論文の内容の要旨

Higher-order quantum computation for equivalence determination of unitary operations

（高階量子計算を用いたユニタリ演算の同値性判別）

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Quantum state discrimination is one of the most fundamental tasks in quantum information since it describes the read-out process of classical information from quantum states in quantum communications. In quantum state discrimination, a given *test state* is guaranteed to be identical to one of a set of candidate quantum states. The task is to determine the identity of the test state by performing an appropriate quantum measurement based on the classical descriptions of the candidate states. Quantum state discrimination also has several connections to the principles of quantum mechanics. Perfect discrimination for non-orthogonal candidate states is impossible whereas perfect discrimination of different sequences of classical bits is always possible. Perfect discrimination of non-orthogonal quantum states leads contradictions, perfect cloning of unknown quantum states and instantaneous communication by an entangled state. A Schematics view of discrimination of quantum states is presented in Fig. 2.

The target of quantum system to be discriminated are not limited to quantum states. Quantum operations are also regarded as a carrier of information as well as quantum states. In discrimination of quantum operations, a black-box that implements a quantum operation which is chosen from a set of candidate quantum operations. The task is to determine which operation is performed by the black-box by applying the operation on an initial quantum state followed by an appropriate measurement. There is an intrinsic difference between discrimination of quantum states and discrimination of quantum operations. It was shown that perfect discrimination is possible for discrimination of a finite number of unitary operations with the finite uses of the operation whereas perfect discrimination is not possible for non-orthogonal quantum states with the finite copies of the state.

In quantum state discrimination, classical descriptions of candidate quantum states are assumed to be given. One of fundamental properties of quantum mechanics is that the exact classical description of unknown quantum states cannot be retrieved from a single copy of the state. It is interesting to consider quantum state discrimination in the setting with weaker resources characteristic in quantum mechanics under the assumption of the existence of quantum computer, that the candidate states are given as two unknown physical states (*reference states*) but their classical descriptions are not given. The two candidate states are labeled one and two, respectively, for distinguishing the two reference states. The task is to determine which of the two reference states is equivalent to the test state, whereas the description of the state is not of interest. We call such a task as *equivalence determination* of quantum states.

We analyze a unitary operation version of the equivalence determination task by using the concept of higher-order quantum computation to investigate properties of discrimination of other quantum objects than states. We consider that three black-boxes (*test box* and *reference boxes*) implementing unknown single-qubit unitary operations, respectively, are given as physical systems. The test box is guaranteed to implement one of the two unitary operations given by the reference boxes. Equivalence determination of unitary operations is a task to determine the reference box implementing the unitary operation of the test box. We assume that the unitary operations of the reference boxes are randomly sampled from the Haar measure of SU(2). We investigate the optimal success probability for this task in the framework of higher-order quantum computation by analyzing quantum implementations of supermaps taking unitary operations as inputs and a binary number indicating the result of discrimination as an output. A schematics view of *equivalence determination* of unitary operations are given in Fig. 2.

As strategies to perform equivalence determination of unitary operations, two different strategies, *parallel strategies* and *ordered strategies* are considered. In parallel strategies, operations between the uses of the black-boxes are not allowed. But in general, arbitrary operations can be inserted between the uses of black-boxes, which we call the ordered strategies. There are several tasks in which the ordered strategies outperform the parallel strategies, but conditions for improvements in ordered strategies are not yet known. Equivalence determination of unitary operations is a problem described by higher-order quantum computation of which inputs are more than two different operations. To the best of our knowledge, such a task considered before is only the problem of implementing so-called *quantum switch*. In quantum mechanics, when the order of operations is changed, the resulting operations may be different since quantum operations do not commute each other in general. Therefore the order of the use of the black-boxes is expected to affect the performance of equivalence determination of unitary operations. As an application, equivalence determination of unitary operations can be useful for checking whether two different quantum circuit representations implement the same quantum dynamics or not.

In Chap. 4, we first consider the case that each of three black-boxes can be used only once both in parallel and ordered uses of the black-boxes. We formulate optimization problems as semidefinite programmings (SDPs) and derive the optimal success probability for equivalence determination by obtaining analytical solutions of the SDPs both in parallel and ordered uses. We show that the ordered strategies do not give improvement in this case. In addition, we find that one of the two reference boxes is not necessary to achieve the optimal success probability since the optimal success probability can be also achieved by a simplified task called comparison of two unitary operations, which is a task to decide whether two black-boxes implement the same unitary operations or not. We also showed that entanglement of an initial state across the systems on which the reference boxes act and the test box acts is necessary to achieve the optimal success probability.

In Chap. 5, we then consider the case that the multiple uses of the reference boxes are allowed. We consider the parallel strategies up to four uses of each of the reference boxes and obtain the optimal success probabilities by numerically solving the corresponding SDPs. We numerically show that the ordered strategies give improvement over the parallel strategies and the optimal success probability varies depending on the deferent orders when one reference box is be used once and the other reference box is be used twice. This result indicates that an appropriate order of the black-boxes in the ordered strategies is necessary to obtain improvement over the parallel strategies.

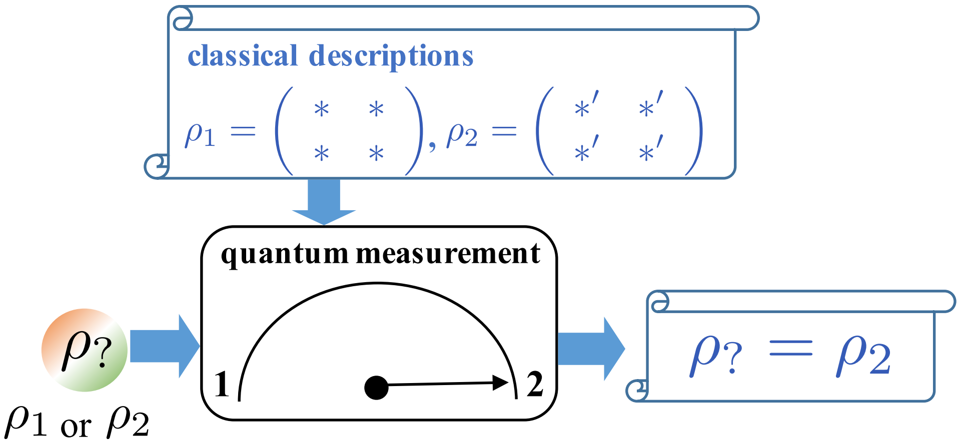

Fig. 1 A schematic representation of quantum state discrimination.

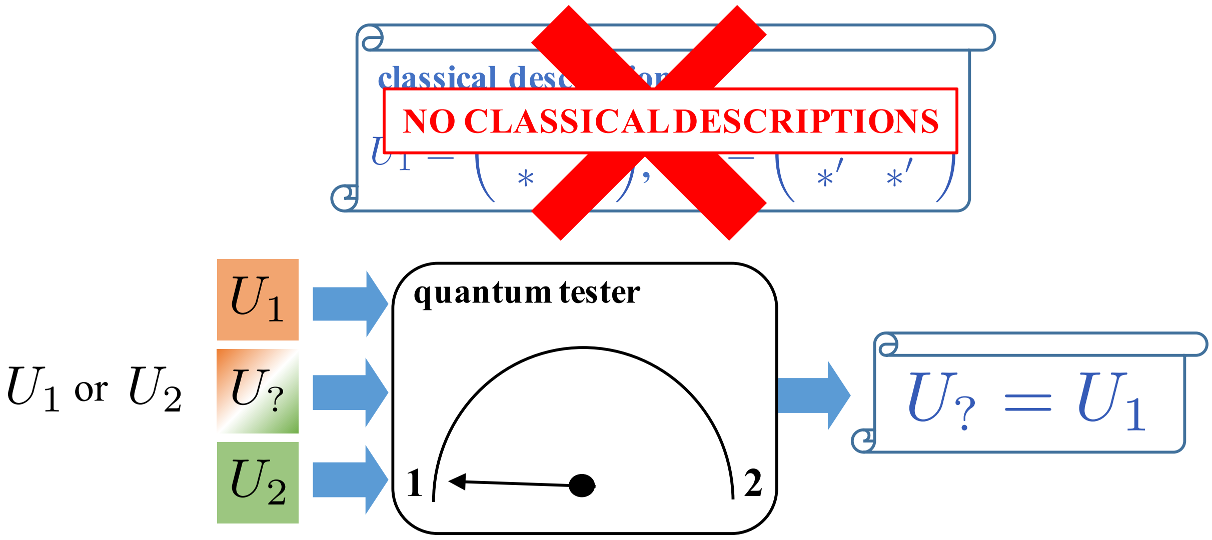

Fig. 2 A schematics view of equivalence determination of unitary operations.