論文の内容の要旨

論文題目 Quantum Annealing: Quantum Statistical Physics applied to Classical Computational Hardness

(量子アニーリング:古典的計算困難性への量子統計物理の応用)

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The main purpose of this Ph.D thesis is to fully address the interrelations of physics and computational complexity through the topic of quantum annealing (QA). While QA was originally proposed as a quantum counterpart for the physics-inspired optimization heuristic of simulated annealing, it was soon proved that the generalization of QA is polynomially equivalent to quantum computers. This opened up possibility for detailed analysis of the interplay between physics and computational complexity, since both fields provided natural tools for analyzing QA from a different perspective. The interrelation between physics and computational complexity could be roughly classified into two types, one where physics affects computational complexity, and other where computational complexity affects physics. QA also provokes interesting problems of *how* we can actually draw border lines between classical and quantum. The three main chapters in this thesis will each deal with one type of the relations mentioned above.

In the first chapter, we will motivate why we would care about the relationship between physics and computational complexity at all in the first place. The structure of this thesis and relations between the chapters will be presented as well.

First we will review computational complexity theory in chapter 2. The field serves as the common language for describing hardness. Basic concepts such as completeness, reductions, and complexity classes will be defined. We will see how computational complexity can have implications for physics *at all*, by introducing the Church-Turing thesis and its "physical" versions, and also the NP hardness assumption, which we will set as a stand point in later chapters.

In chapter 3, we will explain QA and adiabatic quantum computing (AQC). As we mentioned, their relations are subtle and we will explain what we mean exactly by "QA is equivalent to quantum computers *in principle*", which is a commonly used phrase. We also define "stoquas-

ticity" which will become one of the central concepts when we consider classical simulability of QA.

In chapter 4, we address the question "is QA really *not* efficiently simulable by classical algorithms", from the perspective of Monte Carlo methods. While the stoquastic property of the QA Hamiltonian allows us to conduct classical simulations, we show that the equilibration time for the simulations may grow exponentially. We review examples called "obstructions" where it is shown that the classical Monte Carlo algorithm requires exponential time for equilibration while QA will only need polynomial time. Then we will provide arguments and also numerical evidence on how the obstructions may be overcome by using slightly more sophisticated algorithms. We present the seemingly "most likely way of proving classical simulability", which uses the exchange Monte Carlo (EMC) method. Then we construct an example which makes this approach fail, and numerically demonstrate so. This chapter will contain a detailed explanation of the Monte Carlo techniques which we will use in the later chapters. At the same time, we will be seeing common features of such algorithms which requires stoquasticity. This chapter will serve as making clear the fact that stoquasticity indeed lies somewhere between classical and quantum, and demonstrate how difficult it is to prove either equivalence or separation. This will motivate considering the setting for chapter 6, where stoquasticity may indeed have some physically interesting property.

In chapter 5, we address the question "how does physics affect computational complexity", in a classical mechanics set up. We first review known results on physical phase transitions which occur in *problems*. The phase transitions are *spin glass* transitions, which we will discuss its properties. While it may seem obvious that the spin glass transition induces computational hardness, this correspondence (which we will call the RS-RSB/easy-hard correspondence) is not known to generally hold true. In this chapter we focus on the maximum independent set (MIS) problem which is an NP-hard problem, which previous study suggested the break of the correspondence. We construct a new algorithm, which works exponentially more efficiently in some parameter region. The novel algorithm will work efficiently up until the spin glass phase transition point, exhibiting the RS-RSB/easy-hard correspondence. This chapter will show the necessity of an adequate algorithm to explore the relations between computational complexity and physics. This chapter also serves as a guiding principle in the next chapter, when we want to construct average-case hard ensembles for the MIS problem. We use knowledge of average-case complexity introduced in chapter 2, both in this chapter and in the next.

In chapter 6, we address the question "how does computational complexity affect physics", or more precisely, the question "does computational complexity have any implications to physics". We first argue that the NP hardness assumption will lead to a physical prediction for QA, namely that it predicts the existence of an exponentially small energy gap for some Hamiltonians. This by itself is straightforwardly deduced from the NP hardness assumption, and what makes it more interesting is the possibility that the exponentially small energy gap may accompany some physical phenomena. If this is the case, a computational complexity assumption will lead to a physically novel prediction. This is totally possible, and we see this from reviewing two previously proposed physical pictures for explaining the physical backgrounds of the exponentially small energy gaps. We use Monte Carlo techniques introduced in chapter 4 for analyzing the ideal QA procedure in the adiabatic limit, for the MIS problem with unique solutions. Insights and algorithms presented in chapter 5 will be used for constructing hard-on-average MIS problem instances with unique solutions. We will find first order transitions which accompany exponentially small energy gaps. Furthermore, a novel quantum phase will be detected only by the fidelity susceptibility χ_{F}. We confirm that the novel phase is compatible with *neither* of the previously proposed physical pictures. We especially see that the novel phase is not a spin glass phase from observing the spin glass susceptibility for a slightly different ensemble, which the classical limit is known from studies done in chapter 5. While this chapter will leave a big open question about the novel quantum phase, it will show possibility for a novel physical phases arising in a computational setting.

In chapter 7, we will go back through the chapters and discuss possible future directions, and finally conclude this thesis.