審査の結果の要旨

氏名　橋立　洋祐

論文タイトル
Behavioral Decision Theory: An Axiomatic Approach
（行動意思決定理論：公理的アプローチ）

Report on Ph.D. thesis by Yosuke Hashidate

Title of the thesis:
Behavioral Decision Theory: An Axiomatic Approach

Committee:
Akihiko Matsui (Chair)
Michihiro Kandori
Toshihiro Matsumura
Hitoshi Matsushima
Daisuke Oyama

This report contains the appraisals and recommendation regarding the abovementioned thesis, which has been submitted for Ph.D. by Mr. Yosuke Hashidate. The committee unanimously agreed that the thesis had reached the level suitable for Ph.D., and therefore, Ph.D. should be given to Mr. Yosuke Hashidate. The following is a summary of the thesis and appraisals.

This thesis consists of three essays in decision theory, especially axiomatic approaches therein. Each of them adopts an axiomatic approach and is concerning behavioral decision theory that cannot be explained by the standard rational choice theory in economics. The first model is concerning attribute-based inferences. Although this type of inference is often observed in the real life, the fully rational agent with unlimited computational capability does not have to conduct such an inference.

Consider an example of an economist who tries to book a hotel for a conference. Instead of checking all the information contained in the description of the hotels listed on a website, she would pick up some features like price and the distance from the conference site as attributes to save time. The question is how the decision maker reaches a decision and what feature her choice has.

Yosuke paid attention to the trade-off between attributes and showed that based on some axioms, the decision maker's preference exhibits preference for commitment, i.e., the decision maker prefers the smaller choice set than the larger choice set that containes the smaller one, which never occurs under the rational hypothesis.

In order to tackle this problem, Yosuke presented, in addition to the standard axioms, dominance, dissatisfaction, and contemplation. Among them, dominance and contemplation are the same or similar to those used in the existing literature along the same line. The axiom of dissatisfaction is his original. Let us explain. Consider the set X of alternatives and a space $\mathcal{A}$ of menus, each of which is a subset of X. Preferences are on $\mathcal{A}$.

Let A be a menu. Sarver (2008, Econometrica) set forth an axiom called preference for commitment:

$$\{p\} \succeq \{q\} \ \wedge \ p \in A \ \Rightarrow \ A \succeq A \cup \{q\}.$$

Then Sarver came up with an represention of the form:

$$V(A) = \max_{p \in A} \sum_{u \in \mathcal{U}} \left[u(p) - K[\max_{q \in A} u(q) - u(p)] \right],$$

where $\mathcal{U}$ is the set of utility functions and K is a nonnegative scalar.

Yosuke modified this axiom to accomodate the attribute-based inference. Let $\mathbb{A}$ be the set of attributes, and $\mathcal{X} = \times_{i \in \mathcal{A}} \Delta(X_i)$ be the set of the profiles of probability distributions over X_i's. Then the axiom goes as follows.

> Axiom (Dissatisfaction). For any $q \in \mathcal{X}$, if
> - there exist $i, j \in \mathbb{A}$ such that $p_i \succ_i q_i$ for some $p \in A$, and $q_j \succ_j q_j$ for any $p \in A$, and
> - $\{p\} \succeq \{q\}$ for some $p \in A$,
>
> then $A \succeq A \cup \{q\}$.

This requires an explanation. Let us return to the example of hotel booking. Suppose that the decision maker has a choice set A. If there is another alternative q, which is, say, cheaper than any other hotel in A. At the same time, this hotel as a singleton $\{q\}$ is weakly worse than some hotel in A again as a singleton, i.e., $\{p\}$j. Then adding this hotel q to A will weakly make the decision maker worse off.

With these axioms together with the standard ones, Yosuke obtains the representation result:

> A rough description of the theorem (Representation). $\succeq$ satisfies the set of axioms mentioned above if and only if $\succeq$ has a represention of the following form:
>
> $$V(A) = \max_{p \in A} \left[\sum_{i \in \mathbb{A}} u_i(p_i) - \min_{\mu \in \mathcal{M}(u^*(A))} \sum_{i \in \mathcal{A}} \mu_i [\max_{q \in A} u_i(q_i) - u_i(p_i)] \right],$$
>
> where $u^*(A) = (\max p \in A u_(p_i))$, the ideal option in a menu A, and $\mathcal{M}$ is the set of weights over the attributes, which is relative to the ideal point.

The first term in the bracket is a simple sum of utilities from the attributes. The second term corresponds to dissatisfaction. Given attribute i, $[\max_{q \in A} u_i(q_i) - u_i(p_i)]$ is the term that represents the distance between the best and the candidate in attribute i, which is nonpositive. The decision maker tries to tune the weight μ to minimize the weighted sum of the distance, i.e., disssatisfaction.

The analysis is interesting and original. And we think this paper is publishable in a field journal publishing axiomatic decision theory papers.

***

The second model is related to pride (of acting altruistically) and shame and temptation (of acting selfishly). A motivating story is the following: Mr. X and Ms. Y went for a dinner. The dinner costs $20,000$ yen. X obtained the bill. The question is how much X asks Y to pay. X is a poor graduate student, and he is a dictator in this occasion. X thought:

> I wish I could pay the entire amount (due to his pride).
> However, she would understand if I pay this much (temptation).
> Y does not make fun of me if I pay this much (shame).

This way, X cares about Y's thinking about X's act.

The key axiom to the axiomatizing this phenomenon is selfishness and altruism (assume the set of agents is $I = \{1, 2\}$ here):

> Axiom (Selfishness and Altruism). For each $j \in \{1, 2\}$, if $A \cap B = \emptyset$ and $A \succeq_j B$, then
> $$A \succeq_j A \cup B \succeq_j B.$$

This axiom exhibits a trade-off between selfishness and altruism. In fact, $A \succeq_1 A \cup B$ for Agent 1 corresponds to temptation (selfishness), and $A \succeq_2 A \cup B$ for Agent 1 corresponds to shame. Then together with some other axioms, the representaion is roughly given by

$$V(A) = \max_{p \in A} \left[\sum_{i \in I} \alpha_i u(p_i) - \max_{\beta \in \mathcal{B}} [\beta_1 \alpha_1 \{u(p_1) - \bar{u}_1(A)\} + \beta_2 \alpha_2 \{u(p_2) - \bar{u}_2(A)\} \right],$$

where $\bar{u}_i$'s are reference points. Therefore, the term $\{u(p_1) - \bar{u}_1(A)\}$ corresponds to reference-dependent selfishness, and the term $\{u(p_2) - \bar{u}_2(A)\}$ corresponds to reference-dependent altruism.

In terms of originality, this work is a little weaker than the first model. But, we think this part is also publishable in a field journal.

***

The third model is concerning preference for randomization. Again, the expected utility theory for rational decisions implies that if there are two alternatives, then the randomization never goes beyond the two in terms of utility as the expected utility of a mixed lottery is always a convex combination of pure lotteries. However, we often see people randomize their behavior. This model tries to explain this phenomenon by an axiomatic approach.

The axiom he introduces is the following:

> Axiom (Randomization). For any $A, B \in \mathcal{A}$, if for any $\mu \in \Delta(B)$, there exists $\rho \in \Delta(A)$ such that ρ dominates μ, then $A \succeq B$.

This is a weaker requirement than the independence axiom, allowing the decision maker to strictly prefer a mixed lottery to any pure lotteries that constitute it. Then together with some other axioms, Yosuke presents a representation theorem where the preference

is represented by

$$V(A) = \max \rho \in \Delta(A) \sum_{q \in A} u(q) g(p(q)).$$

Notice that if $g(p(q)) = p(q)$, then it is the standard expected utility. This axiom is weaker than Kreps' axiom concerning strategic rationality.

***

Behavioral decision theory would have little discipline unless there is an axiomatic foundation. By appraising axioms, we better understand the decisions made by bounded rational human beings. In this sense, Yosuke's line of research is very important. Although Yosuke's results are not earth-shattering, his solid analysis and interesting representation results are good contributions to the field. Thereby the committee unanimously agreed to give Ph.D. to Mr. Yosuke Hashidate.