博士論文

Essays in Imperfect Competition and Strategic Behavior
(不完全競争市場と企業の戦略的行動に関する研究)

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Contents

Chapter 1

Introduction

1.1 Motivation and selected themes

"Firm" lies at the heart of the economy. The organizations (also called as a company, or a enterprise) bring considerable benefits to the world. People consume products and services which firms provide in a real life. Although there are the bright sides of firms, economists have long paid attention to firms' behavior and incentive, especially in the market with a few competitors. Such market structure is so-called imperfect competition (and/or oligopolistic market), and the oligopoly problem which leads to market failure by the firms having market power is the central issue in economics. The contributions to the issue goes back to the seminal works of Cournot (1838), Bertrand (1883), Edgeworth (1925), Hotelling (1929), and Chamberlin (1933). After the game-theoretic tool which analyzes "Strategic behavior" among a few players was incorporated, the theoretical model of firms have been rapidly and broadly developed. The well-established models apply for describing firm behavior in some industry, providing remarkable predictions which allow us to argue economic policy. Recently, Jean Tirole won the Novel prize at 2014 because of this analysis of market power and regulation. They have tried to answer several questions from general issues to industry- or firm-specific problems regarding monopoly and oligopoly.

The aim of this dissertation is to extend our understanding of imperfect competition and strategic behavior in some circumstances. Specifically, I focus on three topics in the field of industrial organization in this dissertation: multi-store paradox, corporate social responsibility (CSR), and mixed oligopoly.

1.1.1 *Multi-Store Paradox*

Firms produce differentiated goods which are imperfectly substitutable each other. The influential model is proposed by Hotelling (1929) which formulates a so-called linear city model. In the market, consumers are distributed in a line and their standing point is just as a living area. To buy some goods, they visit the stores which are also located in the line. So, one may call the model as a location model. Note that the difference among consumers can be also interpreted as a difference of consumers' taste. The notion of a consumers' taste heterogeneity even now gives foundation of demand functions with product differentiation.

Based on Hotelling (1929) and Salop (1979) which formulates a circular city model, there are substantial works which examine firms' incentive to decide where to locate in the market, which is one of the major area of research on industrial organization. Basically, firms which are located nearby each other or produce similar goods compete severely, which means that each of them sets the lower price to attract more consumers. On the flip side of the intensity of competition within the close area, the more differentiation coming from locations or product positioning lessens price competition, resulting higher prices and profits for firms. From the effect of product differentiation to soften price competition, duopoly firms endogenously choose their locations to maximize the distance between them (D'Aspremont, Gabszewicz, and Thisse, 1979).

In line of this research, firms are mostly assumed to open only one store. Martínez-Giralt and Neven (1988), however, extend their model to allow firms to establish multiple stores. The extension is quite natural,[1] but firms do not open multiple stores and set only one store in equilibrium. Although establishing two stores increases its own market share, it facilitates price competition because the distance from the competitor decreases. The result of this study implies that the latter effect always dominates the former effect. In the context of the location model with a strategic incumbent and entry, Schmalensee (1978) proposes the possibility that the incumbent establishes several stores and crowds the market so as to deter entry. This sheds light on the multi-store problem. However, Judd (1985) points out that the commitment problem of the incumbent exists in the dynamics: since incumbent has an *ex post* incentive to withdraw stores, the entrant who anticipates the *ex post* incentive can enter the market. As a result, the incumbent who anticipates the post-entry as well establishes only one store from the beginning and the new firm enters the market. The results of these studies have huge impact on the literature of endogenous product

[1]Multiproduct firms occupy 87% of total output and 39% of total number of firms at five-digit SIC categories for US manufacturing firms (Bernard, Redding, and Schott, 2010).

differentiation. The representative models can not describe the situation in which firms establish multiple stores, while they do in reality. The theoretical problem became known as multi-store paradox. To tackle this issue, I further incorporate the following two elements and reexamine the incentive to multi-stores, respectively.

First, I consider the location model with strategic multiple incumbents. As in Judd (1985), the simple market structure in which one incumbent and one entrant exist leads to the commitment problem of the incumbent. There is strategic interaction not only between incumbents and an entrant but also among incumbents in the presence of other incumbents. This changes the *ex post* incentive regarding the commitment problem, and shows the possibility that the incumbents establish multiple stores to deter entry. Similar to this notion, Ashiya (2000) and Ishibashi (2003) study the strategic entry deterrence by multiple stores, but they emphasize the multiple entrants under the sequential entry.

Second, in a line with Martínez-Giralt and Neven (1988), I examine the effect of payoff interdependence on the multi-store strategy of firms. The goal of firms is usually assumed that firms purely maximize their own profits. There is however another notion to employ interdependent objective functions for firms' payoff. The firm may maximize the difference between own profit and rival's profit, or partially take rival's profit into account own profit. The fact which supports the idea is, for example, a managerial reward contract where the evaluation of managerial performance is based on the relative as well as absolute performance of managers.[2] The departure from the pure profit maximization can change the previous result and solve the theoretical problem which yields unreasonable predictions.

1.1.2 *Corporate Social Responsibility and self-regulation*

Regarding economic behavior in the market, how to control the negative externalities caused by a firm's activity is also a fundamental problem. The debate goes back to the exchange between Pigou (1932) and Coase (1960) about the market failure under perfectly competitive market due to significant negative externalities, in which their focus is the necessity of the government intervention by a formal regulation. In oligopoly market where each firm has the market power, the distortion created by the market power increases the complexity of this issue, where the first best is not regularly achieved.[3] Moreover, the considerable market power makes it difficult to design and

[2]See Gibbons and Murphy (1990).

[3]Levin (1985) firstly examined the effect of the environmental tax in oligopoly market which is an asymmetric Cournot oligopoly.

implement relevant policy. On the other hand, people are increasingly interested in environmental problems like Global warming, climate change, air pollution, and water pollution.

Against the background, there is a growing interest in voluntary actions by firms that contribute to the environmental problems. Corporate social responsibility (CSR) is a typical and well-know notion which captures such voluntary actions. Although there are several official definitions, European Commission (2002), for instance, define CSR as "*a concept whereby companies integrate social and environmental concerns in their business operations and in their interaction with their stakeholders on a voluntary basis*". Almost ninety percent of Fortune Global 250 firms report their corporate responsibility activities (KPMG, 2013). The practical action from the concept of CSR is sometime called self-regulation or over-compliance. Lyon and Maxwell (2002) summarize several examples which the voluntary agreements take place in Europe and the United States. The typical question in the literature on CSR is why firms voluntarily take certain actions even though they are costly. There are two well-know reasons. One possible explanation is that these activities are connected with advertisement or public reputation of firms, and thus firms earns more profit due to a better reputation (Liu et al., 2015). The other is that polluting firms strategically act and self-regulate because of the threat of future command-and-control regulation (e.g., Maxwell et al., 2000).

In contrast to above two ideas, I propose the possibility that firms adopt CSR activities without green consumers and the threat of entry. The key concept behind the result is a collusive effect through CSR, which relaxes the market competition. The explicit agreement among firms in terms of quantities or prices is prohibited by the regulatory authorities, while coordination about CSR activities though industry associations or firm-to-firm agreements is not. Such joint activities which increase the cost of firms may increase market prices and raise firms' profits.

As in Maxwell et al. (2000), the previous researches in the literature of CSR mainly compare the situation with CSR to without CSR. There are however several types of CSR in practice. For instance, firms commit their goals by setting the absolute target for their total pollution per year. The emission intensity per unit of output is another example. In the context of self-regulation, I investigate the incentive to CSR for each type and find the differences with respect to outputs, profits, and social welfare.

1.1.3 *Mixed Oligopoly*

The last part of this dissertation is to examine Mixed oligopoly. The oligopoly market where private and public firms coexist is called mixed market. Example includes an automobile industry, steel industry, and banking industry in Europe and Japan.[4] A difference between a private and public firm is thought of as a difference of their objectives. Private firms maximize purely their own profit, while public firms take into account consumer benefit and/or public interest. The difference from the private firms changes its own strategy, rivals strategies, and economic implication of oligopoly market. Formally, De Fraja and Delbono (1989) model the public firm such that it maximizes social welfare rather than own profit and investigated the effect of public firm on equilibrium outcomes in the Cournot model. In line of this research, there has been tremendous growth and progress in this field, which reexamines the property of private oligopoly and provides the several insights.

One classical issue in oligopoly theory is a first/second mover advantage. In contrast to Cournot (1838) which examines the situation where both firms simultaneously determine their actions, Stackerberg (1934) advantages our knowledge of firm behavior by considering the case in which firms sequentially modes. Mostly, if the actions of firms are strategic substitute (complement), the first(second)-mover advantage appears.[5] The effect of order of moves is a controversial issue since it directly related to the equilibrium actions of firms and the profits. Moreover, in the presence of the first/second mover advantage, firms endogenously try to obtain this advantage before the market competition. The seminal paper by Hamilton and Slutsky (1990) examine the endogenous order of moves in the market with only private firms and characterized the equilibrium outcomes. I revisit these issues, especially under price competition, by considering mixed oligopoly. In contrast to the previous analysis by Bárcena-Ruiz (2007), I find the sequential move appears in equilibrium and show that the duopoly result by Bárcena-Ruiz (2007) is not robust to a change in the number of firms.

Comparison between price competition and quantity competition is also a key issue in oligopoly theory. Basically, price competition is more competitive, yielding lower profits and larger welfare than quantity competition in the private oligopoly.[6] Ghosh and Mitra (2010) revisit the issue in a mixed duopoly, based on De Fraja and Delbono (1989)'s model. They show that the opposite result holds in terms of profits, while the welfare is still large under the price competition. It implies that

[4]See Bárcena-Ruiz and Garzón (2017), for example.
[5]See, for instance, Gal-Or (1985), Dowrick (1986), and Amir and Stepanova (2006).
[6]See, Vives (1985).

the comparison between price competition and quantity competition is sensitive to the type of firms. Thus, I reexamine price versus quantity competition under the mixed duopoly by incorporating sequential market structure which leads the first/second mover advantage. In contrast to the simultaneous move case investigated by Ghosh and Mitra (2010), the welfare ranking changes and the result depends on whether public is a leader or follower.

1.2 Organization

This dissertation consists of three parts as I mentioned.

- Multi-Store Paradox: Chapter 2 and 3.
- Corporate Social Responsibility: Chapter 4, 5, and 6.
- Mixed Oligopoly: Chapter 7 and 8.

Let us review each chapters in more detail.

In Chapter 2, we revisit the preemptive strategy of setting multiple stores (so-called Spatial Preemption) with one key modification: two incumbents establishing multiple stores. As Judd (1985)'s argument, in a standard case with one incumbent, it is not credible to launch multiple stores as an entry deterrence. We investigate the strategic entry deterrence in the presence of more than one incumbent, based on a location-then-price structure in a variety of a circular city model. We show that the incumbents credibly deter entry by establishing multiple stores, while a monopolistic incumbent can not.

Chapter 3 solves the multi-store paradox by introducing interdependent payoffs between the firms. We show that firms set up multiple stores unless the degree of payoff interdependence is low. We also show that multiple equilibria, namely intertwined and neighboring location equilibria, exist if the degree of payoff interdependence is intermediate.

In Chapter 4, we formulate a model in which whether environmental corporate social responsibility (ECSR) is adopted is chosen and then firms compete in the market. First, we consider emission cap commitment as ECSR. Under quantity competition, ECSR is adopted by the joint-profit-maximising industry association because it serves as a collusive device, although ECSR is not adopted if firms choose it independently. By contrast, under price competition, individual firms voluntarily adopt ECSR but the industry association chooses a higher level of ECSR. Next, we consider emission standard commitment (commitment to per-output emissions) and we find that it is less likely to restrict competition.

In Chapter 5, we compare emission cap commitment that restricts total emissions and emission intensity commitment that restricts emissions per unit of output as measures of self-regulation. The monopolist chooses either emission cap commitment or emission intensity commitment and sets the target level under the constraint that the resulting emissions do not exceed the upper limit. We find that profit-maximizing firms choose emission cap commitment, although emission intensity commitment always yields greater consumer surplus. It is ambiguous whether emission intensity commitment or emission cap commitment yields greater welfare. We present two cases in which emission intensity commitment yields greater welfare. One is the most stringent target case (the target emission level is close to zero), and the other is the weakest target case (the target emission level is close to business-as-usual). Our result suggests that the incentive for adopting emission cap commitment is too large for profit-maximizing firms, and thus, governments should encourage the adoption of emission intensity commitment, especially to achieve a zero-emission society efficiently.

In Chapter 6, we consider a model in which two firms choose whether to adopt environmental corporate social responsibility policies and then face Stackelberg competition under price competition. We show that the first-mover has the advantage, which is in contrast to the second-mover advantage typically seen in standard price competition models.

In Chapter 7, we compare welfare and profits under price and quantity competition in Stackelberg mixed duopolies. Under public leadership, price competition always yields greater profit and welfare than quantity competition. By contrast, under private leadership, the result depends on the nationality of the private firm. When the private firm is domestic (foreign), welfare is greater under quantity (price) competition. However, private firms always earn more under price competition. Introducing the nonnegative profit constraint affects welfare ranking but not profit ranking. These results indicate that profit ranking is fairly robust to the time structure in Stackelberg mixed duopolies, but welfare ranking is not.

In Chapter 8, we investigate endogenous order of moves in a price-setting mixed triopoly. Using the observable delay game, we show that a sequential move occurs in the mixed triopoly. Specifically, one private and a public firm set their prices at period 1 and the other private firm does at period 2 in equilibrium if goods are not significantly differentiated. This is in clear contrast to the mixed duopoly where a simultaneous move game is an unique equilibrium. We also consider a multi-period model and show that the sequential move game still emerges, while a hierarchical

Stackelberg game in which the public firm, one private firm, and the other private move sequentially in order never appears in an equilibrium. This finding suggest that the sequential move game with multiple leaders prevails under price competition even when the public firm exists.

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Chapter 2

Credible Spatial Preemption by Multiple Incumbents*

2.1 Introduction

Strategic entry barrier has attracted much attention by researchers and policymakers. In principle, facing potential entry, a incumbent may strategically act prior to entry and make entry unprofitable. For instance, the incumbent will crowd the market by launching several stores within a certain distance, namely spatial preemption (Schmalensee, 1978). Example includes retailers establishing multiple shopping centers (West, 1992) and hamburger restaurants (Igami and Yang, 2016). However, it is well known that credibility of the preemption, a commitment problem firstly pointed out by Judd (1985), is essential to incumbents facing the threat of entry and that spatial preemption does not work as an entry barrier. Specifically, in a two location model, even if the incumbent has built stores for each location to crowd the market with its stores, the firm will respond to the entry and withdraw either store already established because that strategy softens price competition at the *ex post* state of the market. Anticipating those behaviors of the incumbent, the entrant successfully enters the market. Thus, spatial preemption is not credible so that new entry occurs.

In previous research, Hadfield (1991) shows that a franchise contract is turned into credible commitment. Ishibashi (2003) considers the situation where there are one incumbent and multiple entrants exist. He also assumes that the entrants can choose the timing of entry endogenously . In that situation, the incumbent can deter entry by spatial preemption because there is no incentive

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for each entrant to enter the market prior to the rival's entry. Murooka (2013) demonstrates that, when assuming a liner transportation cost, a incumbent who builds more than two stores could deter entry.

In this paper, we investigate the credibility of spatial preemption in the presence of multiple incumbents using a location model of Salop (1979). Our central concern is to find a new element with regard to credibility for spacial preemption: multiple incumbents establishing multiple stores. We consider a location-then-price competition on a variety of the circular model and also entry-exit structure following Judd (1985). We show that the two incumbents credibly deter the entrant even if a quadratic transportation cost is assumed and the costs of withdrawing stores are zero,[1] while one incumbent can not. Hence, two incumbents establish multiple stores in equilibrium. Intuitively, when there is only one incumbent in the market, the incumbent can substantially lessen price competition by withdrawing its stores. On the other hand, in the two incumbents case, the strategic effect by withdrawing stores is relatively smaller than in the monopoly case because the existence of the other incumbent facilitates price competition in *ex post* state of the market. This is why incumbents continue to maintain their stores to keep their market share. As a result, it credibly creates an entry barrier against the potential entrant and induces incumbents to establish multiple stores on the equilibrium path.

This study is also related to the literature examining firm's incentive for multiple stores. Martínez-Giralt and Neven (1988) formulate the static duopoly model in which firms choose to the number of stores and their locations, and then face price competition. They show that two firms never open multiple stores in equilibrium. Each firm only establishes single store in order to reduce price competition, even if a fixed cost of opening a store is zero. In line with the static case investigated by Martínez-Giralt and Neven (1988), Tabuchi (2012) shows that three or more firms proliferate brands (establish multiple stores) and emphasizes the difference between duopoly and oligopoly in location-then-price competition. In this paper, even two firms have a incentive to establish multiple stores in equilibrium so as to deter entry.

[1] If the exist costs is sufficiently large, the *ex ante* location choice has a huge commitment power so that the entrant does not enter the market. The result of Schmalensee (1978) can be interpreted as an extreme case in line with infinite exit costs, that is, no withdraw options for the incumbent.

2.2 The Model

Consider a variant of the circular city model of Vickrey et al. (1999) and Salop (1979) with sequential entry-exit decisions. There is a circular market of length 1 where consumers are uniformly distributed. The consumers have a unit demand, i.e., each consumes one or zero units of the product. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to v. The utility of the consumers who purchase the product from K_i, which represents i-th store of the firm K, is expressed by $v - \tau(X - K_i)^2 - p_{Ki}$. We assume that v is so large that every consumer consumes one unit of the product. We further assume that firms produce the same physical product, and thus a consumer living at the point $X \in [0, 1]$ chooses the lowest-cost store that minimizes the sum of transport cost and the price, $\tau(X - K_i)^2 + p_{Ki}$, where τ is a positive constant. For simplicity, we assume that $\tau = 1$. Let $\overline{X}_{KiLj}(K \neq L)$ be the consumer whose utility is the same regardless of the firm chosen. We have

$$\overline{X}_{KiLj} = \frac{p_{Ki} - p_{Lj}}{2\tau(K_i - L_j)} + \frac{K_i + L_j}{2}.$$

Similarly, let $\overline{X}_{Kij}$ be the consumer whose utility is indifferent between buying from the ith store of firm K locating at K_i at price p_{Ki} and the jth store locating K_j at price p_{Kj}. We have

$$\overline{X}_{Kij} = \frac{p_{Kj} - p_{Ki}}{2(K_j - K_i)} + \frac{K_j + K_i}{2}.$$

Let D_{Ki} denote the demand of the store K_i. D_{Ki} depends on the locations and prices of stores.

There are two incumbents ($K = A, B$) and one entrant ($K = E$). Incumbent firms can run multiple stores, whereas the entrant can at most one store. All firms sell homogeneous goods. Note that the entrant can open at most one store, E_1 holds if $K = E$. Without loss of generality we assume that $K_1 \leq K_2 \leq ... \leq K_n$. Let $\boldsymbol{K} = (\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\})$ be the list of stores' locations in the market. $K_i = K_j (i \neq j)$ implies that firm K runs one store only at the same point. Henceforth, we assume the locations where firms can take are four equidistant locations, ($K_i \in \{0, 1/4, 1/2, 3/4\}$).[2] Thus, the number of establishing stores per firm could be at most four ($i = 4$). The firm incurs a fixed cost $F > 0$ per store to open. We assume that each firm has an identical production cost functions and the marginal production costs are constant. We normalized the marginal costs as zero. Let p_{Ki} ($K = A, B, E$ $i = 1, 2...n$) be the price of the i-th store of firm K and especially p_{E_1} be the price of the entrant firm.

[2]This assumption is crucial for our analysis. We provide a detail discussion in section 4.

The game runs as follows. In this study, we investigate a subgame-perfect Nach equilibrium.

1st stage: The incumbents ($K = A, B$) independently chooses the number of stores and their locations of the stores.
2nd stage: After observing the incumbent's decision, the entrant ($K = E$) decides whether entry the market or not. When it enters, it chooses the location.
3rd stage: Having observed the potential entrant decision, the incumbent can withdraw any stores established. We assume fixed costs are sunk and additional costs for exit are zero. If the entrant does not enter in the second stage, incumbents continue to keep their locations and proceed to the fourth stage.[3]
4th stage: Given above decisions, each firm decides the price of stores simultaneously.

The game is solved by backward induction.

2.3 Result

Before the main result, we present two benchmark cases: (i) one incumbent with multiple stores and one entrant, which is a modified version of Judd (1985)'s model, and (ii) two single-store incumbents. To focus on our interest, we assume $F < 1/8$ in the following.[4]

2.3.1 Modified Judd (1985)'s model

First, we consider the one incumbent case and show that even if one incumbent can launch several stores, the monopoly firm do only one store and a entrant can always enter the market under $F < 1/8$.

Result 1 *(Monopoly case) If there are one incumbent which can set up multiple stores* ($i = 4$) *and a potential entrant which can establish one store, then establishing multiple stores is not a credible strategy for entry deterrence.*
Proof See the Appendix

[3] If the incumbents know what a potential entrant never in the future, that is, the entrant can commit not to enter the market, they go on the location game with only two firm as well as the static analysis investigated by Martínez-Giralt and Neven (1988). Obviously, each of incumbents withdraw stores except for only one store. Here, we implicitly assume that the potential entrant is able to enter the market even after the third stage. It means that there is a second round of the second and third stage as long as the location choice is not stable. In such case, the incumbents does not collapse their entry barriers and keep their locations if they really want to persuade the entrant. Thus, we assume incumbents continue to maintain their locations.

[4] If $F > 1/8$, then one incumbent can deter entry by establishing just one store so that natural monopoly appears.

The result corresponds to the result of Judd (1985) which can be interpreted as 2 location case in our analysis. The intuition is as follows. Suppose the incumbent firm sets up multiple stores and the entrant enter the market. After the entry, the incumbent faces a decision of whether to withdraws its stores or not. On the one hand, if it withdraws a part of stores and increases the distant between stores, it will lose its market share, but the strategy lessens price competition. The first effect is so-called "direct effect", and the second one is "strategic effect". The more the distance between its stores and rival stores decreases, the more the strategic effect increases. On the other hand, if the incumbent maintains its stores, the strategy gives higher market share but facilitates price competition. Note that, since the entry costs are sunk, the incumbent firm's decision depends only upon the trade-off between increasing market share and relaxing price competition, which affects the equilibrium revenue in the fourth stage. In the monopolistic incumbent case, as the incentive to relax price competition (strategic effect) dominates the incentive to increase the market share (direct effect), the incumbent always withdraws its stores and leaves only one store which is located at the opposite end of the entrant. That is, in response to entry, there is an *ex post* incentive to withdraw the stores. The mechanism is also consistent with the location-then-price game analyzed by Martínez-Giralt and Neven (1988). Thus, anticipating the result, the entrant always enters the market for any locations. Therefore, the incumbent firm runs only one store from the beginning.

2.3.2 Single-store incumbents

In this subsection, we consider the second benchmark case in which there are two incumbents which can establish only one store to discuss the necessity of multiple stores for preemption. Providing the condition on fixed costs is similar to assuming $F < 1/8$ in subsection 3.1.

Suppose there exist two incumbents which can set up single store and one entrant. To find the condition under which the entrant facing duopoly incumbents with single store can enter the market, we should consider the most competitive situation for the entrant. There are two types of locations among the three firms: (i) the incumbents are immediate neighbors or (ii) the incumbent on the opposite side. The most competitive but profitable situation for the entrant occurs when both incumbents establish their stores at the opposite end and the entrant in the middle such as $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0\}, \{1/2\}, 1/4)$. Given the incumbents' locations, the optimal strategy of the entrant is to enter the midpoint of the incumbents to obtain a positive profit, otherwise the entrant faces head-to-head competition leading to zero profits for the entrant. If the incumbents establish their stores at close two locations, the entrant can get the larger market share and higher equilibrium price. Consider the former case and obtain the following condition in terms of fixed costs.

Result 2 *(Single store case) If there exist two incumbents which can set up single store and a potential entrant which can establish one store, then the entrant can always enter the market if* $F < F_1 = \pi_E(\{0\}, \{1/2\}, 1/4)$.

Proof See the Appendix

From a simple calculation, we obtain the equilibrium profit of the entrant under the location. When there are two single-store incumbents, the entrant enter the market if it earns the positive profit.

2.3.3 Duopoly incumbents with multiple stores

We are now ready to proceed to our main analysis that duopoly incumbents establish multiple stores. We present that incumbents with multiple stores can credibly deter entry in an equilibrium, in which both incumbents establish own stores by side, for instance, $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$[5].

First, we show that the incumbents will maintain more than one store in the third stage in contrast to Result 1. To do so, we consider the subgame in which the locations in the first stage is $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4\}, \{1/2, 3/4\}, \cdot)$. Now, two incumbents are identical and take symmetric locations. For the entrant firm, any location is identical, so we assume the entrant enter at 3/4. Given the initial location in the third stage such as $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4\}, \{1/2, 3/4\}, 3/4)$, we investigate the *ex post* incentive in the third stage for the incumbents. We obtain the following lemma:

Lemma 1 *Consider the subgame in the third stage in which locations is* $(\{0, 1/4\}, \{1/2, 3/4\}, 3/4)$*.*
(i) the incumbent facing head-to-head competition with the entrant always withdraws the store at which the entrant also exists.
(ii)the other incumbent will keep both stores even when the rival incumbent withdraws his store.

Proof See the Appendix

Observing the entrant's decision, both incumbents simultaneously decide whether withdrawing their stores or not. The incumbent B always (regardless of whether the incumbent A maintains or withdraws its stores) withdraws its store, B_2, which competes with the entrant at the same location and leads to intense price competition in the whole market. At the location where both

[5]The locations $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{1/4, 1/2\}, \{0, 3/4\}, \emptyset)$, $(\{1/2, 3/4\}, \{0, 1/4\}, \emptyset)$, and $(\{0, 3/4, 1/4, 1/2\}, \emptyset)$ are analogous.

the incumbent and entrant coexist, the competition between them is just like Bertrand competition. That severe competition drastically decreases the equilibrium prices at the neighboring locations. Thus, the incumbent facing head-to-head competition naturally shutdowns its store.

The second point in Lemma 1 is a remarkable result. Given the decision of the rival incumbent as we mentioned above, the other incumbent, firm A, maintains both stores. In Result 1, we showed that one incumbent which establishes multiple stores in the market withdraws its stores except for one store at the opposite end of the entrant in response to entry. This strategy can relax price competition significantly, resulting in higher profits for the incumbent rather than maintaining stores. This means that the strategic effect that relaxes price competition dominates the direct effect that lose the market share. By constant, the relative relationship between them is reversed in this case. Because of the presence of the rival incumbent in addition to the entrant, the strategic effect is relatively decreasing than the direct effect. Therefore, it is more profitable for the incumbent A to maintain its stores, resulting in $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4\}, \{1/2\}, 3/4)$ in this subgame.

Due to the response of the incumbents with multiple stores in the third stage, the entrant will face the tougher *ex post* competition in this case than in the case of single-store incumbents. Thus, the entrant could not possibly enter the market as a result of multiple stores' strategy. Although the following lemma has no additional implication, it is worth noting that the result is necessary for our main result.

Lemma 2 *Consider the subgame in the second stage in which locations is* $(\{0, 1/4\}, \{1/2, 3/4\}, \cdot)$. *Entry is profitable as long as* $F < F_2 = \pi_E(\{0, 1/4\}, \{1/2\}, 3/4)$ $(< F_1)$.

Proof See the Appendix

As we mentioned just above, either incumbent maintains both stores. Thus, the entrant earns at most $\pi_E(\{0, 1/4\}, \{1/2\}, 3/4)$. As long as the fixed cost of the market is smaller than this, the entrant enters the market.

Combining Result 2 and Lemma 2, there is a possibility that the entrant cannot enter the market due to the multiple stores at $F \in (F_2, F_1]$. That is, spacial preemption may appear in an equilibrium if there are two incumbents with multiple stores.

We now turn to the first stage and state our main result that two incumbent credibly deter entry by establishing multiple stores. To show the result, we have to confirm that each incumbent does not have an incentive to deviate from establishing own stores by side in the first stage such as $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$ under $F \in (F_2, F_1]$. One possible candidate to which the incumbents are likely to deviate from $(\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$ is to accommodate the entrant, that is, one incumbent establishes only one store from the beginning. We also solve the subgame by backward induction and compare the equilibrium payoff which the deviating firm obtains in each subgame. Straightforward comparisons yield the following proposition:

Proposition 1 *(Multi-stores incumbents) If there exist two incumbents which can set up multiple stores and a potential entrant with a single store, then incumbents credibly deter entry by establishing multiple stores by side in an equilibrium under $F \in (F_2, F_3]$, where F_3 is the difference of the incumbents' profit between entry deterrence and accommodating entry.*

Proof See the Appendix

We provide some intuition for the result. As in Lemma 1, either incumbent maintains its stores even when the entry occurs. That is, there is no *ex post* incentive to withdraw stores for one of the incumbents. It provides the severe competition in the *ex post* market for the entrant so that it opens the possibility that spacial preemption by multiple stores may appear in an equilibrium. In order for entry deterrence, a significant level of fixed cost is needed, $F_2 < F$, while such large fixed cost reduces the incentive for the incumbents to establish multiple stores from the beginning, $F_3 < F$. In conclusion, we find the range of fixed costs which ensures that spacial preemption is credible.

We show that the existence of multiple incumbents can change the result in the literature of credible spacial preemption and provide further insights on the role of multiple stores. There are however lacks and limitations of our analysis. Lastly, we argue such topics in the next subsection.

2.4 Discussion

2.4.1 Free-riding problem in entry deterrence

We treat the multiple-incumbents model for the analysis of entry deterrence. As is argued in a standard textbook (see Chapter 16 in Belleflamme and Peitz (2010)), several incumbents could lead to the free-riding problem. If entry deterrence can be attained by a proper subset of the incumbents, then outsiders may freely benefit of entry deterrence, resulting in coordination problems. In such case, to make entry barriers under several incumbents, they have to coordinate their *ex ante* decision.

We first provide one feature of our result related to this point. If the incumbents can completely coordinate their actions and that information is revealed, the game with multiple-stores incumbents can be put into the game with a multi-stores monopoly. As we showed in Result 1, the one entrant can not credibly deter entry at all. It implies that non-corporative actions in terms of location choice have an important role to deter entry in this study.

Second, in our model, there is another equilibrium that one incumbent establishes multiple stores and the other incumbent does only one store such as $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0, 1/4, 1/2\}, \{3/4\}, \emptyset)$, bringing credible preemption by multiple stores.

Remark 1 *(Multi-stores incumbent) If there exist two incumbents which can set up multiple stores and a potential entrant with a single store, then incumbents credibly deter entry by establishing multiple stores by either incumbent in an equilibrium under $F \in (F_2, F_4]$, where F_4 is the difference of the incumbent profit between unilateral entry deterrence by multiple stores and accommodating entry.*

Proof They are available upon request.

That is, there would be no need for multiple stores by both incumbents to deter entry in our example. We provide the intuitive proof that we present here. As we showed in Result 2, the incumbent has an incentive to maintain its stores. The mechanism is the same as the subgame in which either incumbent launches multiple stores. The big difference is that the only one firm incurs the huge fixed cost, meaning that there is a substantial incentive for the incumbent to quit entry deterrence and establish only one store as well as the rival incumbent does. We however find the range of fixed costs that credible preemption appears in an equilibrium even when either firm freely rides on the investment of the other incumbent.

2.4.2 Avoidable fixed costs

Our central concern in this paper is to find what is needed for the credible preemption. Some argue that the presence of an avoidable fixed cost facilitates entry of the potential entrant. Bagwell and Ramey (1996) formulate the capacity-then-entry model with the reversible fixed costs and show that the entrant can take the relative advantage to the incumbent if that type of fixed cost is sufficiently large.

In our context, there is an *ex post* advantage if the avoidable fixed cost is incorporated into our model. Specifically, when incumbents shutdown their stores, a part of the fixed costs they pay in the first period is refunded. It drastically changes the *ex post* incentive to withdraw the stores. Although the incumbents have the repayment options, our result still holds as long as the repayment is small. It implies that the *ex post* incentive for the incumbents is weak and that the credible preemption by multiple stores potentially appears in a broad range of markets.

2.4.3 Number of locations

We assume that the location where firms can take are four equidistant locations, $(0, 1/4, 1/2, 3/4)$, in this paper. One may think whether our result is robust to a change in the number of location points. Suppose that there exist six equidistant locations, $(0, 1/6, 1/3, 1/2, 2/3, 5/6)$. Even then, there is an equilibrium path where entry deterrence appears. However, the result is weaker than in the four location cases. There are multiple equilibria in the third stage where the incumbent decide whether or not to withdraw their stores. In a six location case, there is a substantial *ex post* incentive to shutdown for the incumbents simultaneously. For instance, $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, \{E_1\}) = (\{0\}, \{1/3\}, \{2/3\})$ lessen price competition drastically and the either alone would not maintain their stores by itself. The reason for the substantial *ex post* incentive is that $(\{0\}, \{1/3\}, \{2/3\})$ is an ideal location for the three firms. This means that the distance among three firms including the entrant is maximum so that price competition is considerably relaxed. Thus, for any locations, there exits the entry accommodation strategy in an equilibrium in the third stage, and it is payoff dominant strategy. Increasing the number of locations, the similar situation can easily be implemented. Thus, credible spatial preemption will not readily appear.

2.4.4 Linear city model

In this paper, we consider the circular city model based on the seminal works, Salop (1979) and Vickrey et al. (1999) for instance. There is however another typical model to consider the location choice, that is, a linear city model formulated by Hotelling (1929). We argue how the linear city assumption changes our results. For the sake of credible spacial preemption, the direct effect and the strategic effect by withdrawing stores play a substantial role in our mechanism. In the linear city model, firms obtain the absolute location advantage to locate close to the center point, bringing the larger market share and less severe price competition. In such case, if the incumbents establish multiple stores, the location which is relatively close to the center point is targeted by the entrant. Because of the commitment problem coming from the *ex post* incentive to withdraw stores, their stores are took over by the entrant. Hence, establishing only one store at as closely possible as the center point is an optimal strategy for the incumbents. It implies that the feature of the market which has a symmetric environment from a variety of perspective does matter in our analysis.

2.5 Concluding Remarks

In this study, we revisit the preemptive strategy by multiple stores with one key modification: two incumbents establishing multiple stores. We examine the issue using both the four-location-then-price competition on a circular model and entry-exit structure where there are two multi-store incumbents. In previous works, it is not credible for an incumbent to establish multiple stores as an entry deterrence. However, we find that there exists a subgame Nash equilibrium in which two incumbents establishing multiple stores credibly deter a potential entrant under some positive sunk costs. This is because the presence of multiple incumbents changes the *ex post* incentive to withdraw stores. We show that either incumbent will maintain its stores even when the entrant enters the market and the rival incumbent withdraws his stores. It leads to intense price competition in *ex post* market due to the multiple stores, resulting in the possibility of credible spatial preemption. This result also supports several examples of multi-store firms in the market.

This paper restricts the number of incumbent is two and of the possible locations is four. In reality, the number of firms and the possible locations is more flexible. The more general situation is a subject for the future research.

Appendix

Proof of Result 1

We show that, in response to entry, the best reply of firm A in the third stage is to withdraw all stores established in the first stage except for one store which is furthest from the entrant. To do so, we derive the equilibrium profit for each case and compare the values.

Suppose the incumbent establishes four store and the entrant enters at 1/2. First, we consider the case where firm A remains all stores, $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4, 1/2, 3/4\}, \{\emptyset\}, 1/2)$. The demand functions of each store, D_{Ki}, are given by

$$
\begin{aligned}
D_{A_1} &= \overline{X}_{A21} + 1 - \overline{X}_{A41} \\
D_{A_2} &= \begin{cases} \overline{X}_{A23} - \overline{X}_{A21} & \text{if} \quad p_{A_3} \leq p_{E_1} \\ \overline{X}_{E_1A_2} - \overline{X}_{A21} & \text{if} \quad p_{A_3} > p_{E_1} \end{cases} \\
D_{A_3} &= \begin{cases} \overline{X}_{A43} - \overline{X}_{A21} & \text{if} \quad p_{A_3} < p_{E1} \\ \frac{1}{2}(\overline{X}_{A43} - \overline{X}_{A21}) & \text{if} \quad p_{A_3} = p_{E_1} \\ 0 & \text{if} \quad p_{A_3} > p_{E_1} \end{cases} \\
D_{A_4} &= \begin{cases} \overline{X}_{A41} - \overline{X}_{A43} & \text{if} \quad p_{A_3} \leq p_{E_1} \\ \overline{X}_{A41} - \overline{X}_{A_4E_1} & \text{if} \quad p_{A_3} > p_{E_1} \end{cases} \\
D_{E_1} &= \begin{cases} 0 & \text{if} \quad p_{A_3} < p_{E_1} \\ \frac{1}{2}(\overline{X}_{A_4E_1} - \overline{X}_{E_1A_2}) & \text{if} \quad p_{A_3} = p_{E_1} \\ \overline{X}_{A_4E_1} - \overline{X}_{E_1A_2} & \text{if} \quad p_{A_3} > p_{E_1}. \end{cases}
\end{aligned}
$$

The profit functions of firm A and firm E, are expressed as follows.

$$
\begin{aligned}
\pi_A - 4F &= \sum_{i=1}^{4} \pi_{A_i} - 4F = \sum_{i=1}^{4} p_{A_i} D_{A_i} - 4F, \\
\pi_E - F &= \pi_{E_1} - F = p_{E_1} D_{E_1} - F.
\end{aligned}
$$

With these demand functions, firms independently choose their prices to maximize their profits. Since A_3 and E_1 are same location, they compete in only their prices. It means that the two stores are in a Bertrand market structure. Both firms undercut the prices until they converge to zero, $p^*_{A_3} = p^*_{E_1} = 0$.

Given $p^*_{A_3} = p^*_{E_1} = 0$ and $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4, 1/2, 3/4\}, \{\emptyset\}, 1/2)$, the first order conditions with respect to p_{A_1}, p_{A_2}, and p_{A_4} are

$$
\begin{aligned}
(-32p_{A_1} + 16p_{A_2} + 16p_{A_4} + 1)/4 &= 0, \\
(16p_{A_1} - 32p_{A_2} + 1)/4 &= 0, \\
(16p_{A_1} - 32p_{A_4} + 1)/4 &= 0.
\end{aligned}
$$

The second order condition are also satisfied. The first order conditions lead to the following equilibrium prices, $(p^*_{A_1}, p^*_{A_2}, p^*_{A_4}) = (1/8, 3/32, 3/32)$. Therefore, the equilibrium revenue of firm A is $\pi^*_A(\{0, 1/4, 1/2, 3/4\}, \{\emptyset\}, 1/2) = 5/128$.

Second, we investigate the case where firm A withdraws one store. We show firm A earns at most 205/2304. Although there exist four cases which firm A withdraws one store, the firm withdraws A_3 to avoid head-to-head competition, and maximizes the distance between itself and the rival store. Suppose the incumbent A withdraws A_3 store in the third stage. The demand functions of each store, D_{Ki}, are given by

$$\begin{aligned}
D_{A_1} &= \overline{X}_{A_2 1} + 1 - \overline{X}_{A_4 1} \\
D_{A_2} &= \overline{X}_{E_1 A_2} - \overline{X}_{A_2 1} \\
D_{A_4} &= \overline{X}_{A_4 1} - \overline{X}_{A_4 E_1} \\
D_{E_1} &= \overline{X}_{A_4 E_1} - \overline{X}_{E_1 A_2}.
\end{aligned}$$

Given $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4, 3/4\}, \{\emptyset\}, 1/2)$, the profit functions of firm A and firm E are expressed as follows.

$$\begin{aligned}
\pi_A - 4F &= \pi_{A_1} + \pi_{A_2} + \pi_{A_4} - 4F \\
&= p_{A_1} D_{A_1} + p_{A_2} D_{A_2} + p_{A_4} D_{A_4} - 4F \\
&= p_{A_1}\left(\frac{p_{A_2} - p_{A_1}}{2(A_2 - A_1)} + \frac{A_2 + A_1}{2} + 1 - \frac{p_{A_4} - p_{A_1}}{2(A_4 - A_1)} - \frac{A_4 + A_1}{2}\right) \\
&\quad + p_{A_2}\left(\frac{p_{E_1} - p_{A_2}}{2(E_1 - A_2)} + \frac{E_1 + A_2}{2} - \frac{p_{A_2} - p_{A_1}}{2(A_2 - A_1)} - \frac{A_2 + A_1}{2}\right) \\
&\quad + p_4\left(\frac{p_{A_4} - p_{A_1}}{2(A_4 - A_1)} + \frac{A_4 + A_1}{2} - \frac{p_{A_4} - p_{E_1}}{2(A_4 - E_1)} - \frac{A_4 + E_1}{2}\right) - 4F, \\
\pi_E - F &= \pi_{E_1} - F \\
&= p_{E_1} D_{E_1} - F \\
&= p_{E_1}\left(\frac{p_{A_4} - p_{E_1}}{2(A_4 - E_1)} + \frac{A_4 + E_1}{2} - \frac{p_{E_1} - p_{A_2}}{2(E_1 - A_2)} - \frac{E_1 + A_2}{2}\right) - F.
\end{aligned}$$

The first order conditions with respect to p_{A_1}, p_{A_2}, p_{A_4}, and p_{E_1} are

$$\begin{aligned}
\frac{-32p_{A_1}+16p_{A_2}+16p_{A_4}+1}{4} &= 0,\\
\frac{16p_{A_1}-32p_{A_2}+8p_{E_1}+1}{4} &= 0,\\
\frac{16p_{A_1}-32p_{A_4}+p_{E_1}+1}{4} &= 0,\\
\frac{8p^*_{A_2}+8p_{A_4}-32p_{E_1}+1}{4} &= 0.
\end{aligned}$$

The second order condition are also satisfied. The first order conditions lead to the following equilibrium prices, $(p^*_{A_1}, p^*_{A_2}, p^*_{A_4}, p^*_{E_1}) = (17/96, 7/48, 7/48, 5/48)$. Therefore, when firm A withdraws one store, it earns at most $\pi^*_A(\{0, 1/4, 3/4\}, \{\emptyset\}, 1/2) = 205/2304$.

There are two cases: withdrawing two stores and there stores. For each case, we repeat the same calculation. Table 1 summarizes the equilibrium profit of firm A which is maximum value according to the number of remaining stores.

Table 2.1: Equilibrium profit of the incumbent in the fourth stage

$\{\boldsymbol{A}\}$	$\pi^*_A - iF$
$\{0, 1/4, 1/2, 3/4\}$	$5/128 - 4F$
$\{0, 1/4, 1/2\}$	$205/2304 - 4F$
$\{0, 1/4\}$	$347/3456 - 4F$
$\{0\}$	$1/8 - 4F$

NOTE: $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\cdot, \{\emptyset\}, 1/2)$.

To compare the equilibrium profit of firm A, we only have to focus on the equilibrium revenue since the fixed cost is sunk. We can easily see that the withdrawing three stores (keeping only one store at 0) is most profitable for the incumbent A in this subgame in which $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4, 1/2, 3/4\}, \{\emptyset\}, 1/2)$ is an initial location in the third stage. If entrant locates other point, a similar argument can applied for each case.

In addition, the comparison can be made for any subgame in the third stage: the initial number of stores established by the firm A is three or two, or one. From the result, The incumbent, firm A, always withdraws its stores except for one store which is furthest from the entrant observing entry of the potential entrant. Anticipating the result, the entrant can always enter the market except for a natural monopoly. ■

Proof of Result 2

Suppose $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0\}, \{1/2\}, 1/4)$, we obtain the equilibrium profit of the entrant explicitly and show $\pi_E^*(\{0\}, \{1/2\}, 1/4) = F_1 = 121/4096$.

Given that location, the demand functions of each store are

$$\begin{aligned} D_{A_1} &= \overline{X}_{E_1 A_1} + 1 - \overline{X}_{A_1 B_1} \\ D_{B_1} &= \overline{X}_{A_1 B_1} - \overline{X}_{B_1 E_1} \\ D_{E_1} &= \overline{X}_{B_1 E_1} - \overline{X}_{E_1 A_1}. \end{aligned}$$

Based on the demand for each firm, the profit function firm K $(K = A, B, E)$ are expressed as following.

$$\pi_E - F = p_{K_1} D_{K_1} - F$$

The first order conditions with respect to p_{K_1} is

$$\frac{\partial \pi_K}{\partial p_{K_1}} = D_{K_1} + p_{K_1} \frac{\partial D_{K_1}}{\partial p_{K_1}} = 0$$

This leads to the equilibrium prices: $(p_{A_1}^*, p_{B_1}^*, p_{E_1}^*) = (7/64, 7/64, 11/128)$. Substitute these prices into the profit functions, we obtain $\pi_E^*(\{0\}, \{1/2\}, 1/4) = 121/4096$. ■

Proof of Lemma 1

The first part of Lemma 1 is similar to the proof of Result 1 with no withdrawing stores. The firm facing head-to-head competition withdraws its store to avoid Bertrand competition regardless the rival's choice. Thus, we omit the part and assume firm B facing the head-to-head competition withdraws the store at which the entrant also exists. The proof proceeds to the second part. We show that, given $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2\}, 3/4)$, maintaining both stores yields the highest profit. That is, no withdrawing is the best response to $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{\cdot\}, \{1/2\}, 3/4)$. Suppose $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2\}, 3/4)$ and firm A maintains the two stores. The demand functions of each stores are

$$\begin{aligned} D_{A_1} &= \overline{X}_{A_{21}} + 1 - \overline{X}_{A_1 E_1} \\ D_{A_2} &= \overline{X}_{B_1 A_2} - \overline{X}_{A_{21}} \\ D_{B_1} &= \overline{X}_{E_1 B_1} - \overline{X}_{B_1 A_2} \\ D_{E_1} &= \overline{X}_{A_1 E_1} - \overline{X}_{E_1 B_1}. \end{aligned}$$

Therefore, the profit functions are expressed as following.

$$\begin{aligned}\pi_A - 2F &= p_{A_1}D_{A_1} + p_{A_2}D_{A_2} - 2F\\ \pi_B - 2F &= p_{B_1}D_{B_1} - 2F\\ \pi_E - F &= p_{E_1}D_{E_1} - F\end{aligned}$$

The first order condition are

$$\begin{aligned}\frac{\partial \pi_A}{\partial p_{A_1}} &= D_{A_1} + p_{A_1}\frac{\partial D_{A_1}}{\partial p_{A_1}} + p_{A_2}\frac{\partial D_{A_2}}{\partial p_{A_1}} = 0,\\ \frac{\partial \pi_A}{\partial p_{A_2}} &= D_{A_2} + p_{A_2}\frac{\partial D_{A_2}}{\partial p_{A_2}} + p_{A_1}\frac{\partial D_{A_1}}{\partial p_{A_2}} = 0,\\ \frac{\partial \pi_B}{\partial p_{B_1}} &= D_{B_1} + p_{B_1}\frac{\partial D_{B_1}}{\partial p_{B_1}} = 0,\\ \frac{\partial \pi_E}{\partial p_{E_1}} &= D_{E_1} + p_{E_1}\frac{\partial D_{E_1}}{\partial p_{E_1}} = 0.\end{aligned}$$

The second order conditions are satisfied. The first order conditions lead to the following equilibrium prices: $p^*_{A_1} = p^*_{A_2} = 1/10$, $p^*_{B_1} = p^*_{E_1} = 3/40$. Substituting equilibrium prices into the profit function of firm A and E, we obtain the equilibrium profits when firm A maintains two store, $\pi^*_A(\{0,1/4\}, \{1/2\}, 3/4) - 2F = 1/25 - 2F$ and $\pi^*_E(\{0,1/4\},\{1/2\},3/4) - F = 9/400 - F$.

There exist two cases where firm A maintains one store: withdrawing the store at 0 or at 1/4. To obtain the resulting profit for each case, we repeat the same calculation. Table 2 summarizes the equilibrium profit of firm A in this subgame.

Table 2.2: Equilibrium revenue of the incumbent and its strategy

$\{\boldsymbol{A}\}$	$\pi^*_A - iF$
$\{0, 1/4\}$	$1/25 - 2F$
$\{1/4\}$	$147/4096 - 2F$
$\{0\}$	$147/4096 - 2F$

NOTE: $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\cdot, \{1/2\}, 3/4)$.

We investigate whether firm A maintains its store or not. To compare the profit of firm A among thee cases, we only have to focus on the equilibrium revenue of firm A since the fixed costs are sunk. Maintaining is the best reply for firm A in the third stage because $\pi^*_A(\{0,1/4\},\{1/2\},3/4) > \pi^*_A(\{0\},\{1/2\},3/4) = \pi^*_A(\{1/4\},\{1/2\},3/4)$. ■

Proof of Lemma 2

As we show in Lemma 1, given $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2, 3/4\}, 3/4)$, firm A maintain its stores and firm B withdraws one store which faces head-to-head competition. Thus, the equilibrium location in this subgame is $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2\}, 3/4)$. We already obtain the equilibrium profit of the entrant given that location, $\pi_E^*(\{0, 1/4\}, \{1/2\}, 3/4) - F = 9/400 - F$. Thus, given $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2, 3/4\}, 3/4)$, the entrant can enter the market if and only if

$$F < F_2 = \pi_E^*(\{0, 1/4\}, \{1/2\}, 3/4) = 9/400.$$

Moreover, from Result 2, we obtain the following relationship:

$$9/400 = F_2 < F_1 = 121/4096.$$

■

Proof of Proposition 1

We show that given $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$, each incumbent does not have an incentive to deviate from that strategy in the first stage. We already obtained the final state and the equilibrium profit in the subgame with the initial state, $(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1) = (\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$.

To obtain the resulting profit for each subgame, we repeat the same calculation. Table 3 summarizes the initial state, the final state, and the equilibrium profit of firm A in each subgame.

Table 2.3: Equilibrium profit of the incumbent and its strategy

	$(\{\boldsymbol{A}\}, \{\boldsymbol{B}\}, E_1)$		
case	initial	final	$\pi_A^* - iF$
1	$(\{0, 1/4, 1/2, 3/4\}, \{1/2, 3/4\}, \emptyset)$	$(\{0, 1/4, 1/2, 3/4\}, \{1/2, 3/4\}, \emptyset)$	361/28948 - 4 F
2	$(\{0, 1/4, 1/2\}, \{1/2, 3/4\}, \emptyset)$	$(\{0, 1/4, 1/2\}, \{1/2, 3/4\}, \emptyset)$	709/30976 - 3 F
3	$(\{1/4, 1/2, 3/4\}, \{1/2, 3/4\}, \emptyset)$	$(\{0\}, \{1/2\}, 1/4)$	147/4096 - 3 F
4	$(\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$	$(\{0, 1/4\}, \{1/2, 3/4\}, \emptyset)$	1/16 - 2 F
5	$(\{0, 1/2\}, \{1/2, 3/4\}, \emptyset)$	$(\{1/2\}, \{3/4\}, 0)$	147/4096 - 2 F
6	$(\{0, 3/4\}, \{1/2, 3/4\}, \emptyset)$	$(\{0\}, \{3/4\}, 1/2)$	147/4096 - 2 F
7	$(\{1/2, 3/4\}, \{1/2, 3/4\}, \emptyset)$	$(\{1/2\}, \{3/4\}, 1/4)$	147/4096 - 2 F
8	$(\{0\}, \{1/2, 3/4\}, \emptyset)$	$(\{0\}, \{3/4\}, 1/2)$	147/4096 - F
9	$(\{1/2\}, \{1/2, 3/4\}, \emptyset)$	$(\{1/2\}, \{3/4\}, 0)$	147/4096 - F

NOTE: We omit symmetric case which can be formulated by turning above cases clockwise.

As we mentioned above, the deviation incentive is highly ranked in case 8; accommodating entry. Comparing case 4 (entry deterrence) with case 8 (accommodating entry), entry deterrence yields higher profits than accommodating entry if and only if

$$\pi_A(\{0, 1/4\}, \{1/2, 3/4\}, \emptyset) - 2F \geq \pi_A(\{0\}, \{3/4\}, 1/2) - F \Leftrightarrow F \leq F_3 = \frac{109}{4096}.$$

Combining the result with Lemma 2, the entry deterrence of two incumbents is credible if and only if

$$\frac{9}{400} = F_2 < F \leq F_3 = \frac{109}{4096}.$$

■

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Chapter 3

Payoff Interdependence and the Multi-Store Paradox*

3.1 Introduction

Casual observations suggest that a firm often supplies several products that are mutually substitutable. For example, Kellogg, Coca-Cola, and Nisshin supply several varieties of cereals, beverages, and instant noodles. Most major automobile, consumer electronics, and cell phone companies produce various differentiated products. In the spatial context, supermarkets and convenience stores often build multiple stores that may compete with each other. However, two influential works have suggested that these casual observations are not supported by economic theory. Martinez-Giralt and Neven (1988) formulated a model in which duopolists choose to open either one or two stores and then face price competition. They found that firms always choose to open one store in equilibrium, even if the cost of setting up a store is zero. Establishing multiple stores accelerates competition and reduces profits. Therefore, to relax competition, each firm establishes only one store (in the product differentiation context, each firm supplies only one product). In contrast to Schmalensee (1978), Judd (1985) showed that establishing multiple stores does not serve as a commitment to entry deterrence, even when the cost of setting up each store is completely sunk. This problem is called the "multi-store paradox."

In this study, we introduce interdependent payoff functions into the duopoly model of Martinez-Giralt and Neven (1988) and solve this paradox. We assume that each firm i maximizes $\pi_i - \alpha\pi_j$, where π_k is firm k's profit ($k = i, j,\ j \neq i$) and $\alpha \in [0, 1)$ is the degree of interdependence of the

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payoff (i.e., the relative profit maximization approach). We show that firms set up multiple stores unless the degree of interdependence is too small. We also find that when α is neither too high nor too low, two types of equilibria coexist: the equilibria with intertwined stores (i.e., each firm locates its store between the rival's two stores) and those with neighboring stores (i.e., each firm locates its second store next to its first store).

The existence of a neighboring location equilibrium may explain the strategy of Seven-Eleven Japan, the first and largest convenience store chain in Japan. Seven-Eleven Japan follows an approach called "strategic dominance," setting up stores in narrow territories (Seven-Eleven Japan Corporate Profile 2013–2014, p. 15) instead of establishing nationwide store networks. The neighboring location equilibrium is consistent with its strategy.

We now discuss the rationale for employing interdependent objective functions in a general context. First, the evaluation of managerial performance is often based on the relative as well as absolute performance of managers.[1] Outperforming managers often obtain good positions in management job markets. Second, many laboratory (experimental) works have pointed out the importance of relative performance.[2] Third, as Armstrong and Huck (2010) convincingly discussed, concerns about relative profit are closely related to imitative behavior among competing firms, and this imitative behavior is observed frequently.[3] We therefore believe that it is reasonable to consider that firms are not always profit maximizers and to apply this approach to the multi-store problem.

We can also interpret α as indicating the measure of the degree of the toughness of competition, where a higher α indicates tougher competition.[4] Following this interpretation, our result suggests

[1] See Gibbons and Murphy (1990) for empirical evidence.

[2] See Armstrong and Huck (2010). In the real world, people are concerned about relative performance, but not because of the monetary incentives. See Grieco et al. (1993) and Mastanduno (1991) in the context of political science and Ariely (2009) in the context of behavioral economics. The payoff function based on relative wage or relative wealth status has also been intensively discussed in the macroeconomics context. Keynes (1936) discussed the rigidity of nominal wage based on relative wage. See also Akerlof and Yellen (1988) and Corneo and Jeanne (1997, 1999), and Futagami and Shibata (1998). We believe that this is because concern about relative performance is realistic and thus has attracted the interest of many macroeconomists.

[3] See Vega-Redondo (1997) for the model formulation of a related evolutionary game. He considered a quantity-setting model in a homogeneous product market and showed that if firms myopically imitate the most profitable firm's strategy, the industry converges to a highly competitive outcome.

[4] For a general discussion of this approach and useful applications, see Matsumura et al. (2013) and Matsumura and Okamura (2015). The former (latter) discussed the relationship between competition and R&D activities (optimal privatization policy). We can show that given the locations, the ratio between the profit margin (i.e., price minus marginal cost) and the price, known as the Lerner index, is decreasing in α. This index is intensively adopted in the empirical literature as a measure of the intensity of market competition in product markets. Furthermore, Matsumura and Matsushima (2012) showed that collusion is less stable when α is larger under moderate conditions. In this sense, a larger α again indicates a more competitive market.

that tougher competition leads to a multi-store equilibrium, which further accelerates competition.

Some researchers have solved the multi-store paradox. Ishibashi (2003) showed that if there are one incumbent and two or more new entrants, the incumbent may be able to deter entries by establishing two stores. Tabuchi (2012) also showed that a model with three firms solves this problem. However, the driving force in the present study is different from any previous works and our results provide new findings on the multi-store equilibrium.

Murooka's (2013) research is the most closely related to the present study. He also solved the multi-store paradox in a duopoly model.[5] He extended the model of Judd (1985) and investigated entry deterrence by an incumbent, showing that the incumbent maintains multiple stores if it can set up five or more stores before the rival's entry. In his model, it is assumed that the new entrant builds one store and the incumbent builds multiple stores. In the present study, however, we need not assume any asymmetry between the two firms, and we show that both firms build multiple stores.

3.2 The Model

There is a circular market of length 1 where infinitely many consumers lie uniformly. Following Martinez-Giralt and Neven (1988), two firms ($K = A, B$) choose their locations on the unit circle and then choose prices. Each firm runs one or two stores and sells homogeneous goods. Let K_1 and K_2 be the locations of the first and second stores of firm K ($K = A, B$), respectively. Without loss of generality, we assume that $K_1 \leq K_2$ and $A_1 = 0$. $K_1 = K_2$ implies that firm K runs one store only. Let p_{Ki} ($K = A, B,\ i = 1, 2$) be the price of the i-th store of firm K.

The game runs as follows: In the first stage, each firm independently chooses the locations of its two stores. In the second stage, after observing the locations of the stores, each firm K independently chooses the prices at the two stores. The payoff of firm K is given by $U_K = \pi_K - \alpha\pi_L$ ($K, L = A, B,\ L \neq K$), where π_K is the profit of firm K and $\alpha \in [0, 1)$. As we discuss in the Introduction, α indicates the degree of interdependence of the payoff function and/or the toughness of competition in the market. We assume that both firms have an identical production cost function

[5] As pointed out by Hendel and Neiva de Figueiredo (1997), in the context of a spatial model, the strategic effect of investment in a duopoly is much stronger than that in an oligopoly ($n \geq 3$) and duopoly and triopoly models often yield contrasting results. Thus, it is important to solve this paradox under the strongest strategic effect case (duopoly case) because the multi-product case appears under competition (e.g., between Coca-Cola and Pepsi or between Aderans and Artnature.

and that the marginal production cost is constant. We normalize the marginal cost as zero.

The consumers have unit demands; that is, each consumes one or zero units of the product. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to v. We assume that v is so large that every consumer consumes one unit of the product. Because the two firms produce the same physical product, a consumer living at point $X \in [0, 1]$ chooses the lowest-cost store that minimizes the sum of the transport cost and the price, $\tau(X - K_i)^2 + p_{Ki}$, where τ is a positive constant. For simplicity, we assume that $\tau = 1$. Let the demand of store K_i $(K = A, B, i = 1, 2)$ be D_{Ki}. D_{Ki} depends on the locations and prices of stores, and is derived in the next section.

3.3 Equilibrium price

The game is solved by backward induction. We first discuss the second-stage game given the locations of two firms. Following Martinez-Giralt and Neven (1988), we discuss the following two cases.

3.3.1 Intertwined stores

First, we consider the case where $A_1 \leq B_1 \leq A_2 \leq B_2$. Let $\overline{X}_{ij}$ be the consumer whose utility is indifferent to buying from B_i at price p_{Bi} or A_j at price p_{Aj}. We have

$$\overline{X}_{ij} = \frac{p_{Bi} - p_{Aj}}{2\tau(B_i - A_j)} + \frac{B_i + A_j}{2}.$$

The demand of each store is given by

$$\begin{aligned} D_{A1} &= \overline{X}_{11} + (1 - \overline{X}_{21}), \\ D_{A2} &= \overline{X}_{22} - \overline{X}_{12}, \\ D_{B1} &= \overline{X}_{12} - \overline{X}_{11}, \\ D_{B2} &= \overline{X}_{21} - \overline{X}_{22}. \end{aligned}$$

With these demand functions, firms independently choose their prices and maximize their payoffs. The first-order conditions are

$$\begin{aligned}
\frac{2p_{A1}(1+B_1-B_2)+P_{B2}B_1(-1+\alpha)+(1-B_2)(p_{B1}(-1+\alpha)-B_1(1+B_1-B_2))}{2B_1(-1+B_2)} &= 0,\\
\frac{2p_{A2}(B_1-B_2)+P_{B1}(A_2-B_2)(-1+\alpha)+(B_1-A_2)(p_{B2}(-1+\alpha)+(B_2-A_2)(B_1-B_2))}{2(A_2-B_1)(B_2-A_2)} &= 0,\\
\frac{2p_{B1}A_2+P_{A1}(A_2-B_1)(-1+\alpha)+B_1(p_{A2}(-1+\alpha)+A_2(B_1-A_2))}{2B_1(B_1-A_2)} &= 0,\\
\frac{2p_{B2}(-1+A_2)+P_{A2}(-1+B_2)(-1+\alpha)+(A_2-B_2)(p_{A1}(-1+\alpha)+(1-A_2)(-1+B_2))}{2(A_2-B_2)(-1+B_2)} &= 0.
\end{aligned}$$

We can show that U_K is quasi-concave with respect to prices, and thus, the second-order conditions are satisfied. The first-order conditions lead to the following equilibrium prices:

$$\begin{aligned}
p^*_{A1} &= \frac{B_1(-1+B_2)}{Z}\left\{A_2^2[B_1^2(1+\alpha)^2+B_2^2(1+\alpha)^2\right.\\
&\quad -2B_1(1+\alpha)(-1+B_2+B_2\alpha)-2B_2(1+\alpha)-(-1+\alpha)^2]\\
&\quad +A_2[-B_1^2(1+\alpha)^2-B_2^2(1+\alpha)^2+B_1(2B_2(1+\alpha)^2-1-4\alpha+\alpha^2)+B_2(3+\alpha^2)]\\
&\quad \left.-B_1B_2(-1+\alpha)^2\right\},\\
p^*_{A2} &= \frac{(A_2-B_1)(A_2-B_2)}{Z}\left\{A_2^2(1+B_1-B_2)(1+\alpha)[(B_1-B_2)(1+\alpha)-1+\alpha]\right.\\
&\quad -A_2(1+B_1-B_2)(1+\alpha)[(B_1-B_2)(1+\alpha)-1+\alpha]\\
&\quad \left.-B_1(-1+B_2)(-1+\alpha)^2\right\},\\
p^*_{B1} &= \frac{B_1(-A_2+B_1)}{Z}\left\{B_1^2(1+\alpha)(-1+A_2)[1-\alpha+A_2+A_2\alpha]\right.\\
&\quad +B_1(1+\alpha)(-2B_2+1)(-1+A_2)[1-\alpha+A_2+A_2\alpha]\\
&\quad +B_2^2(A_2^2(1+\alpha)^2-2A_2\alpha(1+\alpha)+2(-1+\alpha))\\
&\quad \left.+B_2(-A_2^2(1+\alpha)^2+A_2(3\alpha^2+1)-2\alpha+2)-A_2(-1+\alpha)^2\right\},\\
p^*_{B2} &= \frac{(-1+B_2)(-A_2+B_2)}{Z}\left\{B_2^2A_2(1+\alpha)[-2+A_2+A_2\alpha]\right.\\
&\quad -B_2(1+\alpha)A_2(1+2B_2)[-2+A_2+A_2\alpha]\\
&\quad \left.+B_1^2(A_2^2(1+\alpha)^2-2A_2\alpha(1+\alpha)-(-1+\alpha)^2)+B_1B_2(A_2(1+\alpha)^2-1-4\alpha+\alpha^2)\right\},\\
\text{where }\ Z &= (1+\alpha)\left\{-B_1^2(-1+A_2)(-1+\alpha)^2-2B_1A_2(-1+A_2)[B_2(-3-2\alpha+\alpha^2)-(-1-2\alpha+\alpha^2)]\right.\\
&\quad \left.+B_2^2A_2(A_2(-3-2\alpha+\alpha^2)+4)+2B_2A_2(A_2(1+2\alpha-\alpha^2)-2)A_2^2(-1+\alpha)^2\right\}.
\end{aligned}$$

3.3.2 Neighboring stores

For the second case, suppose that $A_1 \leq A_2 \leq B_1 \leq B_2$.[6] Let $\overline{X}_K$ be the consumer whose utility is indifferent to buying from firm K's first store, located at K_1, at price p_{K1} or its second store, located at K_2, at price p_{K2}. We have

$$\overline{X}_K = \frac{p_{K2} - p_{K1}}{2(K_2 - K_1)} + \frac{K_2 + K_1}{2}.$$

The demand functions for each store are as follows:

$$\begin{aligned} D_{A1} &= \overline{X}_A + (1 - \overline{X}_{21}), \\ D_{A2} &= \overline{X}_{12} - \overline{X}_A, \\ D_{B1} &= \overline{X}_B - \overline{X}_{12}, \\ D_{B2} &= \overline{X}_{21} - \overline{X}_B. \end{aligned}$$

Given these demand functions, firms independently choose their prices and maximize their payoffs. The first-order conditions are

$$\begin{aligned} \frac{2p_{A1}(1 + A_2 - B_2) + 2p_{A2}(-1 + B_2) + A_2(p_{B2}(-1 + \alpha) + (1 + A_2 - B_2)(-1 + B_2))}{2A_2(-1 + B_2)} &= 0, \\ \frac{2p_{A2}B_1 + 2p_{A1}(A_2 - B_1) + A_2(p_{B1}(-1 + \alpha) + (A_2 - B_1)B_1)}{2A_2(A_2 - B_1)} &= 0, \\ \frac{2p_{B1}(A_2 - B_2) - 2p_{B2}(A_2 - B_1) + (B_1 - B_2)(p_{A2}(-1 + \alpha) - (A_2 - B_1)(A_2 - B_2))}{2(A_2 - B_1)(B_1 - B_2)} &= 0, \\ \frac{2p_{B2}(-1 + B_1) - 2p_{B1}(-1 + B_2) + (B_1 - B_2)(p_{A1}(-1 + \alpha) - (-1 + B_1)(-1 + B_2))}{2(B_1 - B_2)(-1 + B_2)} &= 0. \end{aligned}$$

We can show that U_K is quasi-concave with respect to prices, and thus, the second-order

[6] Because of the symmetry of the circular market, a similar principle can also apply in the case where $A_1 \leq B_1 \leq B_2 \leq A_2$.

conditions are satisfied. The first-order conditions lead to the following equilibrium prices:

$$\begin{aligned}
p_{A1}^* &= \frac{(1-B_2)}{L}\left\{-B_1^2(1+\alpha)[A_2(-3-2\alpha+\alpha^2)+4]\right.\\
&\quad +B_1(B_2(1+\alpha)-3+\alpha)(A_2(-3-2\alpha+\alpha^2)+4)\\
&\quad \left.+\,A_2(-1+\alpha)(B_2(1-\alpha^2)+2A_2(1+\alpha)-6+2\alpha)\right\},\\
p_{A2}^* &= \frac{(-A_2+B_1)}{L}\left\{-A_2^2(-3-2\alpha+\alpha^2)[B_1(1+\alpha)-B_2(1+\alpha)+2]\right.\\
&\quad -A_2(1+\alpha)[B_2^2(-3-2\alpha+\alpha^2)+B_2(B_1(3+2\alpha-\alpha^2)+8-4\alpha)+2(-1+\alpha)]\\
&\quad \left.-\,4(-1+B_2)((B_2-B_1)(1+\alpha)-3+\alpha)\right\},\\
p_{B1}^* &= \frac{(-A_2+B_1)}{L}\left\{B_1^2(-3-2\alpha+\alpha^2)[A_2(1+\alpha)-2]\right.\\
&\quad +B_1[-B_2(1+\alpha)(A_2(-3-2\alpha+\alpha^2)+4)+2(-1+\alpha^2)]\\
&\quad \left.+\,2B_2^2(-1+\alpha^2)-2B_2(2A_2(1+\alpha)-7+2\alpha+\alpha^2)+4(A_2(1+\alpha)-3+\alpha)\right\},\\
p_{B2}^* &= \frac{(1-B_2)}{L}\left\{B_2^2(-3-2\alpha+\alpha^2)[A_2(1+\alpha)-2]\right.\\
&\quad +B_2(1+\alpha)[-B_1(A_2(-3-2\alpha+\alpha^2)+4)-(A_2^2(-3-2\alpha+\alpha^2)+A_2(5-6\alpha+\alpha^2)+6-2\alpha)]\\
&\quad +2B_1^2(-1+\alpha^2)+B_1(-3+\alpha)(A_2^2(1+\alpha)^2+A_2(-3-2\alpha+\alpha^2)+2-2\alpha)\\
&\quad \left.-\,4A_2(A_2(1+\alpha)-3+\alpha)\right\},\\
\text{where } L &= (-3-2\alpha+\alpha^2)(B_1(A_2(-3-2\alpha+\alpha^2)+4)+B_2(A_2(3+2\alpha-\alpha^2)-4)-4A_2+4)).
\end{aligned}$$

3.4 Results

We substitute the equilibrium prices into the payoff functions and derive the payoff function as a function of location. In the first stage, each firm K independently chooses the locations of its stores.

We investigate how α affects the equilibrium locations of the stores. The following proposition states the relationship between the equilibrium locations of stores and α:

Proposition 1 *(i) (Single-Store Equilibrium) If* $\alpha \in [0, 3-2\sqrt{2}]$, *then* $(A_1, A_2, B_1, B_2) = (0, 0, 1/2, 1/2)$ *constitutes an equilibrium.*
(ii) (Neighboring Location Equilibrium) If $\alpha \in (3-2\sqrt{2}, \overline{\alpha}_1]$, *then*

$$(A_1, A_2, B_1, B_2) = (0, \frac{-1+6\alpha-\alpha^2}{4+4\alpha}, 1/2, 1/2 + \frac{-1+6\alpha-\alpha^2}{4+4\alpha})$$

constitutes an equilibrium where $\overline{\alpha}_1 \simeq 0.58$.
(iii) (Intertwined Location Equilibrium) If $\alpha \in [\overline{\alpha}_2, 1)$, *then* $(A_1, A_2, B_1, B_2) = (0, 1/2, 1/4, 3/4)$

constitutes an equilibrium where $\overline{\alpha}_2 \simeq 0.19$.

Proof See the Appendix.

Proposition 1(i) implies that the result of Martinez-Giralt and Neven (1988) holds if $\alpha \leq 3 - 2\sqrt{2} \simeq 0.17$. Martinez-Giralt and Neven (1988) showed that when $\alpha = 0$, $(A_1, A_2, B_1, B_2) = (0, 0, 1/2, 1/2)$. In other words, neither firm chooses two stores, even if the cost of setting up a store is zero. Setting up two stores accelerates competition and reduces profits. Therefore, each firm avoids building multiple stores in order to mitigate competition if the firms are close to profit maximizers.

Proposition 1(ii–iii) imply that firms build multiple stores in equilibrium if $\alpha > 3 - 2\sqrt{2}$. Given $A_1 = 0$, a slight increase in A_2 from $A_2 = 0$ accelerates competition between the two firms, reduces the prices of both firms, increases (decreases) the market share of firm A (B), and reduces both firms' profits. Moreover, it reduces firm B's profit more significantly because firm B suffers from both lower prices and a smaller market share, whereas firm A suffers from lower prices but gains from a higher market share. This creates the incentive for establishing two stores.

Proposition 1(ii–iii) imply that multiple equilibria exist if $\alpha \in (\overline{\alpha}_2, \overline{\alpha}_1)$. We explain the intuition as follows. Suppose that $\alpha \in [\overline{\alpha}_2, \overline{\alpha}_1]$. Then, suppose that firm B changes its locations from $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ to $(B_1, B_2) = (1/4, 3/4)$. This deviation increases the market share of firm B and accelerates competition between the two firms. Given $(B_1, B_2) = (1/4, 3/4)$, firm A's relocation from $(A_1, A_2) = (0, (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ to $(A_1, A_2) = (0, 1/2)$ increases the market share of firm A and further accelerates competition between the two firms. This competition-accelerating effect (i.e., price-reducing effect) reduces firm B's profit more significantly because the market share of firm B is larger before the deviation. This may not hold if $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ because the market share of firm B is the same as that of firm A before the deviation. Therefore, the aforementioned relocation of firm A improves its payoff if $(B_1, B_2) = (1/4, 3/4)$. However, if $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$, firm A's payoff does not improve. This leads to multiple equilibria.

Proposition 1(ii) discusses a neighboring location equilibrium where a store of firm A (B) is located next to another store of firm A (B). The existence of a neighboring location equilibrium may explain the strategy of Seven-Eleven Japan. Note that only the neighboring location equilibrium exists when $\alpha \in (3 - 2\sqrt{2}, \overline{\alpha}_2)$ and it is one of the equilibria (both neighboring and intertwined location equilibria exist) when $\alpha \in [\overline{\alpha}_2, \alpha \in (3 - 2\sqrt{2}, \overline{\alpha}_1)]$. Seven-Eleven Japan follows a strategy called "strategic dominance," by which it sets up stores in narrow territories instead of establishing nationwide networks of stores.[7] The neighboring location equilibrium is consistent with this strategy. Similar strategies are observed in many Japanese chain stores (Komoto, 1997).

3.5 Concluding Remarks

In this study, we revisited the multi-store paradox formulated by Martinez-Giralt and Neven (1988) by introducing interdependent payoff functions. We assumed that duopolists are concerned with both their own and their rival's profits. We found that firms set up multiple stores unless the degree of payoff interdependence is low. We also found that multiple equilibria, namely intertwined and neighboring location equilibria, exist if the degree of payoff interdependence is neither too low nor too high. We also showed that our approach can solve the multi-store paradox suggested by Judd (1985), who discussed it in the context of entry deterrence.

If the number of stores of each firm is an exogenous variable, an increase in this number accelerates competition and reduces firms' profits. In this study, we showed that a large degree of interdependence leads to multiple stores in equilibrium. As Matsumura et al. (2013) suggested, we can interpret that the degree of payoff interdependence indicates the degree of the toughness of competition. Following this interpretation, our result suggested that tougher competition leads firms to set up multiple stores in equilibrium. This result suggests the possible inverse causality of the traditional view; tougher competition leads to multiple stores and setting up multiple stores further accelerates competition.

We believe that the payoff interdependence approach or relative performance approach adopted in this study is a useful theoretical framework. If we allow negative α, the model setting enables us to treat competitiveness as a continuous variable, and this model contains a perfectively competitive situation and monopoly as special cases (see Matsumura and Matsushima, 2012; Matsumura *et al.*, 2013). Thus, this approach is useful as a tool to investigate how the toughness of competition affects the equilibrium outcome in broad contexts.

Although the conjectural variation approach has similar properties, this model assumes that firm 1's output affects that of firm 2 and vice versa. This assumption is inconsistent if we respect the conjectural variation model as a static model. Our approach does not have this shortcoming,

[7] Regarding its store location strategy, Seven-Eleven Japan states that "high-density, concentrated store openings . . . are a vital trajectory toward realizing close and convenient stores." (Seven-Eleven Japan Corporate Profile 2013–2014, p. 15).

which is an important advantage. Hence, we believe that it should thus be adopted in broader contexts in the IO literature to solve many other paradoxes in economics.

Appendix

Proof of Proposition 1(i)

We show that when $\alpha \in [0, 3-2\sqrt{2}]$, given $(A_1, A_2) = (0,0)$, the best reply for firm B is $(B_1, B_2) = (1/2, 1/2)$.

Because $A_1 = A_2 = 0$, firm B must establish neighboring locations. Let $p_A^* := p_{A1}^* = p_{A2}^*$. The payoff of firm B, U_B, is expressed as follows:

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B1}^*\left(\frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} + \frac{B_2 + B_1}{2} - \frac{p_{B1}^* - p_A^*}{2B_1} - \frac{B_1}{2}\right) \\ &\quad + p_{B2}^*\left(\frac{p_{B2}^* - p_A^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} - \frac{B_2 + B_1}{2}\right) \\ &\quad - \alpha\left\{p_A^*\left(\frac{p_{B1}^* - p_A^*}{2(B_1)} + \frac{(B_1)}{2} + 1 - \frac{p_{B2}^* - p_A^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2}\right)\right\}. \end{aligned}$$

Let $\Theta = B_2 - B_1$ be the distance between stores B_1 and B_2. The payoff function is rewritten as

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B1}^*\left(\frac{p_{B2}^* - p_{B1}^*}{2\Theta} + \frac{\Theta + 2B_1}{2} - \frac{p_{B1}^* - p_A^*}{2B_1} - \frac{B_1}{2}\right) \\ &\quad + p_{B2}^*\left(\frac{p_{B2}^* - p_A^*}{2(\Theta + B_1 - 1)} + \frac{(\Theta + B_1 + 1)}{2} - \frac{p_{B2}^* - p_{B1}^*}{2\Theta} - \frac{\Theta + 2B_1}{2}\right) \\ &\quad - \alpha\left\{p_A^*\left(\frac{p_{B1}^* - p_A^*}{2(B_1)} + \frac{(B_1)}{2} + 1 - \frac{p_{B2}^* - p_A^*}{2(\Theta + B_1 - 1)} - \frac{\Theta + B_1 + 1}{2}\right)\right\}. \end{aligned}$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-1 + \Theta + 2B_1)(-1 + \alpha)(\Theta^2(-1 + \alpha^2) + \Theta(3 + 2\alpha - \alpha^2) - (-3 + \alpha)^2)}{2(-1 + \Theta)(-3 + \alpha)^2(1 + \alpha)} = 0.$$

The second-order condition is satisfied.

We can show that $\Theta^2(-1 + \alpha^2) + \Theta(3 + 2\alpha - \alpha^2) - (-3 + \alpha)^2$ is strictly negative. Thus, the first-order condition is satisfied if and only if $(-1 + \Theta + 2B_1) = 0$ (i.e., $B_1 + B_2 = 1$). This implies that the locations must be symmetric (i.e., $1/2 - B_1 = B_2 - 1/2$). By substituting this condition into the first-order condition with respect to Θ, we have

$$\frac{\partial U_B}{\partial \Theta} = \frac{-3\Theta^2(-1 + \alpha)^2(1 + \alpha) + 2\Theta(-5 - 5\alpha + \alpha^2 + \alpha^3) - 3 - 19\alpha - 9\alpha^2 + \alpha^3}{8(-3 + \alpha)^2(1 + \alpha)}.$$

We can show that $\partial U_B/\partial\Theta < 0$ for all $\Theta \in [0,1/2]$ if $\alpha \in [0, 3-2\sqrt{2})$. Thus, $\Theta = 0$ is optimal. In addition, $\partial U_B/\partial\Theta = 0$ for $\Theta = 0$ if $\alpha = 3-2\sqrt{2}$. These two conditions ($\Theta = 0$ and $-1+\Theta+2B_1 = 0$) imply that the best reply for firm B is $(B_1, B_2) = (1/2, 1/2)$ when $\alpha \in [0, 3-2\sqrt{2}]$.

By symmetry, given $(B_1, B_2) = (1/2, 1/2)$, $(A_1, A_2) = (0, 0)$ is the best reply for firm A if $\alpha \in [0, 3-2\sqrt{2}]$. ■

Proof of Proposition 1(ii)

We show that when $\alpha \in (3-2\sqrt{2}, \overline{\alpha}_1]$, given $(A_1, A_2) = (0, (-1+6\alpha-\alpha^2)/(4+4\alpha))$, the best reply for firm B is $(B_1, B_2) = (1/2, 1/2 + (-1+6\alpha-\alpha^2)/(4+4\alpha))$. We take the following two steps. First, we show that the above location is firm B's optimal strategy for $\alpha \in (3-2\sqrt{2}, 1)$ in Neighboring. Second, we allow firm B to establish intertwined locations and show that this never improves its payoff for $\alpha \in (3-2\sqrt{2}, \overline{\alpha}_1]$.

Suppose that firm B establishes neighboring locations. Suppose that $\alpha \in (3-2\sqrt{2}, 1)$. Let $\overline{A}_2$ be $(-1+6\alpha-\alpha^2)/(4+4\alpha)$. Given $(A_1, A_2) = (0, \overline{A}_2)$, the payoff function of firm B in Neighboring is

$$
\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p^*_{B1}\left(\frac{p^*_{B2}-p^*_{B1}}{2(B_2-B_1)} + \frac{B_2+B_1}{2} - \frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)} - \frac{B_1+\overline{A}_2}{2}\right) \\
&\quad + p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} + \frac{(B_2+1)}{2} - \frac{p^*_{B2}-p^*_{B1}}{2(B_2-B_1)} - \frac{B_2+B_1}{2}\right) \\
&\quad - \alpha\left\{p^*_{A1}\left(\frac{p^*_{A2}-p^*_{A1}}{\overline{A}_2} + \frac{\overline{A}_2}{2} + 1 - \frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} - \frac{B_2+1}{2}\right)\right. \\
&\quad \left. + p^*_{A2}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)} + \frac{(B_1+\overline{A}_2)}{2} - \frac{p^*_{A2}-p^*_{A1}}{\overline{A}_2} - \frac{\overline{A}_2}{2}\right)\right\}.
\end{aligned}
$$

Let $B_2 - B_1$ be Θ. The payoff function is rewritten as

$$
\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p^*_{B1}\left(\frac{p^*_{B2}-p^*_{B1}}{2\Theta} + \frac{B_2+B_1}{2} - \frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)} - \frac{B_1+\overline{A}_2}{2}\right) \\
&\quad + p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(\Theta+B_1-1)} + \frac{(\Theta+B_1+1)}{2} - \frac{p^*_{B2}-p^*_{B1}}{2\Theta} - \frac{\Theta+2B_1}{2}\right) \\
&\quad - \alpha\left\{p^*_{A1}\left(\frac{p^*_{A2}-p^*_{A1}}{\overline{A}_2} + \frac{\overline{A}_2}{2} + 1 - \frac{p^*_{B2}-p^*_{A1}}{2(\Theta B_1-1)} - \frac{\Theta+B_1+1}{2}\right)\right. \\
&\quad \left. + p^*_{A2}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)} + \frac{(B_1+\overline{A}_2)}{2} - \frac{p^*_{A2}-p^*_{A1}}{\overline{A}_2} - \frac{\overline{A}_2}{2}\right)\right\}.
\end{aligned}
$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-1+\Theta-\overline{A}_2+2B_1)(-1+\alpha)g\left(\overline{A}_2,\Theta,\alpha\right)}{2(-3+\alpha)^2(1+\alpha)(\Theta(\overline{A}_2(-3-2\alpha+\alpha^2)+4)^2} = 0,$$

$$\begin{aligned}
\text{where} \quad g\left(\overline{A}_2,\Theta,\alpha\right) \quad = \quad & \overline{A}_2^3(-1+\alpha^2)(\Theta(-3-2\alpha+\alpha^2)+4)^2 \\
& +\overline{A}_2^2[\Theta^3(-3+\alpha)^2(-1+\alpha)(1+\alpha)^3 \\
& +16\Theta^2(1+\alpha)^2(7+\alpha-5\alpha^2+\alpha^3)(-69+68\alpha-6\alpha^2-12\alpha^3+3\alpha^4) \\
& +4\Theta(-41-24\alpha+36\alpha^2+6\alpha^3-11\alpha^4+2\alpha^5)] \\
& +4\overline{A}_2[2\Theta^3(1+\alpha)^2(3-4\alpha+\alpha^2)-2\Theta(-41-6\alpha+24\alpha^2-10\alpha^3+\alpha^4) \\
& +\Theta(-32-21\alpha+22\alpha^2+4\alpha^3-6\alpha^4+\alpha^5)-4(15-5\alpha-3\alpha^2+\alpha^3)] \\
& +16[\Theta^3(-1+\alpha^2)-2\Theta^2(-2+\alpha)(1+\alpha)+4\Theta(-3+\alpha)+(-3+\alpha)^2].
\end{aligned}$$

The second-order condition is satisfied. We can show that $g\left(\overline{A}_2,\Theta,\alpha\right)$ is strictly negative. Thus, the first-order condition is satisfied if and only if $-1+\Theta-\overline{A}_2+2B_1=0$. By taking another first-order condition, $\partial U_B/\partial\Theta=0$, and substituting the condition $-1+\Theta-\overline{A}_2+2B_1=0$ into it, we have

$$\begin{aligned}
\frac{\partial U_B}{\partial \Theta} \quad = \quad & \frac{1}{8(-3+\alpha)^2(1+\alpha)}\Big\{\overline{A}_2^2(-1+\alpha)^2(1+\alpha)-3\Theta^2(-1+\alpha)^2(1+\alpha)-2\overline{A}_2(-1+\alpha)^2(1+\alpha) \\
& +2\Theta(1+\alpha)(\overline{A}_2(-1+\alpha)^2+\alpha^2-5)-3+19\alpha-9\alpha^2+\alpha^3\Big\}=0.
\end{aligned}$$

The second-order condition is satisfied. The first-order condition leads to the following optimal Θ:

$$\Theta^* = \frac{-1+6\alpha-\alpha^2}{4+4\alpha}.$$

Therefore, the optimal neighboring location is

$$(B_1^*, B_2^*) = (1/2, 1/2+\frac{-1+6\alpha-\alpha^2}{4+4\alpha}).$$

Next, we allow firm B to establish intertwined locations. Given $(A_1,A_2)=(0,\overline{A}_2)$, the payoff function of firm B in Intertwined is

$$\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p^*_{B1}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)}+\frac{B_1+\overline{A}_2}{2}-\frac{p^*_{B1}-p^*_{A1}}{2B_1}-\frac{B_1}{2}\right) \\
&\quad +p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)}+\frac{(B_2+1)}{2}-\frac{p^*_{B2}-p^*_{A2}}{2(B_2-\overline{A}_2)}-\frac{B_2+\overline{A}_2}{2}\right) \\
&\quad -\alpha\left\{p^*_{A1}\left(\frac{p^*_{B1}-p^*_{A1}}{2B_1}+\frac{B_1}{2}+1-\frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)}-\frac{B_2+1}{2}\right)\right. \\
&\quad \left.+\,p^*_{A2}\left(\frac{p^*_{B2}-p^*_{A2}}{2(B_2-\overline{A}_2)}+\frac{B_2+\overline{A}_2}{2}-\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\overline{A}_2)}-\frac{B_1+\overline{A}_2}{2}\right)\right\}.
\end{aligned}$$

The two first-order conditions $\partial U_B/\partial B_1 = 0$ and $\partial U_B/\partial B_2 = 0$ are satisfied when $(B_1, B_2) = ((-1+6\alpha-\alpha^2)/(8+8\alpha), (3+10\alpha-\alpha^2)/(8+8\alpha))$. The second-order conditions are also satisfied.

Finally, we investigate whether the intertwined or the neighboring location is best for firm B. The neighboring location $(B_1, B_2) = (1/2, 1/2+(-1+6\alpha-\alpha^2)/(4+4\alpha))$ is the best reply for firm B if and only if

$$\begin{aligned}
&U_B(0, \frac{-1+6\alpha-\alpha^2}{4+4\alpha}, 1/2, 1/2+\frac{-1+6\alpha-\alpha^2}{4+4\alpha}) = \frac{(3-\alpha)(-1+\alpha)^2}{16(1+\alpha)^2} \\
&\geq U_B(0, \frac{-1+6\alpha-\alpha^2}{4+4\alpha}, \frac{-1+6\alpha-\alpha^2}{8+8\alpha}, \frac{3+10\alpha-\alpha^2}{8+8\alpha}) = \frac{2+11\alpha-26\alpha^2+20\alpha^3-8\alpha^4+\alpha^5}{64(1+\alpha)^3}.
\end{aligned}$$

This holds true if and only if $\alpha \in (3-2\sqrt{2}, \overline{\alpha}_1]$, where $\overline{\alpha}_1$ is a positive solution to the following equation:

$$\frac{(3-\alpha)(-1+\alpha)^2}{16(1+\alpha)^2} = \frac{2+11\alpha-26\alpha^2+20\alpha^3-8\alpha^4+\alpha^5}{64(1+\alpha)^3}.$$

By symmetry, given $(B_1, B_2) = (1/2, 1/2+(-1+6\alpha-\alpha^2)/(4+4\alpha))$, $(A_1, A_2) = (0, (-1+6\alpha-\alpha^2)/(4+4\alpha))$ is the best reply for firm A if $\alpha \in (3-2\sqrt{2}, \overline{\alpha}_1]$. ■

Proof of Proposition 1(iii)

We show that when $\alpha \in [\overline{\alpha}_2, 1)$, given $(A_1, A_2) = (0, 1/2)$, the best reply for firm B is $(B_1, B_2) = (1/4, 3/4)$. We perform the following two steps. First, we show that the above is the optimal strategy in Intertwined. Second, we allow firm B to set up neighboring locations and show that this never improves its payoff in these α.

Suppose that firm B establishes intertwined locations. Given $(A_1, A_2) = (0, 1/2)$, the payoff of firm B, U_B, is

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p^*_{B1}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} + \frac{B_1+\frac{1}{2}}{2} - \frac{p^*_{B1}-p^*_{A1}}{2B_1} - \frac{B_1}{2}\right) \\ &\quad + p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} + \frac{(B_2+1)}{2} - \frac{p^*_{B2}-p^*_{A2}}{2(B_2-\frac{1}{2})} - \frac{B_2+\frac{1}{2}}{2}\right) \\ &\quad -\alpha\left\{p^*_{A1}\left(\frac{p^*_{B1}-p^*_{A1}}{2B_1} + \frac{B_1}{2} + 1 - \frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} - \frac{B_2+1}{2}\right)\right. \\ &\quad \left. + p^*_{A2}\left(\frac{p^*_{B2}-p^*_{A2}}{2(B_2-\frac{1}{2})} + \frac{B_2+\frac{1}{2}}{2} - \frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} - \frac{B_1+\frac{1}{2}}{2}\right)\right\} \end{aligned}$$

The two first-order conditions $\partial U_B/\partial B_1 = 0$ and $\partial U_B/\partial B_2 = 0$ are satisfied when $(B_1, B_2) = (1/4, 3/4)$. The second-order conditions are also satisfied. We now allow firm B to establish neighboring locations. Given $(A_1, A_2) = (0, 1/2)$, the payoff of firm B is

$$\begin{aligned} U^B &= \pi_B - \alpha\pi_A \\ &= p^*_{B1}\left(\frac{p^*_{B2}-p^*_{B1}}{2(B_2-B_1)} + \frac{B_2+B_1}{2} - \frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} - \frac{B_1+\frac{1}{2}}{2}\right) \\ &\quad + p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} + \frac{(B_2+1)}{2} - \frac{p^*_{B2}-p^*_{B1}}{2(B_2-B_1)} - \frac{B_2+B_1}{2}\right) \\ &\quad -\alpha\left\{p^*_{A1}\left(\frac{p^*_{A2}-p^*_{A1}}{1} + \frac{1}{4} + 1 - \frac{p^*_{B2}-p^*_{A1}}{2(B_2-1)} - \frac{B_2+1}{2}\right)\right. \\ &\quad \left. + p^*_{A2}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} + \frac{(B_1+\frac{1}{2})}{2} - \frac{p^*_{A2}-p^*_{A1}}{1} - \frac{1}{4}\right)\right\}. \end{aligned}$$

Let $B_2 - B_1$ be Θ. The payoff function can be expressed by B_1 and Θ:

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p^*_{B1}\left(\frac{p^*_{B2}-p^*_{B1}}{2\Theta} + \frac{\Theta+2B_1}{2} - \frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} - \frac{B_1+\frac{1}{2}}{2}\right) \\ &\quad + p^*_{B2}\left(\frac{p^*_{B2}-p^*_{A1}}{2(B_1+\Theta-1)} + \frac{(B_1+\Theta+1)}{2} - \frac{p^*_{B2}-p^*_{B1}}{2\Theta} - \frac{\Theta+2B_1}{2}\right) \\ &\quad -\alpha\left\{p^*_{A1}\left(\frac{p^*_{A2}-p^*_{A1}}{1} + \frac{1}{4} + 1 - \frac{p^*_{B2}-p^*_{A1}}{2(B_1+\Theta-1)} - \frac{B_1+\Theta+1}{2}\right)\right. \\ &\quad \left. + p^*_{A2}\left(\frac{p^*_{B1}-p^*_{A2}}{2(B_1-\frac{1}{2})} + \frac{(B_1+\frac{1}{2})}{2} - \frac{p^*_{A2}-p^*_{A1}}{1} - \frac{1}{4}\right)\right\}. \end{aligned}$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-3+2\Theta+4B_1)(-1+\alpha)f(\Theta,\alpha)}{8(-3+\alpha)^2(1+\alpha)(\Theta(5-2\alpha+\alpha^2))^2} = 0,$$

$$\begin{aligned} \text{where} \quad f(\Theta,\alpha) \quad &= \quad 2\Theta^3(-1+\alpha)(5-2\alpha+\alpha^2)^2 + \Theta^2(129+48\alpha-15\alpha^2-8\alpha^3-45\alpha^4+24\alpha^5-5\alpha^6) \\ &\quad\quad +16\Theta(-33+33\alpha-32\alpha^2+22\alpha^3-7\alpha^4+\alpha^5)+16(25-26\alpha+11\alpha^2-2\alpha^3). \end{aligned}$$

The second-order condition is satisfied. We can show that $f(\Theta,\alpha)$ is strictly positive. Thus, the first-order condition is satisfied if and only if $-3+2\Theta+4B_1=0$. This implies that the locations must be symmetric (i.e., $3/4-B_1=B_2-3/4$).

By substituting this condition into $\partial U_B/\partial\Theta$, we have

$$\frac{\partial U_B}{\partial \Theta} = \frac{-12\Theta^2(-1+\alpha)^2(1+\alpha)+4\Theta(-9-11\alpha+\alpha^2+3\alpha^3)-15-79\alpha-33\alpha^2+\alpha^3}{32(-3+\alpha)^2(1+\alpha)}.$$

Let $\overline{\alpha}_4$ be the solution for $15-79\alpha+33\alpha^2-\alpha^3=0$ in $\alpha\in[0,1)$. We can show that $\partial U_B/\partial\Theta<0$ for all $\Theta\in[0,1/2]$ if $\alpha\in[0,\overline{\alpha}_4)$. Thus, $\Theta=0$ and $(B_1,B_2)=(3/4,3/4))$ is optimal among neighboring locations if $\alpha\in[0,\overline{\alpha}_4)$. If $\alpha\in[\overline{\alpha}_4,1)$, the first-order condition $\partial U_B/\partial\Theta=0$ leads to the following optimal Θ:

$$\Theta^* = \frac{-9-2\alpha+3\alpha^2}{6(-1+\alpha)^2} + \sqrt{\frac{9+111\alpha-158\alpha^2+94\alpha^3-27\alpha^4+3\alpha^5}{9(-1+\alpha)^4(1+\alpha)}}.$$

To sum up, given $(A_1,A_2)=(0,1/2)$, the optimal location among neighboring locations for firm B is

$$(B_1^*,B_2^*) = \begin{cases} (\frac{3}{4},\frac{3}{4}) \quad \text{if} \quad 0\le\alpha<\overline{\alpha}_4 \\ \left(\frac{(3\alpha^2-8\alpha+9)}{6(\alpha-1)^2} - \sqrt{\frac{(\alpha-3)^2(3\alpha^3-9\alpha^2+13\alpha+1)}{36(\alpha-1)^4(\alpha+1)}}, \frac{\alpha(3\alpha-5)}{3(\alpha-1)^2} + \sqrt{\frac{(\alpha-3)^2(\alpha(3(\alpha-3)\alpha+13)+1)}{36(\alpha-1)^4(\alpha+1)}}\right) \quad \text{if} \quad \overline{\alpha}_4\le\alpha<1. \end{cases}$$

Finally, we investigate whether the intertwined or the neighboring location is best for firm B. The intertwined location $(B_1,B_2)=(1/4,3/4)$ is the best reply for firm B if and only if $U_B(0,1/2,1/4,3/4)\ge U_B(0,1/2,B_1^*,B_2^*)$. This holds true if and only if $\alpha\in[\overline{\alpha}_2,1)$, where $\overline{\alpha}_2$ is a positive solution to the equation

$$U_B(0,1/2,1/4,3/4) = \frac{1-\alpha}{32(1+\alpha)} = \frac{25-69\alpha+31\alpha^2-3\alpha^3}{64(-3+\alpha)^2(1+\alpha)} = U_B(0,1/2,3/4,3/4).$$

By symmetry, given $(B_1,B_2)=(1/4,3/4)$, $(A_1,A_2)=(0,1/2)$ is the best reply for firm A if $\alpha\in[\overline{\alpha}_2,1)$. ■

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Chapter 4

Environmental Corporate Social Responsibility as a Collusive Device*

4.1 Introduction

There are broad arguments about how instrumental differences among environmental policies affect firms' incentives and whether they improve or worsen environmental problems. Traditionally, governments have formed command-and-control regulations, taxes, and subsidies. Recently, however, a new approach, voluntary participation by firms or industry associations, has been introduced in environmental policies.[1] This voluntary approach beyond compliance has some advantages over the traditional mandatory regulations.[2] For example, it could be quickly and flexibly implemented because no conflict exists between policymakers and firms. Although this self-regulation has been widely adopted in recent decades, the effects and mechanism are not well understood either theoretically or empirically. Voluntary emission reduction (i.e. abatement) will increase a company's own cost and thus, might cause a cost disadvantage when the company's rivals do not participate in the voluntary emission reduction cooperatively. In addition, if all firms accept higher costs to engage in the voluntary agreement, who pays for these higher costs? Thus, it is important to investigate why the voluntary approach works in markets and how it affects the economy.

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[1] Because of worldwide political pressures concerning climate change, many polluting companies are voluntarily reducing their energy use or greenhouse gas (GHG) emissions and actively participating in GHG emissions reporting programs. For example, in 2014, 26 major firms in the power generation, cement production, and steel sectors in Korea voluntarily declared they would reduce fine dust. Most recently, EuroVAprint, which is composed of leading European printer and copier manufacturers, has established a voluntary agreement, and its activities are ongoing.

[2] See Vogel (2005), McWilliams et al. (2006), and Calveras et al. (2007).

As voluntary participation in environmental issues, environmental corporate social responsibility (ECSR) has received increasing attention from both natural and social science researchers. Economic researchers have intensively discussed this problem recently (Maxwell et al., 2000; Lyon and Maxwell, 2004; Lambertini and Tampieriz, 2015; Liu et al., 2015) because many listed firms are highly concerned about ECSR (KPMG, 2013). The CDP (2013), for example, reported that major companies, such as ExxonMobil, Walt Disney, Walmart, and Microsoft, use an internal (implicit) carbon price as an incentive and a strategic planning tool. Although ECSR is costly, recent analysis suggests that it can form part of an optimal firm strategy if society rewards social behavior.[3] Some empirical works have suggested that the financial performance of those firms believed to be highly concerned with ECSR is better.[4] There are two well-known reasons why voluntary approaches work in the market. One possible explanation in the field is that ECSR is connected with advertisement or public reputation of firms and thus, it eventually could change consumer preferences and ultimately individual behaviour. If consumers bear at least some of the negative externalities and value ECSR, these firms adopting ECSR could obtain increased demand, and thereby earn more profits (see Liu et al., 2015, and works cited therein). The other is that voluntary agreement can be used as a countermeasure for the regulatory threat by the government, which allows firms to avoid the public regulation (see Maxwell et al., 2000 and Antweiler, 2003).

In this study, however, we extend the body of knowledge on strategic ECSR by demonstrating that adopting ECSR can be profitable for firms even if it neither raises their reputations nor be the countermeasure for the regulation threat. In particular, we shed light on the role of industry associations, which play important roles in the adoption of ECSR.[5]

[3] As McWilliams and Siegel (2001) and Baron (2008) argued, this strategic behavior can be interpreted as a market-driven interaction to maximize the profits induced by the demand side or as a hedge against the risk of future regulation or activism. See also Kitzmueller and Shimshack (2012). Recent works, such as Goering (2014) and Brand and Grothe (2015), have considered a bilateral monopoly and showed that firms voluntarily adopt corporate social responsibility (CSR) to increase their profits. In their model, CSR implies that firms are concerned about consumer surplus.

[4] Margolis et al. (2007) used meta-analysis and detected a modest positive average correlation between CSR and corporate financial performance).

[5] Many industry and economic associations play leading roles in ECSR, such as the Japan Association of Corporate Executives, Japan Business Federation, Japan Iron and Steel Federation, Federation of Electric Power Companies of Japan, and the Federation of German Industries (Bundesverband der Deutschen Industrie), which is an alliance of associations, including many influential industrial sector associations in Germany. In addition, the business community has formed its own organizations specializing in CSR. For example, Business for Social Responsibility (BSR) is a business association founded in 1992 to provide corporations with expertise on the subject and to provide an opportunity for business executives to advance the field and learn from one another. See Carroll and Shabana (2010) for further discussion on BSR practices of business associations.

We discuss two kinds of ECSR. One is emission cap commitment (firms commit to an upper bound of emissions). Committing to reduce total emissions is equivalent to a cap of emission levels. The concept builds on absolute emission targets. Examples include many companies in the energy and semiconductor industries in the US and UK (see Margolick and Russell, 2001; Lee, 2010), among which NRG Energy, a leading energy company in the US, is a typical example (Cardwell, 2014). Furthermore, according to the CSR reports of the Japan Association of Corporate Executives and Japan Business Federation, many major Japanese firms have adopted such commitments. The other kind of ECSR is emission standard commitment (firms commit to emission level per unit of output). Firms belonging to the Federation of Electric Power Companies of Japan, an association of dominant electric companies in 10 areas in Japan, committed to an emission standard before the Great East Japan Earthquake in 2011.[6]

In each of the above two types of ECSR, we formulate the following three-stage duopoly game. In the first stage, each firm or the industry association to which both firms belong chooses the level of commitment as ECSR. In the second stage, the firms compete in the market. In the third stage, they engage in emission abatement activities.

Emission cap commitment yields the following results. In a quantity competition model, the industry association chooses a strictly positive degree of ECSR, although ECSR is not adopted if individual firms choose not to. By contrast, in a price competition model, even individual firms voluntarily adopt a positive degree of ECSR. Nevertheless, the industry association chooses a higher level of ECSR. These findings together suggest that industry associations have a stronger incentive to encourage firms to adopt ECSR than each firm working alone. In addition, we show that ECSR may harm welfare, either in Bertrand competition or Cournot competition, because ECSR restricts competition and raises prices.

Anti-trust legislation prevents collusion in prices or quantities, and thus, prohibits the formation of price or quantity cartels. However, it is unclear whether firms cooperate when choosing their degrees of ECSR in the face of such regulations. Indeed, business and industry associations often play a leading role in the adoption of ECSR by members. For example, many Japanese associations, such as the Japan Association of Corporate Executives, Japan Business Federation, Japan Iron and Steel Federation, and Federation of Electric Power Companies of Japan, emphasize ECSR in their reports and on their websites, and encourage – and often force – member firms to adopt ECSR.

[6] For examples and discussions on emission standards, see Helfand (1991), Farzin (2003), and Lahiri and Ono (2007).

Thus, we believe that cooperation in forming ECSR is quite natural and realistic.

The emission standard commitment has contrasting implications. Even a joint-profit-maximizing industry association might not choose a positive degree of ECSR when it chooses the emission standard. When it chooses this standard, the cap of total emissions is proportional to the output level. Thus, the output-restriction effect of ECSR under the emission standard is weaker than that under the emission cap. This yields contrasting results for the emission standard and emission cap.

This result suggests that ECSR by emission standard is less likely to restrict competition. If the emission standard is adopted by the association, the ECSR is more likely to be formed for the purpose of benevolence or improving industry image, such as advertising, rather than for the purpose enhancing collusion.

This type of ECSR was adopted by the Federation of Electric Power Companies of Japan before the Great East Japan Earthquake. The members of this association were dominant electric companies of from 10 areas in Japan, each with 90–100% market share in its area. Because competition was very weak in the Japanese electric power market, the association had little incentive to induce collusion by ECSR. Therefore, we guess that they adopted this type of ECSR for the purpose of improving industry image.

Regarding the anti-competitive effects of industry associations, similar-taste papers appeared recently. Marshall and Leslie (2012) has shown how third-party organizations themselves can be useful directly maintaining collusion. One of the best example is AC-Treuhand AG, which called itself a consulting firm for industry groups, but was found by the European Commission to simply facilitate collusion by gathering and sharing prices and quantities for different industry participants. Azar et al. (2016) has shown that co-ownership of all firms in a particular market by financial firms such as BlackRock and Vanguard (through their mutual funds) can soften competition. However, the mechanisms in both papers are quite different from that in this study.

The rest of this study is organized as follows. Section 2 presents the basic model of emission cap commitments. Sections 3 and 4 investigate quantity and price competition, respectively, and present our main results. Section 5 shows that emission standard commitments yield contrasting results to emission cap and implicit emission price commitments. Section 6 concludes.

4.2 The Model

We consider a symmetric duopoly model. There are two identical firms, firms 1 and 2, producing homogeneous commodities[7] for which the inverse demand function is given by $P(Q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. We assume that $P(Q)$ is twice continuously differentiable and $P'(Q) < 0$ for all Q as long as $P > 0$. Let $C(q_i) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ be the cost function of firm i, where $q_i \in \mathbb{R}_+$ is the output of firm i. We suppose C is twice continuously differentiable, increasing, and convex for all q_i.[8] We assume that the marginal revenue is decreasing (i.e. $P'(Q) + P''(Q)q_i < 0$). Under quantity competition, this guarantees that the strategies are strategic substitutes and that the second-order condition and the stability condition are satisfied.

There are emissions associated with the production, which yields a negative externality. Firm i's emission level is $g(q_i)-x_i$, where $g : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is emissions associated with production and $x_i (\in \mathbb{R}_+)$ is firm i's abatement level. We assume that g is twice continuously differentiable, increasing, and convex for all q_i.

Firm i $(i = 1, 2)$ adopts emission cap T_i and commits to $g(q_i) - x_i \leq T_i$. Firm i chooses a strictly positive x_i if and only if the emission cap constraint is binding. We regard that firms adopt ECSR if and only if the emission cap constraint is binding, and thus, $x > 0$ in equilibrium.

Firm i's profit is

$$P(Q)q_i - C(q_i) - K(x_i),$$

where the third term represents the abatement cost. We suppose that K is twice continuously differentiable, increasing, and strictly convex for $x_i > 0$. We further assume that $K(0) = K'(0) = 0$. This assumption guarantees that the social optimal level of abatement is never zero, and guarantees that the profit function is smooth.[9]

We examine the following three-stage game. In the first stage, firms non-cooperatively or cooperatively commit to their emission caps. In the non-cooperative ECSR choice case, each firm i independently chooses T_i to maximize its own profit. In the cooperative ECSR choice case, the industry association chooses $T_1 = T_2 = T$ to maximize joint profits. In the second stage, the firms

[7]We can show that Propositions 1 and 2 hold even if we introduce product differentiation that is discussed in Section 4 under moderate conditions. Proposition 1 holds if the strategies are strategic substitutes and the stability condition is satisfied in the quantity-competition stage and Proposition 2 holds even without the condition of the strategic substitute.

[8]We can relax this assumption. Our results hold if $C'' - P' > 0$ for all q_1 and q_2 as long as $P > 0$.

[9]As discussed later, marginal cost is C' when the constraint is not binding and $C' + K'g'$ when it is binding. The assumption guarantees $C = C + K$ and $C' = C' + K'g'$ when $g(q_i) - x_i = T_i$.

face quantity competition. In the third stage, the firms engage in emission abatement activities.[10]

4.3 Quantity Competition

We solve the game by backward induction. First, we discuss the abatement activity. Because firms commit to the emission cap restriction,

$$x_i = \max\{g(q_i) - T_i, 0\}. \tag{4.1}$$

Next, we discuss the second-stage quantity competition, given T_1 and T_2. The firms choose their quantities independently, given T_1 and T_2. Let $q_i^{SQ}(T_i, T_j)$ (**s**econd-stage game equilibrium output under **q**uantity competition) be the equilibrium output of firm i ($i = 1, 2,\ i \neq j$).[11]

There are three possible cases: (i) neither firm faces the emission cap restriction (i.e. $x_1 = x_2 = 0$), (ii) both firms are under the constraint (i.e. $x_1 x_2 > 0$), and (iii) only one firm, firm i, operates under the emission restriction (i.e. $x_i > 0$ and $x_j = 0$).

First, we consider case (i). The profit of firm $i = 1, 2$ for $g(q_i) \leq T_i$ is $\Pi_i(q_i, q_j) = P(Q)q_i - C(q_i)$. Let the superscript UQ denote the equilibrium outcome of this case (**u**nconstrained **q**uantity competition). The equilibrium output, q^{UQ}, is characterized by the following first-order condition:

$$\frac{\partial \Pi_i}{\partial q_i} = P'(Q)q_i + P(Q) - C'(q_i) = 0 \ (i = 1, 2,\ i \neq j). \tag{4.2}$$

The second-order condition $2P' + P'q_i - C'' < 0$ is satisfied. The equilibrium is unique, stable, and symmetric under the assumptions we made in the previous section.[12] If $T_i \geq T^{UQ} := g(q^{UQ})$ ($i = 1, 2$), $x_1 = x_2 = 0$, and thus, we regard no firm as adopting ECSR.

Second, we consider case (ii). As long as the emission cap constraint is binding, the profit function is $\Pi_i(q_i, q_j, T_i) = P(Q)q_i - C(q_i) - K(g(q_i) - T_i)$. The first-order condition is

$$\frac{\partial \Pi_i}{\partial q_i} = P'q_i + P - C' - K'g' = 0 \ (i = 1, 2,\ i \neq j). \tag{4.3}$$

The second-order condition and the stability condition are satisfied. Thus, unique equilibrium exists and is stable.

[10] The second and third stages are interchangeable in our analysis. If x_i is chosen before q_i, firm i chooses q_i such that $T_i = g(q_i) - x_i$ as long as the constraint $g(q_i) - x_i \leq T_i$ is binding. Thus, firm i chooses x_i taking account into the effect on its output. However, the resulting x and q are identical.

[11] Notations we use in this study are summarized in the Appendix A

[12] See Vives (1999).

Differentiating (4.3) leads to

$$\frac{dq_i^{SQ}}{dT_i} = -\frac{(\partial^2\Pi_i/\partial q_i\partial T_i)(\partial^2\Pi_j/\partial q_j^2)}{(\partial^2\Pi_i/\partial {q_i}^2)(\partial^2\Pi_j/\partial {q_j}^2)-(\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i)} > 0, \tag{4.4}$$

$$\frac{dq_j^{SQ}}{dT_i} = \frac{(\partial^2\Pi_i/\partial q_i\partial T_i)(\partial^2\Pi_j/\partial q_j\partial q_i)}{(\partial^2\Pi_i/\partial {q_i}^2)(\partial^2\Pi_j/\partial {q_j}^2)-(\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i)} < 0, \tag{4.5}$$

where we use $\partial^2\Pi_i/\partial q_i\partial T_i = K''g' > 0$, the second-order condition ($\partial^2\Pi_i/\partial q_i^2 = 2P' + P''q_i - C'' - K''(g')^2 - K'g'' < 0$), and the stability condition ($(\partial^2\Pi_i/\partial {q_i}^2)(\partial^2\Pi_j/\partial {q_j}^2) - (\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i) = (2P'+P''q_i-C''-K''(g')^2-K'g'')(2P'+P''q_j-C''-K''(g')^2-K'g'')-(P'+P''q_i)(P'+P''q_j) > 0$). The second-order condition and the stability condition are satisfied under the standard assumptions we made in Section 2.

An increase in T_i increases q_i because it reduces firm i's marginal cost $C'+K'g'$, which indirectly reduces q_j through the strategic interaction. Furthermore, because $|\partial^2\Pi_j/\partial q_j^2| = |2P'+P'q_j-C''-K''g'-K'g''| > |\partial^2\Pi_j/\partial q_j\partial q_i| = |P'+P'q_j|$, we obtain $dq_i^{SQ}/dT_i + dq_j^{SQ}/dT_i > 0$ (the direct effect dominates the indirect effect through strategic interaction).

Third, we consider case (iii). In this case, the equilibrium outputs are characterized by

$$\frac{\partial\Pi_i}{\partial q_i} = P'q_i + P - C' - K'g' = 0, \tag{4.6}$$

$$\frac{\partial\Pi_j}{\partial q_j} = P'q_j + P - C' = 0 \quad (j \neq i) \tag{4.7}$$

In this case, the equilibrium outputs depend only on T_i. Differentiating (4.6) and (4.7) leads to

$$\frac{dq_i^{SQ}}{dT_i} = -\frac{(\partial^2\Pi_i/\partial q_i\partial T_i)(\partial^2\Pi_j/\partial q_j^2)}{(\partial^2\Pi_i/\partial {q_i}^2)(\partial^2\Pi_j/\partial {q_j}^2)-(\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i)} > 0 \tag{4.8}$$

$$\frac{dq_j^{SQ}}{dT_i} = \frac{(\partial^2\Pi_i/\partial q_i\partial T_i)(\partial^2\Pi_j/\partial q_j\partial q_i)}{(\partial^2\Pi_i/\partial {q_i}^2)(\partial^2\Pi_j/\partial {q_j}^2)-(\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i)} < 0. \tag{4.9}$$

Again, an increase in T_i directly increases q_i and reduces q_j through the strategic interaction. Furthermore, because $|\partial^2\Pi_j/\partial q_j^2| = |2P' + P'q_j - C''| > |\partial^2\Pi_j/\partial q_j\partial q_i| = |P' + P'q_j|$, we obtain $dq_i^{SQ}/dT_i + dq_j^{SQ}/dT_i > 0$ (the direct effect dominates the indirect effect through strategic interaction).

We now consider the model in which each firm i independently chooses T_i to maximize its own profit. Let the superscript NQ denote the equilibrium outcome of this game (**N**on-cooperative

ECSR choice under **q**uantity competition). We show that cases (ii) and (iii) never appear in equilibrium, and thus, emission cap constraint is not binding in equilibrium.

Suppose that the constraint for firm i is binding in equilibrium, and thus, $x_i > 0$.

$$\frac{\partial \Pi_i}{\partial T_i} = \frac{\partial \Pi_i}{\partial q_i}\frac{dq_i^{SQ}}{dT_i} + \frac{\partial \Pi_i}{\partial q_j}\frac{dq_j^{SQ}}{dT_i} + K' > 0 \quad (i = 1, 2, \ i \neq j), \tag{4.10}$$

where we use $\partial \Pi_i / \partial q_i = 0$ (first-order condition), $\partial \Pi_i / \partial q_j = P' q_i < 0$, $dq_j^{SQ}/dT_i < 0$, and $K' > 0$. Thus, a marginal increase in T_i increases firm i's profit as long as the constraint is binding. This implies that cases (ii) and (iii) never appear in equilibrium. These discussions lead to the following proposition.

Proposition 1 *Under quantity competition, no firm individually adopts ECSR (i.e. $x^{NQ} = 0$).*

Next, we consider the model in which the industry association chooses $T = T_1 = T_2$ to maximize the industry profit. Let the superscript CQ denote the equilibrium outcome of this game (**C**ooperative ECSR choice under **q**uantity competition).

We show that $T^{CQ} < T^{UQ}$, and thus, case (ii) appears in equilibrium by showing that a marginal decrease in T from T^{UQ} increases the joint profit. Note that the joint profit remains unchanged for $T \geq T^{UQ}$ because any $T \geq T^{UQ}$ yields the same outcome $(q_1, q_2) = (q^{UQ}, q^{UQ})$.

For $T \leq T^{UQ}$, we have

$$\frac{\partial (\Pi_1 + \Pi_2)}{\partial T}\Big|_{T=T^{UQ}} = 2\frac{\partial \Pi_1}{\partial T}\Big|_{T=T^{UQ}} = 2\frac{\partial \Pi_1}{\partial q_2}\Big(\frac{dq_2^{SQ}}{dT_1} + \frac{dq_2^{SQ}}{dT_2}\Big) < 0, \tag{4.11}$$

where we use $\partial \Pi_1 / \partial q_1 = 0$ (first-order condition), $\partial \Pi_1 / \partial q_2 = P' q_1 < 0$, $dq_2^{SQ}/dT_1 + dq_2^{SQ}/dT_2 > 0$ (direct effect dominates indirect effect), and $K'(0) = 0$ (Note that $x_i = 0$ when $T = T^{UQ}$). Thus, a marginal decrease in T_i from T^{UQ} increases the joint profit. In other words, $T \geq T^{UQ}$ is never optimal for the industrial association. These discussions lead to the following proposition.

Proposition 2 *Under quantity competition, the industry association adopts ECSR (i.e. $T^{CQ} < T^{UQ}$, and thus, $x^{CQ} > 0$).*

A marginal decrease in $T_1(T_2)$ from T^{UQ} decreases firm 1's (firm 2's) profit by the second order (envelope theorem), whereas a marginal decrease in $T_2(T_1)$ from T^{UQ} increases firm 1's (firm 2's) profit by the first order. Therefore, a simultaneous decrease in T_1 and T_2 increases the joint profits.

Propositions 1 and 2 indicate that the industry association plays a crucial role in adopting ECSR. Although the firms have no incentive to adopt ECSR, they accept ECSR coordinated by the industry association because it serves as a collusive device that restricts their output, resulting in a higher price.

Finally, we discuss the welfare implications of ECSR. Total social surplus (firm profits plus consumer surplus minus the loss caused by the externality) is given by

$$W = \int_0^Q P(z)dz - \sum_{i=1}^{2} [C(q_i) + K(x_i)] - \eta\Big(\sum_{i=1}^{2} [g(q_i) - x_i]\Big),$$

where $\eta : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is the welfare loss of emissions. We assume that η is twice continuously differentiable, increasing, and convex.

Suppose that the government can choose $T = T_1 = T_2 \leq T^{UQ}$. Given the Cournot competition in the second stage, W is denoted by the following function.

$$W(T) = \int_0^Q P(z)dz - \sum_{i=1}^{2} [C(q_i^{SQ}) + K(g(q_i^{SQ}) - T)] - \eta(2T).$$

We obtain

$$\frac{\partial W}{\partial T} = 2\Big(-P' q_1^{SQ}\Big(\frac{dq_1^{SQ}}{dT_1} + \frac{dq_2^{SQ}}{dT_1}\Big) + K' - \eta'\Big), \tag{4.12}$$

where we use (4.3). The first term in (4.12) represents the welfare-improving effect of output expansion caused by a lesser degree of ECSR ($-P'q_1$ is equal to price-cost margin $P - C' - K'g'$). The second term represents the abatement cost-saving effect. The third term represents the welfare loss caused by an increase in emissions. The sign of the derivative at $T = T^{CQ}$ is negative if η' is large enough. In this case, T^{CQ} ($< T^{UQ}$) is still too large from the viewpoint of social welfare, and it implies that ECSR by industry association improves welfare as long as $W(T)$ is concave. Note that each firm chooses $T = T^{UQ}$ without the industry association. However, if η' is sufficiently small, (4.12) is positive and the degree of ECSR adopted by the industry association is too high for social welfare (i.e. the loss of collusive behavior dominates the emission-reducing effect).

4.4 Price Competition

We now consider Bertrand competition with product differentiation.[13] Assume there are two symmetric firms which produce differentiated products. The direct demand function for product i is given by $D_i(P) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ where $P := (p_1, p_2) \in \mathbb{R}^2_+$ is the price vector. We assume that D is twice

[13] Without product differentiation, there is no pure strategy equilibrium in our setting.

continuously differentiable for all $P > 0$. The demand is downward sloping, $\partial D_i/\partial p_i < 0, i = 1, 2$, and $\partial D_i/\partial p_j > 0, j \neq i$ as long as $D > 0$. The latter condition means that goods are substitutes. In addition, we assume that the direct effect of a price change dominates the indirect effect, $\sum_{j=1}^{2} (\partial D_i/\partial p_j) < 0$ and $\partial^2 D_i/(\partial p_i)^2 + \left|\partial^2 D_i/\partial p_i \partial p_j\right| < 0$. We further assume that demands have increasing differences in (p_i, p_j), $\partial^2 D_i/\partial p_i \partial p_j \geq 0$, which implies that the price setting game is supermodular. These are standard assumptions in the literature on Bertrand competition in differentiated product markets.[14] Except for the demand system, we follow the same structure in the quantity competition analysis.

The emission abatement level x_i is the same as that in the previous section. Here, we discuss the second-stage price competition. The firms choose their prices independently, given T_1 and T_2. Let $p_i^{SP}(T_i, T_j)$ (**s**econd-stage game equilibrium outcome under **p**rice competition) be the equilibrium price of firm i ($i = 1, 2,\ i \neq j$). Similar to the quantity competition analysis, there are three possible cases: (i) neither firm faces the emission cap restriction, (ii) both firms are under the constraint, and (iii) only one firm, firm i, operates under the emission restriction.

First, consider case (i). The profit of firm i for $g(q_i) \leq T_i$ is $\Pi_i(p_i, p_j) = p_i D_i(P) - C(D_i(P))$. Let the superscript UP denote the equilibrium outcome of this case (**u**nconstrained **p**rice competition). The equilibrium price, p_i^{UP}, is characterized by the following first-order condition:

$$\frac{\partial \Pi_i}{\partial p_i} = D_i(P) + p_i \frac{\partial D_i}{\partial p_i} - C' \frac{\partial D_i}{\partial p_i} = 0 \ (i = 1, 2,\ i \neq j). \tag{4.13}$$

We assume that the second-order condition
$\partial D_i/\partial p_i + (1 - (\partial D_i/\partial p_i) C'') \partial D_i/\partial p_i + (p_i - C')\partial^2 D_i/\partial p_i^2 < 0$ is satisfied.[15] Then, a unique, stable, and symmetric equilibrium exists. If $T_i \geq T^{UP} := g(D_i(P^{UP}))$ $(i = 1, 2)$, $x_1 = x_2 = 0$, and thus, we regard no firm as adopting ECSR.

Second, we consider case (ii). As long as the emission cap constraint is binding, the profit function is $\Pi_i(p_i, p_j, T_i) = p_i D_i(P) - C(D_i(P)) - K\left(g\left(D_i(P)\right) - T_i\right)$. The first-order condition is

$$\frac{\partial \Pi_i}{\partial p_i} = D_i(P) + p_i \frac{\partial D_i}{\partial p_i} - C' \frac{\partial D_i}{\partial p_i} - K' g' \frac{\partial D_i}{\partial p_i} = 0 \ (i = 1, 2,\ i \neq j). \tag{4.14}$$

The second-order condition and the stability condition are satisfied.[16] Thus, the unique equilibrium exists and is stable.

[14] See Vives (1999).

[15] The second-order condition might not be satisfied if $\partial^2 D_i/\partial p_i^2$ is positive and quite large, which is satisfied in the case of extremely convex demand. We rule out such a case by imposing the second-order condition.

[16] We show that the stability condition is satisfied in the Appendix B.

Differentiating (4.14) leads to

$$\frac{dp_i^{SP}}{dT_i} = -\frac{(\partial^2\Pi_i/\partial p_i\partial T_i)(\partial^2\Pi_j/\partial p_j^2)}{(\partial^2\Pi_i/\partial {p_i}^2)(\partial^2\Pi_j/\partial {p_j}^2) - (\partial^2\Pi_i/\partial p_i\partial p_j)(\partial^2\Pi_j/\partial p_j\partial p_i)} < 0, \tag{4.15}$$

$$\frac{dp_j^{SP}}{dT_i} = \frac{(\partial^2\Pi_i/\partial p_i\partial T_i)(\partial^2\Pi_j/\partial p_j\partial p_i)}{(\partial^2\Pi_i/\partial {p_i}^2)(\partial^2\Pi_j/\partial {p_j}^2) - (\partial^2\Pi_i/\partial p_i\partial p_j)(\partial^2\Pi_j/\partial p_j\partial p_i)} < 0, \tag{4.16}$$

where we use $\partial^2\Pi_i/\partial p_i\partial T_i = (\partial D_i/\partial p_i)K'g' < 0$, the second-order condition $(\partial^2\Pi_i/\partial p_i^2 = \partial D_i/\partial p_i + (1-(C''+K''(g')^2+K'g'')(\partial D_i/\partial p_i))\partial D_i/\partial p_i + (p_i - C' - K'g')\partial^2 D_i/\partial p_i^2 < 0)$, and the stability condition $((\partial^2\Pi_i/\partial {p_i}^2)(\partial^2\Pi_j/\partial {p_j}^2) - (\partial^2\Pi_i/\partial p_i\partial p_j)(\partial^2\Pi_j/\partial p_j\partial p_i) > 0)$.

An increase in T_i decreases p_i because it reduces firm i's marginal cost $C'+K'g'$, which indirectly reduces p_j through the strategic interaction.

Third, we consider case (iii). In this case, the equilibrium prices are characterized by

$$\frac{\partial\Pi_i}{\partial p_i} = D_i(P) + p_i\frac{\partial D_i}{\partial p_i} - C'\frac{\partial D_i}{\partial p_i} - K'g'\frac{\partial D_i}{\partial p_i} = 0, \tag{4.17}$$

$$\frac{\partial\Pi_j}{\partial p_j} = D_i(P) + p_i\frac{\partial D_i}{\partial p_i} - C'\frac{\partial D_i}{\partial p_i} = 0 \quad (j \neq i) \tag{4.18}$$

In this case, the equilibrium prices depend only on T_i. Differentiating (4.17) and (4.18) leads to

$$\frac{dp_i^{SP}}{dT_i} = -\frac{(\partial^2\Pi_i/\partial p_i\partial T_i)(\partial^2\Pi_j/\partial p_j^2)}{(\partial^2\Pi_i/\partial {p_i}^2)(\partial^2\Pi_j/\partial {p_j}^2) - (\partial^2\Pi_i/\partial p_i\partial p_j)(\partial^2\Pi_j/\partial p_j\partial p_i)} < 0, \tag{4.19}$$

$$\frac{dp_j^{SP}}{dT_i} = \frac{(\partial^2\Pi_i/\partial p_i\partial T_i)(\partial^2\Pi_j/\partial p_j\partial p_i)}{(\partial^2\Pi_i/\partial {p_i}^2)(\partial^2\Pi_j/\partial {p_j}^2) - (\partial^2\Pi_i/\partial p_i\partial p_j)(\partial^2\Pi_j/\partial p_j\partial p_i)} < 0. \tag{4.20}$$

Again, an increase in T_i decreases p_i and indirectly reduces p_j through the strategic interaction.

We now consider the model in which each firm i independently chooses T_i to maximize its own profit. Let the superscript NP denote the equilibrium outcome of this game (**N**on-cooperative ECSR choice under **p**rice competition). We show that cases (i) and (iii) never appear in equilibrium, and thus, emission cap constraint is binding for both firms.

Suppose that the constraint for firm i is not binding in equilibrium, and thus, $x_i = 0$.

$$\frac{\partial\Pi_i}{\partial T_i} = \frac{\partial\Pi_i}{\partial p_i}\frac{dp_i^{SP}}{dT_i} + \frac{\partial\Pi_i}{\partial p_i}\frac{dp_j^{SP}}{dT_i} + K' < 0 \ (i = 1,2, \ i \neq j), \tag{4.21}$$

where we use $\partial\Pi_i/\partial p_i = 0$ (first-order condition), $\partial\Pi_i/\partial p_j = (p_i - C' - K'g')\partial D_i/\partial p_j > 0$, $dp_j^{SQ}/dT_i < 0$, and $K'(0) = 0$. Thus, a marginal decrease in T_i increases firm i's profit as long as the constraint is not binding. This implies that cases (i) and (iii) never appear in equilibrium. These discussions lead to the following proposition.

Proposition 3 *Under Bertrand competition, firms non-cooperatively adopt ECSR (i.e. $x^{NP} > 0$).*

In contrast to the quantity competition model, each firm voluntarily adopts ECSR, which increases its marginal costs of production. An increase in the production cost of firm i raises firm i's price as well as its rival's price through strategic interaction, resulting in an increase in firm i's profit.

We now compare the cooperative and non-cooperative cases under price competition. We consider the model in which the industry association chooses $T = T_1 = T_2$ to maximize the industry profit. If $T \geq T^{UP}$, $x_1 = x_2 = 0$ and prices of both firms do not depend on T. Thus, the joint profits do not depend on T. We assume that for $T \leq T^{UP}$, joint profit is concave with respect to T.

Let the superscript CP denote the equilibrium outcome of this game (**C**ooperative ECSR choice under **p**rice competition). We show that $T^{CP} < T^{NP}$ is in equilibrium by showing that a marginal decrease in T from T^{NP} increases joint profits.

We obtain

$$\begin{aligned} \frac{\partial(\Pi_1 + \Pi_2)}{\partial T}\Big|_{T=T^{NP}} &= 2\frac{\partial \Pi_1}{\partial T}\Big|_{T=T^{NP}} \\ &= 2\left(\frac{\partial \Pi_1}{\partial p_2}\Big(\frac{dp_2^{SQ}}{dT_1} + \frac{dp_2^{SQ}}{dT_2}\Big) + K'\right) = 2\frac{\partial \Pi_1}{\partial p_2}\frac{dp_2^{SQ}}{dT_2} < 0, \end{aligned} \tag{4.22}$$

where we use $\partial\Pi_i/\partial p_i = 0$ (first-order condition), $\partial\Pi_i/\partial p_j = (p_i - C' - K'g')\partial D_i/\partial p_j > 0$, $dp_i^{SQ}/dT_i < 0$, and $(\partial\Pi_i/\partial p_j)(dp_j^{SP}/dT_i) + K' = 0$ when $T_i = T^{NP}$. Thus, the marginal decrease in T_i from T^{NP} increases the joint profit. This implies that T^{NP} is too large from the joint-profit-maximizing viewpoint. These discussions lead to the following proposition.

Proposition 4 *Under price competition, the industry association adopts a higher level of ECSR (i.e. $T^{CP} < T^{NP} < T^{UP}$, and thus, $x^{CP} > x^{NP} > 0$).*

A decrease in T_i raises the price of firm i and increases the profit of firm j. When individual firm i chooses T_i, firm i considers its own profit only and does not take into account this rival's profit-raising effect. Thus, T^{NP} is too large from the viewpoint of joint profit maximization.

We obtain similar welfare implications under quantity competition. When the degree of negative externality of emissions is large, even T^{CP} is too large for social welfare. However, when the degree of negative externality of emissions is small, even $T^{NP}(> T^{CQ})$ is too small for social welfare. In short, ECSR can be either beneficial or harmful for social welfare.

4.5 ECSR by Emission Standard

In this section, we consider ECSR by the emission standard commitment. For simplicity, we assume that without abatement activity, the emission level is proportional to the output level, that is $g(q_i) = eq_i$. We normalize $e = 1$. Note that this specification satisfies the assumptions made in the previous sections.

Firm i $(i = 1, 2)$ adopts the emission standard $t_i \in [0, 1]$ and commits to $(q_i - x_i)/q_i \leq t_i$. We regard firm i as adopting ECSR if $t_i < 1$.

First, we consider quantity competition. In the third stage, each firm i chooses

$$x_i = (1 - t_i)q_i. \tag{4.23}$$

We discuss the second-stage quantity competition. The firms choose their quantities independently, given t_i and t_j. The profit of firm $i = 1, 2$ is $\Pi_i(q_i, q_j, t_i) = P(Q)q_i - C(q_i) - K((1 - t_i) q_i)$. Let $q_i^{SQ}(t_i, t_j)$ be the equilibrium output of firm i $(i = 1, 2,\ i \neq j)$ in this subgame. The equilibrium output, q_i^{SQ}, is characterized by the following first-order condition:

$$\frac{\partial \Pi_i}{\partial q_i} = P'(Q)q_i + P(Q) - C'(q_i) - (1 - t_i)K' = 0 \ (i = 1, 2,\ i \neq j). \tag{4.24}$$

The second-order condition and the stability condition are satisfied under the assumptions discussed in Section 3. Thus, unique equilibrium exists and is stable.

Differentiating (4.24) leads to

$$\begin{aligned}\frac{dq_i^{SQ}}{dt_i} &= -\frac{(\partial^2 \Pi_i/\partial q_i \partial t_i)(\partial^2 \Pi_j/\partial q_j^2)}{(\partial^2 \Pi_i/\partial {q_i}^2)(\partial^2 \Pi_j/\partial {q_j}^2) - (\partial^2 \Pi_i/\partial q_i \partial q_j)(\partial^2 \Pi_j/\partial q_j \partial q_i)} \\ &> 0,\end{aligned} \tag{4.25}$$

$$\begin{aligned}\frac{dq_j^{SQ}}{dt_i} &= \frac{(\partial^2 \Pi_i/\partial q_i \partial t_i)(\partial^2 \Pi_j/\partial q_j \partial q_i)}{(\partial^2 \Pi_i/\partial {q_i}^2)(\partial^2 \Pi_j/\partial {q_j}^2) - (\partial^2 \Pi_i/\partial q_i \partial q_j)(\partial^2 \Pi_j/\partial q_j \partial q_i)} \\ &< 0,\end{aligned} \tag{4.26}$$

where we use $\partial^2\Pi_i/\partial q_i\partial t_i = K' + (1-t_i)K''q_i > 0$, the second-order condition ($\partial^2\Pi_i/\partial q_i{}^2 = 2P' + P''q_i - C'' - (1-t_i)^2K'' < 0$), and the stability condition $((\partial^2\Pi_i/\partial q_i{}^2)(\partial^2\Pi_j/\partial q_j{}^2) - (\partial^2\Pi_i/\partial q_i\partial q_j)(\partial^2\Pi_j/\partial q_j\partial q_i) = (P''q_i + 2P' - C'' - (1-t_i)^2K'')(P''q_j + 2P' - C'' - (1-t_i)^2K'') - (P''q_i + P')(P''q_j + P') > 0)$. Furthermore, because $|\partial^2\Pi_j/\partial q_j^2| = |2P' + P'q_j - C'' - (1-t_j)^2K''| > |\partial^2\Pi_j/\partial q_j\partial q_i| = |P' + P'q_j|$, we obtain $dq_i^{SQ}/dt_i + dq_j^{SQ}/dt_i \geq 0$ (the direct effect dominates the indirect effect through strategic interaction).

We now highlight one important property. Because $K'(0) = 0$ and $x_i = 0$ when $t_i = 1$, $\partial^2\Pi_i/\partial q_i\partial t_i = K' + (1-t_i)K''q_i = 0$ when $t_i = 1$. Thus, we obtain $dq_i^{SQ}/dt_i = dq_j^{SQ}/dt_i = 0$ when $t_i = 1$.

We now discuss the first-stage action. First, we consider the model in which each firm i individually chooses t_i to maximize its own profit. Again, the superscript NQ denote the equilibrium outcome under non-cooperative choice of ECSR.

For any $t_i \in [0, 1)$, we obtain

$$\frac{\partial\Pi_i}{\partial t_i} = \frac{\partial\Pi_i}{\partial q_i}\frac{dq_i^{SQ}}{dt_i} + \frac{\partial\Pi_i}{\partial q_j}\frac{dq_j^{SQ}}{dt_i} + K'q_i^{SQ} > 0, \ (i = 1, 2, \ i \neq j), \tag{4.27}$$

where we use $\partial\Pi_i/\partial q_i = 0$, $\partial\Pi_i/\partial q_j = P'q_i < 0$, $dq_j^{SQ}/dt_i < 0$, and $K'q_i^{SQ} > 0$. Therefore, each firm chooses $t_i = 1$. These discussions lead to the following proposition.

Proposition 5 *Under quantity competition with emission standard commitment, no firm individually adopts ECSR (i.e. $t_i = 1$, and thus, $x^{NQ} = 0$).*

Next we consider the model in which the industry association chooses $t = t_i = t_j$ to maximize the joint profit. We assume that joint profit is concave in t_i. Again, let the superscript CQ denote the equilibrium outcome of this cooperative choice of ECSR. We obtain

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t}\Big|_{t=1} = 2\frac{\partial\Pi_i}{\partial t} = 2\left(\frac{\partial\Pi_i}{\partial q_j}\Big(\frac{dq_j^{SQ}}{dt_i} + \frac{dq_j^{SQ}}{dt_j}\Big) + K'q_i^{SQ}\right) = 0, \tag{4.28}$$

where we use $dq_i^{SQ}/dt_i = dq_j^{SQ}/dt_i = 0$, $x = 0$, and $K'(0) = 0$ when $t = 1$. This implies that $t = 1$ is optimal. This leads to the following proposition.

Proposition 6 *Under quantity competition with emission standard commitment, even the industry association does not adopt ECSR (i.e. $t = 1$, and thus, $x^{CQ} = 0$).*

Some readers might consider that the assumption that joint profit is concave in t is restrictive. We show that

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t}\bigg|_{t=1} = 0$$

but it might imply that $t = 1$ yields locally minimized joint profits rather than maximized ones if the abovementioned assumption is not satisfied. However, it is quite difficult to derive a clear condition guaranteeing this assumption. We now present an example satisfying this assumption.

Suppose that demand is linear ($P = \alpha - Q$), marginal cost is constant and is normalized to zero, and the abatement cost function is quadratic ($K = kx_i^2/2$). Then, we obtain[17]

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t} = \frac{2\alpha^2 k(1-t)\left(1 + k(1-t)^2\right)}{\left(3 + k(1-t)^2\right)^3}. \tag{4.29}$$

This is positive for $t \in [0, 1)$ and zero when $t = 1$. Thus, $t = 1$ (no ECSR) maximizes the joint profits.

Proposition 6 is in sharp contrast to Proposition 1. Even the industry association that maximizes joint profit does not adopt ECSR.[18] Under the emission standard commitment, firm i can emit $t_i q_i$, whereas under the emission cap commitment, the firm can emit T_i independently of q_i. Thus, each firm has a stronger incentive to expand its output under the emission standard commitment. Therefore, the output-restricting effect of ECSR is weaker under the emission standard commitment.

This result suggests that ECSR by emission standard is less likely to restrict competition. If the emission standard is adopted by the industry association, it is more likely to be for the purposes of benevolence or improvement of industry image, like advertising, and not for the purpose of enhancing collusion.

We show that under price competition, we obtain a similar result to Proposition 6. That is, as long as the joint profit is concave with respect to t, $t = 1$ yields joint profit maximization. While the assumption of concavity might be too restrictive, it is quite difficult to derive a clear condition guaranteeing this assumption. We now present an example satisfying this assumption.

Suppose that the demand is given by $P = \alpha - \beta q_i - 0.5\beta q_j$ ($i = 1, 2,\ i \neq j$), then marginal cost is constant and is normalized to zero, and the abatement cost function is quadratic $K = kx_i^2/2$.

[17] The detailed derivation of (4.29) is relegated to Appendix C.

[18] Firms do not choose ECSR when each firm i chooses t_i independently.

Then, we obtain[19]

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t} = \frac{32\alpha^2 k \left(3\beta + 4k(1-t)^2\right)(1-t)}{\left(9\beta + 4k(1-t)^2\right)^3}. \tag{4.30}$$

This is positive for $t \in [0, 1)$ and zero when $t = 1$. Thus, in fact $t = 1$ (no ECSR) maximizes the joint profits.

4.6 Concluding Remarks

In this study, we demonstrate that profit-maximizing industry associations adopt ECSR even when it induces member firms to engage in unprofitable emission abatement activities. This cost increase raises prices or reduces quantities, resulting in an increase in industry profits. Therefore, ECSR can yield collusive behavior that reduces welfare, even though it reduces total emissions.

In addition, we show that whether the effect of restricting competition is significant depends on the type of ECSR. We show that the emission cap commitment has this effect, but the emission standard commitment does not.

A limitation of this study is that we neglect such environmental policies as emission taxes and tradable permits. ECSR may well reduce environmental taxes or relax environmental regulations, which would increase industry profits further. Introducing the government as an active player that implements environmental policies and investigating the relationship between these policies and ECSR are avenues left to future research.

[19] The detailed derivation of (4.30) is relegated to Appendix D.

Appendix

Notation

Table 4.1: Notation in this paper

superscript	under **Q**uantity Competition	
SQ	**S**econd-stage equilibrium output under constraints	$\{q_i^{SQ}\}$
UQ	**U**nconstrainted quantity competition	$\{q_i^{UQ}, T_i^{UQ}\}$
NQ	**N**oncooperative ECSR choice	$\{T_i^{NQ}, x_i^{NQ}\}$
CQ	**C**ooperative ECSR choice	$\{T^{CQ}, x^{CQ}\}$
superscript	under **P**rice Competition	
SP	**S**econd-stage equilibrium output under constraints	$\{p_i^{SP}\}$
UP	**U**nconstrainted price competition	$\{p_i^{UP}, T_i^{UP}\}$
NP	**N**oncooperative ECSR choice	$\{T_i^{NP}, x_i^{NP}\}$
CP	**C**ooperative ECSR choice	$\{T^{CP}, x^{CP}\}$

Stability condition under price competition

$$
\begin{aligned}
&\frac{\partial^2 \Pi_i}{\partial {p_i}^2}\frac{\partial^2 \Pi_j}{\partial {p_j}^2} - \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j}\frac{\partial^2 \Pi_j}{\partial p_i \partial p_j} \\
&= \left(\frac{\partial D_i}{\partial p_i} + \left(1 - \left(C'' + K''(g')^2 + K'g''\right)\frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_i}{\partial p_i} + (p_i - C' - K'g')\frac{\partial^2 D_i}{\partial p_i^2}\right) \cdot \\
&\quad \left(\frac{\partial D_j}{\partial p_j} + \left(1 - \left(C'' + K''(g')^2 + K'g''\right)\frac{\partial D_j}{\partial p_j}\right)\frac{\partial D_j}{\partial p_j} + (p_j - C' - K'g')\frac{\partial^2 D_j}{\partial p_j^2}\right) \\
&- \left(\left(1 - \left(C'' + K''(g')^2 + K'g''\right)\frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_i}{\partial p_j} + (p_i - C' - K'g')\frac{\partial^2 D_i}{\partial p_i \partial p_j}\right) \cdot \\
&\quad \left(\left(1 - \left(C'' + K''(g')^2 + K'g''\right)\frac{\partial D_j}{\partial p_j}\right)\frac{\partial D_j}{\partial p_i} + (p_i - C' - K'g')\frac{\partial^2 D_j}{\partial p_i \partial p_j}\right) > 0.
\end{aligned}
$$

Derivation of (4.29)

To obtain (4.29), we solve the game by backward induction. First, we consider the abatement activity in the third stage. Each firm i chooses

$$x_i = (1 - t_i)q_i.$$

Next, we discuss the second-stage quantity competition. The firms choose their quantities independently, given t_i and t_j. For $t_i < 1$, firm i's profit, Π_i, is

$$Pq_i - \frac{k((1-t_i)q_i)^2}{2}.$$

The first-order condition is

$$\frac{\partial \Pi_i}{\partial q_i} = \alpha - 2q_i - q_j - k(1-t_i)q_i = 0 \ (i = 1, 2, \ i \neq j).$$

We obtain the equilibrium outputs:

$$q_i^{SQ} = \frac{\alpha \left(k\left(1-t_j\right)^2 + 1\right)}{k^2\left(1-t_i\right)^2\left(1-t_j\right)^2 + 2k\left(t_i^2 - 2t_i + t_j^2 - 2t_j + 2\right) + 3}.$$

Substituting these equilibrium quantities into the profit function, we have the following resulting profit:

$$\Pi_i(t_i, t_j) = \frac{\alpha^2\left(2 + k\left(1-t_i\right)\right)\left(k\left(1-t_j\right)^2 + 1\right)^2}{2\left(k^2\left(1-t_i\right)^2\left(1-t_j\right)^2 + 2k\left(t_i^2 - 2t_i + t_j^2 - 2t_j + 2\right) + 3\right)^2}.$$

We now discuss the first-stage action. We consider the model in which the industry association chooses $t = t_i = t_j$ to maximize the joint profit. Again, let the superscript CQ denote the equilibrium outcome of this cooperative choice of ECSR. We obtain

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t} = \frac{\alpha^2 k(1-t)\left(1 + k(1-t)^2\right)}{\left(3 + k(1-t)^2\right)^3} > 0.$$

Derivation of (4.30)

To obtain (4.30), we solve the game by backward induction. The emission abatement level x_i is the same as that under quantity competition. Here, we begin by discussing the second-stage price competition. The firms choose their price independently, given t_i and t_j. For $t_i < 1$, firm i's profit, Π_i, is

$$p_i q_i - \frac{k((1-t_i)q_i)^2}{2}.$$

The first-order condition is

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{2\left(\alpha\left(3\beta + 4k(1-t_i)^2\right) - 3\beta(4p_i - p_j) - 4k(1-t_i)^2(2p_i - p_j)\right)}{9\beta^2} = 0.$$

We obtain the equilibrium prices:

$$p_i^{SP} = \frac{\alpha\left(3\beta + 4k(1-t_i)^2\right)\left(5\beta + 4k(1-t_j)^2\right)}{45\beta^2 + 16k^2(1-t_i)^2(1-t_j)^2 + 28\beta k(2 - (2-t_i)t_i - (2-t_j)t_j)}.$$

Substituting these equilibrium prices into the profit function, we have the following resulting profit:

$$\Pi_i(t_i, t_j) = \frac{4\alpha^2\left(3\beta + 2k(1-t_i)^2\right)\left(5\beta + 4k(1-t_j)^2\right)^2}{\left(45\beta^2 + 16k^2(1-t_i)^2(1-t_j)^2 + 28\beta k(2 - (2-t_i)t_i - (2-t_j)t_j)\right)^2}$$

We now discuss the first-stage action. We consider the model in which the industry association chooses $t = t_i = t_j$ to maximize the joint profit. Again, let the superscript CP denote the equilibrium outcome of this cooperative choice of ECSR. We obtain

$$\frac{\partial(\Pi_i + \Pi_j)}{\partial t} = \frac{32\alpha^2 k \left(3\beta + 4k(1-t)^2\right)(1-t)}{\left(9\beta + 4k(1-t)^2\right)^3} > 0.$$

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Chapter 5

Emission Cap Commitment versus Emission Intensity Commitment as Self-Regulation*

5.1 Introduction

Self-regulatory actions by an industry or firms have received considerable attention from economists and policymakers. In particular, self-regulation has been introduced in environmental policies as a tool to improve the environment, in addition to command-and-control regulation and/or economic incentives, such as emission taxes and tradable permits. Firms publicly initiate pledges to improve their environmental performance and undertake efforts to attain the goals by themselves.[1] A typical question regarding self-regulation is why firms voluntarily take certain actions even though they are costly. The literature on self-regulation suggests that polluting firms strategically act and self-regulate because of the threat of future regulation by regulatory authorities (e.g., Maxwell et al., 2000; Antweiler, 2003; Lyon and Maxwell, 2003; Fleckinger and Glachant 2011). Maxwell et al. (2000) formulated a theoretical model in which firms can choose their levels of voluntary pollution prior to political action by consumers leading to mandatory regulation and showed that self-regulation effectively preempts political entry. Antweiler (2003) empirically tested the effect of green regulatory threat. In addition, private politics, such as a boycott, have been used to explain

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[1]This form of self-regulation is called "unilateral commitments" in which the target is set by the industry or companies themselves. The OECD (1999) categorized voluntary environmental agreements into the three categories of unilateral commitments, public voluntary programs, and negotiated agreements.

the motivation of voluntary actions. Egorov and Harstad (2017) examined the interaction among public regulation, self-regulation, and boycott as private politics and showed the possibility of self-regulation. There is, however, another natural question to self-regulation: what measure should firms adopt as self-regulation?

There are several ways to commit to improve environmental performance.[2] Consider air pollution or carbon dioxide (CO2) emissions. A commitment to decrease the total emissions or to limit the upper bound of emissions per year is a direct measure (emission cap commitment). On the other hand, the emission intensity per unit of output is another popular measure (emission intensity commitment).[3] The choice of commitment device might affect the behavior of firms and resulting welfare.[4]

We formulate a self-regulation model in which the polluting firm self-regulates to preempt mandatory regulation or to avoid consumer activism, and then the firm determines its output and abatement. The model employs a general formulation of a monopoly setting to highlight the difference between emission intensity commitment and emission cap commitment as measures of self-regulation. Our central concerns are which of emission intensity commitment or emission cap commitment is chosen by the pollutant, and the ranking of consumer surplus and social welfare between these two measures of self-regulation. We compare the equilibrium outcomes at the environmental target, which is a common requirement regardless of the measure of self-regulation. Specifically, the environmental target is assumed the total amount of emissions.

We find that the equilibrium abatement investment and output are larger (and thus, consumer surplus is larger) under emission intensity commitment than under emission cap commitment. By contrast, emission cap commitment yields higher profit than does emission intensity commitment. Therefore, profit-maximizing firms choose emission cap commitment as a self-regulation tool. From

[2]For example, major Japanese firms belonging to the Japan Business Federation, Japan Iron and Steel Federation, and Federation of Electric Power Companies of Japan have committed to either upper bound of emissions or emission intensity per unit of output. Moreover, an international survey by KPMG in 2015 showed that nearly 92% of Global Fortune 250 firms issued corporate social responsibility reports in 2015, up from 35% in 1999, and most are concerned with environmental problems.

[3]Japanese electric power companies have committed to CO2 emissions/kWh, not total emissions, as self-regulation. Japanese Ministry of the Environment has declared that it would introduce stricter regulation if this self-regulation were to turn out not to work effectively.

[4]A substantial body of literature on mandatory green regulations compares the different forms of mandatory regulation, including emission tax, emission cap regulation, and emission intensity regulation. Many studies have shown that different policy instruments yield different welfare and environmental consequences, because different policy instruments provide different incentives for firms (Besanko, 1987; Helfand, 1991; Lahiri and Ono, 2007; Kiyono and Ishikawa, 2013; Amir et al., 2017). The studies have mainly focused on the effects of regulations on welfare and emission levels, and have not discussed firms' incentives for adopting a regulation measure.

a welfare perspective, however, it is ambiguous whether emission cap commitment or emission intensity commitment is better. We find that emission intensity commitment is unambiguously better than emission cap commitment in two important cases: in the case with the strictest target (when the emission target is close to zero emissions) and in the case with the loosest target (when the emission target is close to the business-as-usual level). Firms prefer emission cap commitment even when emission intensity commitment is desirable for welfare. Thus, our result suggests that the incentive to adopt emission cap commitment is too strong for profit-maximizing firms as a self-regulation measure and governments should encourage the adoption of emission intensity commitment rather than emission cap commitment, especially to achieve a zero-emission society efficiently.

However, emission cap commitment can yield greater welfare. We show that emission cap commitment can yield greater welfare, depending on the emission target level and the curvature of the abatement cost function. If the target level is far from both zero emission and business-as-usual levels, and the convexity of the abatement cost function is strong, emission cap commitment is better for welfare.

The rest of this study is organized as follows. Section 2 describes the model. Section 3 analyzes and compares the two self-regulation measures of emission intensity commitment and emission cap commitment. Section 4 concludes.

5.2 The Model

We consider the self-regulation model of a polluting monopoly.[5] The firm produces a single commodity for which the inverse demand function is given by $P(q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. We assume that $P(q)$ is twice continuously differentiable and $P'(q) < 0$ for all q as long as $P > 0$. Let $C(q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ be the cost function of the firm, where $q \in \mathbb{R}_+$ is the output of the firm. We suppose C is twice continuously differentiable, increasing, and convex for all q.[6] We assume that the marginal revenue is decreasing (i.e., $2P'(q) + P''(q)q < 0$). This condition guarantees that the second-order condition is satisfied.

Some emissions are associated with production, which yields a negative externality. After emissions have been generated, they can be reduced by the polluting firm through investment in abatement technologies.[7] Thus, the firm's net emissions are $E := g(q) - x$, where $g : \mathbb{R}_+ \mapsto \mathbb{R}_+$

[5] Our results hold in symmetric Cournot oligopolies under the standard conditions (e.g., stability conditions).

[6] We can relax this assumption. Our results hold if $C'' - P' > 0$ for all q as long as $P > 0$.

[7] These are called end-of-pipe technologies. An alternative approach to reduce emissions is to change the production

represents emissions associated with production and $x(\in \mathbb{R}_+)$ is the firm's abatement level. We assume that g is twice continuously differentiable, increasing, and convex for all q.

The firm's profit is

$$P(q)q - C(q) - K(x),$$

where the third term represents the abatement cost. We suppose that K is twice continuously differentiable, increasing, and strictly convex for $x > 0$. We further assume that $K(0) = K'(0) = 0$.[8] This assumption guarantees that the social optimal level of abatement is never zero and that the profit function is smooth.

Total social surplus (firm profits plus consumer surplus minus the loss caused by the externality) is given by

$$W = \pi + CS - \eta(E) = \int_0^q P(z)dz - C(q) - K(x) - \eta(E),$$

where $\eta : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is the welfare loss of emissions.

The firm undertakes self-regulation through emission intensity commitment or emission cap commitment. One might consider that the regulator should impose an emission tax or mandatory regulation on the polluting firm in order to restrict emissions rather than relying on self-regulation. The situation this study considers is similar to that of Segerson and Miceli (1998), Lyon and Maxwell (2003), Glachant (2007), and Brau and Carraro (2011). Self-regulation might be preferable to mandatory regulation, since the former reduces the administrative cost associated with serious mandatory regulation by law or avoids political resistance from regulated industry. Alternatively, if the government plans to impose an emission tax to reduce total emissions to $\bar{E}$ and firms expect the possible introduction of an emission tax following self-regulation, the firms would introduce self-regulation that yields $E = \bar{E}$ to prevent the introduction of such an emission tax.

An alternative interpretation of the environmental target that the polluting firm voluntary commits to is based on consumer activism. Unlike lobbying or political campaigns, consumers who have disutility from negative externalities organize activist groups and start a boycott if their requirements are not met (Egorov and Harstad, 2017). In this case, it is natural to consider that their concern is emission levels.

We assume that the environmental target is exogenously given. In other words, we do not model the regulator and the activist group as players. As discussed earlier in this section, we can describe

process. For a recent discussion of the relationship between mandatory regulation and this type of innovation, see Matsumura and Yamagishi (2017).

[8] The form of the abatement (R&D) cost function is a standard assumption in industrial organization and environmental economics (D'Aspremont and Jacquemin, 1998; Amir et al., 2017).

$\bar{E}$ depending on the administrative cost, political pressure, or the opportunity cost of the consumer boycott. There are, however, several ways to formulate such regimes so that we simply treat the target as an exogenous variable and examine how the firm undertakes self-regulation at each level, $\bar{E}$. We assume that $\bar{E} \in (0, E^B)$ where E^B is the profit-maximizing emission level without a binding emission target (business-as-usual level). Let q^B be the profit-maximizing output without a binding emission target.

The timing of the game is as follows. Given the environmental target, $\bar{E}$, the firm decides whether to use emission intensity commitment or emission cap commitment in the first stage. In the second stage, the firm chooses its output and abatement level to maximize the profit under the self-regulation it committed to in the first stage.

5.3 Analysis

We compare the effects on equilibrium outcomes of the two instruments used in this research.

5.3.1 Emission Intensity Commitment

First, we consider the case in which the firm adopts emission intensity commitment as self-regulation. Let α be the committed upper bound of the emission per unit of output. In the second stage, the firm chooses its output, q, and abatement level, x, to maximize its profit subject to

$$\alpha \geq \frac{E}{q} = \frac{g(q) - x}{q}. \tag{5.1}$$

When the constraint is binding[9], the firm's optimization problem is

$$\max_q P(q)q - C(q) - K(g(q) - \alpha q). \tag{5.2}$$

Let the superscript EI denote the equilibrium outcomes under emission intensity commitment. Define $\pi^{EI}(q; \alpha) := P(q)q - C(q) - K(g(q) - \alpha q)$. The equilibrium output, $q^{EI}(\alpha)$, is characterized by the following first-order condition:

$$\frac{\partial \pi^{EI}}{\partial q} = P'q + P - C' - K'(g' - \alpha) = 0. \tag{5.3}$$

The second-order condition is satisfied. We obtain $x^{EI}(\alpha) = g(q^{EI}(\alpha)) - \alpha q^{EI}(\alpha)$ and $E^{EI}(\alpha) = g(q^{EI}(\alpha)) - x^{EI}(\alpha) = \alpha q^{EI}(\alpha)$.

[9] In this game, the constraint is always binding because $\bar{E} < E^B$.

Differentiating (5.3) leads to

$$\frac{dq^{EI}}{d\alpha} = -\frac{\partial^2\pi/\partial q\partial\alpha}{\partial^2\pi/\partial q^2} > 0, \tag{5.4}$$

where we use $\partial^2\pi/\partial q\partial\alpha = K' + (g' - \alpha)K''q > 0$ and $\partial^2\pi/\partial q^2 = 2P' + P''q - C'' - g''K' - (g' - \alpha)^2K'' < 0$. An increase in α relaxes the emission restriction and reduces the marginal cost of production, which increases q.

In the first stage, the firm sets the emission intensity $\alpha = \bar{\alpha}$ such that $E^{EI}(\bar{\alpha}) = \bar{E}$. Let $(q^{EI}(\bar{E}), x^{EI}(\bar{E}))$ be the pair of equilibrium output and abatement and $W^{EI}(\bar{E})$ be the equilibrium welfare under emission intensity commitment when $\alpha = \bar{\alpha}$.

5.3.2 Emission Cap Commitment

Next, we consider the case in which the firm adopts emission cap commitment. The profit function of the firm is $P(q)q - C(q) - K(g(q) - \bar{E})$. Let the superscript EC denote the equilibrium outcomes under emission cap commitment. Then, the profit function of the firm under emission cap commitment is defined by $\pi^{EC}(q; \bar{E}) := P(q)q - C(q) - K(g(q) - \bar{E})$. The equilibrium output, $q^{EC}(\bar{E})$, is characterized by the following first-order condition:

$$\frac{\partial\pi^{EC}}{\partial q} = P'q + P - C' - K'g' = 0. \tag{5.5}$$

The second-order condition is satisfied. We obtain $x^{EC}(\bar{E}) = g(q^{EC}(\bar{E})) - \bar{E}$. Differentiating (5.5) leads to

$$\frac{dq^{EC}}{d\bar{E}} = -\frac{\partial^2\pi/\partial q\partial\bar{E}}{\partial^2\pi/\partial q^2} > 0, \tag{5.6}$$

where we use $\partial^2\pi/\partial q\partial \ = K''g' > 0$ and $\partial^2\pi/\partial q^2 = 2P' + P''q - C'' - g''K' - g'^2K'' < 0$. Similar to the emission intensity case, an increase in $\bar{E}$ increases q.

5.3.3 Comparison

In this subsection, we compare the two instruments. First, we consider the equilibrium output. Comparing emission intensity commitment with emission cap commitment, we present the following result.

Lemma 1 *The equilibrium output is larger under emission intensity commitment than under emission cap commitment, that is,* $q^{EI}(\bar{E}) > q^{EC}(\bar{E})$.

Proof.

By using (5.3), (5.5), and the emission equivalence, we obtain

$$\begin{aligned}\left.\frac{\partial \pi^{EC}}{\partial q}\right|_{q=q^{EI}} &= K'(g(q^{EI}) - \bar{\alpha} q^{EI}) \left[(g'(q^{EI}) - \bar{\alpha}\right] - K'(g(q^{EI}) - \bar{E}) g'(q^{EI}) \\ &= -K'(g(q^{EI}) - \bar{\alpha} q^{EI})\bar{\alpha} < 0.\end{aligned}$$

This implies that the output level of q^{EI} exceeds the profit-maximizing level under emission cap commitment, because the second-order condition is satisfied. ■

Lemma 1 states that the firm produces more outputs under emission intensity commitment than under emission cap commitment even though the resulting emissions from the pollutant are the same in both regimes. We explain the intuition behind Lemma 1 after presenting Proposition 1.

From Lemma 1 and the emission equivalence, we obtain the following lemma.

Lemma 2 *Emission intensity commitment yields greater net consumer surplus than emission cap commitment, that is,* $CS(q^{EI}(\bar{E}) - \eta(\bar{E}) > CS(q^{EC}(\bar{E})) - \eta(\bar{E})$.

Proof.

It is straightforward from the emission equivalence and Lemma 1. ■

We now present our result on the firm's profit.

Proposition 1 *Emission cap commitment yields higher profit than does emission intensity commitment (i.e.,* $\pi^{EC}(q^{EC}(\bar{E}), \bar{E}) > \pi^{EI}(q^{EI}(\bar{E}), \bar{\alpha})$.

Proof.

Using the resulting profit and the emission equivalence, we obtain

$$\begin{aligned}\pi^{EC}(q^{EC}, \bar{E}) &= P(q^{EC})q^{EC} - C(q^{EC}) - K(g(q^{EC}) - \bar{E}) \\ &> P(q^{EI})q^{EI} - C(q^{EI}) - K(g(q^{EI}) - \bar{E}) \\ &= P(q^{EI})q^{EI} - C(q^{EI}) - K(g(q^{EI}) - \bar{\alpha} q^{EI}) = \pi^{EI}(q^{EI}, \bar{\alpha}),\end{aligned}$$

where the inequality follows from the fact that $\arg\max_{\{q\}} P(q)q - C(q) - K(g(q) - \bar{E}) = q^{EC}$ and $q^{EI} \neq q^{EC}$. ■

We explain the intuition behind Lemma 1 and Proposition 1. Under emission intensity, the firm faces a time-inconsistency problem. In the second stage, given α, an increase in q increases the upper limit of emissions. Therefore, the firm has a stronger incentive to increase its output than under emission cap commitment (Lemma 1). However, this makes it stricter for the required α in the first stage to meet the emission target $\bar{E}$. This reduces the firm's profit. Therefore, the firm's profit is larger under emission cap commitment, which does not yield such a time-inconsistency problem, than under emission intensity commitment.

We now discuss social welfare. Emission intensity commitment is superior for consumer welfare than is emission cap commitment (Lemma 2), but is less profitable for the firm (Proposition 1). Thus, it is generally ambiguous which is socially preferable. Let $W^{EI}(\bar{E})$ and $W^{EC}(\bar{E})$ be the equilibrium welfare under emission intensity commitment and emission cap commitment, respectively. We present two cases in which emission intensity commitment yields greater welfare than emission cap commitment (i.e., $W^{EI}(\bar{E}) > W^{EC}(\bar{E})$). First, we consider the case with the most stringent target case ($\bar{E}$ is close to zero). When the firm is not allowed to pollute in the process of producing output (i.e., $\bar{E} = \bar{\alpha} = 0$), all emissions are reduced by the abatement activities and there are no emissions in the industry. Regardless of the output level, the total emissions are zero if and only if emissions per unit of output are zero. Therefore, when $\bar{E} = 0$, emission cap commitment and emission intensity commitment yield the same outcome. Let q^Z and x^Z be common q and x under the zero-emission constraint (i.e., when $\bar{E} = 0$).

We now present a result when $\bar{E}$ is close to zero.

Proposition 2 *If $\bar{E}$ is sufficiently close to zero, emission intensity commitment yields greater welfare than does emission cap commitment.*

Proof.

For $i = EC, EI$, we obtain

$$\begin{aligned}
\left.\frac{\partial W^i}{\partial \bar{E}}\right|_{\bar{E}=0} &= \frac{\partial W}{\partial q}\frac{dq^i}{d\bar{E}} + \frac{\partial W^i}{\partial x}\frac{dx^i}{d\bar{E}} + \frac{\partial W^i}{\partial E} \\
&= (P(q^Z) - C'(q^Z))\frac{dq^i}{d\bar{E}} - K'(x^Z)\frac{dx^i}{d\bar{E}} - \eta'(0) \\
&= (P(q^Z) - C'(q^Z))\frac{dq^i}{d\bar{E}} - K'(x^Z)\left(g'(q^Z)\frac{dq^i}{d\bar{E}} - 1\right) - \eta'(0) \\
&= (P(q^Z) - C'(q^Z) - K'(x^Z)g'(q^Z))\frac{dq^i}{d\bar{E}} + K'(x^Z) - \eta'(0),
\end{aligned}$$

where we use $g - x = \bar{E}$ (and thus, $dx^i/d\bar{E} = g'(dq^i/d\bar{E}) - 1$), and $(q^{EC}, x^{EC}) = (q^{EI}, x^{EI}) =$

(q^Z, x^Z) when $\bar{E} = 0$. Because $q^Z < q^{EC} < q^{EI}$ for all $\bar{E} > 0$, we obtain

$$\frac{dq^{EI}}{d\bar{E}}\Big|_{\bar{E}=0} > \frac{dq^{EC}}{d\bar{E}}\Big|_{\bar{E}=0}.$$

From (5.5), we obtain $P - C' - K'g' > P + P'q - C' - K'g' = 0$. Under these conditions, we obtain

$$\frac{\partial W^{EI}}{\partial \bar{E}}\Big|_{\bar{E}=0} > \frac{\partial W^{EC}}{\partial \bar{E}}\Big|_{\bar{E}=0}.$$

Because $W^{EI} = W^{EC}$ when $\bar{E} = 0$, we obtain Proposition 2. ■

The intuition behind the result is as follows. As we explained after Proposition 1, given $\bar{\alpha} > 0$, the firm has a stronger incentive to expand its output under emission intensity commitment than under emission cap commitment, because under emission intensity, the firm can increase the upper limit of emissions in the second stage (time-inconsistency problem). However, this problem does not exist when $\bar{E} = \bar{\alpha} = 0$. Therefore, $q^{EC} = q^{EI}$ and $x^{EC} = x^{EI}$ when $\bar{E} = \bar{\alpha} = 0$.

An increase in $\bar{\alpha}$ relaxes the restriction on emissions. This leads to an increase in emissions, resulting in larger disutility from the emissions (emission effect). However, by the assumption of emission equivalence between two regimes, the emission effect is the same for two regimes. An increase in $\bar{E}$ and $\bar{\alpha}$ affects q and x (allocation effect). As stated above, emission intensity commitment yields larger q and x than emission cap commitment does.

Given the emission level, under emission cap commitment, the marginal social cost of the reduction of emissions by reduction of q is P/g' and that by the increase of x is K'. The marginal private cost for meeting the constraint by the reduction of q for the firm is $(P+P'q)/g'$ and that by the increase of x is K'. Thus, both x and q chosen by the firm are too small for social welfare. Given the emission level, under emission intensity, the marginal private cost for meeting the constraint by the reduction of q for the firm is $(P + P'q)/(g' - \alpha)$ and that by the increase of x is K'. When α is small, both x and q chosen by the firm are still too small for social welfare, but both are larger than under emission cap commitment. Therefore, emission intensity commitment is better for social welfare than is emission cap commitment.

We believe that the most stringent case discussed in Proposition 2 is important. Under the Paris Climate Agreement, many countries, such as the UK, France, Germany, and Japan, plan to reduce CO2 emissions drastically by 2050 (about 80% reduction at least against a business-as-usual scenario). To achieve this goal, several industries, such as electric power and transport, an emission constraint that is close to zero emissions might be imposed. Thus, the most stringent case discussed in Proposition 2 might be realistic.

Next, we examine the opposite case, the loosest constraint case in which $\bar{E}$ is close to E^B.

Proposition 3 *Suppose that $\bar{E}$ is sufficiently close to E^B. Emission intensity commitment yields greater welfare than does emission cap commitment.*

Proof.

For $i = EC, EI$, we obtain

$$\begin{aligned}
\frac{\partial W^i}{\partial \bar{E}}\Big|_{\bar{E}=E^B} &= \frac{\partial W}{\partial q}\frac{dq^i}{d\bar{E}} + \frac{\partial W^i}{\partial x}\frac{dx^i}{d\bar{E}} + \frac{\partial W^i}{\partial E} \\
&= (P(q^B) - C'(q^B))\frac{dq^i}{d\bar{E}} - K'(x^B)\frac{dx^i}{d\bar{E}} - \eta'(\bar{E}) \\
&= (P(q^B) - C'(q^B))\frac{dq^i}{d\bar{E}} - \eta',
\end{aligned}$$

where we use $q^{EC} = q^{EI} = q^B$ and $x^{EC} = x^{EI} = 0$ when $\bar{E} = E^B$ and $K'(0) = 0$. Because $q^{EC} < q^{EI} < q^B$ for all $\bar{E} < E^B$, we obtain

$$\frac{dq^{EI}}{d\bar{E}}\Big|_{\bar{E}=E^B} > \frac{dq^{EC}}{d\bar{E}}\Big|_{\bar{E}=E^B}.$$

From (5.5), we obtain $P - C' > 0$. Under these conditions,

$$\frac{\partial W^{EI}}{\partial \bar{E}}\Big|_{\bar{E}=E^B} > \frac{\partial W^{EC}}{\partial \bar{E}}\Big|_{\bar{E}=E^B}.$$

Because $W^{EI} = W^{EC}$ when $\bar{E} = E^B$, we obtain Proposition 3. ■

We explain the intuition behind Proposition 3. Because of emission equivalence, the emission effect is the same between two regimes. When $\bar{E} = E^B$, $q^{EC} = q^{EI} = q^B$ and $x^{EC} = x^{EI} = 0$. Because $K'(0) = 0$, this abatement level is too low for social welfare, and a marginal reduction of emissions by an increase in x is much more efficient than that by a reduction in q for social welfare. In other words, given the emission, q is too large and x is too small for social welfare. A marginal decrease in $\bar{\alpha}$ increases x and reduces q under both emission cap commitment and emission intensity commitment, which improves welfare. The magnitude of this effect is stronger under emission intensity commitment. Note that $q^{EI} > q^{EC}$ and thus, $x^{EI} > x^{EC}$ for $\bar{E} \in (0, E^B)$.

In Propositions 2 and 3, we show that when the target level is close to the strictest and loosest cases, emission intensity commitment is better for social welfare than emission cap commitment. Emission intensity commitment stimulates production and mitigates the problem of suboptimal production and abatement investment, which improves welfare under emission equivalence. Because emission intensity commitment is better for social welfare in the two polar cases, it might be natural to guess that emission intensity commitment is better for any $\bar{E} \in (0, E^B)$. However, this is not true.

Let $\big(x^*(\bar{E}), q^*(\bar{E})\big)$ be the pair of the second–best abatement and output level (social optimum x and q given $E = \bar{E}$). The derivation is as follows. Given $\bar{E}$, the social planner's problem is

$$\begin{aligned} \max_{q,x} \quad & W = \int_0^q P(z)dz - C(q) - K(x) - \eta(\bar{E}) \\ s.t. \quad & \bar{E} = g(q) - x. \end{aligned}$$

The second-best output level, $q^*(\bar{E})$, is characterized by the following first-order condition:

$$\frac{\partial W}{\partial q} = P - C' - K'g' = 0. \tag{5.7}$$

$x^*(\bar{E})$ is derived from $\bar{E} = g(q^*) - x^*$.

As discussed above, $q^{EC}(\bar{E}) < q^*(\bar{E})$ and thus, $x^{EC}(\bar{E}) < x^*(\bar{E})$. Note that $q^{EC}(\bar{E})$ is derived $P + P'q - C' - K'g' = 0$. In the two polar cases (the strictest and loosest cases), $\big(x^{EC}(\bar{E}), q^{EC}(\bar{E})\big) = \big(x^{EI}(\bar{E}), q^{EI}(\bar{E})\big)$. Except for the two polar cases, $\big(x^{EC}(\bar{E}), q^{EC}(\bar{E})\big) < \big(x^{EI}(\bar{E}), q^{EI}(\bar{E})\big)$ holds. As long as $\big(x^{EC}(\bar{E}), q^{EC}(\bar{E})\big) < \big(x^{EI}(\bar{E}), q^{EI}(\bar{E})\big) < \big(x^*(\bar{E}), q^*(\bar{E})\big)$, the outcome under emission intensity commitment is closer to the second-best outcome than that under emission cap commitment, and thus, emission intensity commitment naturally yields greater welfare than emission intensity commitment. However, it is possible that $\big(x^{EI}(\bar{E}), q^{EI}(\bar{E})\big) > \big(x^*(\bar{E}), q^*(\bar{E})\big)$. Because emission intensity can yield excessive production and excessive abatement investment, emission cap commitment might be better than emission intensity commitment for social welfare.

We present an example showing that emission cap commitment could be better than emission intensity commitment for welfare. Suppose that the inverse demand is linear ($P = a - bq$), the marginal production cost is constant (normalized to zero), emissions are proportional to output ($g = eq$), and the abatement cost is quadratic ($K = kx^2/2$). In this example, $E^B = ae/2b$ and $\bar{\alpha} = e$ when $\bar{E} = E^B$.

Straightforward calculation yields the resulting welfare for each regime. Comparing W^{EI} with W^{EC}, we obtain the following result.

Proposition 4 *Suppose that $P = a - bq$, $C = 0$, $g = eq$, and $K = kx^2/2$. Then,*

$$W^{EI} > (<) W^{EC} \quad \textit{if} \;\; k < (>) \tilde{k},$$

where

$$\tilde{k} := \frac{b(2e^2 - 3e\bar{\alpha} + 3\bar{\alpha}^2 + \sqrt{4e^4 + 4e^3\bar{\alpha} + 5e^2\bar{\alpha}^2 - 18e\bar{\alpha}^3 + 9\bar{\alpha}^4})}{2e^2\bar{\alpha}(e - \bar{\alpha})},$$

$\lim_{\bar{E}\to 0} \tilde{k} = \lim_{\bar{E}\to E^B} \tilde{k} = \infty$, *$\tilde{k}$ is U-shaped with respect to $\bar{E}$, and $\tilde{k} \geq \underline{k} := (5b + b\sqrt{89})/2e^2$ for any $\bar{E} \in (0, E^B)$.*

Proof See the Appendix.

Figure 1 shows this result graphically (the case in which $a = 5$, $b = 1$, and $e = 2$).

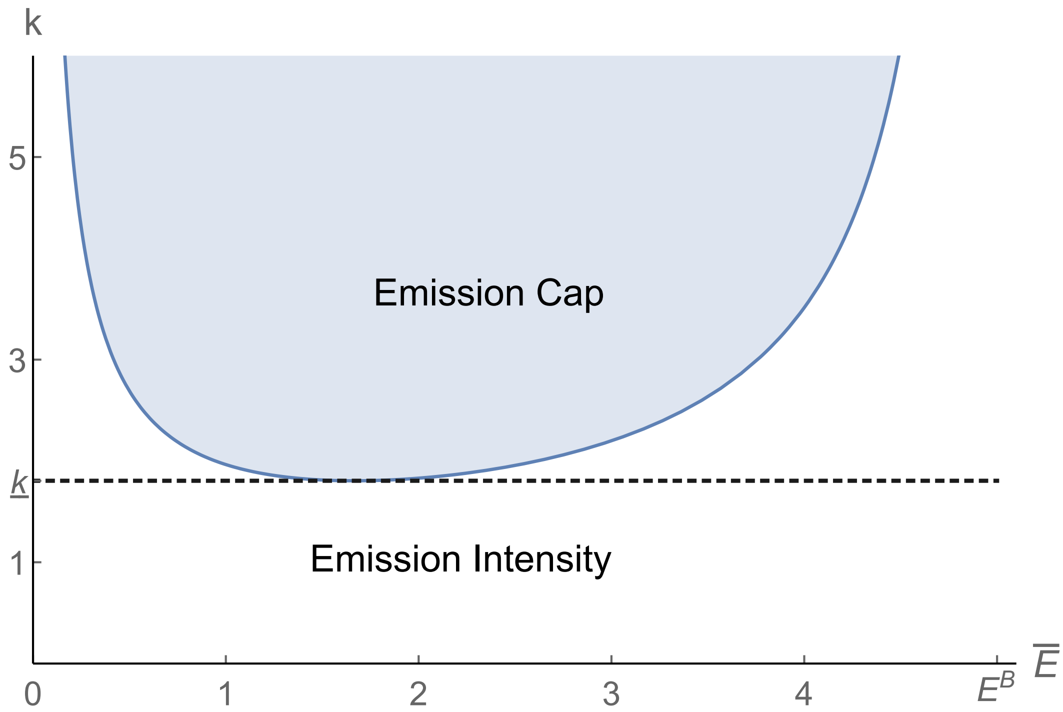

Figure 5.1: Welfare Comparison:

If k is not large, emission intensity commitment yields greater welfare regardless of $\bar{E}$. However, if k is large, emission cap commitment yields greater welfare, because x can be excessive under emission intensity commitment.

As discussed above, production and abatement can be excessive under emission intensity commitment, leading to Proposition 4. Figure 2 shows that x^* can be smaller than x^{EI}, although x^* is larger than x^{EI} regardless of $\bar{E}$ (the case in which $a = 5$, $b = 1$, $k = 3$, and $e = 2$). In other words, the abatement level under emission intensity commitment is too large (and the output level is also too large) to achieve the target level of emissions.

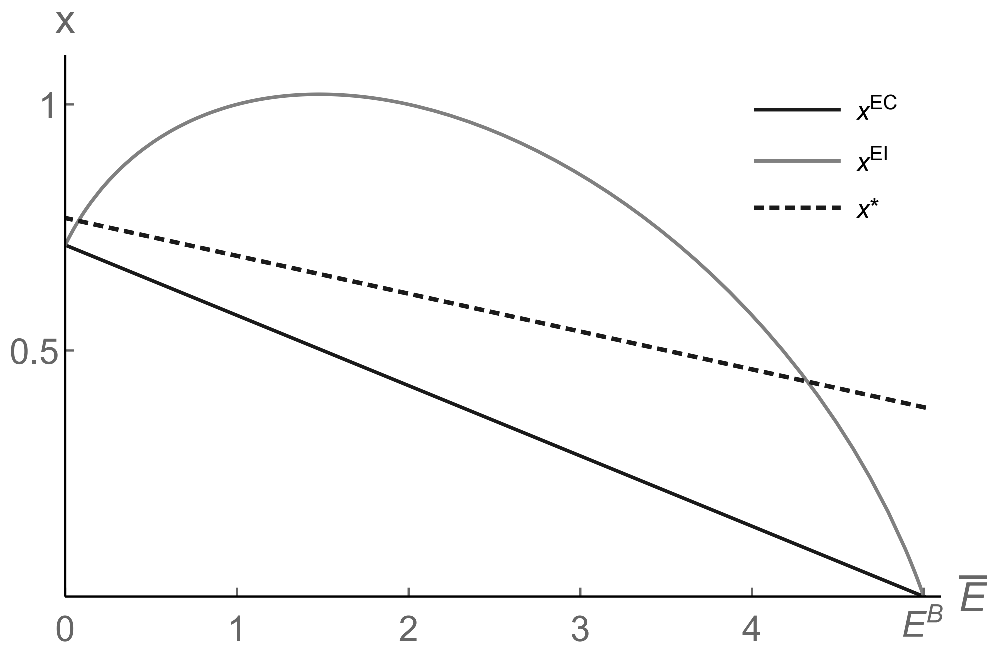

Figure 5.2: Abatement Level Comparison

5.4 Concluding Remarks

In this study, we compare two self-regulation tools, emission cap commitment and emission intensity commitment. We find that profit-maximizing firms always choose emission cap commitment. However, emission intensity commitment always yields greater consumer welfare and can yield greater welfare. Moreover, we present two cases in which emission intensity commitment yields greater welfare than emission cap commitment does: the case of the strictest target, which is close to a zero-emission target, and the case of the loosest target, which is close to business as usual. Our result suggests that the government should encourage the adoption of emission intensity commitment, especially to achieve a zero-emission society efficiently, because firms prefer emission cap commitment to emission intensity commitment, even when it is not desirable for welfare.

Our study neglects any uncertainty of demand or cost. If the firms commit to self-regulation before knowing the demand parameter, an increase of the degree of demand uncertainty increases the advantage of emission intensity commitment over emission cap commitment for both the welfare and profits of the firms. This is because the firms can expand (shrink) their output more flexibly under emission intensity commitment than under emission cap commitment when demand is high (low). We consider this is the reason that some companies, such as Japanese electric power companies, choose emission intensity commitments as their favored form of self-regulation. Comparing the two tools after introducing demand uncertainty is left to future research.

Appendix

Proof of Proposition 4

First, we consider the equilibrium outputs for each regime. From (5.3) and (5.5), we obtain

$$q^{EI} = \frac{a}{2b + k(e-\alpha)}, \quad q^{EC} = \frac{a + ke\bar{E}}{2b + e^2k}.$$

Substituting the equilibrium outputs into total surplus, we obtain

$$\begin{aligned} W^{EI}(\bar{\alpha}) &= \frac{a(a(3b + k(e-\bar{\alpha})^2) - 2\bar{\alpha}\eta(2b + k(e-\bar{\alpha})^2))}{2(2b + k(e-\bar{\alpha})^2)^2}, \\ W^{EC}(\bar{E}) &= \frac{(a^2 + 2ake\bar{E})(3b + ke^2) - \bar{E}(bk\bar{E}(4b + ke^2) + 2\eta(2b + ke^2)^2)}{2(2b + ke^2)^2}. \end{aligned}$$

Using $E^{EI}(\bar{\alpha}) = \bar{\alpha}q^{EI} = \bar{E}$, $W^{EC}(\bar{E})$ can be rewritten as a function of $\bar{\alpha}$. Thus, we obtain

$$W^{EI}(\bar{\alpha}) - W^{EC}(\bar{E}) = \frac{a^2k\bar{\alpha}(e-\bar{\alpha})H}{2(2b + k(e-\bar{\alpha})^2)^2(2b + ke^2)^2}$$

where $H := 4b^2 - k^2e^2\bar{\alpha}(e-\bar{\alpha}) + bk(2e^2 - 3e\bar{\alpha} + 3\bar{\alpha}^2)$. $W^{EI}(\bar{\alpha}) - W^{EC}(\bar{E})$ is positive if and only if $H > 0$ and

$$H > (<) 0 \text{ if } k < (>) \tilde{k} = \frac{b(2e^2 - 3e\bar{\alpha} + 3\bar{\alpha}^2 + \sqrt{4e^4 + 4e^3\bar{\alpha} + 5e^2\bar{\alpha}^2 - 18e\bar{\alpha}^3 + 9\bar{\alpha}^4})}{2e^2\bar{\alpha}(e-\bar{\alpha})}.$$

Remember that $\bar{\alpha}$ is determined by $E^{EI}(\bar{\alpha}) = \bar{E}$, thus, $\tilde{k}$ also depends on the emission target via $\bar{\alpha}$. It implies that $W^{EI}(\bar{\alpha}) > (<) W^{EC}(\bar{E})$ if $k < (>) \tilde{k}$. Because $\lim_{\bar{\alpha}\to 0}\tilde{k} = \lim_{\bar{\alpha}\to e}\tilde{k} = \infty$, we obtain $\lim_{\bar{E}\to 0}\tilde{k} = \lim_{\bar{E}\to E^B}\tilde{k} = \infty$.

Differentiating $\tilde{k}$ with $\bar{\alpha}$, we obtain

$$\frac{\partial\tilde{k}}{\partial\bar{\alpha}} = \frac{b(e - 2\bar{\alpha})(2e^2 + e\bar{\alpha} - \bar{\alpha}^2 + \sqrt{4e^4 + 4e^3\bar{\alpha} + 5e^2\bar{\alpha}^2 - 18e\bar{\alpha}^3 + 9\bar{\alpha}^4})}{\bar{\alpha}^2(e-\bar{\alpha})^2\sqrt{4e^4 + 4e^3\bar{\alpha} + 5e^2\bar{\alpha}^2 - 18e\bar{\alpha}^3 + 9\bar{\alpha}^4}}.$$

Because $\partial\tilde{k}/\partial\bar{\alpha}$ is negative (positive) when $\bar{\alpha} < (>)\ e/2$, $\tilde{k}(\bar{E})$ is U-shaped and is minimized at $\bar{\alpha} = e/2$. Because $\tilde{k}$ is minimized when $\bar{\alpha} = e/2$, we obtain $\underline{k} = (5b + b\sqrt{89})/2e^2$. Note that $\bar{\alpha}(\bar{E})$ is increasing, $\bar{\alpha}(0) = 0$, and $\bar{\alpha}(E^B) = e$. ■

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Chapter 6

Environmental Corporate Social Responsibility : A Note on the First-Mover Advantage under Price Competition*

6.1 Introduction

The industrial organization literature has extensively debated whether the first or second mover earns larger profits. As Gal-Or (1985) and Dowrick (1986) showed in symmetric duopolies, for strategic substitutes (complements), the first mover (second mover) has the advantage under the stability condition. Under standard condition, the strategies are strategic complements under price competition (Vives, 1999). Thus, under price competition, the second mover naturally enjoys an advantage. In this study, we incorporate environmental corporate social responsibility (ECSR) into the standard price competition model and show that this property does not holds.

ECSR has received increasing attention from broad research in social sciences. In particular, economic researchers have discussed this issue intensively (Liu *et al.*, 2015) because many listed firms are highly concerned about ECSR (KPMG, 2013). CDP (2013), for example, reported that some major companies such as ExxonMobil, Walt Disney, Walmart, and Microsoft use an internal (implicit) carbon price as an incentive and strategic planning tool, although their internal carbon prices differ significantly, ranging from 6 to 60 dollars per ton.[1]

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[1]We should say from 0 to 60 dollars because some firms not reported in CDP do not introduce explicit internal

In this study, we investigate Stackelberg competition after firms choose whether to adopt ESCR. We show that only the follower adopts ECSR. This follower's behavior increases the firms' profits, and further increases the leaders' profits. As a result, this price competition model shows a first-mover advantage. This result is in sharp contrast to the standard result showing a second-mover advantage under price competition.

This study contains another implication. It explains why profit-maximizing firms adopt ECSR and may explain why the degree of ECSR is different among firms. There are two well-known reasons why profit-maximizing firms adopt ECSR. One is that ECSR is connected with the advertisement or reputation of firms; thus, it eventually increases the demand, thereby earning the firms more profits (see Liu *et al.* (2015) and the works cited therein). The other is that voluntary agreement may serve as a countermeasure for the regulatory threat by the government, which allows firms to avoid more severe regulation (Maxwell *et al.*, 2000; Antweiler, 2003). Without those assumptions, we show that firms strategically choose ECSR because it serves as a commitment device.

This study is closely related to the literature using a delegation game with strategic reward contracts, which are a popular topic in the management science and industrial organization literature. Fershtman (1985), Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987) made groundbreaking theoretical contributions in this regard. They considered the separation of owners and managers and examined the following two-stage game. In the first stage, the owners write managers' reward contracts and announce them publicly. In the second stage, having read the contracts, the managers maximize payoffs. In these studies, the managers' rewards are proportional either to a linear combination of profits and outputs or to profits and firm revenues. Later, researchers investigated different types of strategic contracts based on relative performance or market share. In fact, management rewards often positively relate to firm revenues, market share, and/or relative performance, as well as their own profits. Because the model formulation in these studies provides an approximation of reality and explains much of the actual firm behaviors, it has become one of the most popular models in industrial organization and management science.

The incentive of such delegation has been analyzed in oligopoly theory, which is mainly focused on models of quantity competition (strategic substitute cases). Clearly, if we consider a model of price competition in this framework, the owners must attribute a negative weight to outputs,

carbon prices.

revenues, market share, or relative profits. It is, however, much less sustained from the empirical perspectives.[2] In our study, we discuss the commitment through adopting an ECSR policy. We believe that our formulation is more natural to address such firm's commitment for many industries than the standard delegation setting.[3]

6.2 The Model

We assume a standard differentiated duopoly with linear demand (Dixit, 1979). The quasilinear utility function of the representative consumer is $U(q_1, q_2) = \alpha(q_1 + q_2) - \beta(q_1^2 + 2\delta q_1 q_2 + q_2^2)/2 + y$, where y is the consumption of an outside good provided competitively with a unit price. Parameters α and β are positive constants, and $\delta \in (0, 1)$ represents the degree of product differentiation, where a smaller δ indicates a larger degree of product differentiation.

Firms 1 and 2 produce differentiated commodities for which the inverse demand function is given by $p_i = \alpha - \beta q_i - \beta\delta q_j$ $(i = 1, 2,\ i \neq j)$, where p_i and q_i are firm i's price and quantity, respectively. The common marginal production cost is constant and normalized to zero. Firm i's emission level producing a negative externality is ηq_i.[4]

The owners of firm i's payoff is π_i, and management's payoff is $V_i = \pi_i - \theta_i \eta q_i$, where $\theta_i (\geq 0)$ is an internal (implicit) emission price, representing the degree of ECSR and determined by the profit-maximizing owner of firm i.[5]

The game runs as follows. In the first stage, the owner of firm i independently chooses θ_i. In the second stage, firms choose their prices sequentially (Stackelberg competition).

[2]For empirical evidence, see Murphy (1985) and Coughlan and Schmidt (1985). These empirical studies suggest that negative weight on these variables is not realistic. Matsumura and Matsushima (2012) showed that firms put positive weight on a location-then-price model because it has a positive effect on location choice, and this effect dominates the negative effect on price choice.

[3]For example, most listed firms in Japan, as well as many Japanese economic associations, such as the Japan Association of Corporate Executives, the Japan Business Federation, the Japan Iron and Steel Federation, and the Federation of Electric Power Companies of Japan emphasize ECSR in their reports and websites.

[4]If we introduce an emission abatement activity that is independent of its production level, we obtain the same qualitative results.

[5]Our analysis is equivalent to the following scenario discussed in Graff Zivin and Small (2005) and Baron (2007). Firm i commits to donating a monetary amount to $\theta_i \eta y_i$ for environmental improvements, the management of firm i maximizes its net profit (the profit after subtracting the cost of donation), and the owner considers the corporate donation and private giving as perfect substitutes.

6.3 Results

First, we discuss the second-stage price competition. Firm 1 chooses its price and then firm 2 chooses its price. The first-order condition for firm 2 is

$$\frac{\partial V_2}{\partial p_2} = \frac{\alpha(1-\delta) + \eta\theta_2 - 2p_2 + \delta p_1}{\beta(1-\delta^2)} = 0.$$

The reaction function for firm 2 is

$$R_2(p_1) = \frac{\alpha(1-\delta) + \eta\theta_2 + \delta p_1}{2}.$$

Firm 1 maximizes $\pi_1(p_1, R_2(p_1)) - \theta_1\eta q_1(p_1, R_2(p_1))$ with respect to p_1. The first-order condition is

$$\frac{\alpha(-2+\delta+\delta^2) + \eta((\delta^2-2)\theta_1 - \delta\theta_2) + (4-2\delta^2)p_1}{2\beta(-1+\delta^2)} = 0.$$

The Stackelberg equilibrium is

$$\begin{aligned}
p_1 &= \frac{\alpha(-2+\delta+\delta^2) + \eta((-2+\delta^2)\theta_1 - \delta\theta_2)}{2(-2+\delta^2)} \\
p_2 &= \frac{\alpha(4-2\delta-3\delta^2+\delta^3) - \eta(\delta(-2+\delta^2)\theta_1 + (-4+\delta^2)\theta_2)}{4(2-\delta^2)} \\
\pi_1 &= \frac{(\alpha(-2+\delta+\delta^2) + \eta((-2+\delta^2)\theta_1 - \delta\theta_2))(\alpha(-2+\delta+\delta^2) - \eta((-2+\delta^2)\theta_1 + \delta\theta_2)))}{8\beta(-2+\delta^2)(-1+\delta^2)}, \\
\pi_2 &= \frac{H}{16\beta(-2+\delta^2)^2(1-\delta^2)},
\end{aligned}$$

where $H := (\alpha(4-2\delta-3\delta^2+\delta^3) + \eta(\delta(2-\delta^2)\theta_1 + (3\delta^2-4)\theta_2))(\alpha(4-2\delta-3\delta^2+\delta^3) - \eta(\delta(-2+\delta^2)\theta_1 + (\delta^2-4)\theta_2)$.

We now discuss the first-stage actions.[6] The owner of firm i chooses θ_i. The first-order conditions are

$$\begin{aligned}
\frac{\partial \pi_1}{\partial \theta_1} &= \frac{\eta^2(2-\delta^2)\theta_1}{4\beta(-1+\delta^2)} = 0, \\
\frac{\partial \pi_2}{\partial \theta_2} &= \frac{\eta(\delta^2(\alpha(4-2\delta-3\delta^2+\delta^3) - \eta\delta(-2+\delta^2)\theta_1) - \eta(16-16\delta^2+3\delta^4)\theta_2)}{8\beta(-2+\delta^2)^2(1-\delta^2)} = 0.
\end{aligned}$$

The equilibrium θ_i is

$$\theta_1 = 0,\ \theta_2 = \frac{\alpha(-1+\delta)\delta^2(\delta^2-2\delta-4)}{\eta(16-16\delta^2+3\delta^4)} := \theta^F > 0. \tag{6.1}$$

[6] In this study, we assume that firms choose θ simultaneously. Our results hold true if firms choose θ sequentially.

The resulting profits are

$$\begin{aligned}
\pi_1 &= \frac{\alpha^2(-1+\delta)(-2+\delta^2)(-8-4\delta+4\delta^2+\delta^3)^2}{2\beta(1+\delta)(4-3\delta^2)(-4+3\delta^2)} := \pi^L \\
\pi_2 &= \frac{\alpha^2(-1+\delta)(-4-2\delta+\delta^2)^2}{4\beta(1+\delta)(4-\delta^2)(-4+3\delta^3)} := \pi^F.
\end{aligned}$$

We obtain the following result.

Proposition 1 *Only the follower adopts ECSR, and this increases both firms' profits. The leader earns the larger profit (i.e., first-mover advantage appears).*

Proof First, we show that $\pi^L > \pi^F$.

$$\pi^L - \pi^F = \frac{\alpha^2(\delta-1)\delta^5\left(5\delta^3+4\delta^2-16\delta-16\right)}{4\beta(\delta+1)\left(4-3\delta^2\right)^2\left(\delta^2-4\right)^2} > 0.$$

Let us now check that ECSR increases both firms' profits. Firm 2 chooses $\theta_2 = \theta^F$ in equilibrium, and thus, firm 2's profit must be larger than that when $\theta_1 = \theta_2 = 0$. Firm 1's profit is less than firms 2's if $\theta_1 = \theta_2 = 0$ (i.e., the second mover has the advantage under standard price competition setting). Because $\pi^L > \pi^F$ and ECSR increases firm 2's profit, it increases firm 1's profit as well. ■

Each firm has an incentive to commit to a higher price in order to induce the rival's higher price. The price leader chooses its price, and then the follower chooses its price. Thus, the leader can directly commit to a higher price without using ECSR. Therefore, choosing a positive θ has no strategic value for the leader. In contrast, for the follower, ECSR has strategic value because the follower can commit to setting a higher price by choosing a positive θ as it increases the marginal cost. Observing a positive θ, the leader expects the higher price of the follower and also sets a price higher than that without the follower's ECSR, because the leader's pricing strategy is strategic complement. Firm 2's strategic behavior (less aggressive pricing rather than profit-maximizing pricing) benefits firm 1 and yields the first-mover advantage.

This result is in sharp contrast to the standard result under price competition. Gal-Or (1985) showed that when two symmetric firms move sequentially under price competition, the follower enjoys the second-mover advantage. Although we assume that the firms are identical, there is a difference between the leader and the follower in terms of adopting ECSR. This generates heterogeneity between the two firms and changes the profit ranking between the first and second movers.

Gal-Or (1987) and Shinkai (2000) also suggested that the standard result shown in Gal-Or (1985) may not hold under an incomplete information game. They assumed that the first mover has informational advantage, and the action of the first mover reveals its information. They showed that this signaling effect may change the profit ranking among the first and second movers. The mechanism of our study is completely different from theirs because we discuss the complete information game.

Ono (1978) considered a duopoly in a homogeneous product market with cost asymmetry. He showed that the firm with lower (higher) cost prefers the role of the leader (follower) if the cost difference is sufficiently large. Van Damme and Hurkens (2004) and Amir and Stepanova (2006) showed that this holds true in a differentiated product market. In contrast, Hirata and Matsumura (2011) presented another duopoly model in a homogeneous product market in which the firm with higher (lower) cost prefers the role of the leader (follower) if the cost difference is sufficiently large. In other words, they have already shown that in some price competition models, there is no clear profit ranking (neither unanimous first-mover nor second-mover advantage exits) under large cost difference. There are two important differences between our model and theirs. One is that we do not assume exogenous cost difference between the two firms. The other is that in our model, both firms prefer the role of the leader.

We now discuss the welfare implications of ECSR. The total social surplus (firm profits plus consumer surplus minus the loss from the externality) is given by

$$SW = p_1q_1 + p_2q_2 + \left[\alpha(q_1+q_2) - \frac{\beta(q_1^2 + 2\delta q_1q_2 + q_2^2)}{2} - (p_1q_1 + p_2q_2)\right] - \eta(q_1+q_2).$$

Without ECSR, the total social surplus is

$$SW^N = \frac{\alpha\left(\alpha\left(5\delta^5 + 23\delta^4 - 28\delta^3 - 96\delta^2 + 32\delta + 96\right) - 8\eta\left(\delta^5 + 3\delta^4 - 6\delta^3 - 14\delta^2 + 8\delta + 16\right)\right)}{32\beta(\delta+1)\left(\delta^2-2\right)^2}$$

When the follower adopts ECSR, the total social surplus is

$$\begin{aligned} SW^E \quad = \quad & \frac{\alpha^2\left(5\delta^9 + 71\delta^8 - 92\delta^7 - 696\delta^6 + 496\delta^5 + 2336\delta^4 - 896\delta^3 - 3200\delta^2 + 512\delta + 1536\right)}{8\beta(\delta+1)\left(4-3\delta^2\right)^2\left(\delta^2-4\right)^2} \\ & - \frac{\alpha\eta\left(\delta^5 + 7\delta^4 - 12\delta^3 - 32\delta^2 + 16\delta + 32\right)}{2\beta(\delta+1)\left(\delta^2-4\right)\left(3\delta^2-4\right)}. \end{aligned}$$

We obtain

$$SW^E - SW^N = \frac{\alpha\delta^2\left(\delta^3 - 3\delta^2 - 2\delta + 4\right)G}{32\beta(\delta+1)\left(4-3\delta^2\right)^2\left(\delta^2-4\right)^2\left(\delta^2-2\right)^2},$$

where $G := 8\eta(3\delta^8 - 6\delta^7 - 34\delta^6 + 44\delta^5 + 136\delta^4 - 96\delta^3 - 224\delta^2 + 64\delta + 128) - \alpha(25\delta^8 - 2\delta^7 - 240\delta^6 + 24\delta^5 + 800\delta^4 - 32\delta^3 - 1088\delta^2 + 512)$. This is positive if and only if

$$\eta > \overline{\eta} := \frac{\alpha\left(25\delta^8 - 2\delta^7 - 240\delta^6 + 24\delta^5 + 800\delta^4 - 32\delta^3 - 1088\delta^2 + 512\right)}{8\left(3\delta^8 - 6\delta^7 - 34\delta^6 + 44\delta^5 + 136\delta^4 - 96\delta^3 - 224\delta^2 + 64\delta + 128\right)} > 0.$$

These lead to the following proposition.

Proposition 2 *ECSR improves welfare if and only if $\eta > \overline{\eta}$.*

There is a tradeoff between the environment and the anti-competitive effect. On one hand, ECSR reduces emissions that yield negative externalities. On the other hand, it raises the prices and reduces consumer surplus. Thus, it does not always have a beneficial effect on welfare. Because the equilibrium level of $\theta^F\eta$, the additional marginal cost due to ECSR, is independent of η, the equilibrium prices (and outputs) are independent of η. From the viewpoint of social welfare, the optimal outputs are decreasing in η. Therefore, the equilibrium outcome is more likely excessive when η is larger. If the degree of negative externalities is significant, the emission-reducing effect dominates the price-raising effect, and thus ECSR benefits welfare.

In energy-intensive industries, such as electricity, steel, cement, and some other heavy industries, the negative externality is significant, and thus the welfare improving effect of ECSR may be prominent. However, in industries with less significant negative externality (industries with low emission per output), it is possible that the price-raising effect dominates the emission-reducing effect. In this context, estimating the degree of negative externalities in industries (e.g., emission per unit of outputs) is important.[7]

Finally, we discuss outcomes for endogenous timing in the second-stage competition (i.e., when the first mover identity is endogenous). Consider the following model. The first stage remains the same as in the basic model; however, in the second stage, firms experience Hamilton and Slutsky's (1990) observable delay game.[8] As Hamilton and Slutsky (1990) showed, if two firms have the same costs, two Stackelberg equilibria (either firm 1 or firms 2 is the leader) exist. As Amir and Stepanova (2006) demonstrated, if the two firms have different costs (in our model if $\theta_1 \neq \theta_2$), the firm with the lower cost becomes the leader.[9] Thus, given $\theta_1 = 0$, firm 2 becomes the follower

[7] For the empirical works on this problem, see Holland *et al.* (2016).

[8] The observable delay game is the most popular model among endogenous timing games and has been adopted extensively in various contexts. See Pal (1998), Bárcena-Ruiz (2007), and Matsumura and Ogawa (2014).

[9] Strictly speaking, two Stackelberg equilibria can exist, but the equilibrium with the lower-cost firm's leadership is risk dominant. In the context of price leadership, Ono (1978) first pointed out that the firm with lower costs becomes the price leader.

unless it chooses $\theta_2 = 0$. If it chooses $\theta_2 = 0$ and may become the leader, the leader's profit when $\theta_1 = \theta_2 = 0$ is less than π^F (because of the second-mover advantage under the standard price competition). If firm 2 becomes the follower, choosing $\theta_2 = \theta^F$ is optimal. Therefore, firm 2's best reply is choosing $\theta_2 = \theta^F$, thus becoming the follower in the subsequent game. Under these conditions, Stackelberg competition appears in equilibrium if the firms' roles are endogenous. Therefore, Stackelberg competition is natural in this context.

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Chapter 7

Comparing Welfare and Profit in Quantity and Price Competition within Stackelberg Mixed Duopolies*

7.1 Introduction

The literature contains extensive comparisons between price and quantity competition. In oligopolies of private firms, price competition is stronger, yielding lower profits and greater welfare than in the case of quantity competition.[1] Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly in which a welfare-maximizing public firm competes against a profit-maximizing private firm.[2] They showed that price competition yields larger profit for the private firm and greater welfare than quantity competition. In other words, welfare ranking is common with private duopolies but profit ranking is the opposite.

The literature on Cournot–Bertrand comparison in mixed oligopolies has become rich and di-

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[1] See Shubik and Levitan (1980) and Vives (1985). Introducing some externality effects can undoubtedly reverse the welfare ranking. For example, if we introduce a negative externality associated with production, a lower output level under quantity competition can yield greater welfare. In this study, we completely neglect this type of technological externality.

[2] Most countries have state-owned public firms with substantial influence on their market competitors. Such mixed oligopolies occur in various industries, such as airlines, steel, automobile, railway, natural gas, electricity, postal services, education, hospital, home loan, and banking. Analyses of mixed oligopolies date back to Merrill and Schneider (1966). Their study and many others in the field assume that a public firm maximizes welfare (consumer surplus plus firm profits), while private firms maximize profits. For examples of mixed oligopolies and recent developments in research in this field, see Chen (2017), Ishida and Matsushima (2009),

verse recently. Haraguchi and Matsumura (2014) showed that Ghosh and Mitra's (2010) result holds, regardless of the nationality of the private firm. Scrimitore (2014) adopted Matsumura's (1998) partial privatization approach and considered the optimal degree of privatization. Her findings showed that under optimal privatization policies, Cournot competition could yield higher profits for private firms than under Bertrand competition while welfare ranking is not (Bertrand yields greater welfare). Haraguchi and Matsumura (2016) investigated an oligopoly model and showed that the profit-ranking can be reverted if the number of private firms is large, whereas the welfare-ranking is not. The aforementioned studies, however, assumed that firms play simultaneous-move games.

Another strand of the literature related to this study is on Stackelberg mixed oligopolies. The literature on mixed oligopolies contains intense discussions of both public leadership and private leadership models.[3] Wang and Mukherjee (2012) and Wang and Lee (2013) considered public leadership in a homogeneous product market and showed that the public leadership benefits total social surplus but is less beneficial for consumer welfare than public monopoly.[4] Gelves and Heywood (2013) found that mergers of public and private firms can improve welfare under public leadership. Pal (1998) showed that welfare is greater in the private leadership game than in the public leadership and simultaneous-move games, and Ino and Matsumura (2010) showed it in a free-entry market. However, all of these studies assumed quantity competition and did not discuss the price–quantity comparison.

In this study, we consider two sequential-move games, public leadership and private leadership games, and revisit the price–quantity comparison in mixed duopolies. In private duopolies with sequential move, Boyer and Moreaux (1987) showed that quantity competition is more profitable

[3]Both public leadership and private leadership models are important for analyzing Japanese financial markets, which are typical examples of mixed oligopolies. Until the 1980s, public enterprises played a leading role in the Japanese economy. It was believed that lending by public financial institutions (e.g., the Development Bank of Japan) had a pump-priming effect on private bank lending. Furthermore, public financing occupied an important position in Japanese financial markets for over 40 years (Horiuchi and Sui, 1993). The Koizumi Cabinet (April 2001–September 2006) changed this by declaring that public firms should play a complementary role to private firms, with the latter leading the markets rather than the former. Consequently, major public institutions were substantially downscaled. This situation can be described by the private leadership model (Ino and Matsumura, 2010, Matsumura and Ogawa, 2017). However, public institutions recently begun to lead Japanese markets once more. Newly established public financial institutions such as the Industrial Revitalization Corporation of Japan, the Enterprise Turnaround Initiative Corporation of Japan, and the Regional Economy Vitalization Corporation of Japan play leading roles in financial markets (Matsumura and Ogawa, 2017). The public leadership model is also a useful means to investigate this situation.

[4]They established another great contribution by showing that each private firm's profit can be increasing with the number of private firms in their mixed oligopolies. For this discussion, see also Matsumura and Sunada (2013).

and price competition provides higher total welfare, whatever the role (leader, follower). Thus, we can naturally suppose that in mixed duopolies, the two sequential-move games also provide the same welfare and profit ranking. This supposition is, however, not correct. The two sequential-move games provide the same profit ranking as the simultaneous-move game, but the private leadership game yields a different welfare ranking from that of the simultaneous-move and public leadership games.

First, we analyze a model in which the private (public) firm is the leader (follower). In the context of mixed oligopolies, the public firm's acceptance of the follower role often improves welfare (Pal, 1998; Matsumura, 2003a; Ino and Matsumura, 2010; Matsumura and Ogawa, 2017). Many stated that the public firm should play a complementary role to private firms, and the role of follower is adequate from this viewpoint. In this study, we show that when the public firm is the follower, quantity competition is stronger than price competition, resulting in a smaller profit for the private firm. This result is in accordance with that in the simultaneous-move game. However, the welfare ranking can be reversed. When the private firm is domestic (foreign), quantity (price) competition yields greater welfare than price (quantity) competition. In other words, the welfare ranking is crucially dependent on the nationality of the private firm.[5]

Next, we analyze a model in which the public (private) firm is the leader (follower). We find the same profit and welfare rankings as in the simultaneous-move game, that is, price competition yields higher profit for the private firm and greater welfare, regardless of the nationality of the private firm.

We then introduce the nonnegative profit constraint for the public firm. The literature on mixed oligopolies shows that the public firm's welfare-maximizing behavior may yield negative profits for the public firm.[6] However, as Estrin and de Meza (1995), Ishida and Matsushima (2009), and Wang and Tomaru (2015) discussed, we often observe that the nonnegative profit constraint is imposed on public firms. We find that the welfare and profit ranking remains unchanged under public leadership. By contrast, under private leadership, the nonnegative profit constraint affects the welfare ranking. This constraint improves welfare with quantity competition, while welfare remains unchanged with price competition. As a result, quantity competition more likely yields

[5]The nationality of private firms is often crucial in shaping mixed oligopolies; refer to the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). For recent developments in this field, refer to Bárcena-Ruiz and Garzón (2005a,b), Han and Ogawa (2008), Lin and Matsumura (2012), and Cato and Matsumura (2015).

[6]See the studies mentioned in footnote 5.

greater welfare than price competition.

Finally, we discuss the endogenous competition structure (endogenous price-quantity choice) as discussed by Singh and Vives (1984). They formulated a two-stage game. In the first stage, each firm simultaneously chooses a price or quantity contract. In the second stage, after observing the rival's choice in the first stage, each firm simultaneously chooses price or quantity according to the first stage choice. They investigated a private duopoly and showed that both firms choose the quantity contract. Cournot competition therefore appears in equilibrium. Matsumura and Ogawa (2012) investigated this endogenous competition structure in a mixed duopoly and showed that both firms choose the price contract. Bertrand competition therefore appears in equilibrium.[7]

We investigate this problem in public and private leadership games.[8] We consider two time lines: one where two firms simultaneously choose a price or quantity contract before facing Stackelberg competition, and the other in which the leader chooses between price or quantity first, before the follower chooses after observing the leader's price or quantity. We find that in both timelines, under both public and private leadership, price competition appears in equilibrium, regardless of whether quantity or price competition yields greater welfare. This indicates that equilibrium competition structure can be inefficient under private leadership.

7.2 Model

We adopt a standard duopoly model with differentiated goods and linear demand (Dixit, 1979).[9] The quasi-linear utility function of the representative consumer is

$$U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{\beta}{2}(q_0^2 + 2\delta q_0 q_1 + q_1^2) + y, \tag{7.1}$$

where q_0 is the consumption of good 0 produced by the public firm, q_1 is the consumption of good 1 produced by the private firm, and y is the consumption of an outside good provided competitively

[7]However, not all studies on mixed oligopolies support this result. Chirco and Scrimitore (2013), Chirco *et al.* (2014), Scrimitore (2013), and Haraguchi and Matsumura (2016) suggested that price competition may fail to be an equilibrium outcome.

[8]Because our primary aim is to revisit price–quantity comparisons in Stackelberg mixed duopolies, we do not endogenize the timing of their action. Although many papers discussed endogenous timing in mixed oligopolies, endogenizing the timing is out of scope of this study. Pal (1998) began the discussion of endogenous timing in mixed oligopolies. Many papers, such as those by Pal (1998), Matsumura (2003a), and Matsumura and Ogawa (2010) showed that private leadership is more likely to appear in mixed oligopolies. However, Matsumura (2003b) and Matsumura and Ogawa (2017) showed examples in which public leadership is more likely to appear in equilibrium.

[9]This demand function is popular in the literature on mixed oligopolies. Refer to Bárcena-Ruiz (2007) and Ishida and Matsushima (2009).

(with a unitary price). Parameters α and β are positive constants and $\delta \in (0,1)$ represents the degree of product differentiation, where a smaller δ indicates a larger degree of product differentiation. The inverse demand functions for goods $i = 0, 1$ with $i \neq j$ are

$$p_i = \alpha - \beta q_i - \beta\delta q_j, \tag{7.2}$$

where p_i is the price of firm i.

The marginal cost of production is constant for both firms. We denote the marginal cost of firm i with c_i, assuming $\alpha > c_0 \geq c_1$. In addition, we assume that α is sufficiently large and that $c_0 - c_1$ is not too large to assure interior solutions in the following games. Firm 0 is a state-owned public firm whose payoff is the domestic social surplus (welfare). This is given by

$$SW = (p_0 - c_0)q_0 + (1-\theta)(p_1 - c_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right], \tag{7.3}$$

where $\theta \in [0,1]$ is the ownership share of foreign investors in firm 1, which is potentially affected by policymakers acting on capital liberalization. Firm 1 is a private firm and its payoff is its own profit:

$$\pi_1 = (p_1 - c_1)q_1. \tag{7.4}$$

7.3 Private leadership: Public firm as the Stackelberg follower

In this section, we analyze a model in which firm 0 (1) is the follower (leader). First, we discuss quantity competition. Firm 1 chooses its quantities and then firm 0 moves after observing q_1. The first-order condition for firm 0 is[10]

$$\frac{\partial SW}{\partial q_0} = \alpha - c_0 - \beta(q_0 + (1-\theta)\delta q_1) = 0. \tag{7.5}$$

From (7.5), we obtain the following reaction function for firm 0:

$$R_0(q_1) = \frac{\alpha - c_0 - \beta(1-\theta)\delta q_1}{\beta}. \tag{7.6}$$

Firm 1 maximizes its profit, $\pi_1(q_1, R_0(q_1))$, with respect to q_1. The first-order condition for firm 1 is

$$\alpha - c_1 - \delta(\alpha - c_0) - 2\beta(1 - (1-\theta)\delta^2)q_1 = 0. \tag{7.7}$$

[10] All of the second-order conditions in this study are satisfied.

Let the superscript "FQ" denote the equilibrium outcome of this game, where "F" means public followership (private leadership) and "Q" means quantity competition. From (7.7), we obtain

$$q_1^{FQ} = \frac{(\alpha - c_1) - (\alpha - c_0)\delta}{2\beta(1-(1-\theta)\delta^2)}. \tag{7.8}$$

Substituting it into (7.6), we obtain

$$q_0^{FQ} = \frac{(\alpha - c_0)(2-(1-\theta)\delta^2) - (\alpha - c_1)(1-\theta)\delta}{2\beta(1-(1-\theta)\delta^2)}. \tag{7.9}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FQ} = \frac{(\alpha - c_1 - (\alpha - c_0)\delta)^2}{4\beta(1-(1-\theta)\delta^2)}, \tag{7.10}$$

$$SW^{FQ} = \frac{H_1}{8\beta(1-(1-\theta)\delta^2)^2}, \tag{7.11}$$

where the Appendix reports H_1 and other constants.

Next, we discuss price competition. The direct demand for good i is given by

$$q_i = \frac{\alpha - \alpha\delta - p_i + \delta p_j}{\beta(1-\delta^2)} \ (i = 1, 2, i \neq j). \tag{7.12}$$

After observing p_1, the follower, firm 0, chooses p_0. The first-order condition is

$$\frac{\partial SW}{\partial p_0} = \frac{c_0 - p_0 + \delta(1-\theta)(p_1 - c_1)}{\beta(1-\theta)} = 0. \tag{7.13}$$

From (7.13), we obtain the following reaction function of firm 0.

$$R_0(p_1) = c_0 + \delta(1-\theta)(p_1 - c_1). \tag{7.14}$$

The leader, firm 1, maximizes its profit, $\pi_1(p_1, R_0(p_1))$. The first-order condition of firm 1 is

$$\frac{\alpha(1-\delta) + \delta c_0 + (1-2(1-\theta)\delta^2)c_1 - 2(1-\delta^2+\theta\delta^2)p_1}{\beta(1-\theta)} = 0. \tag{7.15}$$

Let the superscript "FP" denote the equilibrium outcome of this game, where "P" indicates price competition. From (7.15), we obtain

$$p_1^{FP} = \frac{\alpha(1-\delta) + \delta c_0 + (1-2(1-\theta)\delta^2)c_1}{2-2(1-\theta)\delta^2}. \tag{7.16}$$

Substituting it into (7.14), we obtain

$$p_0^{FP} = \frac{(1-\theta)\delta(\alpha(1-\delta)-c_1)+(2-(1-\theta)\delta^2)c_0}{2-2(1-\theta)\delta^2}. \tag{7.17}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FP} = \frac{(\alpha(1-\delta)+\delta c_0 - c_1)^2}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)}, \tag{7.18}$$

$$SW^{FP} = \frac{H_2}{8\beta(1-\delta^2)(1-(1-\theta)\delta^2)^2}. \tag{7.19}$$

We now compare the welfare and profit levels in these two games. We address how the leader–follower structure affects the profit and welfare rankings in the mixed duopoly.

Proposition 1 *Consider Stackelberg competition with private leadership. (i) Quantity competition yields greater welfare than price competition if and only if the foreign ownership share in the private firm is below the threshold value $\bar{\theta}(\delta) \in (0,1)$. (ii) The private firm obtains greater profit under price competition than under quantity competition regardless of θ.*

Proof. Comparing social surplus under price competition and quantity competition, we obtain

$$SW^{FQ} - SW^{FP} = \frac{(\alpha - c_1 - (\alpha - c_0)\delta)^2\delta^2\left(1-\theta^2\delta^2+2\theta\left(\delta^2-1\right)-\delta^2\right)}{8\beta(1-\delta)(1+\delta)\left(1-(1-\theta)\delta^2\right)^2}.$$

This shows that $SW^{FQ} > SW^{FP}$ holds if $0 < \theta < \bar{\theta}(\delta) = \left(\sqrt{(1-\delta^2)} - (1-\delta^2)\right)/\delta^2$. Similarly, straightforward computations show

$$\pi_1^{FP} - \pi_1^{FQ} = \frac{\delta^2(\alpha - c_1 - (\alpha - c_0)\delta)^2}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)} > 0. \blacksquare$$

As Haraguchi and Matsumura (2014) showed, price competition yields greater welfare and profit for the private firm than quantity competition, regardless of the nationality of the private firm in the simultaneous-move game. Proposition 1(i) is in sharp contrast to the result of the simultaneous-move game, whereas Proposition 1(ii) suggests that the profit ranking is in accordance with it. We explain the intuition behind Proposition 1(i).

As the Stackelberg leader, the private firm has an incentive to make the public firm less aggressive (choosing a smaller output or higher price) to increase its profit. Thus, under quantity (price) competition, the private firm chooses a larger output (higher price) than in the simultaneous-move case. The larger output (higher price) of the private firm improves (reduces) the consumer surplus.

Therefore, welfare is greater under quantity competition than under price competition when the private firm is domestic.

However, when θ is positive, the private firm's higher output increases its profit, thus increasing the outflow to foreign investors, which cancels the welfare gain under quantity competition. This effect is more significant when the foreign ownership share in the private firm is larger. Therefore, the welfare ranking is again reversed in this case.

7.4 Public leadership: Public firm as the Stackelberg leader

In this section, we analyze a Stackelberg model in which firm 0 (1) is the leader (follower). First, we discuss quantity competition. Firm 1 maximizes its own profit given q_0. The first-order condition for firm 1 is given by

$$\frac{\pi_1}{\partial q_1} = \alpha - c_1 - \beta(\delta q_0 + 2q_1) = 0. \tag{7.20}$$

From (7.20), we obtain the following reaction function of firm 1:

$$R_1(q_0) = \frac{\alpha - c_1 - \beta\delta q_0}{2\beta}. \tag{7.21}$$

Considering the reaction function $R_1(q_0)$, firm 0 maximizes domestic welfare. The first-order condition for firm 0 is

$$\frac{1}{4}(4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta - \beta(4 - 3\delta^2 + 2\theta\delta^2)q_0) = 0. \tag{7.22}$$

Let the superscript "LQ" denote the equilibrium outcome of this game, where "L" indicates public leadership (private followership) and "Q" represents quantity competition. From (7.22), we obtain

$$q_0^{LQ} = \frac{4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{7.23}$$

Substituting it into (7.21), we obtain

$$q_1^{LQ} = \frac{2((\alpha - c_1) - \delta(\alpha - c_0))}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{7.24}$$

Substituting these equilibrium quantities into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LQ} = \frac{4((\alpha - c_1) - \delta(\alpha - c_0))^2}{\beta(4 - (3 - 2\theta)\delta^2)^2}, \tag{7.25}$$

$$SW^{LQ} = \frac{H_3}{2\beta\,(4 - (3 - 2\theta)\delta^2)}. \tag{7.26}$$

Next, we discuss price competition. After observing p_0, firm 1 chooses p_1 to maximize its own profit. The first-order condition for firm 1 is

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\alpha(1-\delta) + c_1 + \delta p_0 - 2p_1}{\beta(1-\delta^2)} = 0. \tag{7.27}$$

From (7.27), we obtain the following reaction function for firm 1:

$$R_1(p_0) = \frac{\alpha(1-\delta) + c_1 + \delta p_0}{2}. \tag{7.28}$$

The leader, firm 0, maximizes domestic welfare with respect to p_0. The first-order condition for firm 0 is

$$\frac{\delta(1-2\theta)(\alpha(1-\delta) - c_1) + 2c_0\left(2-\delta^2\right) - (2\delta^2\theta - 3\delta^2 + 4)p_0}{\beta\left(\delta^2 - 1\right)} = 0. \tag{7.29}$$

Let the superscript "LP" denote the equilibrium outcome of this game. From (7.29), we obtain

$$p_0^{LP} = \frac{\delta(1-2\theta)(\alpha(1-\delta) - c_1) + 2c_0\left(2-\delta^2\right)}{4 - \delta^2(3-2\theta)}. \tag{7.30}$$

Substituting it into (7.28), we obtain

$$p_1^{LP} = \frac{\left(2-\delta^2\right)\left(\alpha(1-\delta) + c_0\delta\right) + 2c_1\left(1-(1-\theta)\delta^2\right)}{4 - \delta^2(3-2\theta)}. \tag{7.31}$$

Substituting these equilibrium prices into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LP} = \frac{\left(2-\delta^2\right)^2\left((\alpha - c_1) - (\alpha - c_0)\delta\right)^2}{\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)^2}, \tag{7.32}$$

$$SW^{LP} = \frac{H_4}{2\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)}. \tag{7.33}$$

We now compare welfare and profit levels in these two games.

Proposition 2 *Consider Stackelberg competition with public leadership. (i) Price competition yields greater welfare than quantity competition. (ii) The private firm obtains greater profit under price competition than under quantity competition.*

Proof. Comparing the social surplus and private profit under both types of competition, we obtain

$$SW^{LP} - SW^{LQ} = \frac{\delta^2((\alpha - c_1) - (\alpha - c_0)\delta)^2}{2\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)} > 0$$

$$\pi_1^{LP} - \pi_1^{LQ} = \frac{\delta^4((\alpha - c_1) - (\alpha - c_0)\delta)^2}{\beta(1-\delta^2)\left(4-(3-2\theta)\delta^2\right)^2} > 0. \quad \blacksquare$$

As the Stackelberg leader, the public firm has an incentive to make the private firm more aggressive (choosing larger output or lower price) to improve welfare. Thus, under quantity (price) competition, the public firm chooses a smaller output (lower price) than in the simultaneous-move case. Therefore, the private firm's profit is larger (smaller) than in the simultaneous-move case under quantity (price) competition. Thus, the public firm's strategic behavior as the Stackelberg leader reduces the profit advantage of price competition. Nevertheless, profit ranking is not reversed. A public firm's lower price reduces the resulting output of the private firm, and thereby reduces welfare. Therefore, the public firm still sets a higher price under price competition than the resulting price under quantity competition, and price competition thus yields a larger profit for the private firm.

We now explain the intuition of Proposition 2(i). Under quantity competition, the private firm's larger output improves welfare, while (given the output of the private firm) a smaller output from the public firm reduces welfare. Under price competition, the lower prices of both the public and private firms improve welfare. Thus, the welfare-improving effect of the public firm's strategic behavior is stronger under price competition than under quantity competition. Therefore, the welfare ranking is not reversed.

Table 7.1: Time structure and welfare and profit rankings

	Public Follower	Public Leader	Simultaneous move
Social Welfare	$SW^{FP} \gtreqless SW^{FQ}$	$SW^{LP} > SW^{LQ}$	$SW^{P} > SW^{Q}$
Profit	$\pi^{FP} > \pi^{FQ}$	$\pi^{LP} > \pi^{LQ}$	$\pi^{P} > \pi^{Q}$

NOTE: $SW^{FP} \geq (<) SW^{FQ}$ depends on the degree of foreign penetration, θ. See Proposition 1 in detail. The result under simultaneous move is obtained by Ghosh and Mitra (2010).

7.5 Nonnegative profit constraint

In the previous sections, we allowed the public firm to choose the price or quantity without any constraint. As a result, the public firm's equilibrium profit can be negative. However, we often observe that the nonnegative profit constraint is imposed on public firms. The equilibrium profit is positive under private leadership if firms face price competition, and this constraint is not binding. By contrast, the equilibrium profit of the public firm is negative under private leadership when firms face quantity competition and $\theta > 0$. In addition, the equilibrium profit of the public firm is negative under public leadership if $\theta > 1/2$, regardless of whether the firms face price or quantity competition.

We find that under public leadership, introducing the nonnegative constraint in the public firm's profit affects neither the profit nor welfare ranking between price and quantity competition (i.e., welfare is greater and the private firm's profit is larger under price competition than under quantity competition).[11] However, under private leadership, introducing this constraint increases the private firm's profit and improves welfare when firms face quantity competition, whereas both remain unchanged when firms face price competition. Therefore, imposing the nonnegative constraint on the public firm's profit strengthens the profit and welfare advantage of quantity competition.

We now proceed to the formal analysis of private leadership. First, we consider quantity competition. Given q_1, firm 0 chooses its quantity to maximize social welfare subject to the nonnegative profit condition:

$$\max_{q_0} SW \quad \text{subject to } \pi_0 \geq 0.$$

We then obtain the following reaction function:

$$R_0(q_1) = \begin{cases} \dfrac{\alpha - c_0 - \beta\delta q_1}{\beta} & \text{if} \quad 0 \leq q_1 \leq \overline{q}_1 := \dfrac{\alpha - c_0}{\beta\delta(1-\theta)} & (7.34) \\ \dfrac{(\alpha - c_0 - \beta\delta(1-\theta)q_1}{\beta} & \text{if} \quad \overline{q}_1 < q_1. & (7.35) \end{cases}$$

(7.34) is derived from the zero profit condition (i.e., the constraint is binding) and (7.35) is derived from the first-order condition without constraint (i.e., the constraint is not binding). Intuitively, when firm 1 chooses a smaller output ($q_1 \leq \overline{q}_1$), firm 0 produces a larger output to improve social welfare, resulting in a negative profit if there is no constraint. However, the nonnegative profit constraint applies to firm 0 and cannot choose the optimal output. Therefore, the constraint determines firm 0's output.

Firm 1 maximizes its profit, $\pi_1(q_1, R_0(q_1))$, with respect to q_1. We obtain

$$\overline{q}_1 - q_1^{FQ} = \frac{(\alpha - c_0)(2 - \delta^2(1-\theta)) - (\alpha - c_1)\delta(1-\theta)}{2\beta\delta(1-(1-\theta)\delta^2)(1-\theta)} > 0.$$

Because $\pi_1(q_1, R_0(q_1))$ is concave with respect to q_1 in the private leadership game without the nonnegative profit constraint, it is decreasing in q_1 for $q_1 \geq q_1^{FQ}$. This implies that $q_1 \leq \overline{q}_1$ in equilibrium because $q_1^{FQ} < \overline{q}_1$. Under the nonnegative profit condition, firm 0's reaction function is the former case. Thus, the first-order condition of firm 1 is

$$(\alpha - c_1) - \delta(\alpha - c_0) - 2q_1\beta(1-\delta^2) = 0. \tag{7.36}$$

[11] The formal proof is available upon request from the authors.

Let the superscript "FQcon" denote the equilibrium outcome in the private leadership quantity competition game with the nonnegative profit constraint. From (7.36), we obtain

$$q_1^{FQcon} = \frac{\alpha - c_1 - \delta(\alpha - c_0)}{2\beta(1-\delta^2)}. \tag{7.37}$$

Substituting this into (7.34), we obtain

$$q_0^{FQcon} = \frac{(\alpha - c_0)(2-\delta^2) - (\alpha - c_1)\delta}{2\beta(1-\delta^2)}. \tag{7.38}$$

Substituting these equilibrium quantities into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FQcon} = \frac{((\alpha - c_1) - (\alpha - c_0)\delta)^2}{4\beta(1-\delta^2)} \tag{7.39}$$

$$SW^{FQcon} = \frac{H_5}{8(1-\delta^2)}. \tag{7.40}$$

Next, we consider price competition. From (7.14), we obtain $R_0(p_1) \geq c_0$ as long as $p_1 \geq c_1$. Because firm 1 never chooses $p_1 < c_1$, firm 0's profit is never negative, even without the nonnegative profit constraint, and this constraint is therefore not binding. Comparing social surplus and the private firm's profit under both types of competition with the nonnegative profit condition, we obtain

$$SW^{FQcon} - SW^{FP} = \frac{\delta^2\left(1 - 2\delta^2\theta^3 - \left(1 - 3\delta^2\right)\theta^2 - \delta^2\right)((\alpha - c_1) - (\alpha - c_0)\delta)^2}{8\beta(1-\delta)(1+\delta)\left(1 - (1-\theta)\delta^2\right)^2} > 0 \tag{7.41}$$

$$\pi_1^{FP} - \pi_1^{FQcon} = \frac{((\alpha - c_1) - (\alpha - c_0)\delta)^2\delta^2(1-\theta)}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)} > 0. \tag{7.42}$$

These discussions lead to the following result.

Proposition 3 *Suppose that firm 0 cannot choose an output that yields a negative profit. Under private leadership with quantity competition, the private firm's profit is smaller and welfare is greater when firms face quantity competition than when firms face price competition, regardless of* θ.

As we stated above, under private leadership, the public firm's profit is negative when firms face quantity competition. Imposing the nonnegative profit constraint makes the public firm less aggressive. Expecting this less aggressive behavior, the private leader expands its output, resulting in a welfare gain. Under quantity competition, the private firm's profit increases due to the less competitive situation brought about by the nonnegative profit constraint. The profit ranking, however, does not change.

Proposition 3 has an important policy implication. Under private leadership, it is beneficial to impose a nonnegative profit constraint on the public firm if firms face quantity competition. However, when firms face price competition, this constraint does not matter.

7.6 Endogenous competition structure

In this section, we endogenize the choice of strategic variable (either price or quantity contract). We consider the following timeline:

(a) First, both the leader and the follower choose a price or quantity contract. After observing the price-quantity choices, they face Stackelberg competition.

(b) First, the leader chooses either the price or quantity level. After observing the leader, the follower chooses price or quantity.

In other words, the roles of the leader and the follower are given exogenously but firms can choose a price or quantity contract.

In both scenarios, given the leader's price or quantity, the follower's demand function that maps the follower's price to its quantity is fixed. Therefore, the follower's choice of price and quantity does not affect the equilibrium outcome and thus the follower is indifferent between the two. However, the leader's choice significantly affects the equilibrium outcome. As Singh and Vives (1984) showed, the firm's demand elasticity is higher when the rival chooses the price than when it chooses the quantity. In our context, the follower's demand is more sensitive to its price when the leader chooses the price. We explain the intuition. Given the leader's price, a reduction in the follower's price reduces the leader's output; by definition, the price remains unchanged. Given the leader's quantity, a reduction in the follower's price reduces the leader's price; by definition, the output remains unchanged. Therefore, given the leader's quantity, the follower's price reduction automatically reduces the rival's price and thus, the follower's demand is less sensitive to its own price given the leader's quantity.

If the leader chooses the price (quantity), price (quantity) competition occurs because, as we discuss above, the follower's choice between the price and quantity does not matter. We showed earlier that price competition always provides the private firm with a higher profit. Therefore, under private leadership, the private firm chooses the price, resulting in price competition. We also showed that price competition yields greater welfare under public leadership. Therefore, under public leadership, the public firm chooses the price, resulting in price competition. Under these conditions, price competition occurs if we endogenize the competition structure, regardless of public or private leadership. This suggests that under private leadership, it is possible that the equilibrium competition structure will be inefficient.[12]

7.7 Concluding remarks

In this study, we revisit the welfare and profit comparison between price and quantity competition in mixed duopolies. We consider sequential-move games and find that welfare can reverse when the public firm is the follower, while price competition is always better for welfare when the public firm is the leader. In addition, we find that foreign ownership share plays an important role when the public firm is the follower. Finally, we endogenize the competition structure and find that price competition appears regardless of whether the public or private firm is the leader. We do not consider any government strategic policies such as tax-subsidies, privatization, and trade policies in this study. These policies are intensively discussed in the literature and incorporating these into our analysis remains for future research.[13]

[12] In the simultaneous-move games, price competition yields greater welfare in both private and mixed duopolies, and the equilibrium outcome is price competition. In this sense, the outcome is efficient in mixed duopolies. Our result clearly shows that this is not true under private leadership.

[13] For recent developments in studies of these policies in mixed markets, see Cato and Matsumura (2015) and the works cited therein.

Appendix

$H_1 = (\alpha - c_0)^2(\delta^4(1-\theta)^2 + \delta^2(6\theta - 5) + 4) + 2(\alpha - c_0)(\alpha - c_1)\delta(3\delta^2\theta^2 - 6\delta^2\theta + 3\delta^2 + 2\theta - 3) + (\alpha - c_1)^2(3 - 3\delta^2\theta^2 - (2 - 6\delta^2)\theta - 3\delta^2)$,

$H_2 = (\alpha - c_0)^2((\theta^2 - 6\theta + 5)\delta^4 + (6\theta - 9)\delta^2 + 4) - 2(\alpha - c_0)(\alpha - c_1)\delta(\theta^2(4\delta^2 - 3)\delta^2 - 2\theta(4\delta^4 - 5\delta^2 + 1) + 4\delta^4 - 7\delta^2 + 3) + (\alpha - c_1)^2(\theta^2(4\delta^2 - 3)\delta^2 - 2\theta(4\delta^4 - 5\delta^2 + 1) + 4\delta^4 - 7\delta^2 + 3)$,

$H_3 = 4(\alpha - c_0)^2 + (3 - 2\theta)(\alpha - c_1)^2 - 2(\alpha - c_0)(\alpha - c_1)(3 - 2\theta)\delta$,

$H_4 = (\alpha - c_0)^2(2 - \delta^2)^2 + 2(\alpha - c_0)(\alpha - c_1)\delta(2\theta(1 - \delta^2) + 2\delta^2 - 3) - (\alpha - c_1)^2(2\theta(1 - \delta^2) + 2\delta^2 - 3)$,

$H_5 = (\alpha - c_0)^2(4 - (1 + 2\theta)\delta^2) - 2(\alpha - c_0)(\alpha - c_1)(3 - 2\theta)\delta + (\alpha - c_1)(3 - 2\theta))$,

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Chapter 8

Endogenous Timing in a Price-Setting Mixed Triopoly*

8.1 Introduction

The literature on oligopoly theory contains numerous studies of firms' endogenous order of moves. Depending on the context, there is a first-mover or second-mover advantage thorough strategic effects,[1] which firms try to obtain in advance. In a seminal paper on this issue, Hamilton and Slutsky (1990) first allowed firms to choose order of moves in the observable delay game and showed that a simultaneous (resp. sequential) move game appears in the equilibrium if firms' reaction functions slope up (down). The earlier work has been used to analyze the endogenous choice of the leader/follower role in several different settings to derive general insights. For example, the market with private firms and the state-owned public firm has received substantial interest from economists. Such market is called a mixed market. A difference between a private and public firm is thought of as a difference of their objectives. Private firms only consider their own profit, while public firms pursue the non-profit goal such as the public interest. Although the real goal for the public sector is unclear, the difference among types of firms may change their behavior in reality.[2]

Pal (1998) revisited the observable delay game, formulated by Hamilton and Slutsky (1990), in a quantity-setting mixed oligopoly and showed that the public follows all the private firms

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[1]See, Gal-Or (1985) or Amir and Stepanova (2006) for example.

[2]In the literature on the mixed oligopolies, private firms simply maximize their own profit but public firms care about social surplus. For the recent development in this field, see, Chen (2017), Heywood and Ye (2017), and Lee, Matsumura and Sato (2017).

in the equilibrium. This result is contrast to private oligopoly, where all firms set their output simultaneously.

Bárcena-Ruiz (2007), which is closely related to this paper, examined the firms' order of move in a price setting mixed duopoly and showed that a simultaneous move appears in equilibrium. This result is also the opposite of private duopoly, where firms set prices sequentially. His analysis, however, is limited to the case of duopoly. One natural question is whether or not the simultaneous move structure is robust to a change in the number of private firms.

In this paper, we study endogenous order of moves in a price-setting mixed triopoly with differentiated products. One welfare maximizing public firm competes against two private firms which maximize their own profits. We find that the sequential move which is called a Stackelberg in this paper, in which the public firm and one private firm choose their prices at period 1 and the other private firm does at period 2, emerges in equilibrium. This is in clear contrast with the result of Bárcena-Ruiz (2007) which presents the simultaneous move game as an equilibrium structure.

Furthermore, we discuss a multi-period model.[3] We show that another type of a sequential move game (so-called a hierarchical Stackelberg), where the public firm chooses its price at first period, one private firm does at second period, and the other private firm does at the third period never appears in an equilibrium, while the Stackelberg game still occurs.

Contributions of this paper are twofold. First, because the most of the markets are constructed from the oligopoly market, our result suggests that the Stackelberg game (the sequential game with multiple leaders) prevails under price competition even when the public firm exists. This is directly linked to the issue of endogenous order of moves and to identify the market structure for competition policies. Second, the result of this paper emphasizes that what variables (price or quantity) firms set is still important in the mixed oligopoly to consider the equilibrium market structure.

To make our discussion clear, it is useful to mention what happens in the endogenous price-quantity model formulated by Singh and Vives (1990), where each firm can commit to a quantity or price strategy before competition stage. They showed that in a private duopoly quantity contract is the dominant strategy for both firms. Matsumura and Ogawa (2012) and Haraguchi and Matsumura (2016) have analyzed endogenous competition structure in a mixed oligopoly with differentiated products. The first (second) work shows that in the case of a mixed duopoly (oligopoly), the private firm chooses prices (may choose quantities) endogenously. These results showed that the importance

[3]See, Matsumura(1999).

of the number of private firms in endogenous competition structure in mixed oligopolies. The relationship between this study and Bárcena-Ruiz (2007) parallels to the relations between Haraguchi and Matsumura(2016) and Matsumura and Ogawa (2012), which emphasize the importance of the number of private firms in endogenous competition structure. There are, however, major difference between Haraguchi and Matsumura (2016) and our study. The former endogenized price-quantity choice but the latter investigate endogenous timing. In addition, endogenous timing model can extend aforementioned hierarchical Stackelberg model, while the endogenous price-quantity model cannot apply with respect to this discussion.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes endogenous order of moves for a two period case. Section 4 discusses multi-period model and Section 5 concludes.

8.2 The Model

We adopt a standard differentiated triopoly with a linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is:

$$U(q_i) = \alpha \sum_{i=0}^{2} q_i - \beta(\sum_{i=0}^{2} q_i^2 + \delta \sum_{i=0}^{2} \sum_{i \neq j} q_i q_j)/2 + y,$$

where q_i is the consumption of good i produced by firm i $(i = 0, 1, 2)$, and y is the consumption of an outside good that is provided competitively (with a unit price). Parameters α is positive constant, and $\delta \in (0, 1)$ represents the degree of product differentiation: a smaller δ indicates a larger degree of product differentiation. The demand functions are given by:

$$q_i = \frac{\alpha(1-\delta) - p_i + \delta \sum_{j \neq i} p_j}{\beta(1-\delta)(1+2\delta)} \quad (i \neq j; \quad i, j = 0, 1, 2). \tag{8.1}$$

For simplicity, we assume ll firms share the same constant marginal cost of production of firms and it is given by $c \in [0, \alpha)$.[4] The profit function of firm i is given by:

$$\pi_i = (p_i - c)q_i \quad (i \neq j; \quad i, j = 0, 1, 2).$$

The firm i $(i = 1, 2)$ are private and they choose the price, p_i $(i = 1, 2)$, that maximizes their own profit. The firm 0 is state-owned public enterprise and chooses the price, p_0, that maximizes social

[4]The results in this paper holds even when there is a cost asymmetry between a public and private firms, e.g., $c_0 > c$.

welfare which is given by:

$$
\begin{aligned}
SW &= \sum_{i=0}^{2}(p_i - c)q_i + \left[\alpha \sum_{i=0}^{2} q_i - \frac{\beta\left(\sum_{i=0}^{2} q_i^2 + \delta \sum_{i=0}^{n}\sum_{i\neq j} q_i q_j\right)}{2} - \sum_{i=0}^{2} p_i q_i\right] && (8.2)\\
&= (\alpha - c)\sum_{i=0}^{2} q_i - \frac{\beta\left(\sum_{i=0}^{2} q_i^2 + \delta \sum_{i=0}^{n}\sum_{i\neq j} q_i q_j\right)}{2}. && (8.3)
\end{aligned}
$$

Following Hamilton and Slutsky (1990), we consider a two stage game in the context of a mixed triopoly. In the first stage, the public and private firms simultaneously decide whether to set prices at period 1 or at period 2.[5] Let $t_i \in \{1, 2\}$ denote the time period chosen by firm i $(i = 0, 1, 2)$. After observing their decisions (t_0, t_1, t_2), firms set their prices according to the commitments in the second stage. If all firms choose the same period, firms make price decisions simultaneously. If the timing decisions are different among firms, they move sequentially in the price setting period. To obtain a sub-game perfect Nash-equilibrium, we solve this game backwards.

8.3 Results for two time periods

Before characterizing a Nash equilibrium of the full game, we explore the second stage given the order of moves, (t_0, t_1, t_2).

8.3.1 Simultaneous game

First, we discuss simultaneous game, in which all firms choose prices at a same period. The first order conditions for public and private firms are;

$$
\begin{aligned}
\frac{\partial SW}{\partial p_0} &= \frac{(1-\delta)c - p_0(1+\delta) + \delta\sum_{i=1}^{2} p_i}{\beta(1+2\delta)(1-\delta)} = 0,\\
\frac{\partial \pi_i}{\partial p_i} &= \frac{(1-\delta)\alpha + (1+\delta)c - 2(1+\delta)p_i + \delta\sum_{j\neq i} p_j}{\beta(1+2\delta)(1-\delta)} = 0 \quad (i \neq 0),
\end{aligned}
$$

respectively. The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$
\begin{aligned}
R_0(p_i) &= \frac{(1-\delta)c + \delta\sum_{i=1}^{2} p_i}{1+\delta}, && (8.4)\\
R_i(p_j) &= \frac{(1-\delta)\alpha + (1+\delta)c + \delta\sum_{j\neq i} p_j}{2(1+\delta)} \quad (i \neq 0) && (8.5)
\end{aligned}
$$

[5] We extend our baseline model to the model with more than two periods in section 4.

These functions lead to the equilibrium prices of the public and the private firms as follows:

$$\begin{aligned} p_0^N &= \frac{2\delta(1-\delta)\alpha + (2+\delta+\delta^2)c}{2+3\delta-\delta^2}, \\ p^N &= \frac{(1-\delta^2)\alpha + (1+3\delta)c}{2+3\delta-\delta^2}, \end{aligned}$$

where superscript N denotes the equilibrium outcomes of the simultaneous move game, respectively. Substituting these equilibrium prices into payoff functions, we obtain the resulting welfare and profit of the private firm:

$$SW^N = \frac{(\alpha-c)^2(10+34\delta+21\delta^2-16\delta^3-\delta^4)}{2\beta(1+2\delta)(2+3\delta-\delta^2)^2}, \tag{8.6}$$

$$\pi^N = \frac{(\alpha-c)^2(1-\delta)(1+\delta)^3}{\beta(1+2\delta)(2+3\delta-\delta^2)^2}. \tag{8.7}$$

8.3.2 Sequential games

In this subsection, we consider four different Stackelberg games: (i) public firm as a Stackelberg leader (i.e., $(t_0, t_1, t_2) = (1, 2, 2)$), (ii) public firm as a Stackelberg follower (i.e., $(t_0, t_1, t_2) = (2, 1, 1)$), (iii) one private firm as a Stackelberg leader (i.e., $(t_0, t_1, t_2) = (2, 1, 2)$ or $(2, 2, 1)$), and (iv) one private firm as a Stackelberg follower (i.e., $(t_0, t_1, t_2) = (1, 1, 2)$ or $(1, 2, 1)$). Let superscript LFF, FLL, FLF, and LLF denote the equilibrium outcome in case (i), case (ii), case (iii), and case (iv) respectively.[6] We use the subscript $\{l, f\}$ to indicate a private leader and private follower respectively. The equilibrium outcomes for these four games can be obtained in a standard manner, which is provided in Appendix A. Table 1 presents the set of equilibrium payoffs for each case.

Table 8.1: Payoff matrix for 2 periods case

$0\backslash 1$	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	(SW^N, π^N, π^N)	$(SW^{LLF}, \pi_f^{LLF}, \pi_l^{LLF})$
$t_0 = 2$	$(SW^{FLL}, \pi^{FLL}, \pi^{FLL})$	$(SW^{FLF}, \pi_f^{FLF}, \pi_l^{FLF})$

Firm 2 chooses $t_2 = 1$.

$0\backslash 1$	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	$(SW^{LLF}, \pi_l^{LLF}, \pi_f^{LLF})$	$(SW^{LFF}, \pi^{LFF}, \pi^{LFF})$
$t_0 = 2$	$(SW^{FLF}, \pi_l^{FLF}, \pi_f^{FLF})$	(SW^N, π^N, π^N)

Firm 2 chooses $t_2 = 2$.

[6] For instance, in case (i), the public firm is *leader* and privates are *follower* so that we denote case (i) by LFF.

8.3.3 Equilibrium outcomes in the observable delay game

In this subsection, we analyze the first stage and characterize a Nash equilibrium of our full game. For the analysis of the first stage, we start from the welfare comparison among the five games.

Lemma 1 *The equilibrium welfare has the following relationships:*
(i) $SW^{FLL} < SW^N < SW^{LFF}$ *and (ii)* $SW^{FLF} < SW^{LLF}$.

Proof See the Appendix B.

We provide the intuition behind Lemma 1-(i). When private firms move at 1 period and act as a leader in the second stage, they set higher prices than do in the simultaneous game. This leads to the lower total production so that social welfare is worse off, $SW^{FLL} < SW^N$. When the public firm is a leader, the public firm becomes more aggressive and sets lower price than does in the simultaneous move. Through the strategic effect, the equilibrium prices decrease and the total production increases. This improves social welfare, $SW^N < SW^{LFF}$. These two effects yield Lemma 1-(ii). From Lemma 1, we obtain the following lemma.

Lemma 2 *It is a dominant strategy for the public firm to choose* $t_0 = 1$.

It implies that the public firm never accepts the role of a follower. But more importantly, the observable delay game in which public and private firms play can be reduced to the game in which only private firms play given $t_0 = 1$. This reduces the complexity of the observable delay game played by more than two players.

Next, we compare the equilibrium profit. From Lemma 2, the possible equilibria in the first stage are a simultaneous move game, case (i), and case (iii). Focusing on these three cases, we obtain the following lemma.

Lemma 3 *The equilibrium profit has the following relationships:*
(i) $\pi_f^{LLF} < (>) \pi^N$ *if* $\delta < (>) \delta^*$, *(ii)* $\pi^{LFF} < \pi_l^{LLF}$ *and (iii)* $\pi_l^{LLF} < \pi_f^{LLF}$,
where $\delta^* \simeq 0.699417$.

Proof See the Appendix C.

We provide the intuition behind Lemma 3. As we mentioned above, when the public firm is a leader, it sets a lower price to induce the lower prices in equilibrium. Thus, from the perspective of the private firm, the private avoids to be a follower, $\pi_f^{LLF} < \pi^N$ and $\pi^{LFF} < \pi_l^{LLF}$. There is, however, another aspect related to the private profit, that is, the extent of competition between private firms.

It is well-known results in the literature on first/second-mover advantage that a sequential move game relaxes the industry competition, and second mover advantage exits under price competition. Comparing π_f^{LLF} with π^N, we find the threshold $\delta^* \in (0,1)$ such that $\pi_f^{LLF} = \pi^N$. It means that the effect of second mover's advantage under price competition offsets the effect of aggressive behavior by the public firm. When the degree of product differentiation is sufficiently large, the effect is so large that the profit of the private follower might be large, $\pi_f^{LLF} > \pi^N$. Lemma 3-(iii) represents second-mover advantage in the sequential move game.

Before characterizing the Nash equilibrium of the full game, we state the result of endogenous timing in a price-setting mixed duopoly to emphasize our result.[7]

Result *In a mixed duopoly, a simultaneous move game, where both public and private firm set prices at period 1, appears in equilibrium.*

In the mixed duopoly case, firms set prices simultaneously in equilibrium. The intuition is as follows: (i) if the public firm is a leader, the public firm commits to lower price than its in the simultaneous case to induce private firm's aggressive pricing in the second period. This public firm's lower pricing increases welfare but decreases private profit. Thus, the public firm would want to be a leader. (ii) if the private firm is a leader, a private firm commits higher price than its in the simultaneous case to induce public firm's less aggressive pricing in the second period. This private firm's higher pricing yields higher private firm's profit but the welfare would be worse off. Therefore, the private firm also want to be a leader. Finally, a simultaneous move game appears in a mixed duopoly.

Now, we present our main results for two time periods. The results of Lemma 2 and 3 allow us to derive the equilibrium outcomes in the first stage. From Lemma 2, the public firm always selects $t_0 = 1$. Thus, the equilibrium outcome depends only upon the choice of the private firms. From Lemma 3, we obtain the following proposition.

Proposition 1 *In a mixed oligopoly where one public firm and two private firms exist,*
(i) a simultaneous move game, where all firms choose prices in the period 1, appears if $\delta \in (0, \delta^*)$*;*
(ii) a Stackelberg game (a sequential game), where the public firm and one private firm choose their prices at period 1 and the other private firm does at period 2, appears if $\delta \in (\delta^*, 1)$.
Proof

[7] This is showed in the Proposition 2 in Bárcena-Ruiz (2007).

From Lemma 3 both private firms set prices in the period 1 if $\delta \in (0, \delta^*)$. This implies (i). From Lemma 3-(i), only one private firm sets price in the period 2 if $\delta \in (\delta^*, 1)$. This implies (ii).

In the mixed duopoly case, a simultaneous move, which all firms move at period 1, is the unique equilibrium. In the mixed triopoly case, however, the simultaneous move game may fail to be an equilibrium outcome. We explain the intuition behind the results. By Lemma 2, it is a dominant strategy for the public firm to choose $t_0 = 1$. It implies that the public firm never accept the role of follower. On the other hand, private firms may have an incentive to be a follower if goods are not sufficiently differentiated. In such case, the competition among firms in the same period becomes severe because goods are substantially substituted. This is preferable for the public firm but leads lower profit of private firms. As we discussed just after Lemma 3, the sequential move structure under price competition relaxes an industry competition through the strategic effect. Thus, the one private might move at period 2 in equilibrium.The deviation incentive from the simultaneous move game is much larger if there are larger number of private firms in the market. Most of the market is oligopolistic and contains more than one private firm. It implies that the Stackelberg game prevail under price competition even when the public enterprise exists. Note that private firms would not become a follower at the same time. If both privates are followers, there is no strategic interaction between private firms to soften the market competition, and the public firm becomes aggressive to induce of private firms' lower pricing.

8.4 Discussion for more than two periods

If there are more than two periods, a hierarchical Stackelberg game in which all firms sequentially set their price can appear in an equilibrium. In that case, firms play as a leader, follower, or intermediate. We consider the oligopoly game with three firms so that there is at most one intermediate. There are different kinds of sequential move games in addition to the games defined in Section 3.2: (v) public firm as a Stackelberg leader in a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (1, 2, 3)$ or $(1, 3, 2)$), (vi) public firm as an intermediate a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (2, 1, 3)$ or $(2, 3, 1)$), (vii) public firm as a Stackelberg follower a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (3, 1, 2)$ or $(3, 2, 1)$). Let superscript LMF, MLF, and FLM denote the equilibrium outcome in case (v), case (vi), and case (vii), respectively.[8] We use the subscript $\{l, m, f\}$ to indicate a private leader, private intermediate, and private follower, respec-

[8]For instance, in case (v), the public firm is *leader*, one private is *intermediate*, and the other private is *follower* so that we denote case (v) by LMF.

tively. The equilibrium outcomes for these four games can be obtained in a standard manner, which is provided in Appendix D.

For the analysis of the first stage, we begin by the welfare comparison among

Lemma 4 *The equilibrium welfare has the following relationships:*
(i) $SW^{FLM} < SW^{LLF} < SW^{LMF}$ *and (ii)* $SW^{MLF} < SW^{LLF}$*.*

Proof See the Appendix E.

The intuition behind Lemma 4 is similar to Lemma 1. On the one hand, when the private firm is a leader in the second stage, it takes into account the reaction of the follower firm and sets the higher price to increase its profit. This leads to the lower total outputs and decreases social welfare. The existence of a private intermediate accelerates the distortion of private leadership because the private intermediate also takes the same strategy. Moreover, the private leader predicts the intermediate's reaction and raises its price as well, $SW^{FLM} < SW^{LLF}$. On the other hand, When the public firm is a leader in the second stage, the public charges a lower price to make a follower firm set lower price. This leads to increase the total production and improve social welfare. If there is a private intermediate, the positive effect of the public leadership accumulates just like the opposite side of the private leader, $SW^{LLF} < SW^{LMF}$. If the public firm is the intermediate, the public firm can make the private follower lower prices, while the private leader would induce higher prices. The latter effect diminishes the positive effect of the former one, $SW^{MLF} < SW^{LLF}$.

From Lemma 1 and Lemma 4, we obtain the following lemma.

Lemma 5 *Suppose the mixed triopoly with three periods. It is a dominant strategy for the public firm to choose* $t_0 = 1$*.*

The result is fairly straightforward, but it is worth noting that the preference of the public firm for order of moves is robust to the existence of the intermediate. As seen in section 3, this result reduces the candidate for a subgame perfect equilibrium. Indeed, the remaining candidate is only (v) public firm as a Stackerberg leader a hierarchical Stackelberg game.

Next, we find the property of the resulting profit by emphasizing additional case (v). Then, we obtain the following lemma.

Lemma 6 *The equilibrium profit has the following relationships:*
(i) $\pi_f^{LMF} < (>) \pi_l^{LLF}$ *if* $\delta < (>) \delta^{**}$ *and (ii)* $\pi_m^{LMF} < \pi_l^{LLF}$*,*

where $\delta^{**} \simeq 0.968794$.

Proof See the Appendix F.

We provide the intuition behind Lemma 6. As we have seen above, when the public firm is leader, it sets lower price to increase total outputs. The effect of the public leadership accelerates when there is an intermediate so that the equilibrium profits of the private firms decrease, $\pi_f^{LMF} < \pi_l^{LLF}$. However, if the products are not significantly differentiated, the first-order competitive effect in a same period is larger so that the competitive effect offsets the effect of aggressive behavior by the public firm. Comparing π_f^{LMF} with π_l^{LLF}, we find the threshold $\delta^{**} \in (0,1)$ such that $\pi_f^{LMF} = \pi_l^{LLF}$. It means that, if the products are close to homogeneous goods, the private firms prefer to move to the different period in which no other firms are, $\pi_f^{LMF} > \pi_l^{LLF}$. If the private firm is an intermediate, the private intermediate would make the follower raise his price, while the public leadership hinders those effect. Thus, the profit of the private leader definitely larger than of the private intermediate, $\pi_m^{LMF} < \pi_l^{LLF}$.

Lastly, we analyze the first stage of this game. From Lemma 5, the public firm always prefers to choose $t_0 = 1$. Thus, the equilibrium outcome in the first stage depends only upon the private firm's behavior. From Lemma 2,3, and, 6, we obtain the following proposition.

Proposition 2 *Suppose that there are three periods,*

(i) the simultaneous move game, where all firms choose prices at period 1, appears if $\delta \in (0, \delta^*)$*,*

(ii) the sequential move game, where public firm and one private firm set their prices at period 1 and the other private firm does at the following period, appears if $\delta \in (\delta^*, \delta^{**})$*,*

(iii) the sequential move game, where a public firm and one private firm set their prices at 1 period and the other private does at period 3, appears if $\delta \in (\delta^{**}, 1)$*.*

Proof

From Lemma 2 and 6, both private firms set prices in the period 1 if $\delta \in (0, \delta^*)$. This implies (i). From Lemma 3 and 6, one private firm becomes follower if $\delta \in (\delta^*, \delta^{**})$. This implies (ii). From Lemma 3 and 6, one private follower set prices in the period 3 if $\delta \in (\delta^{**}, 1)$. This implies (iii). ■

We show that the sequential move equilibrium still emerges, while a hierarchical Stackelberg equilibrium never appears in an equilibrium. We discuss the intuition of our result. By Lemma 5, the public firm has a strict preference to be a leader. We explain private firms' incentive for order of moves in detail. There are two opposite effects in the three period case:(i) As in the two period model, the public firm becomes aggressive in the case with public leadership and this public aggressive pricing makes private firms' profit worse off. (ii) As is well known in the literature of price competition, both leader and follower firms' profits in Stackelberg competition are higher than its in the simultaneous competition because of the strategic effect. The effect gets larger when products are not differentiated. Thus, either private firm becomes a follower for $\delta > \delta^*$, resulting in a Stackelberg game in the equilibrium. The positive effect also exits in a hierarchical Stackelberg game. However, the effect of the public aggressive pricing reduces the profit-improving effect in that game. Therefore, the profit of a private intermediate in a hierarchical Stackelberg game cannot be higher than the private leader's profit in a Stackelberg game. Thus, a hierarchical Stackelberg equilibrium never appears in an equilibrium.

8.5 Concluding Remarks

We investigate an endogenous order of moves in a price setting mixed triopoly. The study shows that, in contrast to the mixed duopoly, firms may set prices sequentially in the mixed triopoly. This result suggests that the number of private firms is important in the price setting mixed market. We also study multi period case and show that firms set prices sequentially, where public leads private firms. However, public follows private firms in the quantity-setting competition. These results suggest that the role of public sector depends on the competition structure(prices or quantities).

In the mixed oligopolies, the nationality of private firms is important.[9] For instance, Matsumura (2003) discussed that the result of Pal (1998) depends on the nationality of private firm and showed that public leader is endogenously achieved. As it can be seen from this result, the nationality of private firms in the timing game is important. The extensions for such direction are forthcoming challenges.

[9] See, Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Cato and Matsumura (2012), Lin and Matstumura (2012).

Appendix A

Public firm as a Stackelberg leader: $(t_0, t_1, t_2) = (1, 2, 2)$

First, we discuss a Stackelberg model in which the public firm chooses its price at period 1 and private firms at period 2. That is, the public firm is a leader and privates are follower. We solve the game backwards and start with the followers' problems. The private firms choose their actions after observing the leader's action. Their strategy are (8.5). Anticipating their actions, the public firm chooses its action to maximize social welfare, $SW(p_0, R_i(p_0), R_j(p_0))$. The first-order condition of the public firm is given by

$$\begin{aligned}\frac{\partial SW}{\partial p_0} &= \sum_{i=0}^{2}(p_i - c)\left(\frac{\partial q_i}{\partial p_0} + \sum_{k=1}^{2}\frac{\partial q_i}{\partial p_k}\frac{\partial R_k(p_0)}{\partial p_0}\right) \\ &= \frac{2\alpha\delta(1-\delta^2) + c(4+6\delta-\delta^2-\delta^3) - (4+8\delta-\delta^2-3\delta^3)p_0}{\beta(1-\delta)(2+\delta)^2(1+2\delta)} = 0.\end{aligned}$$

We obtain the equilibrium price of the public firm:

$$p_0^{LFF} = \frac{2\alpha\delta(1-\delta^2) + c(4+6\delta-\delta^2-\delta^3)}{4+8\delta-\delta^2-3\delta^3}.$$

Substitute it into (8.5), we obtain the equilibrium price of the private firm

$$p^{LFF} = \frac{\alpha(4+4\delta-7\delta^2-\delta^3) + c(4+12\delta+5\delta^2-5\delta^3)}{2(4+8\delta-\delta^2-3\delta^3)}.$$

Let SW^{LFF} and π^{LFF} be the equilibrium welfare and profit of the private firm in this game, respectively. We obtain

$$SW^{LFF} = \frac{(\alpha-c)^2(36+88\delta+15\delta^2-43\delta^3)}{8\beta(1+2\delta)(4+8\delta-\delta^2-3\delta^3)}, \tag{8.8}$$

$$\pi^{LFF} = \frac{(\alpha-c)^2(1-\delta^2)(4+8\delta+\delta^2)^2}{4\beta(1+2\delta)(4+8\delta-\delta^2-3\delta^3)^2}. \tag{8.9}$$

Public firm as a Stackelberg follower: $(t_0, t_1, t_2) = (2, 1, 1)$

We discuss a Stackelberg model in which the public firm chooses its price at period 2 and all private firms at period 1. We solve the game backwards and start with the follower's problem. The public chooses its price after observing the leaders' actions. Its strategy is (8.4). The leader firms, the private firms, choose their actions to maximize own profits, $\pi_i(R_0(p_i, p_j), p_i, p_j)$ $(i \neq j;\ i, j = 1, 2)$.

The first-order condition of firm i is given by

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial R_0(p_i, p_j)}{\partial p_i} + \frac{\partial q_i}{\partial p_i}\right) + q_i \\ &= \frac{\alpha(1-\delta^2) + c(1+3\delta-\delta^2) - 2(1+2\delta)p_i + \delta(1+2\delta)p_j}{\beta(1-\delta^2)(1+2\delta)} = 0.\end{aligned}$$

We obtain the equilibrium price of the private firm:

$$p^{FLL} = \frac{\alpha(1-\delta^2) + c(1+3\delta-\delta^2)}{(2+\delta)(1+2\delta)}.$$

Substitute it into (8.4), we obtain the equilibrium price of the public firm:

$$p_0^{FLL} = \frac{2\delta(1-\delta)\alpha + (2+\delta)c}{(2+\delta)(1+2\delta)}.$$

Let SW^{FLL} and π^{FLL} be the equilibrium welfare and profit of the private firm in this game, respectively. We obtain

$$SW^{FLL} = \frac{(\alpha-c)^2(10+14\delta-19\delta^2+4\delta^3)}{2\beta(2+\delta)^2(1+2\delta)^2}, \tag{8.10}$$

$$\pi^{FLL} = \frac{(\alpha-c)^2(1-\delta^2)}{\beta(2+\delta)^2(1+2\delta)^2} \tag{8.11}$$

One private firm as a Stackelberg leader: $(t_0, t_1, t_2) = (2, 1, 2)$ or $(2, 2, 1)$

We discuss a Stackelberg model in which one private firm sets its price at period 1 and the others at period 2. The followers choose their price after observing the leader's action. From (8.4) and (8.5), we obtain

$$\begin{aligned}p_0(p_i) &= \frac{\alpha\delta(1-\delta) + c(2+\delta-\delta^2) + \delta(2+3\delta)p_i}{2+4\delta+\delta^2}, \\ p_j(p_i) &= \frac{\alpha(1-\delta^2) + c(1+3\delta) + \delta(1+2\delta)p_i}{2+4\delta+\delta^2} \quad (i \neq j; \quad i, j = 1, 2).\end{aligned}$$

The leader firm, private firm i, chooses its price to maximize own profits, $\pi_i(p_0(p_i), p_i, p_j(p_i))$. The first-order condition of firm i is given by

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial p_0(p_i)}{\partial p_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial p_j(p_i)}{\partial p_i}\right) + q_i \\ &= \frac{\alpha(2+3\delta-2\delta^2-3\delta^3) + c(2+9\delta+6\delta^2+5\delta^3) - 4(1+3\delta+\delta^2-2\delta^3)p_i}{\beta(1-\delta)(1+2\delta)(2+4\delta+\delta^2)} \\ &= 0.\end{aligned}$$

We obtain the equilibrium price of the private leader:

$$p_l^{FLF} = \frac{\alpha(2+3\delta-2\delta^2-3\delta^3)+c(2+9\delta+6\delta^2+5\delta^3)}{4(1+3\delta+\delta^2-2\delta^3)}.$$

Substitute it into (8.12) and (8.12), we obtain

$$\begin{aligned} p_0^{FLF} &= \frac{\alpha\delta(8+20\delta-3\delta^2-24\delta^3-\delta^4)+c(8+32\delta+40\delta^2+15\delta^3-4\delta^4-7\delta^5)}{4(1+3\delta+\delta^2-2\delta^3)(2+4\delta+\delta^2)}, \\ p_f^{FLF} &= \frac{\alpha(4+6\delta-5\delta^2-6\delta^3+\delta^4)+c(4+18\delta+17\delta^2-6\delta^3-5\delta^4)}{4(1+\delta-\delta^2)(2+4\delta+\delta^2)}. \end{aligned}$$

Let SW^{FLF}, π_l^{FLF}, and π_f^{FLF} be the equilibrium welfare, private leader's profit, and private follower's profit in this game, respectively. We obtain

$$SW^{FLF} = \frac{(\alpha-c)^2H_1}{32\beta(1+2\delta)^2(1+\delta-\delta^2)^2(2+4\delta+\delta^2)^2}, \tag{8.12}$$

$$\pi_l^{FLF} = \frac{(\alpha-c)^2(1-\delta)(2+5\delta+3\delta^2)^2}{8\beta(1+2\delta)^2(1+\delta-\delta^2)(2+4\delta+\delta^2)}, \tag{8.13}$$

$$\pi_f^{FLF} = \frac{(\alpha-c)^2(1-\delta)(1+\delta)^3(4+6\delta-\delta^2)^2}{16\beta(1+2\delta)^2(1+\delta-\delta^2)^2(2+4\delta+\delta^2)^2} \tag{8.14}$$

where $H_1 = 160+1344\delta+4272\delta^2+5836\delta^3+1547\delta^4-4004\delta^5-3102\delta^6+268\delta^7+627\delta^8+108\delta^9$.

One private firm as a Stackelberg follower : $(t_0,t_1,t_2) = (1,1,2)$ or $(1,2,1)$

We discuss a Stackelberg model in which both the public and one private firm set their price at period 1 and the other private at period 2. The follower, private firm j ($j \neq i$; $i,j = 1,2$), chooses its price after observing the leaders' actions. The follower's strategy is given by (8.5). The leader firms, the public and private firm i, choose their prices to maximize their payoff, $SW(p_0,p_i,R_j(p_0,p_i))$ and $\pi_i(p_0,p_i,R_j(p_0,p_i))$, respectively. The first-order conditions are given by

$$\begin{aligned} \frac{\partial SW}{\partial p_0} &= \sum_{k=0}^{2}(p_k-c)\left(\frac{\partial q_k}{\partial p_0}+\frac{\partial q_k}{\partial p_j}\frac{\partial R_j(p_0,p_i)}{\partial p_0}\right) \\ &= \frac{\alpha\delta(1-\delta)+c(4+3\delta-5\delta^2)-(4+8\delta+\delta^2)p_0+\delta(4+7\delta)p_i}{4\beta(1-\delta)(1+\delta)(1+2\delta)} = 0 \\ \frac{\partial \pi_i}{\partial p_i} &= (p_i-c)\left(\frac{\partial q_i}{\partial p_i}+\frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_0,p_i)}{\partial p_i}\right)+q_i \\ &= \frac{\alpha(1-\delta)(2+3\delta)+c(2+5\delta+2\delta^2)-2(2+4\delta+\delta^2)p_i+\delta(2+3\delta)p_0}{2\beta(1-\delta)(1+\delta)(1+2\delta)} \\ &= 0. \end{aligned}$$

Solving the system of equations, we obtain the equilibrium prices of the public firm and the private leader:

$$
\begin{aligned}
p_0^{LLF} &= \frac{\alpha\delta(12+22\delta-11\delta^2-23\delta^3)+c(16+52\delta+46\delta^2+9\delta^3+4\delta^4)}{16+64\delta-68\delta^2-2\delta^3-19\delta^4} \\
p_l^{LLF} &= \frac{2\alpha(4+10\delta-11\delta^3-3\delta^4)+c(8+44\delta+68\delta^2+20\delta^3-13\delta^4)}{16+64\delta-68\delta^2-2\delta^3-19\delta^4}.
\end{aligned}
$$

Substitute it into (8.5), we obtain the equilibrium price of the private follower:

$$
p_f^{LLF} = \frac{\alpha(8+28\delta+18\delta^2-24\delta^3-25\delta^4-5\delta^5)+2c(4+26\delta+57\delta^2+45\delta^3+2\delta^4-7\delta^5)}{(1+\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)}
$$

Let SW^{LLF}, π_l^{LLF}, and π_f^{LLF} be the equilibrium welfare, private leader's profit, and private follower's profit in this game, respectively. We obtain

$$
SW^{LLF} = \frac{(\alpha-c)^2 H_2}{2\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)^2}, \tag{8.15}
$$

$$
\pi_l^{LLF} = \frac{2(\alpha-c)^2(1-\delta)(2+3\delta)^2(2+4\delta+\delta^2)^3}{\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)^2}, \tag{8.16}
$$

$$
\pi_f^{LLF} = \frac{(\alpha-c)^2(1-\delta)(8+36\delta+54\delta^2+30\delta^3+5\delta^4)^2}{2\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)} \tag{8.17}
$$

where $H_2 = 640+6016\delta+22896\delta^2+43920\delta^3+40672\delta^4+7508\delta^5-16109\delta^6-10161\delta^7+285\delta^8+1107\delta^9$.

Appendix B

Proof of Lemma 1

From (8.6), (8.8), (8.10), (8.12), and (8.15), we can directly compare the equilibrium welfare. We obtain

$$
\begin{aligned}
SW^{LFF}-SW^{N} &= \frac{2(\alpha-c)^2(1-\delta)^3\delta^2\,(1+2\delta)}{\beta(-3\delta^3-\delta^2+8\delta+4)\left(\delta^2-3\delta-2\right)^2} > 0, \\
SW^{N}-SW^{FLL} &= \frac{(\alpha-c)^2(1-\delta)^2\delta^2\left(-3\delta^3+3\delta^2+10\delta+4\right)}{\beta(2-\delta)^2(2\delta+1)^2\left(\delta^2-3\delta-2\right)^2} > 0, \\
SW^{LLF}-SW^{FLF} &= \frac{(\alpha-c)^2(1-\delta)\delta^2 H_3}{32\beta(1+\delta)(2\delta+1)^2\left(\delta^2-\delta-1\right)^2\left(\delta^2+4\delta+2\right)^2\left(H_4\right)^2} > 0
\end{aligned}
$$

where
$H_3 = 3564\delta^{15}+37731\delta^{14}+191374\delta^{13}+631828\delta^{12}+911470\delta^{11}-845859\delta^{10}-5135848\delta^9-7489972\delta^8-3513408\delta^7+3560208\delta^6+6991488\delta^5+5439936\delta^4+2464512\delta^3+677632\delta^2+105472\delta+7168$,
$H_4 = -19\delta^4-2\delta^3+68\delta^2+64\delta+16$.

Appendix C

Proof of Lemma 3

From (8.7), (8.9), (8.16), and (8.17), we directly compare the equilibrium profit. We obtain

$$\begin{aligned}&\pi^N - \pi_f^{LLF}\\ =\ &\frac{(\alpha-c)^2(1-\delta)\delta^2(3\delta+2)\left(-7\delta^3-9\delta^2+4\delta+4\right)\left(-8\delta^5-13\delta^4+39\delta^3+102\delta^2+72\delta+16\right)}{\beta(1+\delta)\left(\delta^2-3\delta-2\right)^2\left(19\delta^4+2\delta^3-68\delta^2-64\delta-16\right)^2}.\end{aligned}$$

$\pi^N - \pi_f^{LLF}$ is positive if and only if $(-7\delta^3 - 9\delta^2 + 4\delta + 4)$ is positive and

$$-7\delta^3 - 9\delta^2 + 4\delta + 4 > (<)\ \ 0 \text{ if } \delta < (>)\delta^* \simeq 0.699417.$$

We also have

$$\begin{aligned}&\pi_l^{LLF} - \pi^{LFF}\\ =\ &\frac{(\alpha-c)^2(1-\delta)\delta^2 H_5}{\beta(\delta+1)(2\delta+1)\left(3\delta^3+\delta^2-8\delta-4\right)^2\left(19\delta^4+2\delta^3-68\delta^2-64\delta-16\right)^2} > 0,\end{aligned}$$

$$\begin{aligned}&\pi_f^{LLF} - \pi_l^{LLF}\\ =\ &\frac{(\alpha-c)^2(1-\delta)\delta^3(7\delta+4)\left(\delta^2+4\delta+2\right)^2}{\beta(\delta+1)(2\delta+1)\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2} > 0\end{aligned}$$

where $H_5 = -199\delta^{12} + 3636\delta^{11} + 17056\delta^{10} + 133344\delta^9 - 45349\delta^8 - 105024\delta^7 - 65832\delta^6 + 42128\delta^5 + 98064\delta^4 + 72704\delta^3 + 28416\delta^2 + 5888\delta + 512$.

Appendix D

Public firm as a Stackelberg leader a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (1, 2, 3)$ or $(1, 3, 2)$

First, we discuss a Stackelberg model in which the public firm chooses its price at period 1, one private firms does at period 2, and the other private firm does at period 3. That is, the public firm is a leader, one private is intermediate, and the other private is follower. The follower firm, firm j $(j \neq i;\ \ i, j = 1, 2)$, choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (8.5). Anticipating the follower's action, the private firm i as an intermediate chooses its action to maximize own profit, $\pi_i(p_0, p_i, R_j(p_0, p_i))$. The first-order condition of the private intermediate is given by

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_0,p_i)}{\partial p_i}\right) + q_i \\
&= \frac{c\left(2\delta^2+5\delta+2\right) + (3\delta+2)(\alpha(1-\delta)+\delta p_0) - 2\left(\delta^2+4\delta+2\right)p_i}{2\beta(1-\delta)(\delta+1)(2\delta+1)} = 0.
\end{aligned}
$$

We obtain the optimal strategy of the intermediate:

$$
R_i(p_0) = \frac{(3\delta+2)(\alpha(1-\delta)+\delta p_0) + c\left(2\delta^2+5\delta+2\right)}{2\left(\delta^2+4\delta+2\right)}.
$$

The public firm chooses its action to maximize social welfare, $SW\left(p_0, R_i(p_0), R_j\left(p_0, R_i\left(p_0\right)\right)\right)$, anticipating their actions. The first-order condition of the public firm is given by

$$
\frac{\partial SW}{\partial p_0} = \sum_{k=0}^{2}(p_k - c)\left(\frac{\partial q_k}{\partial p_0} + \frac{\partial q_k}{\partial p_i}\frac{\partial R_i(p_0)}{\partial p_0} + \frac{\partial q_k}{\partial p_j}\left(\frac{dR_j\left(p_0, R_i\left(p_0\right)\right)}{dp_0}\right)\right) = 0.
$$

We obtain the equilibrium price of the public firm:

$$
p_0^{LMF} = \frac{H_6}{71\delta^6 + 292\delta^5 + 128\delta^4 - 576\delta^3 - 800\delta^2 - 384\delta - 64}
$$

where $H_6 = \alpha\delta(43\delta^5 + 153\delta^4 + 84\delta^3 - 120\delta^2 - 128\delta - 32) + c(28\delta^6 + 139\delta^5 + 44\delta^4 - 456\delta^3 - 672\delta^2 - 352\delta - 64)$.

Substitute it into $R_i\left(p_0\right)$ and $R_j\left(p_0, R_i\left(p_0\right)\right)$, we obtain the equilibrium prices of the private intermediate and follower: $p_m^{LMF} = R_i\left(p_0^{LMF}\right)$ and $p_f^{LMF} = R_j\left(p_0^{LMF}, R_i\left(p_0^{LMF}\right)\right)$. Let SW^{LMF}, π_m^{LMF}, and π_f^{LMF} be the equilibrium welfare, profit of the private intermediate, and of private follower in this game, respectively. We obtain

$$
\begin{aligned}
& SW^{LMF} \\
=\; & \frac{(\alpha-c)^2\left(333\delta^7 + 1347\delta^6 + 1177\delta^5 - 1827\delta^4 - 4324\delta^3 - 3360\delta^2 - 1184\delta - 160\right)}{2\beta(\delta+1)(2\delta+1)\left(71\delta^6 + 292\delta^5 + 128\delta^4 - 576\delta^3 - 800\delta^2 - 384\delta - 64\right)}, \quad (8.18) \\
& \pi_m^{LMF} \\
=\; & \frac{2(\alpha-c)^2\left(-\delta^3 - 3\delta^2 + 2\delta + 2\right)\left(-21\delta^5 - 2\delta^4 + 116\delta^3 + 168\delta^2 + 88\delta + 16\right)^2}{\beta(\delta+1)(2\delta+1)\left(-71\delta^6 - 292\delta^5 - 128\delta^4 + 576\delta^3 + 800\delta^2 + 384\delta + 64\right)^2}, \quad (8.19) \\
& \pi_f^{LMF} \\
=\; & \frac{(\alpha-c)^2(1-\delta)\left(-35\delta^6 - 50\delta^5 + 192\delta^4 + 536\delta^3 + 504\delta^2 + 208\delta + 32\right)^2}{\beta(\delta+1)(2\delta+1)\left(-71\delta^6 - 292\delta^5 - 128\delta^4 + 576\delta^3 + 800\delta^2 + 384\delta + 64\right)^2}. \quad (8.20)
\end{aligned}
$$

Public firm as an intermediate a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (2, 1, 3)$ **or** $(2, 3, 1)$

We discuss a Stackelberg model in which the public firm chooses its price at period 2, one private firms does at period 1, and the other private firm does at period 3. That is, the public firm is an intermediate, one private is leader, and the other private is follower. The follower firm, firm j ($j \neq i;\ \ i, j = 1, 2$), choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (8.5). Anticipating the follower's action, the public firm as an intermediate chooses its action to maximize social welfare, $SW(p_0, p_i, R_j(p_0, p_i))$. The first-order condition of the public intermediate is given by

$$\frac{\partial SW}{\partial p_0} = \sum_{k=0}^{2}(p_k - c)\left(\frac{\partial q_k}{\partial p_0} + \frac{\partial q_k}{\partial p_j}\frac{\partial R_j(p_0)}{\partial p_0}\right) = 0$$

We obtain the optimal strategy of the public intermediate:

$$R_0(p_i) = \frac{c\left(-5\delta^2 + 3\delta + 4\right) + \delta(\alpha(1-\delta) + (7\delta + 4)p_i)}{\delta^2 + 8\delta + 4}.$$

The private leader chooses its action to maximize own profit, $\pi_i\left(R_0(p_i), p_i, R_j\left(R_0\left(p_i\right), p_i\right)\right)$, anticipating their actions. The first-order condition of the private firm is given by

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial R_0(p_i)}{\partial p_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{dR_j\left(R_0\left(p_i\right), p_i\right)}{dp_i}\right) + q_i = 0.$$

We obtain the equilibrium price of the private leader:

$$p_l^{MLF} = \frac{\alpha\left(3\delta^4 + 11\delta^3 - 10\delta - 4\right) + c\left(17\delta^4 + 3\delta^3 - 30\delta^2 - 22\delta - 4\right)}{2(\delta+1)(2\delta+1)\left(5\delta^2 - 4\delta - 4\right)}$$

Substitute it into $R_0\left(p_i\right)$ and $R_j\left(R_i\left(p_i\right), p_i\right)$, we obtain the equilibrium prices of the public intermediate and private follower: $p_0^{MLF} = R_0\left(p_l^{MLF}\right)$ and $p_f^{MLF} = R_j\left(R_0\left(p_l^{MLF}\right), p_l^{MLF}\right)$. Let SW^{MLF}, π_l^{MLF}, and π_f^{MLF} be the equilibrium welfare, profit of the private leader, and of private follower in this game, respectively. We obtain

$$SW^{MLF} = \frac{(\alpha - c)^2 H_7}{4\beta(\delta+1)^2(2\delta+1)^2\left(-5\delta^2 + 4\delta + 4\right)^2\left(\delta^2 + 8\delta + 4\right)}, \tag{8.21}$$

$$\pi_l^{MLF} = \frac{(\alpha - c)^2(1-\delta)\left(3\delta^3 + 14\delta^2 + 14\delta + 4\right)^2}{4\beta(\delta+1)^2(2\delta+1)^2\left(\delta^2 + 8\delta + 4\right)\left(-5\delta^2 + 4\delta + 4\right)}, \tag{8.22}$$

$$\pi_f^{MLF} = \frac{4(\alpha - c)^2(1-\delta)\left(\delta^4 + \delta^3 - 12\delta^2 - 14\delta - 4\right)^2}{\beta(\delta+1)(2\delta+1)\left(-5\delta^2 + 4\delta + 4\right)^2\left(\delta^2 + 8\delta + 4\right)^2}. \tag{8.23}$$

where $H_7 = 333\delta^9 + 2925\delta^8 + 1996\delta^7 - 8324\delta^6 - 11888\delta^5 + 806\delta^4 + 11080\delta^3 + 8488\delta^2 + 2688\delta + 320$.

Public firm as a follower a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (3, 1, 2)$ **or** $(3, 2, 1)$

We discuss a Stackelberg model in which the public firm chooses its price at period 3, one private firms does at period 1, and the other private firm does at period 2. That is, the public firm is a follower, one private is leader, and the other private is intermediate. The public firm choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (8.4). Anticipating the follower's action, the private firm j $(j \neq i;\ \ i, j = 1, 2)$ as an intermediate chooses its action to maximize own profit, $\pi_j(R_0(p_i, p_j), p_i, p_j)$. The first-order condition of the private intermediate is given by

$$\frac{\partial \pi_j}{\partial p_j} \quad = \quad (p_j - c)\left(\frac{\partial q_j}{\partial p_j} + \frac{\partial q_j}{\partial p_0}\frac{\partial R_0(p_i, p_j)}{\partial p_j}\right) + q_j = 0$$

We obtain the optimal strategy of the private intermediate:

$$R_j(p_i) = \frac{(1-\delta^2)\alpha - (\delta^2 - 3\delta - 1)c + (2\delta^2 + \delta)p_i}{4\delta + 2}.$$

The private leader chooses its action to maximize own profit, $\pi_i\left(R_0\left(p_i, R_j\left(p_i\right)\right), p_i, R_j(p_i)\right)$, anticipating their actions. The first-order condition of the private firm is given by

$$\frac{\partial \pi_i}{\partial p_i} \quad = \quad (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{dR_0\left(p_i, R_j\left(p_i\right)\right)}{dp_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_i)}{\partial p_i}\right) + q_i = 0.$$

We obtain the equilibrium price of the private leader:

$$p_l^{FLM} = \frac{\alpha\left(-\delta^3 - 2\delta^2 + \delta + 2\right) + c\left(-3\delta^3 + 7\delta + 2\right)}{2(2\delta+1)\left(2-\delta^2\right)}.$$

Substitute it into $R_0\left(p_i, R_j\left(p_i\right)\right)$ and $R_j(p_i)$, we obtain the equilibrium prices of the private follower and private intermediate: $p_0^{FLM} = R_0\left(p_l^{FLM}, R_j\left(p_l^{FLM}\right)\right)$ and $p_f^{FLM} = R_j(p_l^{FLM})$. Let SW^{FLM}, π_l^{FLM}, and π_m^{FLM} be the equilibrium welfare, profit of the private leader, and of private intermediate in this game, respectively. We obtain

$$SW^{FLM} = \frac{(\alpha - c)^2\left(5\delta^6 + 92\delta^5 + 11\delta^4 - 380\delta^3 - 128\delta^2 + 384\delta + 160\right)}{32\beta(2\delta+1)^2\left(\delta^2 - 2\right)^2}, \tag{8.24}$$

$$\pi_l^{FLM} = \frac{(1-\delta)(\delta+1)(\delta+2)^2(\alpha - c)^2}{8\beta(2\delta+1)^2\left(2-\delta^2\right)}, \tag{8.25}$$

$$\pi_m^{FLM} = \frac{(1-\delta)(\delta+1)\left(\delta^2 - 2\delta - 4\right)^2(\alpha - c)^2}{16\beta(2\delta+1)^2\left(\delta^2 - 2\right)^2}. \tag{8.26}$$

Appendix E

From (8.8), (8.15), (8.18), (8.21), and (8.24), we can compare the equilibrium welfare. We obtain

$$\begin{aligned}
& SW^{LLF} - SW^{FLM} \\
= \;& \frac{(\alpha - c)^2 \delta^2 H_8}{32\beta(\delta+1)(2\delta+1)^2\left(\delta^2-2\right)^2\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2} > 0, \\
& SW^{LMF} - SW^{LLF} \\
= \;& \frac{2(\alpha - c)^2\delta^2\left(2\delta^3+\delta^2-2\delta-1\right)\left(-51\delta^4-52\delta^3+36\delta^2+56\delta+16\right)^2}{\beta\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2\left(71\delta^6+292\delta^5+128\delta^4-576\delta^3-800\delta^2-384\delta-64\right)} > 0, \\
& SW^{LLF} - SW^{MLF} \\
= \;& \frac{(\alpha - c)^2\delta^2(3\delta+2)^2 H_9}{4\beta(\delta+1)^2(2\delta+1)^2\left(-5\delta^2+4\delta+4\right)^2\left(\delta^2+8\delta+4\right)\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2} > 0,
\end{aligned}$$

where
$H_8 = -1805\delta^{13} + 27\delta^{12} - 4823\delta^{11} - 73127\delta^{10} - 64732\delta^9 + 271340\delta^8 + 460240\delta^7 - 89584\delta^6 - 627520\delta^5 - 346944\delta^4 + 152576\delta^3 + 227072\delta^2 + 86016\delta + 11264$,
$H_9 = -1057\delta^{13} - 18391\delta^{12} - 68012\delta^{11} - 13540\delta^{10} + 251952\delta^9 + 319880\delta^8 - 114184\delta^7 - 473928\delta^6 - 274112\delta^5 + 90208\delta^4 + 183424\delta^3 + 93696\delta^2 + 22016\delta + 2048$.

Appendix F

Proof of Lemma 6

From (8.16), (8.19), and (8.20), we directly compare the equilibrium profit. We obtain

$$\pi_l^{LLF} - \pi_f^{LMF} > (<) \;\; 0 \text{ if } \delta < (>) \delta^{**} \simeq 0.968794.$$

The exact vale of the difference between the resulting profits is too complex. The inequality and the threshold value are obtain by Mathematica (available upon request). We also have

$$\begin{aligned}
& \pi_l^{LLF} - \pi_m^{LMF} \\
= \;& \frac{8(\alpha - c)^2(1-\delta)\delta^2(3\delta+2)^3\left(\delta^2+4\delta+2\right) H_{10}}{\beta\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2\left(-71\delta^6-292\delta^5-128\delta^4+576\delta^3+800\delta^2+384\delta+64\right)^2} \\
> \;& 0
\end{aligned}$$

where $H_{10} = -527\delta^{11} + 5237\delta^{10} + 24561\delta^9 + 14658\delta^8 - 63676\delta^7 - 115848\delta^6 - 45632\delta^5 + 58496\delta^4 + 79680\delta^3 + 41088\delta^2 + 10240\delta + 1024$.

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