論文の内容の要旨

Cooperation in Various Classes of Repeated Games
(種々の繰り返しゲームにおける協力)

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Repeated games have been broadly applied to economic models to show that inefficient outcomes can be avoided through repetition. Studies on repeated games usually impose the following assumptions: (a) The number of repetitions is infinite. (b) Each player's opponents are fixed and are not replaced by others throughout the game. (c) Players choose their actions simultaneously. (d) Each player can perfectly observe opponents' previous actions. Although several studies have examined repeated games without these assumptions, many questions remain unsolved. Therefore, we consider repeated games where some of these assumptions are not satisfied, and investigate whether and how players can cooperate in such games. This paper consists of four chapters, each of which studies an independent model of repeated games. Our model in each chapter can be applied to explain economic phenomena in the real world.

In chapter 1, we examine two-person overlapping generations (OLG) games. The model in these games does not satisfy (b) in the above assumptions. In OLG games, players in each generation take part in the repeated game for a sufficiently long time, and then are gradually replaced by their successors in the next generation. In the versions of folk theorems in the OLG games proved by Kandori [16], and in subsequent studies, it is required that any (per-period) average equilibrium payoffs are included in the feasible and individually rational one-shot payoff set V. In contrast, in our model of two-person OLG games, it is shown that there exist subgame perfect equilibria where some players can attain possibly high average payoffs outside V. We also guarantee that players who give high one-shot payoffs to other players do not have to decrease their own payoffs, on average, when their lifespan is sufficiently long. Consequently, our result establishes the new folk theorem, which differs from those in prior studies.

For this result, the essential logic we must consider is how to punish the player who failed to reward the other player, who should have obtained high payoffs outside V. In OLG games, a player who received low payoffs outside V earlier in his life in order to contribute to another player can be rewarded before his retirement by the other younger player. In our model of two-person OLG games, players in each generation are classified into three types, each with different roles: a contributed player, a contributor, and an inspector. A contributed player and an inspector in a generation have the same action set in the stage game. A contributed player rewards the inspector of the previous generation, who has a different action set in his earlier life, depending on the inspector's past play. The contributed player attains high one-shot payoffs for a sufficiently long time in his later life from the contributor, which guarantees that his total average payoff can be outside V. A contributor rewards the contributed player in his earlier life, accepting possibly low payoffs. If the contributed player had deviated in his earlier life, the contributor decreases his reward. An inspector gives the appropriate payoff to the contributor, depending on whether the contributor rewarded the contributed player sufficiently. By constructing such chains of rewards and punishments lasting over generations, we prevent any deviation for each player. When the lifespans of players are sufficiently long, we can make the contributor's loss arbitrarily small, on average, and he can also attain a considerably higher average payoff in V.

In chapter 2, we study a specific two-person repeated pure coordination game of overlapping generations, with alternating moves. This model does not satisfy assumptions (b) and (c). In the pure coordination stage game, all players receive an identical payoff, given an action profile. In our stage game, two investors decide independently on whether to invest in project A or B. We assume that investors cannot obtain the profit when they invest in different projects. We further assume that the one-shot profit of investing in A by both investors is higher than that of investing in B. We consider the repeated game of overlapping generations with alternating moves, where each investor participates for finite periods. Each investor k's opponent is k－1 during the first half of his life, and then is replaced by k+1. We assume that once each investor has made his decision after entering the game, he cannot revise his decision during his life. We further assume that the one-shot profit inflates or deflates at some rate in every period. For given parameters, we characterize all subgame perfect equilibrium profiles, including the off-equilibrium paths. It is shown that the set of equilibria is discontinuous with respect to the parameters.

We first show that when the growth rates of one-shot payoffs and investors' lifespans are relatively small, all investors choose A on the path. This result holds because, in this case, the weight of investor k's payoff in his earlier life is greater than that in his later life. When investor k－1 chose A, k can receive a higher payoff by making the same choice, regardless of what k+1 chooses in the future. It is then interesting to investigate what happens if an investor chooses B by mistake. Unfortunately, it is shown that if the efficiency from choosing A rather than B is relatively small, all successive investors choose B after it has been chosen once. When k－1 chooses B, investor k can receive a higher payoff by making the same choice, regardless of what k+1 chooses. The efficient outcome can never be restored after someone chooses the inefficient action. For this result, the equilibrium with efficient outcomes in our model are not robust against players' mistakes for some range of parameters.

Second, we show that when the growth rates of one-shot payoffs and investors' lifespans are relatively large, inefficient outcomes can arise, even on the path of equilibria. In particular, the lower bound of players' (per-period) average payoffs in equilibrium can be arbitrarily small, and much smaller than the one-shot minimax payoff, when we select appropriate parameters. In this case, the weights of investors' payoffs in their later life are greater than those in their earlier life because their one-shot payoff grows exponentially over time. For each investor k, what k+1 will choose becomes more important than the choice of k－1. If investor k+1 chooses a different action to that of k, k's average payoff becomes very small because the weight of each investor's payoff in his earlier life is smaller than that in his later life. When k+1 mixes actions appropriately, conditional on k and k－1's realized actions, k becomes indifferent between A and B after any history, and obtains the payoff that is inefficient. Note that inefficient equilibria appear in the case when future payoffs increase.

In chapter 3, we consider the two-person prisoner's dilemma with finite alternating repetition. This model does not satisfy (a) or (c). In the two-person prisoner's dilemma, two players, ROW and COL, choose either cooperation (C) or defection (D), where D strongly dominates C, and the payoff from cooperation is higher than that from defection by both players. When rational players finitely repeat this game, they apply backward induction and always choose D throughout their life, in equilibria. One reasonable way to avoid such myopic actions for a player in finitely repeated games is to make him afraid that his opponent may have to imitate his previous action. That is, his opponent always cooperates if he did so in the previous period, and punishes him if not. Such a strategy profile is called a Tit-for-Tat (TFT) strategy. Kreps et al. [17] considered the symmetric two-person prisoner's dilemma with simultaneous moves, where one player has no choice but to play a TFT strategy. They showed that all sequential equilibria are almost efficient when the number of repetitions is sufficiently high. One of the other representative strategy profiles that can make payoffs in finitely repeated games efficient is the

Grim-Trigger (GT) strategy. According to this strategy, the player cooperates at first, and then continues to cooperate until his opponent deviates. Once a player deviates, he continues to punish the deviator throughout his life. Fudenberg and Maskin [12] studied this strategy in general two-person games with finite repetition, and proved the folk theorem by assuming that both players have to play the GT strategy and that the number of repetitions is sufficiently high.

In our model of alternating repetition, ROW can revise his action only in odd periods, whereas COL can do so only in even periods. At the beginning of the game, nature stochastically decides whether ROW is rational and able to choose his action freely, or has to play either the GT or TFT strategy. Only ROW observes his type directly, while the probability of his type is common knowledge. We examine whether and how players can attain efficient payoffs in this model when they repeat the game for a sufficiently long time. Despite the negative results in several studies (see chapter 3), which show that inefficient outcomes that never arise in simultaneous move games sometimes cannot be excluded from the set of equilibrium payoffs in asynchronous move games, we show that *all* sequential equilibria remain almost efficient, even though the structure of repetition in our model is asynchronous.

The intuitive reason for this result is as follows. First, it is optimal for COL to always choose C, except in the last stage if ROW definitely plays either the Grim-Trigger or Tit-for-Tat strategy. Second, if rational ROW chooses D when COL chooses C, ROW's rationality becomes common knowledge, and players continue to choose D in all subsequent stages, yielding low continuation payoffs. However, if rational ROW chooses C when COL does the same, COL cannot determine ROW's type, and the players can obtain higher continuation payoffs from cooperation. Therefore, it is optimal for COL to cooperate, except in the last few stages if rational ROW also cooperates. Because ROW rationally expects this, he also cooperates, except in the last few stages if COL does so as well, which guarantees the existence of cooperative equilibria with efficient payoffs.

In chapter 4, we consider the two-person repeated prisoner's dilemma with private monitoring. This model does not satisfy (d). At the end of every period, each player receives a noisy private signal with a certain probability, depending on his opponent's current action instead of observing it directly. We assume that the probability distribution of private signals changes over time, depending on calendar dates. We also admit the case where the distribution of signals in any period is different from that in any other period. It is proved that efficient sequential equilibria exist with sufficiently patient players when the monitoring is almost perfect in every period. In order to prove this, we construct the strategy called the Markov strategy, which only depends on each player's own action, the private signal in the previous period, and the calendar date. We observe that when a player's opponent adopts a Markov strategy, he is indifferent between C and D in every period and, therefore, cannot change his continuation payoff by himself. When players are sufficiently patient and the monitoring technology is almost perfect in every period, such a strategy exists for the payoff vector between the payoff of (C, C) and (D, D), which ensures that efficient payoffs are attainable in this game. This argument also applies to the game with alternating repetition, which does not satisfy (c).