論 文 の 内 容 の 要 旨

論文題目　Quantum Approaches to the Modeling of Stock Markets
The Recovery of Stylized Facts with Quantum Oscillators
(株式市場のモデル化に対する量子的アプローチ
－量子振動子による定型化された事実の再現－)

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This doctoral dissertation presents my work in the past three years as a PhD student. It describes the motivation, objective, methods, results and enlightenment of my research about quantum approaches to the modeling of stock markets.

Physical modeling, as well as mathematical analysis, has become indispensable tools in financial studies. The mathematical methods pay attention to the phenomenological behaviors of historical economic data, while physical models shed more light on the microscopic mechanics of the economic systems. The coin of the word ``econophysics" in the mid-1990s started the official union of physics and economics. The major idea of the modeling of financial markets including stock markets in econophysics is stochastic process, which is based on the theory of statistical physics. Although large number of differently developed stochastic models has succeeded in describing the market variables such as stock price, return volatility and trading volume, the underlying mechanics was rarely discussed. On the other hand, agent based model (ABM), which starts its modeling from the microscopic rules predefined, has become popular with the development of computational technology. However, both of these two methods are time-consuming due to thousands of steps of simulation. Fortunately, quantum mechanics has been becoming a novel alternate for financial study with the expectation to overcome the before mentioned defects. In this dissertation, quantum approaches are proposed to explain two of the most important stylized facts in the stock markets: universal distributions of price return and volatility clustering.

In the quantum approaches, the motion of a stock price is modeled by the dynamics of wave function for a quantum particle. The uncertainty in the future stock price makes it possible to apply the squared wave function as the probability density function (PDF) of the price return. The dynamics of the wave function is described by Schrodinger equation (SE) Eq.(1), which can be reduced to a time-independent version as the eigen equation of energy for the stationary markets with invariant Hamiltonian.

$$i\hbar\frac{\partial}{\partial t}\psi(r,t)=\hat{H}\psi(r,t) \tag{1}$$

where i is the imaginary unit, $\hbar$ is the reduced Planck constant, $\hat{H}$ is the Hamiltonian for the price return, r is the price return of the stock price, $\psi(r,t)$ is the corresponding wave function.

Different from the previous related models that used a quantum harmonic oscillator directly, we proposed a financially interpretable generalized form of the Hamiltonian for a stationary market, by analyzing the instantaneous order excess in the market. The potential can then be presented by an anharmonic oscillator with

$$V(r)=\frac{\gamma+c}{2}r^2-\frac{kc}{4}r^4 \tag{2}$$

where γ, c and k are parameters measuring the behaviors of the market participants market makers, contrarians and trend followers assumed in this model. It is demonstrated that the introduction of a quartic term in the potential as a description of the risk aversion brought PDF with sharper crest and fatter tails (Fig. 1).

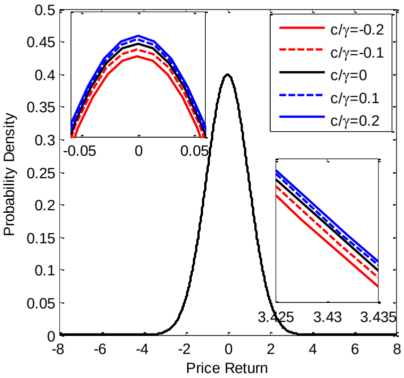

Fig. 1 Theoretical PDF for different values of c/γ (Ground-state Modeling).

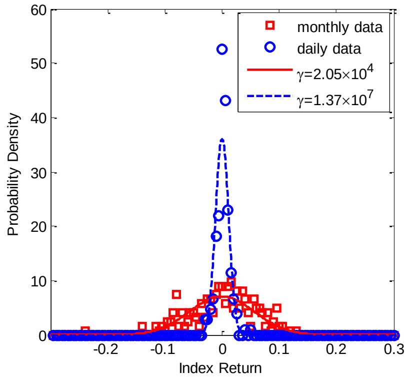

Fig. 2 The fitting of monthly and daily Nikkei 225 Index from 1/4/1996 to 6/1/2016 with c = 0.2 and

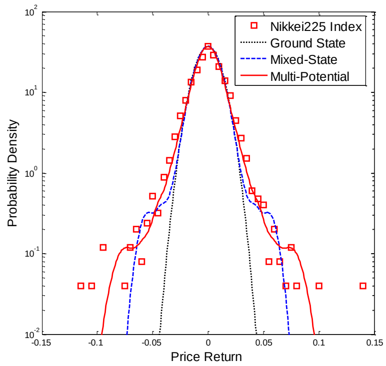

Fig. 3 The fitting of daily Nikkei 225 Index with or without considering of mixed states and multi potentials.

However, the distinct difference in the PDF of the data with different frequencies (Fig. 2) should be attributed to the existence of different energy levels and even the dynamics of the parameters in the potential for long time period. Fig. 3 shows the fitting results of different methods, noted Ground-state Modeling, Mixed-state Modeling and Multi-potential Modeling respectively. More applications into other markets (Table 1) indicate that the stock price stays at the ground state most of the time, but can jump into high energy levels followed by quick return back to the ground state. This dynamical pattern results in the leptokurtic distributions of the price return. When considering data with high frequency and covering long time period, combination of different potential parameters are also required.

Table 1
Adjusted γ with corresponding values of standard deviation σ and kurtosis κ for different market data using different modeling methods. The standard deviations of probability distributions have been required to be consistent with the market indices. The multi-potential modeling gives also consistent kurtoses.

Market indices	Data		Ground-state M.		Mixed-state M.		Multi-potential M.		
	σ	κ_D	γ	κ_{M1}	γ	κ_{M2}	γ_1	γ_2	κ_{M3}
Nikkei 225	0.0154	8.34	3.75×10^6	3.0003	8.50×10^6	6.69	3.40×10^7	3.88×10^6	8.34
SSE composite index	0.0171	7.96	2.44×10^6	3.0003	5.65×10^6	6.70	1.75×10^7	2.73×10^6	7.96
S&P 500	0.0123	10.76	9.10×10^6	3.0005	2.10×10^7	6.69	4.32×10^8	6.75×10^6	10.76

The generality of the potential form Eq. (2) derived from the mechanics of the stock price makes the model applicable to illiquid market such as trend following dominant market (TFDM). The calculated wave functions of TFDM tells us that the corresponding PDF are bimodal with two peaks located on both side of zero return instead of the Gaussian-like distributions of liquid markets. It means that crashes or bubbles are more likely to happen in TFDMs. In order to verify the modeling results, we suggest a simplified method according to Granville's rules to filter data for the moments when the market is dominated by trend followers. We filtered out the price return of the time when the moving averages (MAs) go across the price series. Fig. 4 shows the consistence of the theoretical PDF and the probability distribution of filtered data. The original price series are shuffled to remove the trends. Then we apply the data filtering method and obtain the probability distributions of the filtered data. It is shown in Fig. 5 that the filtering method we proposed is reliable as it retains the statistical properties of the original data different with random time series. In addition, although the general results for different markets with MAs of different lengths are similar, there exist some patterns of the preference of MAs in different markets. In Fig. 6, the value of standard deviation can be regarded as a measurement of the utility frequency of the MA, which means the MA corresponding to larger standard deviation is more frequently used by the chartists. Then it can be seen that the chartists pay more attentions to the MAs of longer terms in Nikkei 225 and S&P 500, while MAs of shorter terms are frequently used by those in SSE Composite Index. For all the markets, there exist preference circles of the MAs.

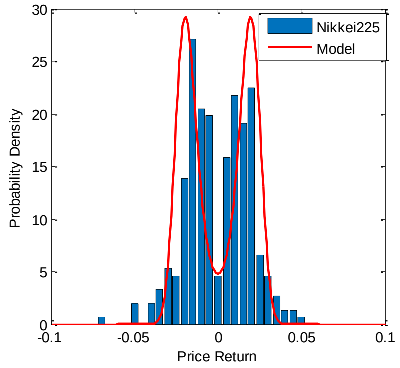

Fig. 4 Theoretical PDF and histogram of filtered data for TFDM.

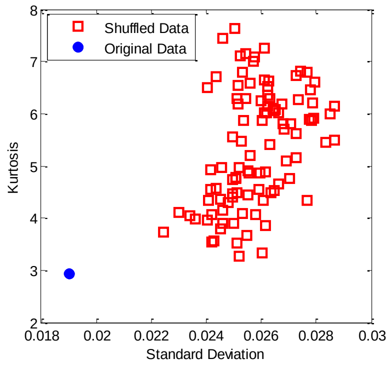

Fig. 5 Statistical properties of the probability distributions for filtered data from original index data and 100 shuffled random data series.

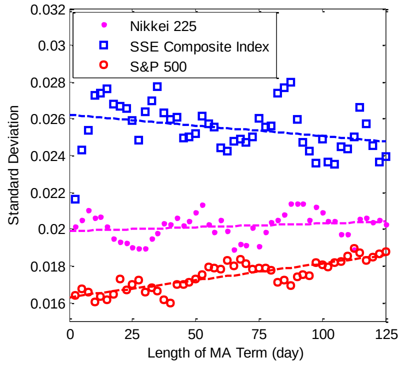

Fig. 6 Standard deviations of the filtered data for different markets, according to the length of MA term.

In the study of the dynamics of stock price, a quantum model is proposed to describe the volatility clustering in the stock price (Fig. 7). As in previous classical models, where the volatility clustering has been studied by autocorrelation of price series as shown in Fig. 8, we define a correlation function of wave functions at different time by wave functions and operators. It is checked that volatility clustering is closely related to the autocorrelation of squared price return (or absolute price return). We apply quantum kicked models to describe the volatility of price return, which is also an extension of the quantum anharmonic oscillator model for the stationary stock market.

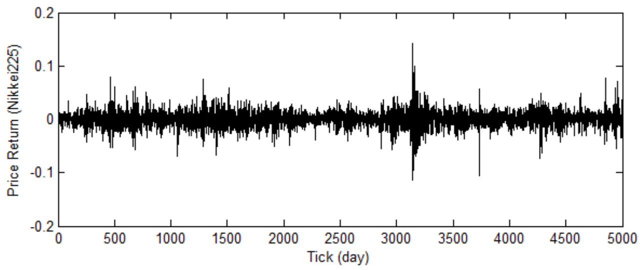

Fig. 7 Return series of daily Nikkei 225 close price series with distinct volatility clustering.

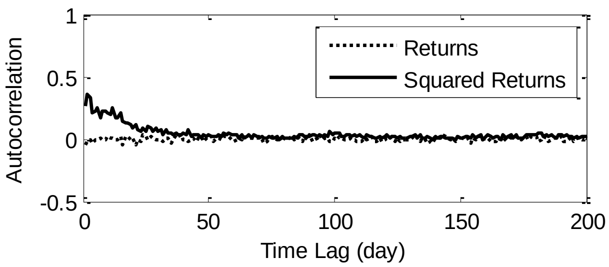

Fig. 8 Autocorrelations of price return series and the corresponding squared returns.

As the financial markets are driven by human whose behaviors are unpredictable, the classical models that based on the determinate theories may be not the best choice for the study of finance. With the assumption that the dynamics of stock price is similar to that of a quantum "particle", we used the wave function and Schrodinger equation to describe the fluctuation of stock price. In order to overcome the shortcomings of previous relevant studies, we proposed a financially interpretable model for the stock price based on the analysis of the fluctuation of excess order in the market. It was demonstrated that the introduction of risk aversion term can somehow help reproduce the leptokurtosis of the distribution, but the emergence of sharp peak and fat tails is mainly attributed to the existence of excited states of the stock price. It is one of the most important contributions of this work as there was no such study before. The utilization of wave functions avoids the introduction of noise term in classical models and shed some light on the nature of stock price. One more advantage of our model is that it is not only applicable to the efficient markets (liquid markets), but also other extreme markets (illiquid markets) such as TFDMs. As there is no raw data for TFDMs, a data filtering method based on Granville's rules was firstly proposed.

The structure of the thesis is arranged as follows. In Chapter 1, the background including fundamental knowledge of stock markets and quantum mechanics is introduced, followed by the objectives of this thesis. Chapter 2 proposes a stationary quantum oscillator model to describe the stock price with reasonable financial interpretation. It is demonstrated that this model can recover the leptokurtic distributions of price return without noise term used in classical stochastic processes. The parameters of the model are adjusted to the real market data in Chapter 3, where an original data filtering method is proposed for illiquid market that is unstable and rarely observed in reality. In Chapter 4, the stationary modeling result is extended to a dynamical one, followed by the analysis of autocorrelations of price return and volatility in order to explain volatility clustering. Conclusions and future plans are addressed in Chapter 5. I hope this work will be a contribution to the quantum modeling of stock markets.