

博士論文（要約）

Behavioral Decision Theory: An Axiomatic Approach

（行動意思決定理論: 公理的アプローチ）

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Behavioral Decision Theory: An Axiomatic Approach

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This dissertation contains three essays in decision theory. Each topic is related to various behavioral regularities that cannot be identified by rational choice theory in economics. Chapter 1 studies choices with attribute-based inferences. Chapter 2 studies choices with social image that stems from the trade-off under altruism and selfishness. Chapter 3 studies stochastic choices that stems from deliberate randomization such as tossing a coin in mind.

CHAPTER 1

Attribute-based inferences are often used in real-life decision-making. Classically, Krantz et al. (1971) provide an axiomatic characterization of separably additive attribute-based utility representations. Krantz et al. (1971) show that, given an attribute space, there exists a set of attribute functions that represents attribute-wise rankings, and preferences are represented by a separably additive utility representation. Keeney and Raiffa (1976) refer to the importance of how decision makers determine the optimal weight on the attribute space. In general, there exists trade-offs across attributes. The trade-offs are closely related to the resulting choice behaviors. Moreover, such a trade-off makes it difficult to determine the optimal weight on the attribute space, as well as to make a choice.

In the literature of marketing, for example, attribute-based inferences often lead to preferences reversals, i.e., violations of WARP (*Weak Axiom of Revealed Preference*). That is, an irrelevant third alternative affect a decision-making between two alternatives. For example, the Attraction Effect (Huber et al. (1982)) and the Compromise Effect (Simonson (1989)) are well-known behavioral regularities as violations of WARP.

Chapter 1 presents a theory of attribute-based inferences. In Chapter 1, we consider the trade-off across attributes as a type of *subjective uncertainty* in terms of preferences over menus. We explore plausible axioms for attribute-based inferences about preferences over menus: *Dominance*, *Dissatisfaction*, and *Contemplation*. We have considered the new axioms to capture how the trade-off across attributes affects decision-making, and how the decision maker contemplate the weight on the attribute space. We find that the trade-off across attributes is related to a class of preferences for commitment (preferring smaller menus).

These key axioms, *Dominance*, *Dissatisfaction*, and *Contemplation*, along with other basic axioms, characterize a dissatisficing-averse utility representation in attribute-based inferences. The utility representation depicts the decision maker who determines the optimal weight on the objective attribute space to minimize the deviation from each attribute-best option. We apply the duality result into the utility representation, by showing that exploring the *best* option on the Pareto frontier in each menu on the attribute-based utility space is equivalent to exploring the *optimal* weight on the attribute space. In terms of attribute-based inferences, a relationship between raw preferences and reasoned choices is discussed. Moreover, Chapter 1 considers a pair of preferences over menus and choice correspondences to characterize ex-post choices of the dissatisficing-averse utility representation. Finally, Chapter 1 characterizes the ex-post choice, by relaxing WARP (*Weak Axiom of Revealed Preference*).

Axioms

Let $X \equiv \prod_{i=1}^n X_i$ be a finite set of all alternatives, where X_i is a domain of attribute i of alternatives. This domain corresponds to an attribute i 's evaluation for alternatives. We assume that an attribute space \mathbb{A} is finite, i.e., $\mathbb{A} = \{1, \dots, n\}$ and $|\mathbb{A}| = n$. The elements of X_i are denoted by $x_i, y_i, z_i \in X_i$. For each i , let $\Delta(X_i)$ be a set of probability distributions over X_i , endowed with the Euclidean metric d . Since each X_i is finite, the topology generated by the Euclidean metric d is equivalent to the weak* topology on $\Delta(X_i)$. The elements of $\Delta(X_i)$ are denoted by $p_i, q_i, r_i \in \Delta(X_i)$.¹

An *option* is denoted by $\mathbf{p} \equiv (p_1, \dots, p_n)$. Let $\mathcal{X} \equiv \prod_{i=1}^n \Delta(X_i)$ be a set of all options. Let \mathcal{A} be the set of all non-empty closed and compact subsets of \mathcal{X} , endowed with the Hausdorff metric. The Hausdorff metric is defined by

$$d_h(A, B) = \max \left\{ \max_{\mathbf{p} \in A} \min_{\mathbf{q} \in B} d(\mathbf{p}, \mathbf{q}), \max_{\mathbf{p} \in B} \min_{\mathbf{q} \in A} d(\mathbf{p}, \mathbf{q}) \right\}.$$

Menus are denoted by $A, B, C \in \mathcal{A}$. Define the convex combinations in the standard manner: For any $A, B \in \mathcal{A}$ and for any $\lambda \in [0, 1]$, $\lambda A + (1 - \lambda)B \equiv \{\lambda \mathbf{p} + (1 - \lambda)\mathbf{q} \mid \mathbf{p} \in A, \mathbf{q} \in B\}$.

The primitive of the model is a binary relation \succeq over \mathcal{A} . The binary relation \succeq describes the decision maker's preference over menus. The asymmetric and symmetric parts of \succeq are denoted by \succ and \sim respectively.

We state the axioms in Chapter 1. First, we state the standard requirements in decision theory. Next, we introduce the new axioms: *Dominance* and *Dissatisfaction*, by relaxing the axiom of *Strategic Rationality*. Finally, we provide a weaker version of *Independence*.

¹A lottery p_i is interpreted as follows. For example, suppose that a university has application letters of Ph.D. applicants with TOEFL iBT scores. However, the university is still uncertain how well Ph.D. applicants can speak English. The lottery p_i captures a prediction of the university for a candidate's attribute-based evaluation.

Axiom (Standard Preferences): \succeq is (i) a *weak order*, (ii) *continuous*, and (iii) *non-degenerate*:

(i) (Weak Order): \succeq is *complete* and *transitive*.

(ii) (Continuity): The sets $\{A \in \mathcal{A} \mid A \succeq B\}$ and $\{A \in \mathcal{A} \mid B \succeq A\}$ are closed (in the Hausdorff metric d_h).

(iii) (Strict Non-Degeneracy): There exist $\mathbf{p}, \mathbf{q} \in \mathcal{X}$ such that $\{\mathbf{p}\} \succ \{\mathbf{q}\}$.

Axiom (Separability): For any $p_i, q_i \in \mathcal{X}_i$ and $r_{-i}, r'_{-i} \in \mathcal{X}_{-i}$,

$$\{(p_i, r_{-i})\} \succeq \{(q_i, r_{-i})\} \Rightarrow \{(p_i, r'_{-i})\} \succeq \{(q_i, r'_{-i})\}.$$

Definition 1. For any $p_i, q_i \in \mathcal{X}_i$,

$$p_i \succsim_i q_i \Leftrightarrow \{(p_i, r_{-i})\} \succeq \{(q_i, r_{-i})\},$$

for any $r_{-i} \in \mathcal{X}_{-i}$.

By using the definition of $(\succsim_i)_{i \in \mathbb{A}}$, we provide the following monotonic condition.

Axiom (Dominance): If for any $\mathbf{q} \in \mathcal{X}$, there exists $\mathbf{p} \in A$ such that for any $i \in \mathbb{A}$, $p_i \succsim_i q_i$, then $A \sim A \cup \{\mathbf{q}\}$.

Axiom (Dissatisfaction): For any $\mathbf{q} \in \mathcal{X}$, if there exist $i, j \in \mathbb{A}$ such that

(i) for some $\mathbf{p} \in A$, $p_i \succ_i q_i$; and

(ii) for any $\mathbf{p} \in A$, $q_j \succ_j p_j$,

and $\{\mathbf{p}\} \succeq \{\mathbf{q}\}$ for some $\mathbf{p} \in A$, then

$$A \succeq A \cup \{\mathbf{q}\}.$$

Axiom (Contemplation): \succeq satisfies the following two conditions:

(i) (No Need for Contemplation): For any $A, B \in \mathcal{A}$, $\mathbf{p} \in \mathcal{X}$, and $\lambda \in [0, 1]$,

$$A \succeq B \Rightarrow \lambda A + (1 - \lambda)\{\mathbf{p}\} \succeq \lambda B + (1 - \lambda)\{\mathbf{p}\}.$$

(ii) (Contemplation Seeking): For any $A, B \in \mathcal{A}$, if there exist $i, j \in \mathbb{A}$ such that

(a) for any $\mathbf{p} \in A$, there exists $\mathbf{q} \in B$ such that $q_i \succ_i p_i$; and

(b) for any $\mathbf{q} \in B$, there exists $\mathbf{p} \in A$ such that $p_j \succ_j q_j$,

then, for any $\lambda \in [0, 1]$,

$$\lambda A + (1 - \lambda)B \succeq A.$$

Result

We state the main result. Let $\Lambda(\mathbb{A})$ be the set of all non-empty compact subsets of non-negative measures on \mathbb{A} . Given a menu $A \in \mathcal{A}$, let $\mathbf{u}^*(A) := (\max_{\mathbf{p} \in A} u_i(p_i))_{i \in \mathbb{A}}$ be the ideal option of the menu A .

Theorem 1. *The following statements are equivalent:*

- (a) \succeq on \mathcal{A} satisfies Standard Preferences, Separability, Dominance, Dissatisfaction, and Contemplation.
- (b) There exists a pair $\langle \mathcal{U}, \mathcal{M} \rangle$ where $\mathcal{U} = (u_1, \dots, u_i, \dots, u_n)$ is a set of non-constant utility functions where $u_i : \Delta(X_i) \rightarrow \mathbb{R}$, and \mathcal{M} , a set of non-negative measures on \mathbb{A} defined by $\mathcal{M} : \mathbb{R}^n \rightarrow \Lambda(\mathbb{A})$, such that \succeq is represented by $V : \mathcal{A} \rightarrow \mathbb{R}$ defined by

$$V(A) = \max_{\mathbf{p} \in A} \left[\sum_{i \in \mathbb{A}} u_i(p_i) + \min_{\mu \in \mathcal{M}(\mathbf{u}^*(A))} \sum_{i \in \mathbb{A}} \mu_i \left(\max_{\mathbf{q} \in A} (u_i(q_i) - u_i(p_i)) \right) \right],$$

and the following conditions hold:

- (i) \mathcal{M} is consistent: for each $\mu, \mu' \in \mathcal{M}$ and $\mathbf{p} \in \mathcal{X}$, $\sum_{i \in \mathbb{A}} \mu_i u_i(p_i) = \sum_{i \in \mathbb{A}} \mu'_i u_i(p_i)$;
- (ii) \mathcal{M} is minimal: for any compact subset \mathcal{M}' of \mathcal{M} , the function V' obtained by replacing \mathcal{M} with \mathcal{M}' no longer represents \succeq .

CHAPTER 2

Social preference is one of the key topics in behavioral economics. In experiments on social preferences, subjects often exhibit pro-social behaviors. In a recent experimental study, Dana et al. (2006) consider an extended version of dictator games. They provide a two-stage decision problem for dictators. At the first stage, dictators have two options. The one is to proceed to the standard dictator games to share \$10 between a dictator and a recipient. The other is to exit with \$9 for dictators and nothing for recipients. The key point in this experiment is that recipients do not know about dictators' choices at the first stage. Dana et al. (2006) find that about one-third of subjects were willing to "exit" a \$10-dictator game, and they take \$9 instead. This type of behaviors is not consistent with behavioral economic models.

Dillenberger and Sadowski (2012) is the first literature to apply the framework of preferences over menus to the study of social image. The trade-off between altruism and selfishness in mind is related to *subjective uncertainty*. The key point in their study is that, compared with the choice at the ex-ante stage, the choice at the ex-post stage, i.e., choosing an allocation from the menu chosen at the ex-ante stage is more altruistic, since that choice is publicly observed. In general, however, the subjective criterion of social image is *opportunity-dependent*.

Chapter 2 develops a unified model in other-regarding preferences and reference-dependent preferences, by eliciting an endogenous reference point as a criterion of social-image. The objective of Chapter 2 is to identify underlying criterion of altruism and selfishness, in terms of preferences over menus. Chapter 2 presents a theory of reference-dependent utilitarian, by anticipating self-image in altruism and selfishness in one's mind. The contribution of Chapter 2 is to provide an axiomatic foundation for reference-dependent pro-social behaviors, in which a "hypothetical" reference point is endogenously formed from the elements in a menu. To do so, Chapter 2 relaxes the axiom of *Strategic Rationality* in Kreps (1979), and to capture reactions from reference points, Chapter 2 also relaxes the axiom of *Independence*. We have considered how the trade-off between the subjective criterion of altruism and selfishness affects choice behaviors. In Chapter 2, we uniquely identify the attitude toward pure altruism, reference-dependent criterion of social image, and parameters for reference points, respectively. Moreover, we provide a comparative attitude toward reference points as self-image in altruism and selfishness.

Axioms

We introduce notation for social decision-making briefly. Let 1 denote the decision maker, and S be the finite set of other agents. Let $I = \{1\} \cup S$ be the set of *all* agents. Let Z be a finite set of outcomes. $\Delta(Z)$ is the set of all lotteries with finite support. Let $(\Delta(Z))^I$ be the set of all *allocations*. The elements of $(\Delta(Z))^I$ are denoted by $\mathbf{p} = (p_1, \dots, p_n) = (p_1, p_S)$ where $S = \{2, \dots, n\}$. The lottery p_i is an allocation for an agent i . Let \mathcal{A} be the set of all non-empty compact subsets of $(\Delta(Z))^I$ endowed with the Hausdorff metric d_h . The Hausdorff metric is defined by

$$d_h(A, B) = \max \left\{ \max_{\mathbf{p} \in A} \min_{\mathbf{q} \in B} d(\mathbf{p}, \mathbf{q}), \max_{\mathbf{p} \in B} \min_{\mathbf{q} \in A} d(\mathbf{p}, \mathbf{q}) \right\},$$

where d is the Euclidean metric. Menus are denoted by $A, B, C \in \mathcal{A}$.

The primitive of the model is a binary relation \succeq over \mathcal{A} . The asymmetric and symmetric parts of \succeq are denoted by \succ and \sim , respectively.

We state the axioms in Chapter 2. First, we state the standard requirements in decision theory. Next, we provide the axiom of *Consistency* for the decision maker to evaluate other agents' allocation by using her own preference. Moreover, we introduce the new axioms on social image: *Dominance* and *Selfishness and Altruism*, by relaxing the axiom of *Strategic Rationality*. Finally, we provide a weaker version of *Independence*.

Axiom (Standard Preferences): \succeq satisfies completeness, transitivity, continuity, and non-degeneracy.

- (i) (Completeness): For any $A, B \in \mathcal{A}$, $A \succeq B$ or $B \succeq A$.

- (ii) (Transitivity): For any $A, B, C \in \mathcal{A}$, if $A \succeq B$ and $B \succeq C$, then $A \succeq C$.
- (iii) (Continuity): The sets $\{A \in \mathcal{A} \mid A \succeq B\}$ and $\{A \in \mathcal{A} \mid B \succsim A\}$ are closed (in the Hausdorff metric d_h).
- (iv) (Non-Degeneracy): There exists $A, B \in \mathcal{A}$ such that $A \succ B$.

We induce two binary relations on $(\Delta(Z))^j$ ($j \in \{1, S\}$), individual preference \succsim_1 and social preference \succsim_S . For each $j \in \{1, S\}$, the asymmetric and symmetric parts of \succsim_j are denoted by \succ_j and \sim_j respectively.

Definition 2. \succsim_1 and \succsim_S are defined as follows.

- (i) For all $p_1, q_1 \in \Delta(Z)$, $p_1 \succsim_1 q_1$ if $\{(p_1, r_S)\} \succeq \{(q_1, r_S)\}$ for some $r_S \in \Delta(Z)^S$.
- (ii) For all $p_S, q_S \in \Delta(Z)^S$, $p_S \succsim_S q_S$ if $\{(r_1, p_S)\} \succeq \{(r_1, q_S)\}$ for some $r_1 \in \Delta(Z)$.

We provide a consistency condition between \succsim_1 and \succsim_S .

Axiom (Consistency): For any $p_S, q_S \in \Delta(Z)^S$, if $p_i \succsim_1 q_i$ for any $i \in S$, then $p_S \succsim_S q_S$.

We consider the following induced binary relations over menus. \succeq_1 and \succeq_S on \mathcal{A} are defined as follows.

Definition 3. For each $j \in \{1, S\}$, we say that $A \succeq_j B$ if for any $\mathbf{q} \in B$ and $\mathbf{p} \in A$, $p_j \succsim_j q_j$.

For each $j \in \{1, S\}$, the asymmetric and symmetric parts of \succeq_j are denoted by \succ_j and \sim_j respectively. These definitions (\succeq_1, \succeq_S) are suitable extensions of individual preference \succsim_1 and social preference \succsim_S .

Axiom (Dominance): For any $A, B \in \mathcal{A}$, if $A \succeq_1 B$ and $A \succeq_S B$, then

$$A \succeq B.$$

The next axiom is a key axiom of this chapter.

Axiom (Selfishness and Altruism): For each $j \in \{1, S\}$, if $A \cap B = \emptyset$ and $A \succeq_j B$, then

$$A \succeq_j A \cup B \succeq_j B.$$

Finally, we relax the axiom of *Independence*.

Axiom (Singleton Independence): For any $A, B \in \mathcal{A}$, $\mathbf{p} \in \Delta(Z)^I$, and $\lambda \in [0, 1]$,

$$A \succeq B \Rightarrow \lambda A + (1 - \lambda)\{\mathbf{p}\} \succeq \lambda B + (1 - \lambda)\{\mathbf{p}\}.$$

Axiom (Contemplation Aversion): For any $A, B \in \mathcal{A}$ and $\lambda \in [0, 1]$,

$$(\forall j \in \{1, S\}) A \succeq_j B \Rightarrow A \succeq \lambda A + (1 - \lambda)B.$$

Result

We state the main result. Let $\Gamma(\{1, S\})$ be the set of all non-empty compact subsets of non-negative measures on $\{1, S\}$.

Theorem 2. *The following statements are equivalent:*

(a) \succeq satisfies Standard Preferences, Consistency, Dominance, Selfishness and Altruism, Singleton Independence, and Contemplation Aversion.

(b) There exists a four-tuple $(u, \alpha, \mathcal{B}, \gamma)$ where u is a non-constant function $u : \Delta(Z) \rightarrow \mathbb{R}$, α is a vector such that $\alpha_1 > 0$, and $\alpha_i \geq 0$ for each $i \in S$ with $\sum_{i \in S} \alpha_i = 1$, $\mathcal{B} : \mathbb{R}^2 \rightarrow \Gamma(\{1, S\})$ is a compact set of non-negative measures on $\{1, S\}$, and γ is the vector such that $\gamma_j \in [0, 1]$ for each $j \in \{1, S\}$ such that \succeq is represented by a function $V : \mathcal{A} \rightarrow \mathbb{R}$ defined by

$$V(A) = \max_{\mathbf{p} \in A} \left[\sum_{i \in I} \alpha_i u(p_i) + \max_{\beta \in \mathcal{B}(\bar{\mathbf{u}}(A))} \left[\beta_1 (\alpha_1 (u(p_1) - \bar{u}_1(A)) + \beta_S (\sum_{i \in S} \alpha_i u(p_i) - \bar{u}_S(A))) \right] \right],$$

where $\bar{\mathbf{u}}(A) = (\bar{u}_1(A), \bar{u}_S(A))$, $\bar{u}_1(A) = \gamma_1 \max_{\mathbf{q} \in A} u(q_1) + (1 - \gamma_1) \min_{\mathbf{r} \in A} u(r_1)$, $\bar{u}_S(A) = \gamma_S \max_{\mathbf{q} \in A} \sum_{i \in S} \alpha_i u(q_i) + (1 - \gamma_S) \min_{\mathbf{r} \in A} \sum_{i \in S} \alpha_i u(r_i)$, and the following conditions hold: for each $j \in \{1, S\}$,

- (i) \mathcal{B} is consistent: for each $\beta, \beta' \in \mathcal{B}$ and $\mathbf{p} \in \Delta(Z)^I$, $\sum_{j \in \{1, S\}} \beta_j u_j(p_j) = \sum_{j \in \{1, S\}} \beta'_j u_j(p_j)$ where $u_1(p_1) = \alpha_1 u(p_1)$ and $u_S(p_S) = \sum_{i \in S} \alpha_i u(p_i)$;
- (ii) \mathcal{B} is minimal: for any compact subset \mathcal{B}' of \mathcal{B} , the function V' obtained by replacing \mathcal{B} with \mathcal{B}' no longer represents \succeq .

CHAPTER 3

Recently, the study of stochastic choices has been rapidly developing. One might have the following question: “why do human behaviors seem to be stochastic?” In decision theory, the reasons for this question are categorized into the following three topics. Note that the intersections between the topics are non-empty.

1. Learning
2. Limited Attention
3. Deliberate Randomization

In the first topic of learning, decision makers privately obtain some information that is not observed by decision analysts. As a result, the resulting choice behaviors seem to be stochastic. In the second topic of attention, decision makers might randomly change the focus for each decision problem. In fact, the attention itself is subjective. As a result, the resulting choice behaviors seem to be stochastic.

In this dissertation, we focus on the study of the third topic: *deliberate randomization*. By using the framework of preferences over menus, we elicit an attitude toward the effect of subjective randomization in one's mind.

Chapter 3 presents a theory of preferences for randomization, especially, “deliberate randomization.” The contribution of Chapter 3 is threefold. First, we elicit a subjective belief of deliberate randomization from deterministic preferences. The key axiom in our axiomatic analysis is a *monotonic* condition for deliberate randomization, stated as *Randomization*. We show that *Randomization*, along with other axioms, axiomatically characterizes a random anticipated utility representation in which the decision maker's subjective belief for deliberate randomization is identified. Second, we identify a class of *preferences for randomization* ranging from the *desire to randomization* to the *aversion to randomization*. Third, we show that the subjective belief for the effect of randomization in one's mind is closely related to several cognitive or psychological effects. Especially, we apply preferences for randomization into subjective partitional learning to capture preferences for delay. We also provide an axiomatic analysis for costs of thinking to identify the attitude toward randomization uniquely.

Axioms

We introduce notation briefly. Let $X \subseteq \mathbb{R}^n$ be a *convex* and *compact* set of all alternatives, endowed with the Euclidean metric d . Each $n \in \mathbb{N}$ is interpreted as an attribute of alternatives. The elements in X are denoted by $x, y, z \in X$. Let $\Delta(X)$ be the set of all probability distributions on X with finite support. The elements in $\Delta(X)$ are denoted by $p, q, r \in \Delta(X)$. In this paper, it is postulated that each lottery is an *option*.² Let \mathcal{A} denote the set of all non-empty *compact* subsets of $\Delta(X)$, endowed with the Hausdorff metric. The Hausdorff metric is defined by

$$d_h(A, B) = \max \left\{ \max_{p \in A} \min_{q \in B} d(p, q), \max_{p \in B} \min_{q \in A} d(p, q) \right\}.$$

The elements in \mathcal{A} are called *menus*, which are denoted by $A, B, C \in \mathcal{A}$. We use the convex combination on menus in the standard manner: for any $A, B \in \mathcal{A}$ and $\lambda \in [0, 1]$, $\lambda A + (1 - \lambda)B = \{ \lambda p + (1 - \lambda)q \mid p \in A, q \in B \}$.

The primitive of the model is a binary relation \succeq over \mathcal{A} , which describes the decision maker's choice of sets of lotteries. As usual, the asymmetric and symmetric parts of \succeq are denoted by \succ and \sim , respectively.

We state the axioms in Chapter 3. First, we state the standard requirements in decision theory. Next, we provide the key axiom on “deliberate randomization” by relaxing the axiom of *Strategic Rationality*. Finally, we provide a weaker version of *Independence*.

²The decision maker might not know about true values of alternatives.

Axiom (Standard Preferences): \succeq satisfy (i) a *weak order*, (ii) *continuity*, and (iii) *non-degeneracy*:

- (i) (Weak Order): \succeq is *complete* and *transitive*.
- (ii) (Continuity): The sets $\{A \in \mathcal{A} \mid A \succeq B\}$ and $\{A \in \mathcal{A} \mid B \succeq A\}$ are closed in the Hausdorff metric.
- (iii) (Non-Degeneracy): There exists $A, B \in \mathcal{A}$ such that $A \succ B$.

We introduce the key axiom of *Randomization*. To do so, we define the following. Let \succeq be a binary relation on $\Delta(\Delta(X))$, the set of all probability distributions on $\Delta(X)$. The asymmetric and symmetric parts of \succeq are denoted by \triangleright and \simeq , respectively. Let δ_x be a *degenerate* lottery (Dirac measure at x), which gives x with certainty. For any $p \in \Delta(X)$, let c_p be the *certainty equivalent* of p , i.e., an element in X with $p \simeq \delta_{c_p}$.

Definition 4. For any $\rho, \mu \in \Delta(X)$ such that $\rho = \rho_1 \circ p_1 \oplus \dots \oplus \rho_m \circ p_m$ and $\mu = \mu_1 \circ q_1 \oplus \dots \oplus \mu_k \circ q_k$, we say that ρ *dominates* μ if

$$\delta_{c_{\rho_1 p_1 + \dots + \rho_m p_m}} \geq \delta_{c_{\mu_1 q_1 + \dots + \mu_k q_k}}.$$

The main axiom is stated as follows.

Axiom (Randomization): For any $A, B \in \mathcal{A}$, if for any $\mu \in \Delta(B)$, there exists $\rho \in \Delta(A)$ such that ρ *dominates* μ , i.e., $\rho \triangleright_{\text{dom.}} \mu$, then $A \succeq B$.

We provide a weaker version of the axiom of *Independence*.

Axiom (Singleton Independence): For any $p, q \in \Delta(X)$, any $r \in \Delta(X) \setminus \{p, q\}$, and any $\lambda \in [0, 1]$,

$$\{p\} \succeq \{q\} \Leftrightarrow \lambda\{p\} + (1 - \lambda)\{r\} \succeq \lambda\{q\} + (1 - \lambda)\{r\}.$$

Results

We state the “anticipated” utility representation of optimal random choices. Given a menu $A \in \mathcal{A}$, let $\Delta(A)$ be the set of all probability measures on A , i.e., $\sum_{x \in A} \rho(x) = 1, \rho(x) \in [0, 1]$ for any $x \in A$.

Theorem 3. *The following statements are equivalent:*

- (a) \succeq satisfies Standard Preferences, Randomization, and Singleton Independence.
- (b) There exists a pair $\langle u, g \rangle$ where u is a non-constant function $u : \Delta(X) \rightarrow \mathbb{R}$ and g is a continuous

and strictly increasing function $g : [0, 1] \rightarrow [0, 1]$ where $g(0) = 0$ and $g(1) = 1$ such that \succeq is represented by $V : \mathcal{A} \rightarrow \mathbb{R}$ defined by

$$V(A) = \max_{\rho \in \Delta(A)} \sum_{q \in A} u(q)g(\rho(q)).$$

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