figure 3.11: The dotted line presents the solution A in Fig.(3.10) for any g . This line is obtained by solving the equation (3.45) with the assumption that g does not depend on ϕ . (See also eq.(3.42) and fig.(3.6)) The solid line shows the relation between g and ϕ . To find a true vacuum, we should find a point at which both of these two lines meet. Seeing this graph, we can easily understand why no phase transition is observed in massive SQCD. In general, we tend to think that the vacuum lies at X , which corresponds to a solution for $f(\varphi) = 0$.

3.3 Summary of chapter 3

In the first half of this chapter we have reviewed ref.[28] as an introduction to the dynamical analysis of supersymmetric QCD. In the latter half, we have reviewed ref.[26] in which we have developed a new method for the analysis of gaugino condensation and the generation of the non-perturbative potential. Our analyses are almost consistent with the previous analyses. Moreover, a non-trivial assumption, that were made in [28], is examined. In the large N limit we have shown that the phase transition of gaugino condensation does not occur in massive case but does occur in massless case.

Chapter 4

Dynamical analysis in supergravity theories

In general, one may think there are four different ways of introducing gaugino condensates into supergravity.

First, in the component Lagrangian method[29], one takes the standard Lagrangian of supersymmetric Yang-Mills theory coupled to supergravity and replaces the gaugino bilinear with a constant of the order of the condensation scale μ^3 . Such a procedure has the drawback of discarding the back-reaction of other fields, hence one cannot determine in this way whether the condensate really forms. The formation of the condensate and its magnitude here are simply assumed implicitly relying on the observation made in the global version of the model. However, as we have assumed that gravitational corrections can play an important role, the internal consistency of this approach is not clear. (There is no reason to believe that dynamical properties of the global supersymmetric models are not changed in supergravity models.)

Second, a refinement of this approach is considered[11]. Taking into account a possible dependence of condensate on some fields leads to the superpotential method. Using these one can then construct the gaugino induced non-perturbative corrections to the original superpotential of the model. For example, the belief that the condensate dissolves in the weak coupling limit leads to a superpotential which decays exponentially with the increasing value of the dilaton field. One may then search for minima of the effective theory and determine the true value of the gaugino condensate that leads to the supersymmetry breaking in supergravity theories.

The third method is the effective Lagrangian approach[8]. Generalizing the global effective potential written by the composite superfields, one can obtain the supergravitational version of the effective potential which is of course consistent to the global theory in the $M_p \rightarrow \infty$ limit. In supergravity models the effective scalar potential is:

$$V = g_{ij^*} F^i F^{j^*} - 3e^G \quad (4.1)$$

where $G = K + \log(|W|^2)$ and $g_{ij^*} = \partial^2 K / \partial z^i \partial z^{j^*}$.

The fourth method is the Nambu-Jona-Lasinio like approach first developed by G.G.Ross et al[30]. One of the major advantages of this method is the clear relation between the constituent Lagrangian and the effective Lagrangian with gaugino condensation. The driving force, that makes gaugino condensate as the coupling becomes strong, is now clear. The stability of the moduli potential is also modified, which we think as another advantage of this method.

4.1 Review of supergravity

In this section, we review the general supergravity Lagrangian to fix our notations. Notations are almost the same as ref.[4] except for the flat space-time metric, which we use $g_{\mu\nu} \sim \text{diag}(1, -1, -1, -1)$. Derivation of this Lagrangian is too much complicated to be discussed here[31]. For simplicity, in this section, we use the $M = 1$ unit.

The general supergravity lagrangian is generally characterized by three functions; the Kähler potential $K(\phi, \phi^*)$, superpotential $W(\phi)$, and kinetic function $f(\phi)$ for vector multiplets. The Kähler potential $K(\phi, \phi^*)$ is a function of scalar fields ϕ and ϕ^* , while superpotential $W(\phi)$ and the kinetic function $f(\phi)$ depend on scalar fields with definite chirality. Here $\mathcal{W}\mathcal{W}$ is the kinetic term for the gauge fields. Using these functions, the general form of the supergravity lagrangian can be written by a superfield formalism,

$$S = \frac{-3}{\kappa^2} \int d^8 z \text{Exp} \left(-\frac{1}{3} \kappa^2 K_0 \right) + \int d^8 z \mathcal{E} \left[W_0 + \frac{1}{4} f_0 \mathcal{W}\mathcal{W} \right] + h.c. \quad (4.2)$$

We can expand it in components which contains scalar field ϕ , chiral fermion χ , gauge boson A_μ , and gauge fermion λ as well as the vierbein e_μ^a and the gravitino ψ_μ .

$$\begin{aligned}
\mathcal{L}_{\text{SUGRA}} = & -\frac{1}{2}eR + eg_{ij}\tilde{D}_\mu\phi^i\tilde{D}^\mu\phi^{*j} - \frac{1}{2}eg^2D_{(a)}D^{(a)} \\
& + ieg_{ij}\bar{\chi}^j\tilde{\sigma}^\mu\tilde{D}_\mu\chi^i + e\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\tilde{\sigma}_\nu\tilde{D}_\rho\psi_\sigma \\
& - \frac{1}{4}ef_{(ab)}^R F_{\mu\nu}^{(a)}F^{\mu\nu(b)} + \frac{1}{8}e\epsilon^{\mu\nu\rho\sigma}f_{(ab)}^I F_{\mu\nu}^{(a)}F_{\rho\sigma}^{(b)} \\
& + \frac{i}{2}e\left[\lambda_{(a)}\sigma^\mu\tilde{D}_\mu\bar{\lambda}^{(a)} + \bar{\lambda}_{(a)}\tilde{\sigma}^\mu\tilde{D}_\mu\lambda^{(a)}\right] - \frac{1}{2}f_{(ab)}^I\tilde{D}_\mu\left[e\lambda^{(a)}\sigma^\mu\bar{\lambda}^{(b)}\right] \\
& + \sqrt{2}egg_{ij}X_{(a)}^{*j}\chi^i\lambda^{(a)} + \sqrt{2}egg_{ij}X_{(a)}^i\bar{\chi}^j\bar{\lambda}^{(a)} \\
& - \frac{i}{4}\sqrt{2}eg\partial_i f_{(ab)}D^{(a)}\chi^i\lambda^{(b)} + \frac{i}{4}\sqrt{2}eg\partial_i f_{(ab)}^*D^{(a)}\bar{\chi}^i\bar{\lambda}^{(b)} \\
& - \frac{1}{4}\sqrt{2}e\partial_i f_{(ab)}\chi^i\sigma^{\mu\nu}\lambda^{(a)}F_{\mu\nu}^{(b)} - \frac{1}{4}\sqrt{2}e\partial_i f_{(ab)}^*\bar{\chi}^i\tilde{\sigma}^{\mu\nu}\bar{\lambda}^{(a)}F_{\mu\nu}^{(b)} \\
& + \frac{1}{2}egD_{(a)}\psi_\mu\sigma^\mu\bar{\lambda}^{(a)} - \frac{1}{2}egD_{(a)}\bar{\psi}_\mu\tilde{\sigma}^\mu\lambda^{(a)} \\
& - \frac{1}{2}\sqrt{2}eg_{ij}\tilde{D}_\nu\phi^{*j}\chi^i\sigma^\mu\tilde{\sigma}^\nu\psi_\mu - \frac{1}{2}\sqrt{2}eg_{ij}\tilde{D}_\nu\phi^j\bar{\chi}^i\tilde{\sigma}^\mu\sigma^\nu\bar{\psi}_\mu \\
& - \frac{i}{4}e\left[\psi_\mu\sigma^{\nu\rho}\sigma^\mu\bar{\lambda}_{(a)} + \bar{\psi}_\mu\tilde{\sigma}^{\nu\rho}\tilde{\sigma}^\mu\lambda_{(a)}\right]\left[F_{\nu\rho}^{(a)} + \hat{F}_{\nu\rho}^{(a)}\right] \\
& + \frac{1}{4}eg_{ij}\left[i\epsilon^{\mu\nu\rho\sigma}\psi_\mu\sigma_\nu\bar{\psi}_\rho + \psi_\mu\sigma^\sigma\bar{\psi}^\mu\right]\chi^i\sigma_\sigma\bar{\chi}^i \\
& - \frac{1}{8}e\left[g_{ij}g_{kl} - 2R_{ij}{}^{*}{}_{kl}\right]\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l \\
& + \frac{1}{16}e\left[2g_{ij}f_{(ab)}^R + f^{R(cd)-1}\partial_i f_{(bc)}\partial_j f_{(ad)}^*\right]\bar{\chi}^j\tilde{\sigma}^\mu\chi^i\bar{\lambda}^{(a)}\tilde{\sigma}_\mu\lambda^{(b)} \\
& + \frac{1}{8}e\nabla_i\partial_j f_{(ab)}\chi^i\chi^j\lambda^{(a)}\lambda^{(b)} + \frac{1}{8}e\nabla_i\partial_j f_{(ab)}^*\bar{\chi}^i\bar{\chi}^j\bar{\lambda}^{(a)}\bar{\lambda}^{(b)} \\
& + \frac{1}{16}ef^{R(cd)-1}\partial_i f_{(ac)}\partial_j f_{(bd)}\chi^i\lambda^{(a)}\chi^j\lambda^{(b)} \\
& + \frac{1}{16}ef^{R(cd)-1}\partial_i f_{(ac)}^*\partial_j f_{(bd)}^*\bar{\chi}^i\bar{\lambda}^{(a)}\bar{\chi}^j\bar{\lambda}^{(b)} \\
& - \frac{1}{16}eg^{ij}\partial_i f_{(ab)}\partial_j f_{(cd)}^*\lambda^{(a)}\lambda^{(b)}\bar{\lambda}^{(c)}\bar{\lambda}^{(d)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{16} e \lambda_{(a)} \sigma^\mu \bar{\lambda}^{(a)} \lambda_{(b)} \sigma_\mu \bar{\lambda}^{(b)} \\
& + \frac{i}{4} \sqrt{2} e \partial_i f_{(ab)} \left[\chi^i \sigma^{\mu\nu} \lambda^{(a)} \psi_\mu \sigma_\nu \bar{\lambda}^{(b)} - \frac{1}{4} \bar{\psi}_\mu \bar{\sigma}^\mu \chi^i \lambda^{(a)} \lambda^{(b)} \right] \\
& + \frac{i}{4} \sqrt{2} e \partial_i f_{(ab)}^* \left[\bar{\chi}^i \bar{\sigma}^{\mu\nu} \bar{\lambda}^{(a)} \bar{\psi}_\mu \bar{\sigma}_\nu \lambda^{(b)} - \frac{1}{4} \psi_\mu \sigma^\mu \bar{\chi}^i \bar{\lambda}^{(a)} \bar{\lambda}^{(b)} \right] \\
& - e e^{K/2} \left\{ W^* \psi_\mu \sigma^{\mu\nu} \psi_\nu + W \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu \right\} \\
& + \frac{i}{2} \sqrt{2} e e^{K/2} D_i W \chi^i \sigma^\mu \bar{\psi}_\mu + D_i W^* \bar{\chi}^i \bar{\sigma}^\mu \psi_\mu \\
& - \frac{1}{2} e e^{K/2} D_i D_j W \chi^i \chi^j + D_i W^* D_j W^* \bar{\chi}^i \bar{\chi}^j \\
& + \frac{1}{4} e e^{K/2} g^{ij*} D_j W^* \partial_i f_{(ab)} \lambda^{(a)} \lambda^{(b)} + D_i W \partial_j f_{(ab)}^* \bar{\lambda}^{(a)} \bar{\lambda}^{(b)} \\
& - e e^K \left[g^{ij*} (D_i W) (D_j W^*) - 3 W^* W \right], \tag{4.3}
\end{aligned}$$

where $f^R \equiv \text{Ref}$ and $f^I \equiv \text{Im}f$. Indices i, j, \dots represent species of chiral multiplets, and $(a), (b), \dots$ are indices for adjoint representation of gauge groups which are raised and lowered by $f_{(ab)}^R$ and its inverse. Notice that the Kähler potential K and the superpotential W in the total lagrangian (4.3) can be arranged into the following form;

$$G \equiv K + \ln(W^* W). \tag{4.4}$$

Here, covariant derivatives are defined as:

$$\tilde{D}_\mu \phi^i \equiv \partial_\mu \phi^i - g A_\mu^{(a)} X_{(a)}^i, \tag{4.5}$$

$$\begin{aligned}
\tilde{D}_\mu \chi^i & \equiv \partial_\mu \chi^i + \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} \chi^i + \Gamma_{jk}^i \tilde{D}_\mu \phi^j \chi^k - g A_\mu^{(a)} \frac{\partial X_{(a)}^i}{\partial \phi^j} \chi^j \\
& - \frac{1}{4} \left(K_j \tilde{D}_\mu \phi^j - K_{j*} \tilde{D}_\mu \phi^{*j} \right) \chi^i - \frac{i}{2} g A_\mu^{(a)} \text{Im} F_{(a)} \chi^i, \tag{4.6}
\end{aligned}$$

$$\begin{aligned}
\tilde{D}_\mu \lambda^{(a)} & \equiv \partial_\mu \lambda^{(a)} + \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} \lambda^{(a)} - g f^{abc} A_\mu^{(b)} \lambda^{(c)} \\
& + \frac{1}{4} \left(K_j \tilde{D}_\mu \phi^j - K_{j*} \tilde{D}_\mu \phi^{*j} \right) \lambda^{(a)} + \frac{i}{2} g A_\mu^{(b)} \text{Im} F_{(b)} \lambda^{(a)}, \tag{4.7}
\end{aligned}$$

$$\begin{aligned}
\tilde{D}_\mu \psi_\nu & \equiv \partial_\mu \psi_\nu + \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} \psi_\nu \\
& + \frac{1}{4} \left(K_j \tilde{D}_\mu \phi^j - K_{j*} \tilde{D}_\mu \phi^{*j} \right) \psi_\nu + \frac{i}{2} g A_\mu^{(b)} \text{Im} F_{(b)} \psi_\nu. \tag{4.8}
\end{aligned}$$

$F_{\mu\nu}^{(a)}$ is the field strength tensor for the gauge boson $A_\mu^{(a)}$, and $\hat{F}_{\mu\nu}^{(a)}$ is defined as

$$\hat{F}_{\mu\nu}^{(a)} \equiv F_{\mu\nu}^{(a)} - \frac{i}{2} \left(\psi_\mu \sigma_\nu \bar{\lambda}^{(a)} + \bar{\psi}_\mu \bar{\sigma}_\nu \lambda^{(a)} + \psi_\nu \sigma_\mu \bar{\lambda}^{(a)} + \bar{\psi}_\nu \bar{\sigma}_\mu \lambda^{(a)} \right). \quad (4.9)$$

Differentiation by the scalar field ϕ^i is symbolically represented by the index i ;

$$(\dots)_i \equiv \partial_i(\dots) \equiv \frac{\partial(\dots)}{\partial\phi^i}, \quad (\dots)_{i^*} \equiv \partial_{i^*}(\dots) \equiv \frac{\partial(\dots)}{\partial\phi^{*i}}. \quad (4.10)$$

In terms of eq.(4.10), derivatives of the superpotential are defined:

$$D_i W \equiv W_i + K_i W, \quad (4.11)$$

$$\begin{aligned} D_i D_j W &\equiv W_{ij} + K_{ij} W + K_i D_j W + K_j D_i W \\ &\quad - K_i K_j W - \Gamma_{ij}^k D_k W. \end{aligned} \quad (4.12)$$

Metric of the Kähler manifold g_{ij^*} is defined by varying the Kähler potential K by the scalar fields ϕ^i and ϕ^{*j} ;

$$g_{ij^*} \equiv \frac{\partial^2 K}{\partial\phi^i \partial\phi^{*j}}, \quad (4.13)$$

and g^{ij^*} is its inverse,

$$g_{ij^*} g^{ik^*} = \delta_{j^*}^{k^*}, \quad g_{ij^*} g^{kj^*} = \delta_i^k. \quad (4.14)$$

From this metric, connection Γ_{ij}^k and curvature $R_{ij^*kl^*}$ is given by:

$$\Gamma_{ij}^k \equiv g^{kl^*} \frac{\partial}{\partial\phi^{*i}} g_{jl^*}, \quad (4.15)$$

$$R_{ij^*kl^*} \equiv \frac{\partial}{\partial\phi^i} \frac{\partial}{\partial\phi^{*j}} g_{kl^*} - g^{mn^*} \left(\frac{\partial}{\partial\phi^{*j}} g_{ml^*} \right) \left(\frac{\partial}{\partial\phi^i} g_{kn^*} \right). \quad (4.16)$$

Using the connection given in eq.(4.15), covariant derivative ∇_i is defined as

$$\nabla_i V_j \equiv \frac{\partial}{\partial\phi^i} V_j - \Gamma_{ij}^k V_k. \quad (4.17)$$

(Here, V_i is a function of scalar fields with index i .) Next, we will comment on $X^{(a)i}$ and $D^{(a)}$. $X^{(a)i}(\phi)$ is the Killing vector associated with the Kähler metric g_{ij} . That is, with the field transformation

$$\phi^i \rightarrow \phi^{i'} = \phi^i + X^{(a)i}(\phi)\epsilon, \quad (4.18)$$

$$\phi^{i*} \rightarrow \phi^{i*'} = \phi^{i*} + X^{*(a)i}(\phi^*)\epsilon, \quad (4.19)$$

(where ϵ is an infinitesimal parameter), the Lie derivative $\delta_X^{(L)}$ of g_{ij} vanishes;

$$\begin{aligned} \delta_X^{(L)} g_{i\bar{j}} &\equiv X^{(a)\bar{k}} \frac{\partial}{\partial \phi^{\bar{k}}} g_{i\bar{j}} + g_{i\bar{k}} \frac{\partial}{\partial \phi^{\bar{j}}} X^{(a)\bar{k}} + g_{j\bar{k}} \frac{\partial}{\partial \phi^{\bar{i}}} X^{(a)\bar{k}} \\ &= \nabla_{\bar{i}} X_j^{(a)} + \nabla_{\bar{j}} X_i^{(a)} \\ &= 0, \end{aligned} \quad (4.20)$$

where $X_i^{(a)} \equiv g_{i\bar{j}} X^{(a)\bar{j}}$, and the index \bar{i} represents both i and i^* . From the above equation, we obtain two equations for the Killing vectors $X^{(a)i}(\phi)$ and $X^{*(a)i}(\phi^*)$;

$$\nabla_i X_j^{(a)} + \nabla_j X_i^{(a)} = 0, \quad (4.21)$$

$$\nabla_i X_{j^*}^{(a)} + \nabla_{j^*} X_i^{(a)} = 0. \quad (4.22)$$

The former equation is automatically satisfied, while the latter allows one to write down the Killing vectors $X^{(a)i}(\phi)$ and $X^{*(a)i}(\phi^*)$ as derivatives of some function $D^{(a)}$ (which is called Killing potential);

$$X^{(a)i}(\phi) = -ig^{ij*} \frac{\partial}{\partial \phi^{j*}} D^{(a)}, \quad (4.23)$$

$$X^{*(a)i}(\phi^*) = ig^{ij*} \frac{\partial}{\partial \phi^i} D^{(a)}. \quad (4.24)$$

Solving eq.(4.22), eq.(4.23) and eq.(4.24), we can obtain the Killing vectors $X^{(a)i}(\phi)$, $X^{*(a)i}(\phi^*)$, and the Killing potential $D^{(a)}$. Let us define $F^{(a)}$, which is an analytic function of ϕ^i , as:

$$F^{(a)} \equiv -ig^{ij*} \frac{\partial D^{(a)}}{\partial \phi^{j*}} \frac{\partial K}{\partial \phi^i} + iD^{(a)}. \quad (4.25)$$

If we take the the minimal Kähler potential $K^{min} = \phi^i \phi^{*i}$, the Killing vectors and the Killing potential is given in the following forms;

$$X^{(a)i}(\phi) = -iT_{ij}^a \phi^j, \quad (4.26)$$

$$X^{*(a)i}(\phi^*) = i\phi^{*j} T_{ji}^a, \quad (4.27)$$

$$D^{(a)} = \phi^{*i} T_{ij}^a \phi^j, \quad (4.28)$$

where T_{ij}^a is a generator of gauged Lie group. For the local SUSY transformation, variations of each component field are given by

$$\delta e_\mu^a = -i(\xi \sigma^a \bar{\psi}_\mu + \bar{\xi} \bar{\sigma}^a \psi_\mu), \quad (4.29)$$

$$\delta \phi^i = \sqrt{2} \xi \chi^i, \quad (4.30)$$

$$\begin{aligned} \delta \chi^i &= i\sqrt{2} \sigma^\mu \left(\bar{D}_\mu \phi^i - \frac{1}{\sqrt{2}} \psi_\mu \chi^i \right) - \Gamma_{jk}^i \delta \phi^j \chi^k \\ &\quad + \frac{1}{4} (K_j \delta \phi^j - K_{j*} \delta \phi^{j*}) \chi^i - \sqrt{2} F^i \xi \\ &\quad + \frac{1}{2\sqrt{2}} \xi g^{ij*} \partial_{j*} f_{(ab)}^* \bar{\lambda}^{(a)} \bar{\lambda}^{(b)}, \end{aligned} \quad (4.31)$$

$$\delta A_\mu^{(a)} = i(\xi \sigma_\mu \bar{\lambda}^{(a)} + \bar{\xi} \bar{\sigma}_\mu \lambda^{(a)}), \quad (4.32)$$

$$\begin{aligned} \delta \lambda^{(a)} &= \hat{F}_{\mu\nu}^{(a)} \sigma^{\mu\nu} \xi - \frac{1}{4} (K_j \delta \phi^j - K_{j*} \delta \phi^{j*}) \lambda^{(a)} - ig D^{(a)} \xi \\ &\quad + \frac{1}{2\sqrt{2}} \xi f^{R(ab)-1} \partial_i f_{(bc)} \chi^i \lambda^{(c)} - \frac{1}{2\sqrt{2}} \xi f^{R(ab)-1} \partial_{i*} f_{(bc)}^* \bar{\chi}^i \bar{\lambda}^{(c)} \end{aligned} \quad (4.33)$$

$$\begin{aligned} \delta \psi_\mu &= 2\bar{D}_\mu \xi - \frac{i}{2} \sigma_{\mu\nu} \xi g_{ij*} \chi^i \sigma^\nu \bar{\chi}^j + \frac{i}{2} (e_\mu^a e_{\nu a} + \sigma_{\mu\nu}) \xi \lambda_{(a)} \sigma^\nu \bar{\lambda}^{(a)} \\ &\quad - \frac{1}{4} (K_j \delta \phi^j - K_{j*} \delta \phi^{j*}) \psi_\mu + ie^{K/2} W \sigma_\mu \bar{\xi}. \end{aligned} \quad (4.34)$$

with

$$F^i \equiv e^{K/2} g^{ij*} D_{j*} W^*. \quad (4.35)$$

Notice that F^i given in eq.(4.35) corresponds to the auxiliary field in chiral multiplet in the global SUSY case. (Do not confuse F^i with $F^{(a)}$ defined in eq.(4.25).)

4.1.1 Duality invariant Lagrangian

Let us consider a very important extension of supergravity: superstring inspired supergravity. In this paper, the word "Duality" is used in an old fashioned way. We believe that the string theory provides a most natural framework for gauge unification since, at tree level, all coupling constants are given in terms of the string coupling constant and the vacuum expectation values of the moduli fields. At scales below the string scale M_s , these couplings will evolve according to their renormalization group equations, and for a simple gauge group G_a the one-loop running gauge coupling is given by:

$$\frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a \log \frac{\Lambda^2}{M_a^2} + \Delta_a \quad (4.36)$$

where b_a is the $N=1$ β -function coefficient and M_a is the renormalization scale below which the coupling begins to run.

The existence of an infinite number of massive states, which is called Kaluza-Klein modes and winding modes, give rise to a string threshold correction Δ_a which is relevant to the determination of the coupling constant at the string scale. These threshold corrections can be calculated explicitly in some definite models including the world-sheet string amplitudes involving external gauge fields and moduli [35, 36]. Another indirect way to determine the threshold corrections is by imposing target space moduli invariance and the cancellation of target moduli anomalies [37].

In general four-dimensional string-inspired models, moduli fields parametrize the geometry and complex structure of the compactified dimensions. For example, the size of the three complex planes of the compactified six-dimensional manifold, and other (1,1) moduli corresponds to the relative shape of these planes. The (1,2) moduli gives the complex structure of the three subtori. For the three (1,1) moduli representing the size of the tori their real part is given by the compactified radius, i.e. $Re T_i = R^2$, while the imaginary part is an axion-like field. In this paper we will only consider the dependence of the effective potential on (1,1) and (1,2) moduli that transform under duality as an element of the group $SL(2, Z)$, i.e. $T \rightarrow (aT - ib)/(icT + d)$ with $ad - bc = 1$ and $a, b, c, d \in Z$. In most cases this is a subgroup for moduli fields in orbifold compactification.

The scale M_a , below which the coupling constant starts to run, is in

general a moduli dependent quantity,

$$M_a^2 = \sum_i (T_{Ri})^{\alpha_i} M_s^2 \quad (4.37)$$

where $T_{Ri} = (T + \bar{T})_i$ and the constant α_i is model and gauge dependent. In the case of single overall (1,1) moduli $T \equiv T_i$ with $\alpha \equiv \sum \alpha_i = -1$, one obtains the naive field theoretical expression for $M_a = (ReSReT)^{\frac{1}{2}}$. The contribution to the gauge coupling constant from the moduli fields in M_a can be calculated from anomalous triangle diagrams with two gauge bosons and several moduli fields as external legs and massless gauginos and charged fermionic matter fields circulating inside loops.

The threshold term in eq.(4.36) is given by

$$\begin{aligned} \Delta_a &= \sum_i (b'_a{}^i - k_a \delta_{GS}^i) \ln |\eta(T_i)|^4 \\ b'_a{}^i &= \frac{1}{16\pi^2} \left(C(G_a) - \sum_{R_a} h_{R_a} T(R_a) \right) (1 + 2n_{R_a}^i) \end{aligned} \quad (4.38)$$

where $n_{R_a}^i$ is the modular weight for a chiral matter superfield in the representation R_a with respect to the i -moduli, $\eta(T) = \exp(-\pi T/12) \prod_{n=1}^{\infty} (1 - \exp(-2n\pi T))$ is the Dedekind η function. In the case of an overall modulus, we can simplify:

$$b'_a = b_a - 2 \sum_{R_a} h_{R_a} T(R_a) (1 + n_{R_a}) \quad (4.39)$$

with $n_{R_a} = \sum_i n_{R_a}^i$.

The universal Green-Schwartz coefficient δ_{GS} is needed to cancel, using the Green-Schwartz mechanism, the gauge independent part of the target space modular anomaly. The threshold contribution of the massive fields, $b'_a{}^i - k_a \delta_{GS}^i$, is in general non-vanishing if at least one of the orbifold twists leaves the i -plane unoriented. In this sector, the massive spectrum is $N=2$ space-time supersymmetric and $b'_a{}^i - k_a \delta_{GS}^i$ is proportional to the $N=2$ β -function coefficient which is in general non-zero. On the other hand, if all orbifold twists rotate a specific plane, then the spectrum is $N=4$ supersymmetric and $b'_a{}^i - k_a \delta_{GS}^i = 0$, giving no threshold contribution. In this case the gauge coupling constant is independent of the T_i moduli and one has that $b'_a{}^i/k_a$ must be equal for all gauge groups.

In order to have a duality invariant gauge coupling constant the α coefficient has to be:

$$\alpha^i = \frac{k_a \delta_{GS}^i - b_a'^i}{b_a} \quad (4.40)$$

As a result, the one-loop gauge coupling constant can be written in an explicit duality invariant form:

$$\frac{1}{g_a^2(\Lambda)} = \frac{k_a}{g_s^2} + b_a \ln \frac{\Lambda^2}{M_s^2} + \sum_i y(b_a'^i - k_a \delta_{GS}^i) \ln [T_{Ri} |\eta(T_i)|^4] \quad (4.41)$$

where the gauge coupling constant at the unification scale is now defined as $g_s^2 = \text{Re} S + \sum_i k_a \delta_{GS}^i \ln T_{Ri}$. For a non-zero Green-Schwartz term there will be a mixing term between the dilaton and other moduli fields in the Kähler potential, then dilaton should transform in such a way that renders g_s duality invariant.

The 4D superstring model below the Planck (or compactification) scale is given by an $N=1$ supergravity model and it is specified once the Kähler potential G and the gauge kinetic function f are given. The gauge kinetic function receives non-trivial threshold corrections and its real part gives the gauge coupling constant, $g^2 = \text{Re} f$. The Kähler potential consists of two parts, K and W , where K is a real-analytic function of the chiral superfields which defines the normalization of the kinetic terms while W is the superpotential. In this model, the Kähler potential and the gauge kinetic function are given by:

$$\begin{aligned} K &= -\ln \left(S + \bar{S} + 2 \sum_i k_a \delta_{GS}^i \ln T_{Ri} \right) - \sum \ln(T + \bar{T})_i \\ f &= S + 2 \sum_i (b_a'^i - k_a \delta_{GS}^i) \ln [\eta(T_i)]^2 \end{aligned} \quad (4.42)$$

Notice that the Kähler potential includes the Green-Schwartz term and that the dilaton transforms under duality in a non-trivial way:

$$S \rightarrow S + 2 \sum_i k_a \delta_{GS}^i \ln(icT_i + d) \quad (4.43)$$

so that $Y \equiv S + \bar{S} + 2 \sum_i k_a \delta_{GS}^i \ln T_{Ri}$ is duality invariant. To make the Kähler potential G modular invariant, superpotential should transform with modular weight -3 .

4.2 Review of the general analysis

There has recently been considerable attention focused on the study of supersymmetric models of elementary particle interactions. This is especially true in the context of grand unification theories, where remarkable studies have been done in the hope of solving the gauge hierarchy problem or unifying the gravitational interaction within the superstring formalism. Supersymmetric extension of the gravity(supergravity) seems necessary when one introduces soft breaking terms and makes the cosmological constant vanish at the same time. In supergravity models, spontaneous breaking of local supersymmetry or super-Higgs mechanism may generate soft supersymmetry breaking terms that allow to fulfill such phenomenological requirements. However, the super-Higgs mechanism implies the existence of a supergravity breaking scale, intermediate between the Planck scale(M_P) and the weak scale(M_W). The intermediate scale is expected to be of $O(10^{13}\text{Gev})$. Here we expect that this intermediate scale is implemented by the mechanism of gaugino condensation in the hidden sector which couples to the visible sector by gravitational interactions. The effective action for gaugino condensation is well studied by many authors[6, 32].

Before discussing the large N expansion of supergravity, we will review the general approaches to the dynamical properties. In this section, we mainly follow ref.[32] and study gaugino condensation in the hidden sector in a modular invariant way. Gaugino condensation is, in general, believed to be a potential source of hierarchical supersymmetry breaking and the source of a non-trivial potential for the dilaton S , whose real part corresponds to the tree level gauge coupling constant. In the effective Lagrangian approach, however, we cannot find a stable potential for dilaton without introducing multiple gaugino condensations. However, if we include hidden matter fields with multiple hidden gauge groups, we can obtain a reasonable value for S and soft breaking terms.

First we consider gaugino condensation without matter fields. the strategy is as follows.

- Derive the general form of the corresponding scalar potential and the minimization conditions.
- Consider in a separate way the case of one, two or more gaugino condensations.

(Multiple gaugino condensation can lead to a stable dilaton potential, but to produce realistic soft terms we need some extensions, for example, inclusion of hidden matter fields. In this sense, hidden matter fields are necessary for realistic model building.)

The process of gaugino condensation in the context of a pure Yang-Mills $N=1$ supergravity theory has been pretty well understood for a long time. It can be described conveniently by an effective superpotential $W^{np}(U)$ of the chiral composite superfield U whose scalar component corresponds to the gaugino composite bilinear field. Assuming that the form of W^{np} is the same as pure supersymmetric Yang-Mills theory, W^{np} reads:

$$W^{np} = aU(f + \frac{2}{3}\beta \log U) \quad (4.44)$$

Here a is some constant. In this case f would be modular dependent and may contain the threshold corrections from underlying string theory. As has been demonstrated in refs. [[40, 41, 42]], it is equivalent to work with the explicit form of W^{np} (4.44) or with the resulting superpotential after substituting the minimum condition $\partial W^{np}/\partial U = 0$:

$$\begin{aligned} W^{np} &= d e^{\frac{3}{2\beta}f} \\ &= d \frac{e^{-\frac{3kS}{2\beta}}}{\prod_{i=1}^3 [\eta(T_i)]^2 - \frac{3k}{4\pi^2\beta} \delta_i^{GS}} \end{aligned} \quad (4.45)$$

where $d = -\beta/6e$. If the gauge group is not simple, i.e. $G = \prod_a G_a$, then $W^{np} = \sum_a W_a^{np}$. Using the Kähler potential (4.42), the scalar potential is given by:

$$\begin{aligned} V &= \frac{2}{Y \prod_{i=1}^3 T_{Ri}} \left[|Y W_S - W|^2 \right. \\ &\quad + \sum_{i=1}^3 \frac{Y}{Y + \frac{1}{4\pi^2} \delta_i^{GS}} \left| \left(W + \frac{1}{4\pi^2} \delta^{GS} i W_S \right) - \frac{1}{2} T_{Ri} W_{T_i} \right|^2 \\ &\quad \left. - 3|W|^2 \right] \end{aligned} \quad (4.46)$$

Using this scalar potential and numerical methods, the property of the vacuum state was studied. Here we only show the results of the previous papers [32]

- With simple gauge group
In this case, we cannot find the stable vacuum state. The asymptotic minimum appears at infinity, that is called "runaway vacuum".
- With multiple gauge groups
If k_a is different from each other, we can find a stable minimum. However, such a vacuum does not have phenomenologically acceptable values of moduli fields.
- With hidden matters with multiple gauge groups
Including multiple hidden matter fields, we can obtain a phenomenological vacuum state. But one should not think that the inclusion of hidden matter is essential, because such an extra parameter region generally makes it easy to adjust the vacuum parameters by hand.

4.3 Large N expansion

In this section we use large N expansion to analyze the vacuum state of supergravity theories[27]. The extension from the previous analysis (analysis for global models) is straightforward.

4.3.1 Introduction

In the previous chapter we have analyzed the large N expansion and gap-equations for global supersymmetric models. The key idea of the analysis was to consider an intermediate scale effective Lagrangian which couples to R-axion superfield.

In this section, we extend the analysis of global supersymmetric models with R-symmetry to its local version with Weyl symmetry. This Weyl symmetry is always present when one consider the superstring inspired models and this symmetry needs a compensator superfield. This compensator plays almost the same role as R-symmetry compensator in the global models.

4.3.2 Gaugino condensation in supergravity

In the standard superfield formalism of the locally supersymmetric action, we have the Lagrangian:

$$S = \frac{-3}{\kappa^2} \int d^8z E \exp\left(-\frac{1}{3}\kappa^2 K_0\right) + \int d^8z \mathcal{E} \left[W_0 + \frac{1}{4}f_0 \mathcal{W}\mathcal{W}\right] + h.c. \quad (4.47)$$

Here we set $\kappa^2 = 8\pi/M_p^2$. In the usual formalism of minimal supergravity, the Weyl rescaling is done in terms of component fields. However, in order to understand the anomalous quantum corrections to the classical action, we need a manifest supersymmetric formalism, in which the Weyl rescaling is also supersymmetric. It is easy to see that the classical action(4.47) itself is not super-Weyl invariant. However, the super-Weyl invariance can be recovered with the help of a chiral superfield φ (Weyl compensator).

For the classical action (4.47), the Kähler function K_0 , the superpotential W_0 and the gauge coupling f_0 are modified in the following way[38]:

$$\begin{aligned} K_0 &\rightarrow K = K_0 - 6\kappa^{-2} \text{Re} \log \varphi \\ W_0 &\rightarrow W = \varphi^3 W_0 \\ f_0 &\rightarrow f = f_0 + \xi \log \varphi \end{aligned} \quad (4.48)$$

ξ is the constant which is chosen to cancel the super-Weyl anomaly. The super-Weyl transformation contains an R-symmetry in its imaginary part, so we can think that this is a natural extension of [30] in which a compensator for the R-symmetry played a crucial role in obtaining the gap equation.

Let us examine the simplest case. We include some scale factor Λ and set the form of W_0 and f_0 as:

$$\begin{aligned} W_0 &= \lambda \Lambda^3 \\ \text{Re} f_0 &= \frac{1}{g_0^2} \end{aligned} \quad (4.49)$$

and rescale the field φ as:

$$\bar{\varphi} = \Lambda \varphi \quad (4.50)$$

Finally we have:

$$\begin{aligned} K &= K_0 - 6\kappa^{-2} \text{Re} \log \left(\frac{\tilde{\varphi}}{\Lambda} \right) \\ W &= \lambda \tilde{\varphi}^3 \\ f &= \frac{1}{g_0^2} + \xi \log \left(\frac{\tilde{\varphi}}{\Lambda} \right) \end{aligned} \quad (4.51)$$

The sign of λ should respect the condition $\lambda\xi > 0$ so that the gap-equation can develop non-trivial solution. We can obtain a constraint equation from the equation of motion for the auxiliary component h_φ of the super-Weyl compensator. The relevant part of the Lagrangian is:

$$e^{-1} L_{AUX} = \left[\frac{\partial W}{\partial \varphi} - \frac{1}{4} \frac{\partial f}{\partial \varphi} \lambda \lambda \right] h_\varphi \quad (4.52)$$

From the equation of motion, constraint is now written as:

$$\frac{\partial W}{\partial \varphi} = \frac{1}{4} \frac{\partial f}{\partial \varphi} \lambda \lambda \quad (4.53)$$

for the rescaled field, this can be written as:

$$\lambda \tilde{\varphi}^3 - \frac{\xi}{12} g^2 \lambda^\alpha \lambda_\alpha = 0 \quad (4.54)$$

The tree level scalar potential for this minimal model is:

$$\begin{aligned} V_0 &= -3\kappa^2 |W|^2 \\ &= -3\kappa^2 \lambda^2 |\tilde{\varphi}^3|^2 \end{aligned} \quad (4.55)$$

The equation of motion for the auxiliary field(4.54) suggests that eq.(4.55) can be interpreted as a four-fermion interaction of the gaugino:

$$-\frac{1}{48} \kappa^2 g^4 \xi^2 |\lambda^\alpha \lambda_\alpha|^2 \quad (4.56)$$

This four-fermion interaction becomes strong as $\frac{1}{g^2} = \text{Ref}$ reaches 0. The strong coupling point is:

$$\tilde{\varphi}_s = \Lambda e^{-\frac{1}{g_0^2 \xi}} \quad (4.57)$$

In the effective Lagrangian analysis, one usually assumes that this point is the true vacuum. We show the vacuum state of three types of analysis (effective Lagrangian analysis, G.G. Ross type and large N expansion) in Fig.(4.2).

By using the tadpole method we can obtain a gap equation:

$$\lambda \tilde{\varphi}^3 \times \left(1 - 4G^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 = \frac{\xi^2 \kappa^2 g^4 n_g}{4} \\ m_\lambda^2 = \frac{\kappa^4 \xi^2 g^4 \lambda^2 |\tilde{\varphi}^3|^2}{4} \end{cases} \quad (4.58)$$

where ξ is determined by the anomaly constraint and n_g is the dimension of the gauge group ($n_g = N_c^2 - 1$). Here, it is proportional to N_c . This equation is a good approximation when we take $N_c \rightarrow \infty$ limit. (The leading contribution is enhanced by an extra N_c factor. See fig.(4.1))

The solution for the gap equation(4.58) is plotted in Fig.(4.2). We can see that there is always a solution for non-zero gaugino condensation. Now let us consider the difference between our result and ref.[30]. In ref.[30], the solution for the gap equation is estimated after fixing the coupling constant at g_c which is introduced by hand. It is true that the effective potential is singular at $\tilde{\varphi}_s$ (4.57), but without introducing the cut-off for the strength of the four fermion coupling $|\lambda\lambda|^2$, we can find a stable solution for (4.58) at finite value.(see Fig (4.2))

For a second example, we include the dilaton superfield S . Now f_0 is not a constant and depends on the field S :

$$f_0 = S \quad (4.59)$$

And the Kähler potential for the dilaton superfield is:

$$K_0 = -\kappa^{-2} \log(S + \bar{S}) \quad (4.60)$$

Here we should include the effect of the dilaton field in the scalar potential. The tree level scalar potential is:

$$V_0 = h_S (G^{-1})_S^S h^S - 3\kappa^2 |W|^2 \quad (4.61)$$

The auxiliary field for S is:

$$\begin{aligned} h_S &= \kappa^2 \left[\frac{1}{2} \frac{W}{S + \bar{S}} + \frac{1}{4} f_S \lambda^\alpha \lambda^\alpha \right] \\ &= \frac{\kappa^2}{4} W \frac{1 + 12 S_R \xi^{-1}}{S_R} \end{aligned} \quad (4.62)$$

Here we set $G = K + \ln(\frac{1}{4}|W|^2)$ and $S_R = (S + \bar{S})/2$. The tree level potential can be given in a simple form

$$V_0 = \lambda^2 A |\tilde{\varphi}^3|^2 \quad (4.63)$$

where

$$A = \frac{1}{16} \kappa^2 \left[\left(1 + \frac{12S_R}{\xi} \right)^2 - 3 \right]. \quad (4.64)$$

The tree level potential for $\tilde{\varphi}$ has no stable supersymmetry breaking solution. This is consistent with an observation that gaugino condensation as usually parameterized does not occur in models with a single gauge group in the hidden sector. However, we have argued that it is essential to go beyond tree level to include non-perturbative effects in the effective potential which may allow non-trivial minimum even in the simplest case of a single hidden sector gauge group. This non-perturbative sum is readily obtained by computing the one-loop correction to V , or directly from the gap-equation. In this case, the gap equation is given by the same constraint equation (4.54), but gaugino mass term is modified.

$$\lambda \tilde{\varphi}^3 \times \left(1 - 4G^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_\lambda^2} \right) = 0$$

$$\begin{cases} G^2 = \frac{\lambda g^4 \xi^2 g^4 n_g A}{36} \\ m_\lambda^2 = \frac{A^2 \xi^2 g^4 \lambda^2 g^2 |\tilde{\varphi}^3|^2}{36} \\ n_g = N_c - 1 \end{cases} \quad (4.65)$$

This equation is, however, too complicated to find a solution. We thus forced to employ some simplification of this analysis, for example, fix the four-fermi interaction coefficient[30] or to approximate the solution at which g^2 goes to infinity, i.e. $f(S) = 0$ [32]. Here we only refer to the paper[30] in which the stability of the potential is well discussed after fixing the four-fermi interaction coefficient. In [30], duality invariant Lagrangian is also considered and it is found that this type of approach takes into account some additional non-perturbative effects [39] and stabilizes the dilaton potential with only one gauge group.

To see the basic mechanism involved, let us reconsider eq.(4.63). This potential suggests that, once gaugino condensate and φ gets non-zero value, S cannot run away to infinity. On the other hand, it is obvious from fig.(4.2)

that eq.(4.65) always have non-trivial (condensating) solution for any S . Moreover, in the effective Lagrangian analysis one consider that the point C in fig.(4.2) is the solution for non-trivial gaugino condensation. As S becomes large, C moves toward the origin as the function of e^{-S} so one tends to think that the potential (4.63) is destabilized. On the other hand, if one uses Nambu-Jona-Lasinio type approach[30], one is lead to the solution B in fig.(4.2) and finds that the potential is indeed stable. However, taking large N limit, we can find that these two solutions corresponds to different kinds of approximations for the true solution at A, which is different from both B and C. Because the solution A is very similar to the solution B and does not behave as e^{-S} , we may think that the potential is stabilized at the true vacuum A.

4.4 Summary of chapter 4

In this chapter we have reviewed the general construction of the supergravity Lagrangian and then we have introduced the general analysis of the non-perturbative properties of supergravity theory which is induced by low energy dynamics of gaugino condensation. After a brief review of the previous approaches, we examined these known results from another point of view. By using large N expansion, we showed that the effective Lagrangian analysis and Nambu-Jona-Lasinio type approach can be regarded as different kinds of approximations to the exact solution.

In the large N limit, we can easily understand why the dilaton potential is stabilized in Nambu-Jona-Lasinio type approach but not stabilized in the effective Lagrangian approach. We also found a stable and exact solution in the large N limit. It is important that we can find a stabilized dilaton potential if we take into account a certain non-perturbative effect from supergravity.

$$\left[e^{\frac{\kappa/2\xi}{2m^2}} g^2 \text{ (circle with cross) }^{\lambda\lambda} \right] + \left[\langle \phi \rangle \right]$$

$$O(g^2 N_c n_g) \equiv O(g_0^2)$$

$$\left[e^{\frac{\kappa/2\xi}{2m^2}} g^2 \text{ (circle with cross and dashed line) }^{\lambda\lambda} \right]$$

$$O(g^4 N_c n_g^2) \equiv O(g_0^4 / N_c)$$

Figure 4.1: The gap equation (4.58) is written in a diagrammatic form. One of the typical higher correction is shown in the second diagram.

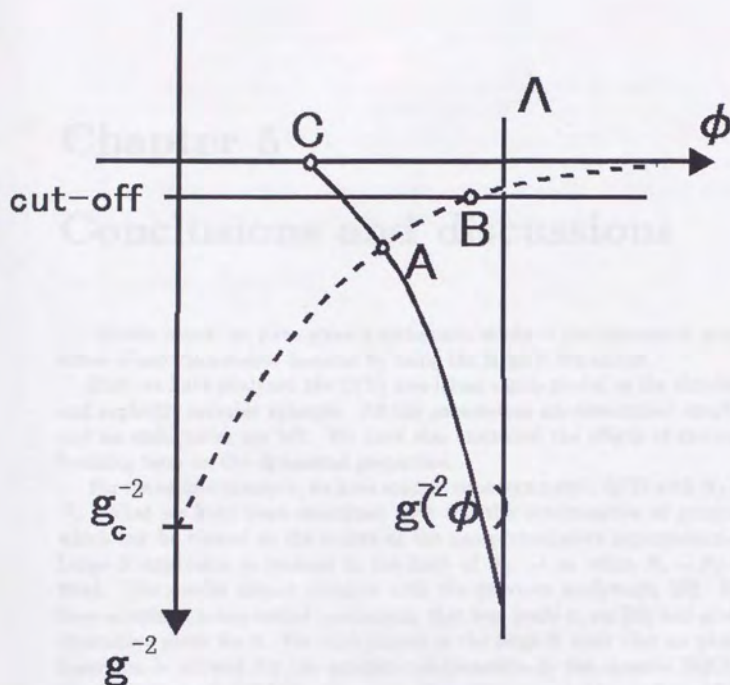


Figure 4.2: The stationary point of the scalar potential(dotted line) and the relation between g and ϕ (solid line) are plotted in the same graph. The only "exact" solution lies at A. If we include a cut-off for the four-fermi interaction coefficient by hand, we will find a vacuum at B(see Appendix). C is the strong coupling point, where Ref vanishes and the gauge coupling g goes to infinity. In many cases, we use C for the solution for the non-trivial gaugino condensation(The relation between effective potential analysis and the condition $f = g^{-2} = 0$ is well discussed in ref.[38]).

Chapter 5

Conclusions and discussions

In this paper, we have given a systematic study of the dynamical properties of supersymmetric theories by using the large N expansion.

First we have analyzed the $O(N)$ non-linear sigma model as the simplest and explicitly solvable example. All the parameters are determined exactly and no ambiguities are left. We have also examined the effects of the soft breaking term on the dynamical properties.

For the second example, we have studied supersymmetric QCD with $N_f < N_c$. What we have been concerned with was the condensation of gaugino which can be viewed as the source of the non-perturbative superpotential. Large N expansion is realized in the limit of $N_f \rightarrow \infty$ while $N_c - N_f$ is fixed. The results almost coincide with the previous analyses[6, 28]. We have examined a non-trivial assumption that was made in ref.[28] and given alternative prove for it. We have proved in the large N limit that no phase transition is allowed for the gaugino condensation in the massive SQCD. However, in massless SQCD, phase transition does occur in the weak coupling region. This can change the asymptotic form of the runaway potential. To be more precise, the flat directions are partially lifted and the potential does not run away.

For the third example, we have analyzed superstring motivated supergravity theories which is the main theme of our paper. In the large N limit, we found that the analyses derived from effective (confined) theory[32] and one from Nambu-Jona-Lasinio type approach[30] correspond to the different kinds of approximations for the exact solution. (This is explicitly visualized in fig.(4.2).) We have used the constituent fields. This type of approach

is very important when we think of the phenomenological implications of supersymmetry breaking mechanism on the moduli stabilization[39] and the phase transitions in the early universe[43].

If we use the composite Lagrangian, we should ignore a certain type of non-perturbative effect from supergravity[39]. It is shown that the dilaton potential is stabilized if we take into account this non-perturbative effect.

If we use the Lagrangian written in terms of the composite (confined) fields and extend it to higher energy scales, some singularity should appear[43] and it is difficult to understand the phase transition.

Acknowledgment

We thank K.Fujikawa for many helpful discussions.

Appendix A

Introduction to large N expansion

This appendix is devoted to the elementary introduction to the large N expansion. Here we mainly follow the 8th chapter of ref.[44].

A.1 ϕ^4 theory

Let us start with the easiest theory and show the fundamental ingredients of the large N expansion. The dynamical variables of the theory are a set of N scalar fields ($\phi^a, a = 1, \dots, N$) with dynamics defined by the Lagrangian:

$$L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{2} \mu_0^2 \phi^a \phi^a - \frac{1}{8} \lambda_0 (\phi^a \phi^a)^2 \quad (\text{A.1})$$

Here the sum on repeated indices is implied. Since we are going to stop the investigation before evaluation any Feynman integrals, we might as well keep the dimension of space-time arbitrary but keep $D \leq 4$.

To find the explicit order estimate of large N expansion, we have written down the first few diagrams in fig.(A.1). The first (tree) diagram is $O(\lambda_0)$ and the second is $O(\lambda_0^2 N)$ because there are N possible choices for the internal index. The third diagram is, however, only $O(\lambda_0^2)$ since the internal indices are fixed.

The explicit factor of N in the second diagram may seem to make the large N limit nonsensical, but this is easily rectified. Let us define a new

parameter g_0 as:

$$g_0 \equiv \lambda_0 N \quad (\text{A.2})$$

and declare that we wish to study the limit of Large N with fixed g_0 . Then the first diagram is estimated to be $O(g_0/N)$. The second is the same order in N , but the third is $O(g_0^2/N^2)$. This is the main idea of the large N expansion. In any case, implicitly or explicitly, we must decide what parameters to hold fixed as N becomes large.

However, there are clearly an infinite number of graphs proportional to $1/N$, times various powers of g_0 . For example we have shown two such diagrams in fig.(A.2).

To keep all these diagrams straight, we can simplify the theory by introducing an auxiliary field σ and altering the Lagrangian:

$$L \rightarrow L + \frac{1}{2} \frac{N}{g_0^2} \left(\sigma - \frac{1}{2} \frac{g_0}{N} \phi^a \phi^a \right)^2 \quad (\text{A.3})$$

This added term has no effects on the dynamics of the theory. The Euler-Lagrange equation for the auxiliary field σ is:

$$\sigma = \frac{1}{2} \frac{g_0}{N} \phi^a \phi^a \quad (\text{A.4})$$

Although the dynamics defined by our new Lagrangian are the same as those defined by the old one, the Feynman rules are different. By elementary algebra we can calculate the Lagrangian:

$$L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a + \frac{1}{2} \mu_0^2 \phi^a \phi^a + \frac{1}{2} \frac{N}{g_0^2} \sigma^2 - \frac{1}{2} \sigma \phi^a \phi^a \quad (\text{A.5})$$

In the new formalism, the non-trivial coupling is extremely simplified. The only non-trivial interaction is the $\sigma \phi \phi$ and all the factors of $1/N$ come from the σ propagator ig_0^2/N . Every line on a closed ϕ loop must always carry the same index, and this index must always summed over, so every closed ϕ loop always gives a factor of N . Counting powers of $1/N$ now becomes much easier, but things can be made easier yet by considering an effective action S_{eff} . An effective action S_{eff} is obtained in two ways, in terms of Feynman graphs or in terms of functional integrals. The description of Feynman diagrams is shown in fig.(A.3). In terms of functional integrals, the quantum theory

is defined by integrations of all fields. The effective action is obtained by integrating over the ϕ s only:

$$e^{iS_{eff}(\sigma)} = \int \prod_a [d\phi^a] e^{iS(\phi^a, \sigma)} \quad (\text{A.6})$$

Whichever way we describe S_{eff} , one thing about it is obvious; every term in it is proportional to N .

To go further, let us define a new shifted field:

$$\sigma' \equiv \sigma - \sigma_0 \quad (\text{A.7})$$

where σ_0 is a constant chosen such that $\sigma = \sigma_0$ is a stationary point of S_{eff} ,

$$\left. \frac{\delta S_{eff}}{\delta \sigma'} \right|_{\sigma'=0} = 0. \quad (\text{A.8})$$

After this redefinition, we obtain a following Lagrangian.

$$\begin{aligned} L = & \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{2} \mu_1^2 \phi^a \phi^a + \frac{1}{2} \frac{N}{g_0} \sigma'^2 \\ & - \frac{1}{2} \sigma' \phi^a \phi^a + \frac{N}{g_0} \sigma_0 \sigma' \end{aligned} \quad (\text{A.9})$$

plus an irrelevant constant. Here we defined

$$\mu_1^2 \equiv \mu_0^2 + \sigma_0. \quad (\text{A.10})$$

Using this Lagrangian, we can calculate an effective potential. The equation for the stationary point of this effective potential gives us a gap equation for σ_0 (See chapter 2).

A.2 More on large N expansion

In this section, we consider large N expansion in SQCD and supergravity models. (Of course, what we consider here are the models that are analyzed in this paper.)

In SQCD, things are very simple. If we make $N_f \rightarrow \infty$ while $n_g N_f = \text{const.}$, the only contribution that contains large N_f factor comes from the coupling between auxiliary field and gaugino bilinear:

$$G^2 F_\phi \lambda \lambda \quad (\text{A.11})$$

where we set $G^2 \equiv \beta g^2/v$ and β contains N_f . We should only define

$$G^2 n_g \equiv G_0^2 \quad (\text{A.12})$$

and declare that we wish to study the limit of large N_f with fixed G_0^2 . An effective action is calculated in this limit with a very simple form. Each diagram that contains any extra internal interaction contains extra power of $1/N_f$. Thus what we should concern is the first diagrams of fig.(3.5). In this case, the solid line represents gaugino propagator.

The next example is the supergravity model. This model is not so simple as SQCD, because in this case we should consider large N_c limit. In general, large N_c limit of QCD like theories suffer a problematic contribution from the planar diagrams. (The notion of planar diagrams is not difficult to understand. If there were no extra N_c factor in the gap equation (4.58), the second diagram in fig.(4.1) should be the same order as the first one. It is easy to find other contributions of the same order. In general, such contributions are labeled by the Euler characteristic and called planar contributions.) Things are much simpler if there is an extra N_c factor. We can declare that $g^2 N_c n_g \equiv \text{const.}$ then we can find that planar contributions becomes small. (See fig.(4.1).)

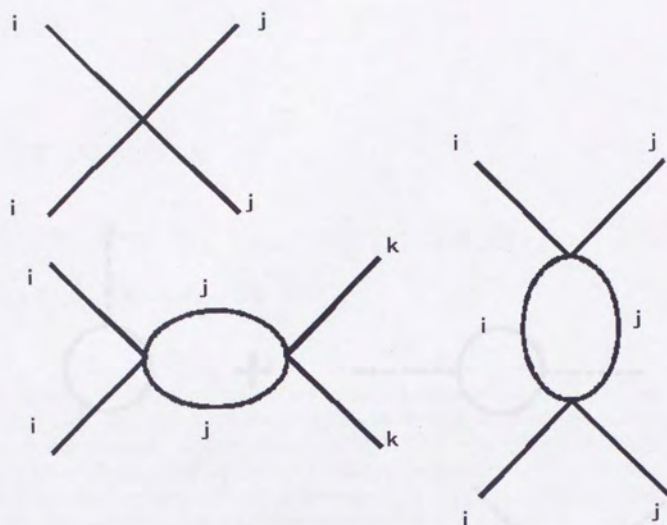


Figure A.1:

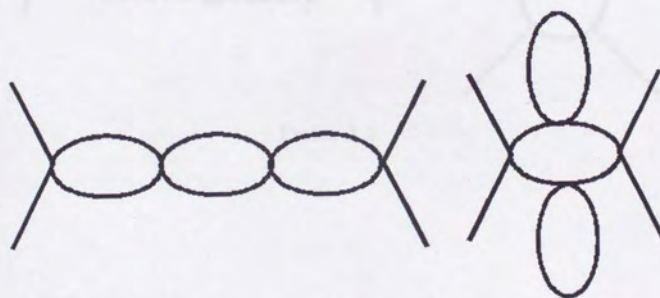


Figure A.2:

Appendix B

Review of the approach by G.G. Ross et al

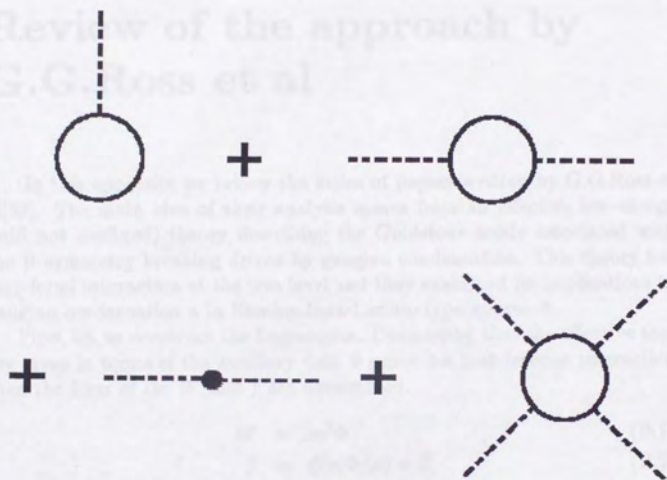


Figure A.3:

Appendix B

Review of the approach by G.G.Ross et al

In this appendix we review the series of papers written by G.G.Ross et al[30]. The main idea of their analysis comes from an effective low-energy (still not confined) theory describing the Goldstone mode associated with the R-symmetry breaking driven by gaugino condensation. This theory has four-fermi interaction at the tree level and they examined its implications to gaugino condensation a la Nambu-Jona-Lasinio type approach.

First, let us construct the Lagrangian. Demanding that the effective theory given in terms of the auxiliary field Φ generates four-fermion interaction then the form of the W and f are determined.

$$W = m^2 \Phi \quad (\text{B.1})$$

$$f = \xi \ln(\Phi/\mu) + S \quad (\text{B.2})$$

where m and μ are mass parameters, ξ is a dimensionless constant and S is a dilaton superfield. Here we take $M_p = 1$ for simplicity. The classical equation of motion for auxiliary component of Φ (ϕ, χ, h) is:

$$\frac{1}{2} \frac{\partial W}{\partial \phi} = \frac{e^{K/2}}{4} \frac{\partial f}{\partial \phi} \lambda \lambda \quad (\text{B.3})$$

This gives the relation:

$$\phi = \frac{e^{K/2} \xi}{2m^2} \lambda \lambda \quad (\text{B.4})$$

which means that the field ϕ can be an order parameter for gaugino condensation. The four-fermion term appears if we consider the tree level (supergravity) potential:

$$V_0 = 3e^{-G} + h_S f^S (G^{-1})_S^S \quad (\text{B.5})$$

where h_S denotes the F-component of the chiral superfield S given by:

$$h_S = e^{-G/2} G_S + \frac{1}{4} \lambda \lambda - G_S^{jk} \bar{\chi}_j \chi_k - \frac{1}{2} \bar{\chi}_S G_j \chi^j \quad (\text{B.6})$$

Including all the components, we can write the tree level potential explicitly:

$$\begin{aligned} V_0 &= \frac{1}{4} \left(1 + \frac{S + \bar{S}}{\xi} \right)^2 e^{-K} |W|^2 \\ &= \frac{1}{4} \left(1 + \frac{S + \bar{S}}{\xi} \right)^2 e^{-K} m^4 \phi^2 \end{aligned} \quad (\text{B.7})$$

In terms of gaugino field, we can obtain the four-fermion interaction term.

$$L_{4fermi} = \frac{1}{16(Ref)^2} \left(1 + \frac{S + \bar{S}}{\xi} \right)^2 |\lambda \lambda|^2 \quad (\text{B.8})$$

where the factor of Ref in the denominator appears because we have rescaled the gaugino fields appearing in this equation to have canonical kinetic terms.

The form we have derived depends on the parameters ξ , m and μ . One can determine ξ by demanding that the low-energy effective Lagrangian is anomaly free under the R-symmetry transformation. The mass scale m should be identified with the Planck mass. Different choices of μ are possible and we take $\mu = \Lambda_{GUT}^3 / m_{Planck}^2$. This choice of μ is justified in superstring motivated analysis. A supersymmetry breaking solution to the mass gap $\partial(V_0 + V_1)/\partial\phi = 0$ is dynamically favored for a large coupling constant. The apparent singularity when Ref approaches zero and the potential unbounded from below are not physically reasonable. On dimensional grounds we introduce a cut-off at the scale $Ref = \Lambda_c$ which corresponds to the effective four-gaugino interaction $|\lambda\lambda|^2/\Lambda_c^2$. (See Fig.(4.2).)

We can easily extend this model to include other moduli fields and string threshold corrections. Here we give the explicit form of typical Lagrangian:

$$K = -\ln \left(S_R + 2 \sum_i k_a \delta_{GS}^i \ln T_{Ri} \right) - \sum_i \ln T_{Ri}$$

$$\begin{aligned}
W &= W_0 + W_m \\
f_0 &= S + 2 \sum_i (b_a^i - k_a \delta_{GS}^i) \ln[\eta(T_i)]^2
\end{aligned} \tag{B.9}$$

where W_0 is the scalar potential due to the gaugino condensate and W_m is the matter superpotential.

If we demand that the effective theory given in terms of the auxiliary superfield Φ generates four-fermi interaction then the form of the W and f are uniquely determined.

$$\begin{aligned}
W_0 &= m^2 \Pi_i \eta^{-2}(T_i) \Phi \\
f &= f_0 + \xi \ln \frac{\Phi}{\mu}
\end{aligned} \tag{B.10}$$

From the classical equation of motion the scalar component of the auxiliary superfield Φ is given in terms of the gaugino bilinear by:

$$\phi = \frac{e^{-K/2} \xi}{2m^2 \Pi_i \eta_i^{-2}} \lambda \lambda \tag{B.11}$$

For a pure gauge theory in the hidden sector the tree level scalar potential is now written by:

$$\begin{aligned}
V_0 &= m_{\frac{3}{2}}^2 B_0 \\
B_0 &= \left(1 + \frac{Y}{\xi}\right)^2 + \sum_i \frac{Y}{Y + a_i} \left(a_i - \frac{a_i}{\xi}\right)^2 \frac{T_{Ri}}{4\pi^2} |G_2(T_i)|^2 - 3
\end{aligned} \tag{B.12}$$

where G_2 is the Einstein modular form with weight 1/2 and a_i is defined as $a_i = 2(k_a \delta_{GS}^i - b_{ai}')$. The gravitino mass is given by:

$$m_{\frac{3}{2}} = \frac{1}{4} e^K \Pi_i |\eta(T_i)|^{-4} |\phi|^2 \tag{B.13}$$

It is clear that a non-zero gravitino mass is only possible for non-vanishing VEV of ϕ . The one-loop radiative corrections may be calculated using Coleman-Weinberg one-loop effective potential.

$$V_1 = \frac{1}{32\pi^2} \text{Str} \int d^2 p p^2 \ln(p^2 + M^2) \tag{B.14}$$

After introducing the cut off parameter for *Ref* and fixing the form of four-fermion interaction term as $|\lambda\lambda|^2/\Lambda_c^2$, the extremum conditions are solved.

Here we do not describe further detailed analysis on this phenomenological applications because our aim in this appendix is to introduce a main idea of the method developed by G.G.Ross et al.

Bibliography

- [1] S. J. Lee, *Phys. Rev. D* **37**, 1993 (1988).
- [2] S. J. Lee and J. H. Kim, *Phys. Rev. D* **38**, 1993 (1988).
- [3] S. J. Lee and J. H. Kim, *Phys. Rev. D* **38**, 1993 (1988).
- [4] J. H. Kim and J. H. Kim, *Phys. Rev. D* **38**, 1993 (1988).
- [5] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [6] H. A. Kadir, S. J. Lee, S. J. Lee, G. G. Ross, G. G. Ross, *Phys. Rev. D* **38**, 1993 (1988).
- [7] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [8] S. J. Lee, S. J. Lee, S. J. Lee, and S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [9] S. J. Lee, S. J. Lee, and S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [10] S. J. Lee, and J. H. Kim, *Phys. Rev. D* **38**, 1993 (1988).
- [11] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [12] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [13] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [14] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [15] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [16] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [17] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [18] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [19] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).
- [20] S. J. Lee, *Phys. Rev. D* **38**, 1993 (1988).

Bibliography

- [1] H.P.Nills, Phys.Rep.110(1984)1
- [2] S.Deser and B.Zumino, Phys.Rev.Lett.38(1977)1433
- [3] M.Dine, A.E.nelson, Y.Shirman, Phys.Rev.D51(1995)1362
- [4] J.Wess and J.Bagger, Supersymmetry and Supergravity (1992)
Princeton University press
- [5] H.P.Nills, Phys.Rev.Lett.115(1982)193
- [6] D.Amati, K.Konishi, Y.Meurice, G.C.Rossi, G.Veneziano,
Phys.Rep.162(1988)169
- [7] N.V.Krasnikov, Phys.Lett.B193(1987)37
- [8] J.A.Casas, Z.Lalak, C.Munoz and G.G.Ross,
Nucl.Phys.B347(1990)243;
J.A.Casas, Z.Lalak and C.Munoz, Nucl.Phys.B399(1993)623
- [9] M.B.Greem and J.H.Schwarz and E.Witten, Superstring Theory (1987)
Cambridge Univ. Press
- [10] E.Witten, Nucl.Phys.B188(1981)513; Nucl.Phys.B202(1982)253
- [11] M.Dine, R.Rohm, N.Seiberg and E.Witten,
Phys.Lett.B156(1985)55
- [12] G.D.Coughlan, G.Germain, G.G.Ross and G.Segré
Phys.Lett.B198(1987)465;
P.Binétury and M.K.Gaillard, Phys.Lett.B253(1991)119;

- Z.Larak, A.Niemeyer, G.Giraldi and R.Grimm,
Nucl.Phys.B453(1995)100
- [13] S.Kachru, hep-th/9603059
E.Witten, hep-th/9604030
K.R.Dienes and A.E.Faraggi, Phys.Rev.Lett.75(1995)2646
- [14] T.Matsuda, J.Phys.A28(1995)3809
- [15] K.Fujikawa and W.Lang, Nucl.Phys.B88(1975)61
- [16] T.Matsuda, J.Phys.G22(1996)1127
- [17] T.Matsuda, J.Phys.G22(1996)119
- [18] D.J.Gross, A.Neveu, Phys.Rev.D10(1974)3235
- [19] A.M.Polyakov, Phys.Lett.59B(1975)79
- [20] E.Witten, Phys.Rev.D16(1977)2991
- [21] A.C.Davis, P.Salomonson, J.W.Holten,
Nucl.Phys.B208(1982)484
- [22] O.Alvarez, Phys.Rev.D17(1978)1123
- [23] A.M.Polyakov Gauge Fields and Strings,
Harwood Academic Publishing(1987)
- [24] V.G.Koures, K.T.Mahathappa, Phys.Rev.D43(1991)3428
- [25] O.Aharony, J.Sonnenschein, M.E.Peskin and S.Yankielowicz,
Phys.Rev.D52(1995)6157
L. Alvarez-Gaume, M. Marino, hep-th/9606168
- [26] T.Matsuda, J.Phys.G22(1996)L29
- [27] T.Matsuda, Phys.Rev.D54(1996)7650
- [28] I.Affleck, M.Dine, N.Seiberg,
Nucl.Phys.B241(1984)493; Nucl.Phys.B256(1985)557

- [29] S.Ferrara, L.Gairardello and H.P.Nills, Phys.Lett.125B(1983)457
- [30] A.de la Maccora and G.G.Ross,
Nucl.Phys.B404(1993)321; Phys.Lett.B325(1994)85;
Nucl.Phys.B443(1995)127
- [31] E.Cremmer, S.Ferrara, L.Girardello and A.Van Proyen,
Nucl.Phys.B212(1983)413
- [32] J.P.Derendinger, L.E.Ibàñez and H.P.Nills, Phys.Lett.B155(1985)65
M.Dine, R.Rohm, N.Seiberg and E.Witten, Phys.Lett.B156(1985)55
C.Kounnas and M.Porrati, Phys.Lett.B191(1987)91
S.Ferrara, N.Magnoli, T.R.Taylor and G.Veneziano,
Phys.Lett.B245(1990)243
A.Font, L.E.Ibàñez, D.Lust and F.Quevedo, Phys.Lett.B245(1990)401
H.P.Nills and M.Olechowski, Phys.Lett.B248(1990)268
P.Binèturey and M.K.Gaillard, Phys.Lett.B253(1991)119;
Nucl.Phys.B358(1991)194
B.de Carlos, J.A.Casas and C.Muñoz, Nucl.Phys.B399(1993)623
- [33] J.Polchinski, Phys.Rev.D26(1982)3674
- [34] S.Weinberg, Phys.Rev.D7(1973)2887
R.Miller, Phys.Lett.124B(1983)59; Nucl.Phys.B241(1984)535
- [35] L.Dixon, V.Kaplunovsky and J.Louis, Nucl.Phys.B355(1991)649
G.Lopes Cardeso and B.Ovrut, Nucl.Phys.B369(1992)351
- [36] I.Antoniadis, E.Gava and K.S.Narain, Nucl.Phys.B383(1992)93;
Phys.Lett.B283(1992)209
I.Antoniadis, K.S.Narain and T.R.Taylor, Phys.Lett.B267(1991)37
- [37] V.P.Nair, A.Shapere, A.Strominger and F.Wilczek,
Nucl.Phys.B287(1987)402
B.Sathiapalan, Phys.Rev.Lett.58(1987)1597
J.J.Atick and E.Witten, Nucl.Phys.B310(1988)291
W.Lerche, D.Lüst and N.P.Warner, Phys.Lett.B231(1989)417
M.Cvetic, J.Molera and B.Ovrut, Phys.Lett.B248(1990)83

- [38] V.Kaplunovsky and J.Louis, Nucl.Phys.B422(1994)57
- [39] B. De Carlos and M. Moretti, Phys.Lett.B341(1995)302
- [40] J.A.Casas, Z.Lalak, C.Munoz and G.G.Ross, Nucl.Phys.B347(1990)243
- [41] D.Lust and T.R.Taylor, Phys.Lett.B253(1991)335
- [42] B.de Carlos, J.A.Casas and C.Munoz, Phys.Lett.B263(1991)248
- [43] T.Matsuda, Phys.Lett.B383(1996)28
- [44] S.Coleman, Aspects of symmetry(1985), Cambridge

