学位論文

Techniques and phenomenology of radiative corrections from Kaluza-Klein modes in Scherk-Schwarz mechanism

(Scherk-Schwarz機構におけるKaluza-Kleinモードによる輻射補正のテクニックと現象論)

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Abstract

Supersymmetry (SUSY) has been attractive to the physicists searching for the physics beyond the standard model. However there is a tension between the Large Hadron Collider (LHC) results and the low scale SUSY scenario. This situation can be accommodated by the Scherk-Schwarz mechanism, that is a SUSY breaking mechanism by the twisted boundary condition in extra dimensional space since it generates the compressed spectrum of the supersymmetric particles. At the tree-level the degeneracy is exact and it is lifted by the radiative corrections.

In this thesis, we calculate the gaugino and sfermion mass corrections in a general setup, 5D SUSY gauge theory compactified over S^1/Z_2 orbifold. It is not trivial that the corrections are small enough to maintain the compressed spectrum, since the 5D gauge theory is not renormalizable. Furthermore there are an infinite number of loop diagrams since the 4D effective Lagrangian has Kaluza-Klein (KK) modes. We regularize the divergence by the KK-regularization scheme and find that the linear and higher divergence don't appear in the mass correction and there remain logarithmic divergence and constant.

We also discuss the compact SUSY model, a realistic application of the Scherk-Schwarz mechanism. Using the results in the general setup, we evaluate the gaugino and the stop mass corrections from the gauge and the Yukawa interactions on the brane. We find that the model has valid phenomenology in certain parameter region of the model.

Contents

1 Introduction

Issues in the standard model of particle physics

The standard model of particle physics is successful but has both experimental and theoretical issues. Therefore the standard model should be regarded as an effective field theory, a low energy limit of a more fundamental theory. Under this circumstance, the search for physics beyond the standard model (BSM) has been one of the primary subjects for many particle physicists.

Among the issues in the standard model, let us first focus on the hierarchy problem [1,2]; for the physical Higgs mass squared

$$m_{\text{phys}}^2 \sim m_{\text{bare}}^2 + \frac{\lambda}{16\pi^2}\Lambda^2 \tag{1.1}$$

to be $(125\text{GeV})^2$, there must be a weird cancellation between m_{bare}^2, a parameter of the theory, and the loop correction $\sim \Lambda^2/16\pi^2$, if the UV cutoff Λ is as large as Planck scale or GUT scale. Here λ in the numerator is just an order one coupling constant. This problem arises from the difference between the electroweak scale ~ 100 GeV and the UV scale $\gtrsim 10^{15}$ GeV. In other words, we need to tune the parameter of the theory very precisely to fit the experimental result, and thus this problem is also known as the fine-tuning problem. To tackle this what has been paid much attention to is the supersymmetry.

Supersymmetry

Supersymmetry (SUSY), a symmetry which exchanges a boson and a fermion, is one of the most attractive tools for BSM (for a review [3]). If the SUSY is exact, for each particle in the standard model there must be a corresponding particle (supersymmetric particle, or sparticles in short) which has the same mass. Since there are sparticles, there are corresponding loop corrections for each loop correction in the standard model which has the opposite sign, and thus the quadratic divergence to the Higgs mass squared correction vanishes and there is no more hierarchy problem.

However, there is obviously no experimental sign for the sparticles, for example there is no scalar particle which has the same mass as the electron. Fortunately, we can break the supersymmetry and ameliorate the hierarchy problem at the same time, if for example we assume the SUSY is spontaneously broken at some energy higher than experimental scale. In this case, the SUSY breaking effect is called soft (and parameters of the SUSY breaking terms in the effective Lagrangian are called soft parameters) and doesn't violate the cancellation of quadratic divergence in Higgs mass, since the correction to the Higgs mass now becomes the logarithmic divergence;

$$m_{\text{phys}}^2 \sim m_{\text{bare}}^2 + \frac{\lambda}{16\pi^2} M_{\text{soft}}^2 \ln\left(\frac{\Lambda}{M_{\text{soft}}}\right). \tag{1.2}$$

We don't need fine-tuning if the supersymmetric scale M_{soft} is less than about a few TeV. This idea of the weak scale SUSY as a solution to the hierarchy problem was suggested as early as in 1980 [4].

MSSM, CMSSM, and little hierarchy problem

Among the supersymmetric extensions of the standard model, the bench mark model is called the minimal supersymmetric standard model (MSSM) (see [3] for details of the model). It is minimal in the sense of its matter contents; there are the standard model particles and their super-partners, plus one more Higgs $SU(2)_L$ doublet in order for the anomaly cancellation and to implement the Yukawa coupling under the supersymmetric condition. Even though it has minimal matter content, there are about a hundred more parameters than in the standard model because of the introduction of sparticles and the soft SUSY breaking parameters. Therefore the search for SUSY has been often discussed within the framework of the Minimal Supergravity (mSUGRA) or the Constrained MSSM (CMSSM) [5–7], which has only five parameters.

However, in the framework of MSSM (and CMSSM) the SUSY breaking scale must be larger than a few TeV for the following reasons. The first reason is that there is a tree level upper bound $M_Z \sim 91$ GeV for the Higgs mass, while the Higgs has been found at 125 GeV [8,9]. Therefore we need to push the Higgs mass by the radiative corrections [10–15] from large SUSY breaking. The other reason is that SUSY has not been found at any experiment, even at the TeV-scale Large Hadron Collider (LHC). This large SUSY is a very bad situation for the community not only because it requires more experimental effort to find evidence for SUSY, but also because it again generates the hierarchy problem (called the little hierarchy problem).

Compressed spectrum of sparticles

To overcome the little hierarchy problem, the idea of the compressed spectrum has been suggested [16–19]. The key idea of the compressed spectrum is to assume all the sparticles has similar masses so that the decay of sparticles generated by the collider is very difficult to distinguish with the standard model background. Basically, the search for sparticles relies on the events that have missing transverse energy since the LSP, the lightest supersymmetric particle, cannot be seen at the detector. The standard model also has energy-missing events due to the neutrino or the instruments nature, and the lower the missing energy the exponentially more the events happens. If the sparticles have similar masses, the associate standard model particles has small energy by the kinematics and thus that kinds of events are almost invisible. Therefore the compressed SUSY scenario is still consistent with the LHC result even if it has low scale ($M_{\text{soft}} \sim$ TeV) SUSY scale (see Fig. 1). This was first just an idea that doesn't have theoretical reasoning, until the compact SUSY model has been proposed.

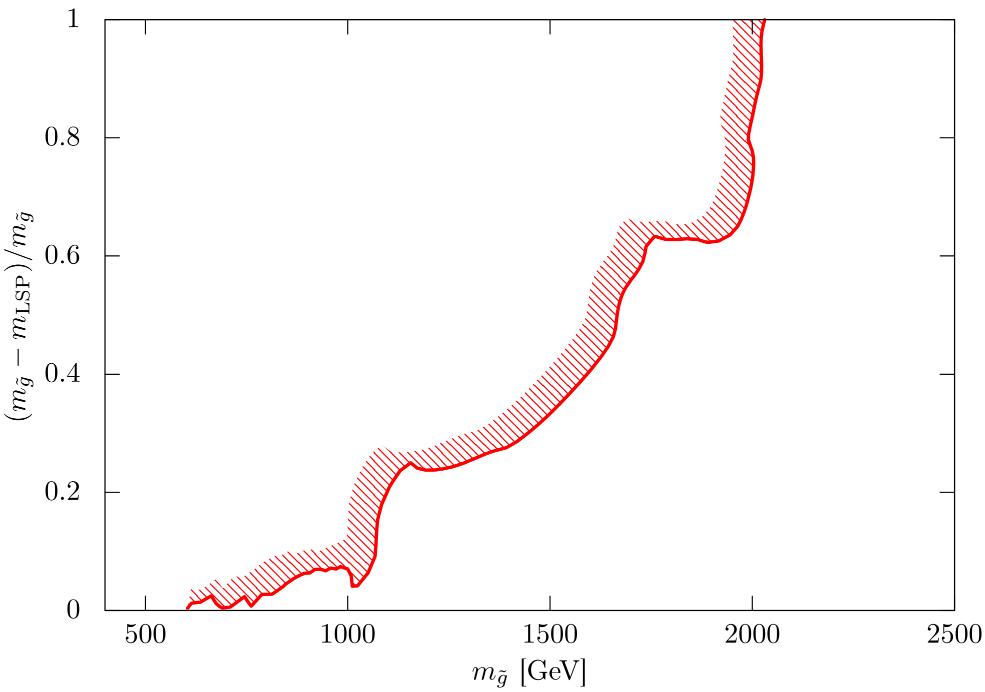


Figure 1: The horizontal axis is the gluino mass and the vertical axis represents the degeneracy between the gluino and the LSP. The region above the red curve is excluded while the bottom part of the figure (degenerate region) is still allowed. This figure is made by modifying the fig.13b in [20].

There are also other possibilities to solve the little hierarchy problem, such as the anthropic principle (like in [21–35]), the R-parity violation models [36–38], and the stealth SUSY scenario [39, 40]. We in this thesis focus on the compressed spectrum idea.

The Scherk-Schwarz mechanism and the compact SUSY model

It was shown that SUSY model with compressed spectrum can be built by using the Scherk-Schwarz mechanism [41, 42], where SUSY is broken by twisted boundary conditions of an extra-dimension (S^1/Z_2, in the simplest case). The twist of the boundary condition can be parametrized by one parameter $\alpha \in [0, 1/2]$, and the mass spectrum of the supersymmetric particles is nearly degenerate (compressed). At tree-level, all the soft masses of gaugino and sfermions are the same ($= \alpha/R$, where R is the radius of the extra-dimension), and the degeneracy is broken by radiative corrections.

A realistic model has been proposed [43–45] that uses this mechanism. This model is phenomenologically attractive not only because it can solve the tension between the LHC results and the hierarchy problem as explained above, but also because it has only four parameters (α, R, μ and the cutoff of the model Λ) and thus very predictive. Furthermore, thanks to the A-term we can explain naturalness problem more easily [46]. We call this model the compact SUSY model, and to investigate the model is the primary purpose of this thesis. Although the Higgs sector of the compact SUSY model has been analyzed at 1-loop in [47–49], the other radiative corrections have not been done yet. It is not trivial that the compact SUSY model can retain the degenerate spectrum if the radiative corrections are taken into account, especially since the 5D gauge theory is UV sensitive (in contrast to the Higgs fields which live on a brane in the model). We first study the radiative corrections of the sparticles, the gaugino and the sfermion at 1-loop in a general 5D SUSY gauge theory compactified over S^1/Z_2, and then apply the results to the compact SUSY model. To discuss the full phenomenology, we also need radiative correction from the Yukawa interaction on the brane (Higgs fields live on the brane in the model). We show that the degenerate spectrum holds in some region of the parameter space.

The structure of this paper

This thesis is organized as follows;

- We are now at the end of section 1 and this is the introduction part, where we have briefly reviewed the issues of the standard model and the motivation of supersymmetry that leads to the compact SUSY model.

- In section 2, we write down the Lagrangian for the 5D supersymmetric Yang Mills theory, which the compact SUSY model is based on. Although it is a non-trivial task to write down the Lagrangian, we already have results of higher dimensional supersymmetric Lagrangian with the familiar $\mathcal{N}=1$, $D=4$ superspace notation. We derive the component expression and check the $SU(2)_R$ symmetry for the later calculations.

- In section 3, we describe the Scherk-Schwarz mechanism, that is an essential concept for the compact SUSY mode. Although the naive compactification of 5D supersymmetric theory over S^1/Z_2 leads to 4D $\mathcal{N}=1$ supersymmetric theory, if we introduce the twisted boundary conditions around the S^1 we can break the remaining supersymmetry and obtain the softly broken SUSY with degenerate spectrum. We explain this mechanism both by using the component expression, and by the Radion mediation. We also consider the fields on branes and its interactions.

- In section 4, we calculate the 1-loop radiative correction to the gaugino and sfermion masses in the general setup shown in section 3. Since this part is rather technical, we only show the results and put the detailed calculation in the Appendices A, B and C; A for the notation, B for the derivation of the integration formulae, C for the detailed calculation of the 4D Lagrangian and the Feynman diagrams.

- In section 5, we discuss the compact SUSY model, which is an extension of the standard model which utilizes the Scherk-Schwarz mechanism. After defining the model setup, we show the 1-loop spectrum of the theory using the results in section 4, and calculating the correction from the Yukawa couplings on the brane. After that we discuss the implication of the 1-loop results and the phenomenology of the compact SUSY.

- Section 6 is the conclusion and supplementary comments and future outlooks, such as further phenomenological discussion on the compact SUSY model, and the realization of the Scherk-Schwarz mechanism and the compact SUSY model in the framework of the superstring theory.

Some of the results in section 4 and 5 will be on a journal as a collaboration work [50].

2 Lagrangian for 5D SUSY Yang Mills Theory

In this section, we write down the Lagrangian for the 5D $\mathcal{N}=1$ supersymmetric theory since it is the basis of the compact SUSY model. Although it's a non-trivial task to construct this Lagrangian, there is fortunately a familiar 4D superspace notation [51]. Starting with this Lagrangian, we derive the component expressions in a 5D Lorentz-symmetry-manifest way and check the $SU(2)_R$ invariance for the later discussion. Similar computation can be seen in [49], but note that the notation is a little bit different.

2.1 Notation

Our purpose is first to write down the 5D SUSY gauge theory Lagrangian with 4D $\mathcal{N}=1$ superspace notation. From a 4D point of view, there seems to be $\mathcal{N}=2$ SUSY, and thus the gauge field A_μ must be combined with two weyl fermions λ_1, λ_2 and one complex scalar fields χ if we consider a multiplet with minimal helicity. These combination (vector multiplet) can be written by a pair of a 4D vector superfield and a chiral superfield as follows;

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda}_1 - i\bar{\theta}^2\theta\lambda_1 + \frac{1}{2}\theta^4 D, \tag{2.1}$$

$$\chi = \frac{A_5 + i\Sigma}{\sqrt{2}} + \sqrt{2}\theta\lambda_2 + \theta^2 F_\chi, \tag{2.2}$$

which are adjoint representation of gauge group G, i.e.

$$X = X^a T^a, \quad X = V, \chi, A_M, \lambda_i, D, \Sigma, F_\chi, \tag{2.3}$$

where T^a is the generator of G. For the component expression, we have taken Wess Zumino gauge.

In addition to the vector multiplet, we can add a Hyper multiplet which can be written in 4D superspace notation as

$$\begin{aligned} \Phi &= \phi_1 + \sqrt{2}\theta\psi_1 + \theta^2 F_1, \\ \Phi^C &= \phi_2 + \sqrt{2}\theta\psi_2 + \theta^2 F_2, \end{aligned} \tag{2.4}$$

where Φ is a chiral superfield which is a fundamental representation of G, and Φ^C is also a chiral superfield but an anti-fundamental representation of G. Or, we can add multiple (say, F) numbers of Hyper multiplets, which has $SU(F)$ global symmetry. (Φ and Φ^C are fundamental and anti-fundamental representation, respectively.) We don't assign any indices for the flavor to keep the notation quiet.

Using the above notation, we summarize the 5D Lagrangian in $G = SU(N)$, $SU(F)$ flavors case. To keep the discussion less messy, we divide the Lagrangian into three parts, the Vector part, Hyper part and gauge fixing term;

$$\mathcal{L}_5 = \mathcal{L}_5^{\text{Vector}} + \mathcal{L}_5^{\text{Hyper}} + \mathcal{L}_5^{\text{gauge-fix}}, \tag{2.5}$$

and, in the following subsections, work on part by part to derive the component expression starting from the 4D superspace formalism.

2.2 Vector Part

In 4D superspace notation, $\mathcal{L}_5^{\text{Vector}}$ consists of two parts;

$$\mathcal{L}_5^{\text{Vector}} = \mathcal{L}_5^{\text{Vector},1} + \mathcal{L}_5^{\text{Vector},2}, \tag{2.6}$$

where

$$\mathcal{L}_5^{\text{Vector},1} = \frac{1}{16kg^2}\text{Tr}\left\{\int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}\right\}, \tag{2.7}$$

$$\begin{aligned}\mathcal{L}_5^{\text{Vector},2} = -\frac{1}{2kg^2}\int d^4\theta \ \text{Tr}\Big\{&(\partial_5 - \sqrt{2}ig\chi^\dagger)e^{-2gV}(\partial_5 - \sqrt{2}ig\chi)e^{2gV} \\ &- \frac{1}{2}\partial_5 e^{-2gV}\partial_5 e^{2gV} + g^2(\chi\chi + \chi^\dagger\chi^\dagger)\Big\}.\end{aligned} \tag{2.8}$$

As usual, k is defined by the trace of the generator of the group

$$\text{Tr}\left[T^a T^b\right] = k\delta^{ab}, \tag{2.9}$$

and $\mathcal{W}_\alpha$ is the gauge invariant chiral superfield out of V;

$$\begin{aligned}\frac{1}{g}\mathcal{W}_\alpha &= -\frac{1}{4g}\bar{D}^2 e^{-2gV} D_\alpha e^{2gV} \\ &= -i\lambda_{1\alpha} + \theta_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta\theta_\beta\partial_{[\mu}A_{\nu]} + \theta^2\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}_1^{\dot{\alpha}}.\end{aligned} \tag{2.10}$$

The first part $\mathcal{L}_5^{\text{Vector},1}$ is the usual 4D gauge kinetic term, whereas the second one $\mathcal{L}_5^{\text{Vector},2}$ is χ's 4D kinetic term and 5D parts. The combination of them makes 5D Lorentz, SUSY and gauge invariant Lagrangian for the Vector multiplet.

In component expression, each part of the Lagrangian becomes

$$\mathcal{L}_5^{\text{Vector},1} = -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - i\bar{\lambda}_1^a\bar{\sigma}^\mu\mathcal{D}_\mu\lambda_1^a + \frac{1}{2}(D^a)^2, \tag{2.11}$$

$$\begin{aligned}\mathcal{L}_5^{\text{Vector},2} = &-\frac{1}{2}F^{a\mu5}F^a_{\mu5} - i\lambda_2^a\sigma^\mu\mathcal{D}_\mu\bar{\lambda}_2^a + \lambda_2^a\mathcal{D}_5\lambda_1^a - \bar{\lambda}_1^a\mathcal{D}_5\bar{\lambda}_2^a \\ &+ igf^{abc}\left(\lambda_1^a\Sigma^b\lambda_2^c - \bar{\lambda}_1^a\Sigma^b\bar{\lambda}_2^c\right) \\ &- \frac{1}{2}\mathcal{D}_\mu\Sigma^a\mathcal{D}^\mu\Sigma^a - \mathcal{D}_5\Sigma^a D^a + \left|F_\chi^a\right|^2.\end{aligned} \tag{2.12}$$

Here, the covariant derivative for the adjoint representation field is defined as

$$\mathcal{D}_M X^a = \partial_M X^a + gf^{abc}A_M^b X^c, \quad M = \mu, 5, \quad X^a = \lambda_1^a, \lambda_2^a, \Sigma^a. \tag{2.13}$$

When combining the two portions of the Lagrangian, it is useful to define

$$D'^a = D^a - \mathcal{D}_5 \Sigma^a. \tag{2.14}$$

This helps us eliminate D^a linear term and get 5D Lorentz explicit form for Σ^a. Thus, we obtain

$$\begin{aligned}
\mathcal{L}_5^{\text{Vector}} &= \mathcal{L}_5^{\text{Vector},1} + \mathcal{L}_5^{\text{Vector},2} \\
&= -\frac{1}{4} F^{aMN} F^a_{MN} - \frac{1}{2} \mathcal{D}_M \Sigma^a \mathcal{D}^M \Sigma^a - \left(-\lambda_2, \bar{\lambda}_1\right) i\Gamma^M \mathcal{D}_M \begin{pmatrix} \lambda_1 \\ -\bar{\lambda}_2 \end{pmatrix} \\
&\quad + g f^{abc} \left(\lambda_2^a \Sigma^b \lambda_1^c - \bar{\lambda}_1^a \Sigma^b \bar{\lambda}_2^c\right) + \left|F_\chi^a\right|^2 + \frac{1}{2} (D'^a)^2.
\end{aligned} \tag{2.15}$$

To write the gaugino kinetic term, we have used

$$i\Gamma^M \mathcal{D}_M = \begin{pmatrix} \mathcal{D}_5 & i\sigma^\mu \mathcal{D}_\mu \\ i\bar{\sigma}^\mu \mathcal{D}_\mu & -\mathcal{D}_5 \end{pmatrix}. \tag{2.16}$$

We take the chiral representation for the gamma matrices in this entire paper. See Appendix A for more detail of the notation.

2.3 Hyper Part

In 4D superspace notation, $\mathcal{L}_5^{\text{Hyper}}$ consists of two parts;

$$\mathcal{L}_5^{\text{Hyper}} = \mathcal{L}_5^{\text{Hyper},1} + \mathcal{L}_5^{\text{Hyper},2}, \tag{2.17}$$

where

$$\mathcal{L}_5^{\text{Hyper},1} = \int d^4\theta \left[\Phi^\dagger e^{-2gV} \Phi + \Phi^C e^{2gV} \Phi^{C\dagger}\right], \tag{2.18}$$

$$\mathcal{L}_5^{\text{Hyper},2} = \int d^2\theta \, \Phi^C \left(\partial_5 - \sqrt{2} i g \chi\right) \Phi + \text{h.c.}. \tag{2.19}$$

The first part $\mathcal{L}_5^{\text{Hyper},1}$ is the familiar 4D kinetic terms for Φ and Φ^C, and the second one $\mathcal{L}_5^{\text{Hyper},2}$ is the 5D part. In component expression,

$$\begin{aligned}
\mathcal{L}_5^{\text{Hyper},1} &= -\left(\mathcal{D}^\mu \phi_1\right)^\dagger \mathcal{D}_\mu \phi_1 - \left(\mathcal{D}^\mu \phi_2^\dagger\right)^\dagger \mathcal{D}_\mu \phi_2^\dagger - i\bar{\psi}_1 \bar{\sigma}^\mu \mathcal{D}_\mu \psi_1 - i\psi_2 \sigma^\mu \mathcal{D}_\mu \bar{\psi}_2 + |F_1|^2 + |F_2|^2 \\
&\quad - \sqrt{2} i g \left(\phi_1^\dagger \lambda_1 \psi_1 - \bar{\psi}_1 \bar{\lambda}_1 \phi_1 + \phi_2 \bar{\lambda}_1 \bar{\psi}_2 - \psi_2 \lambda_1 \phi_2\right) - g \left(\phi_1^\dagger D \phi_1 - \phi_2 D \phi_2^\dagger\right),
\end{aligned} \tag{2.20}$$

and

$$
\begin{aligned}
\mathcal{L}_5^{\text{Hyper},2} = & F_2\left[\partial_5 - ig\left(A_5 + i\Sigma\right)\right]\phi_1 + \phi_2\left[\partial_5 - ig\left(A_5 + i\Sigma\right)\right]F_1 \\
& - \psi_2\left[\partial_5 - ig\left(A_5 + i\Sigma\right)\right]\psi_1 \\
& + \sqrt{2}ig\left(\psi_2\lambda_2\phi_1 + \phi_2\lambda_2\psi_1\right) - \sqrt{2}ig\phi_2 F_\chi \phi_1 + \text{h.c.}.
\end{aligned} \tag{2.21}
$$

The covariant derivative for the fundamental representation field is defined as

$$
\mathcal{D}_M X = \partial_M X - igA_M X, \quad M = \mu, 5, \quad X = \phi_1, \phi_2^\dagger, \psi_1, \bar{\psi}_2. \tag{2.22}
$$

When combining the two portions of the Lagrangian, it is useful to define

$$
F_1'^* = F_1^\dagger + \left(-\mathcal{D}_5 + g\Sigma\right)\phi_2, \tag{2.23}
$$

$$
F_2'^* = F_2^\dagger + \left(\mathcal{D}_5 + g\Sigma\right)\phi_1, \tag{2.24}
$$

to clear the F_i linear terms and to get 5D Lorentz invariant kinetic terms for ϕ_i, as similar to the Σ^a case. Therefore we obtain

$$
\begin{aligned}
\mathcal{L}_5^{\text{Hyper}} &= \mathcal{L}_5^{\text{Hyper},1} + \mathcal{L}_5^{\text{Hyper},2} \\
&= -\left(\mathcal{D}^M\phi_1\right)^\dagger \mathcal{D}_M\phi_1 - \left(\mathcal{D}^M\phi_2^\dagger\right)^\dagger \mathcal{D}_M\phi_2^\dagger - \left(\psi_2, \bar{\psi}_1\right) i\Gamma^M \mathcal{D}_M \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} + \left|F_1'\right|^2 + \left|F_2'\right|^2 \\
&\quad - g^2\left(\Sigma\phi_1\right)^\dagger\left(\Sigma\phi_1\right) - g^2\left(\Sigma\phi_2^\dagger\right)^\dagger\left(\Sigma\phi_2^\dagger\right) - g\left(\psi_2\Sigma\psi_1 + \bar{\psi}_1\Sigma\bar{\psi}_2\right) \\
&\quad - \sqrt{2}ig\left(\phi_1^\dagger, \phi_2\right)\left[\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}\psi_1 + \begin{pmatrix} \bar{\lambda}_2 \\ \bar{\lambda}_1 \end{pmatrix}\bar{\psi}_2\right] + \text{h.c.} \\
&\quad - g\left(\phi_1^\dagger D'\phi_1 - \phi_2 D'\phi_2^\dagger\right) - \sqrt{2}ig\left(\phi_2 F_\chi \phi_1 - \phi_1^\dagger F_\chi^\dagger \phi_2^\dagger\right).
\end{aligned} \tag{2.25}
$$

2.4 Summary

After integrating out the auxiliary fields F_χ, D^a, we obtain the following Lagrangian;

$$
\begin{aligned}
\mathcal{L}_5 = -\frac{1}{4}F^{aMN}F^a_{MN} + \mathcal{L}_5^{\text{gauge-fix}} - \frac{1}{2}\mathcal{D}_M\Sigma^a\mathcal{D}^M\Sigma^a - \left(-\lambda_2, \bar{\lambda}_1\right) i\Gamma^M\mathcal{D}_M \begin{pmatrix} \lambda_1 \\ -\bar{\lambda}_2 \end{pmatrix} \\
- \left(\mathcal{D}^M\phi_1\right)^\dagger \mathcal{D}_M\phi_1 - \left(\mathcal{D}^M\phi_2^\dagger\right)^\dagger \mathcal{D}_M\phi_2^\dagger - \left(\psi_2, \bar{\psi}_1\right) i\Gamma^M\mathcal{D}_M \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} \\
- g^2\left(\Sigma\phi_1\right)^\dagger\left(\Sigma\phi_1\right) - g^2\left(\Sigma\phi_2^\dagger\right)^\dagger\left(\Sigma\phi_2^\dagger\right) - g\left(\psi_2\Sigma\psi_1 + \bar{\psi}_1\Sigma\bar{\psi}_2\right) \\
+ gf^{abc}\left(\lambda_2^a\Sigma^b\lambda_1^c - \bar{\lambda}_1^a\Sigma^b\bar{\lambda}_2^c\right) \\
- \sqrt{2}ig\left(\phi_1^\dagger, \phi_2\right)\left[\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}\psi_1 + \begin{pmatrix} \bar{\lambda}_2 \\ \bar{\lambda}_1 \end{pmatrix}\bar{\psi}_2\right] + \text{h.c.} \\
- \frac{g^2}{2}\left(\phi_1^\dagger T^a\phi_1 - \phi_2 T^a\phi_2^\dagger\right)^2 - 2g^2\left(\phi_1^\dagger T^a\phi_2^\dagger\right)\left(\phi_2 T^a\phi_1\right)
\end{aligned}
\tag{2.26}
$$

$$
\begin{aligned}
= \mathcal{L}_{5,A_M} + \mathcal{L}_{5,\Sigma} + \mathcal{L}_{5,\lambda} + \mathcal{L}_{5,\phi} + \mathcal{L}_{5,\psi} \\
+ \mathcal{L}_{5,\phi\Sigma\Sigma\phi} + \mathcal{L}_{5,\psi\Sigma\psi} + \mathcal{L}_{5,\lambda\Sigma\lambda} + \mathcal{L}_{5,\phi\lambda\psi} + \mathcal{L}_{5,\phi\phi\phi\phi}.
\end{aligned}
\tag{2.27}
$$

In the second equality, we have defined $\mathcal{L}_{5,*}$ for later convenience.

2.5 $SU(2)_R$ symmetry

$\mathcal{L}_5$ has to have the $SU(2)_R$ symmetry. In component expression the $SU(2)_R$ transformation is

$$
\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} \rightarrow \begin{pmatrix} \phi_1' \\ \phi_2'^\dagger \end{pmatrix} = U\begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix}, \tag{2.28}
$$

$$
\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} \rightarrow \begin{pmatrix} \lambda_1' \\ -\lambda_2' \end{pmatrix} = U\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}, \tag{2.29}
$$

where

$$
SU(2) \ni U = e^{i\theta^a\frac{\sigma^a}{2}} = \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix}, \quad \alpha, \beta \in \mathbf{C},\ |\alpha|^2 + |\beta|^2 = 1, \tag{2.30}
$$

and the other components are singlet under $SU(2)_R$ symmetry [1]. This $SU(2)_R$ symmetry plays a key role when we consider the Scherk-Schwarz mechanism in the next section.

[1] Here we consider the Lagrangian from which the auxiliary fields D^a, F^a_χ and F_1, F_2 are already removed by the equations of motion. Actually the $SU(2)_R$ transformation of the auxiliary fields are quite difficult to decide. The $SU(2)_R$ invariance of the Vector part of Lagrangian can be seen if we notice the D^a and F_χ form a triplet [52]. However it is difficult to see the invariance of the Hyper part since the transformation of the auxiliary fields F_1 and F_2 are not trivial.

Therefor we devote the following discussion to check the $SU(2)_R$ invariance of $\mathcal{L}_{5,\lambda}$, $\mathcal{L}_{5,\phi\lambda\psi}$, $\mathcal{L}_{5,\lambda\Sigma\lambda}$, $\mathcal{L}_{5,\phi\phi\phi\phi}$ (the invariance of other terms are trivial). To this end, it is useful to find other $SU(2)_R$ doublets. First, since the fundamental representation of $SU(2)$ and anti-fundamental representation of $SU(2)$ are equivalent, following combinations

$$\mathbf{\Phi}^c = \begin{pmatrix} \phi_2 \\ -\phi_1^\dagger \end{pmatrix}, \quad \begin{pmatrix} \bar{\lambda}_2 \\ \bar{\lambda}_1 \end{pmatrix} \tag{2.31}$$

are also $SU(2)_R$ doublets. Second, using a pair of symplectic Majorana fields;

$$\Upsilon_1 = \begin{pmatrix} \lambda_1 \\ -\bar{\lambda}_2 \end{pmatrix}, \quad \Upsilon_2 = \begin{pmatrix} \lambda_2 \\ \bar{\lambda}_1 \end{pmatrix} \tag{2.32}$$

we can show that $(\Upsilon_1, -\Upsilon_2)^T$ is also a $SU(2)_R$ doublet, i.e.

$$\begin{pmatrix} \Upsilon_1 \\ -\Upsilon_2 \end{pmatrix} \to \begin{pmatrix} \Upsilon_1' \\ -\Upsilon_2' \end{pmatrix} = U \begin{pmatrix} \Upsilon_1 \\ -\Upsilon_2 \end{pmatrix}. \tag{2.33}$$

The rest of the work is just to arrange the terms so they are represented by the above doublets.

$SU(2)_R$ invariance of $\mathcal{L}_{5,\lambda}$

The kinetic term for the gaugino can be easily rewritten using the above defined symplectic Mojorana fields;

$$\begin{aligned}
\mathcal{L}_{5,\lambda} &= -\left(-\lambda_2, \bar{\lambda}_1\right) i\Gamma^M \mathcal{D}_M \begin{pmatrix} \lambda_1 \\ -\bar{\lambda}_2 \end{pmatrix} = -\overline{\Upsilon}_1 i\Gamma^M \mathcal{D}_M \Upsilon_1 = -\overline{\Upsilon}_2 i\Gamma^M \mathcal{D}_M \Upsilon_2 \\
&= -\frac{1}{2}\sum_{i=1,2} \overline{\Upsilon}_i i\Gamma^M \mathcal{D}_M \Upsilon_i = -\frac{1}{2}\left(\overline{\Upsilon}_1, -\overline{\Upsilon}_2\right) i\Gamma^M \mathcal{D}_M \begin{pmatrix} \Upsilon_1 \\ -\Upsilon_2 \end{pmatrix}.
\end{aligned} \tag{2.34}$$

Since $(\Upsilon_1, -\Upsilon_2)$ and its Dirac conjugate are doublets, the invariance is manifest in this form.

$SU(2)_R$ invariance of $\mathcal{L}_{5,\phi\lambda\psi}$

The 3-point interaction terms between the sfermion ϕ, the fermion ψ and the gaugino λ can be rewritten as follows

$$\begin{aligned}
\mathcal{L}_{5,\phi\lambda\psi} &= -\sqrt{2}ig\left(\phi_1^\dagger, \phi_2\right)\left[\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}\psi_1 + \begin{pmatrix} \bar{\lambda}_2 \\ \bar{\lambda}_1 \end{pmatrix}\bar{\psi}_2\right] + \text{h.c.} \\
&= -\sqrt{2}ig\left(\phi_1^\dagger, \phi_2\right) T^a \overline{\Psi}^c \begin{pmatrix} \Upsilon_1^a \\ -\Upsilon_2^a \end{pmatrix} + \text{h.c.}.
\end{aligned} \tag{2.35}$$

Here, Ψ^c is defined as

$$\Psi^c = \begin{pmatrix} \psi_2 \\ \bar{\psi}_1 \end{pmatrix}. \tag{2.36}$$

$SU(2)_R$ invariance of $\mathcal{L}_{5,\lambda\Sigma\lambda}$

The gaugino-gaugino-Σ interaction terms are

$$\begin{aligned}\mathcal{L}_{5,\lambda\Sigma\lambda} &= igf^{abc}\left(\lambda_1^a\Sigma^b\lambda_2^c - \bar{\lambda}_1^a\Sigma^b\bar{\lambda}_2^c\right) \\ &= \frac{ig}{2}f^{abc}\left(\overline{\Upsilon}_1^a, -\overline{\Upsilon}_2^a\right)\Sigma^b\begin{pmatrix}\Upsilon_1^c \\ -\Upsilon_2^c\end{pmatrix},\end{aligned} \tag{2.37}$$

and its invariance is manifest.

$SU(2)_R$ invariance of $\mathcal{L}_{5,\phi\phi\phi\phi}$

The sfermion 4-point interaction terms can be expressed using only the $SU(2)_R$ doublets as follows;

$$\mathcal{L}_{5,\phi\phi\phi\phi} = -\frac{g^2}{2}\left(\phi_1^\dagger T^a\phi_1 - \phi_2 T^a\phi_2^\dagger\right)^2 - 2g^2\left(\phi_1^\dagger T^a\phi_2^\dagger\right)\left(\phi_2 T^a\phi_1\right) \tag{2.38}$$

$$= -g^2\left[|\mathbf{\Phi}^c T^a\mathbf{\Phi}|^2 - \frac{1}{2}|\mathbf{\Phi}^\dagger T^a\mathbf{\Phi}|^2\right]. \tag{2.39}$$

The invariance is again trivial to see.

3 $S^1/\mathbf{Z}_2$ orbifold compactification of 5D SUSY Yang Mills Theory

In the previous section, we have written down the Lagrangian for the 5D SUSY gauge theory with matter fields described as Hyper multiplets. Since we are interested in realistic model, we need to compactify the extra 5th dimension. If the shape of the extra dimension is assumed to be S^1, the 4D theory has $\mathcal{N}=2$ SUSY that cannot contain chiral fermions which is a crucial defect for phenomenology. If we take the S^1/Z_2 orbifold compactification, we can obtain chiral fermions and the 4D effective theory has $\mathcal{N}=1$ SUSY. Still we need to break one more SUSY, and here comes the Scherk-Schwarz mechanism [41, 42], where the SUSY is softly broken by the twisted boundary condition.

In this section, we first review the Scherk-Schwarz mechanism and derive the 4D effective Lagrangian with out component notation for the later analysis of the compact SUSY model.

3.1 The Scherk-Schwarz mechanism

Let y be a coordinate of the extra dimensional circle S^1 and consider the parity and translation that act on y as;

$$\mathcal{P} : y \to -y, \quad \mathcal{T} : y \to y + 2\pi R, \tag{3.1}$$

where R is the radius of the circle S^1 ($\mathcal{P}$ is same as the Z_2 but we just follow the convention). The algebra is given by

$$\mathcal{P}^2 = 1, \quad \mathcal{P}\mathcal{T}\mathcal{P} = \mathcal{T}^{-1}. \tag{3.2}$$

Let us further introduce the transformation rules for the fields under these symmetries. Let $\phi_{i=1\sim N}$ be a N dimensional representation and its transformation under $\mathcal{P}$ and $\mathcal{T}$ be

$$\mathcal{P} : \phi_i(y) \to \phi'_i(-y) = P_i^{\,j}\phi_j(y), \quad \mathcal{T} : \phi_i(y) \to \phi'_i(y + 2\pi R) = T_i^{\,j}\phi_j(y). \tag{3.3}$$

We impose the boundary condition as follows;

$$\mathcal{P} : P_i^{\,j}\phi_j(-y) = \phi_i(y), \quad \mathcal{T} : T_i^{\,j}\phi_j(y - 2\pi R) = \phi_i(y). \tag{3.4}$$

Let us first consider the one dimensional representation of the algebra. Since $\mathcal{P}^2 = 1$, $P = \pm 1$ and from $\mathcal{P}\mathcal{T}\mathcal{P} = \mathcal{T}^{-1}$ we obtain $T = \pm 1$. In the case of $T = -1$, there is no mass-less particle in the spectrum which is not desireble for phenomenology and thus we take $T = +1$. Therefore we have two kinds of the fields, distinguished by the Z_2 charge P, the even parity $P = +1$ and the odd parity $P = -1$ fields and each can be expanded as follows;

$$\phi(y) = \sum_{n=0}^{\infty} c_n \cos\left(\frac{n}{R} y\right) \quad (P = +1), \tag{3.5}$$

$$\phi(y) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n}{R} y\right) \quad (P = -1). \tag{3.6}$$

Let us next consider the two dimensional representation. The irreducible representation is given by

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} \cos 2\pi\alpha & \sin 2\pi\alpha \\ -\sin 2\pi\alpha & \cos 2\pi\alpha \end{pmatrix} = e^{i\sigma_2(2\pi\alpha)}. \tag{3.7}$$

This is equivalent to

$$P' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T' = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}. \tag{3.8}$$

under the unitary matrix $U = \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. We embed this representation on to the $SU(2)_R$ doublets and show that the $\mathcal{N} = 1$ SUSY is softly broken by imposing the boundary condition in the next subsection. Whereas $SU(2)_R$ singlets are 1-dimensional representation and don't contribute to the SUSY breaking.

3.2 KK expansion

In this subsection, we derive the KK expanded 4D Lagrangian in the Scherk-Schwarz mechanism, that is we would like to perform the following y integral;

$$\mathcal{L}_4 = \int_0^{2\pi R} dy\ \mathcal{L}_5. \tag{3.9}$$

We define

$$\hat{\alpha} = \frac{\alpha}{R}, \quad \hat{n} = \frac{n}{R}, \quad \hat{m} = \frac{m}{R}, \tag{3.10}$$

to tidy up the presentation of the equations. We also introduce notation $\mathcal{L}_{d,*} = \mathcal{L}_{d,*}^{\text{free}} + \mathcal{L}_{d,*}^{\text{gauge-int}}$, for $d = 4, 5$ and $* = \phi, \psi, A_\mu, A_5, \Sigma, \lambda$ to express which part of Lagrangian we are focusing on.

3.2.1 Σ^a

We start from the simplest case, a $SU(2)_R$ singlet real scalar field Σ^a. Since the $A_5^a(y)$ has the odd parity, the chiral superfield χ has odd parity and so Σ^a does to be consistent with the SUSY. Therefore the boundary condition is

$$\mathcal{T}: \quad \Sigma^a(y-2\pi R) = \Sigma^a(y) \tag{3.11}$$

$$\mathcal{P}: \quad -\Sigma^a(-y) = \Sigma^a(y), \tag{3.12}$$

and the KK-expansion is given by

$$\Sigma^a(y) = \sum_{n=1}^{\infty} \Sigma_n^a \frac{\sin \hat{n}y}{\sqrt{\pi R}}. \tag{3.13}$$

Since the 5D Lagrangian is

$$\begin{aligned}\mathcal{L}_{5,\Sigma}^{\text{free}} &= -\frac{1}{2}\partial^M \Sigma^a \partial_M \Sigma^a = \frac{1}{2}\Sigma^a[\partial^2 + \partial_5^2]\Sigma^a \\ &= \frac{1}{2\pi R}\sum_{n,m=1}^{\infty} \Sigma_m^a \left[\partial^2 - \hat{n}^2\right] \Sigma_n^a \sin \hat{m}y \sin \hat{n}y,\end{aligned} \tag{3.14}$$

the 4D Lagrangian is

$$\mathcal{L}_{4,\Sigma}^{\text{free}} = \sum_{n=1}^{\infty} \frac{1}{2}\Sigma_n^a \left[\partial^2 - \hat{n}^2\right] \Sigma_n^a. \tag{3.15}$$

We have got the KK-modes Lagrangian. There is no massless degree of freedom for parity odd field.

3.2.2 $A_M^a = (A_\mu^a, A_5^a)$

Let us next consider the gauge field. Since they are singlets under $SU(2)_R$, the boundary condition is

$$\mathcal{T}: \quad A_M(y-2\pi R) = A_M(y), \tag{3.16}$$

$$\mathcal{P}: \quad A_\mu(-y) = +A_\mu(y), \quad A_5(-y) = -A_5(y). \tag{3.17}$$

Therefore the KK expansion is

$$A_\mu^a(y) = \sum_{n=0}^{\infty} A_{\mu,n}^a \frac{\eta_n \cos \hat{n}y}{\sqrt{\pi R}}, \quad A_5^a(y) = \sum_{n=1}^{\infty} A_{5,n}^a \frac{\sin \hat{n}y}{\sqrt{\pi R}}. \tag{3.18}$$

We take R_ξ gauge,

$$\mathcal{L}_5^{\text{gauge-fix}} = \frac{1}{2\xi}\left(\partial^\mu A_\mu + \xi\partial^5 A_5\right)^2, \tag{3.19}$$

and then the 5D Lagrangian is

$$\begin{aligned}
\mathcal{L}_{5,A}^{\text{free}} &= -\frac{1}{4}\partial^{[M}A^{N]}\partial_{[M}A_{N]} + \mathcal{L}_5^{\text{gauge-fix}} \\
&= \frac{1}{2}A_\mu\left[g^{\mu\nu}(\partial^2+\partial_5^2) + \partial^\mu\partial^\nu\right]A_\nu + A_5\partial^2 A_5 - A_\mu\partial^\mu\partial_5 A_5 \\
&\qquad - \frac{1}{2\xi}A_\mu\partial^\mu\partial^\nu A_\nu + A_\mu\partial^\mu\partial_5 A_5 + \frac{1}{2}\xi A_5\partial_5^2 A_5 \\
&= \frac{1}{2}A_\mu\left[g^{\mu\nu}(\partial^2+\partial_5^2) + \partial^\mu\partial^\nu(1-\xi^{-1})\right]A_\nu + \frac{1}{2}A_5\left[\partial^2+\xi\partial_5^2\right]A_5.
\end{aligned} \tag{3.20}$$

Substituting the KK expansion, we obtain the 4D Lagrangian given by

$$\mathcal{L}_{4,A} = \sum_{n=0}^{\infty}\frac{1}{2}A_{\mu,n}\left[g^{\mu\nu}(\partial^2-\hat{n}^2) + \partial^\mu\partial^\nu(1-\xi^{-1})\right]A_{\nu,n} \tag{3.21}$$

$$+\sum_{n=1}^{\infty}\frac{1}{2}A_{5,n}\left[\partial^2-\xi\hat{n}^2\right]A_{5,n}. \tag{3.22}$$

We take $\xi = 1$ (Feynman gauge) in later calculation.

3.2.3 ϕ

Let us next consider the $SU(2)_R$ doublet, ϕ, and show that the SUSY is softly broken by the twisted boundary condition. The 5D Lagrangian is

$$\mathcal{L}_{5,\phi}^{\text{free}} = -\left(\partial^M\phi_1\right)^\dagger\partial_M\phi_1 - \left(\partial^M\phi_2^\dagger\right)^\dagger\partial_M\phi_2^\dagger, \tag{3.23}$$

$$= \mathbf{\Phi}^\dagger\left(\partial^2+\partial_5^2\right)\mathbf{\Phi}, \tag{3.24}$$

where we have used the following notation to express the doublet;

$$\mathbf{\Phi} := \begin{pmatrix}\phi_1 \\ \phi_2^\dagger\end{pmatrix}. \tag{3.25}$$

The boundary condition is

$$\mathcal{P}: \quad \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\mathbf{\Phi}(-y) = \mathbf{\Phi}(y), \tag{3.26}$$

$$\mathcal{T}: \quad e^{-i\sigma_2 2\pi\alpha}\mathbf{\Phi}(y-2\pi R) = \mathbf{\Phi}(y) \tag{3.27}$$

If we define

$$\tilde{\mathbf{\Phi}} = \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} = U\mathbf{\Phi} \qquad \Longleftrightarrow \qquad \mathbf{\Phi} = U^\dagger \tilde{\mathbf{\Phi}}, \tag{3.28}$$

$$\text{with} \qquad U = U(y) = e^{i\sigma_2 \hat{\alpha} y} = \begin{pmatrix} \cos\hat{\alpha}y & \sin\hat{\alpha}y \\ -\sin\hat{\alpha}y & \cos\hat{\alpha}y \end{pmatrix} \tag{3.29}$$

the boundary condition becomes much easier;

$$\mathcal{P}: \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tilde{\mathbf{\Phi}}(-y) = \tilde{\mathbf{\Phi}}(y), \tag{3.30}$$

$$\mathcal{T}: \quad \tilde{\mathbf{\Phi}}(y - 2\pi R) = \tilde{\mathbf{\Phi}}(y). \tag{3.31}$$

Here we have used the relation

$$\begin{aligned} &U(y) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U^\dagger(-y) = e^{i\sigma_2\hat{\alpha}y}\sigma_3 e^{i\sigma_2\hat{\alpha}y} = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ &U(y)e^{-i\sigma_2 2\pi\alpha}U^\dagger(y - 2\pi R) = e^{i\sigma_2\hat{\alpha}y}e^{-i\sigma_2 2\pi\alpha}e^{-i\sigma_2\hat{\alpha}(y-2\pi R)} = 1. \end{aligned} \tag{3.32}$$

Therefore, we can perform the KK expansion easily;

$$\tilde{\mathbf{\Phi}} = \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \tilde{\phi}_{1,n} \frac{\eta_n}{\sqrt{\pi R}} \cos\hat{n}y \\ \tilde{\phi}_{2,n}^\dagger \frac{1}{\sqrt{\pi R}} \sin\hat{n}y \end{pmatrix}. \tag{3.33}$$

Here $\tilde{\phi}_{2,0} \equiv 0$. The Lagrangian in terms of $\tilde{\mathbf{\Phi}}$ is

$$\begin{aligned} \mathcal{L}_{5,\phi}^{\text{free}} &= \tilde{\mathbf{\Phi}}^\dagger U \left(\partial^2 + \partial_5^2\right) U^\dagger \tilde{\mathbf{\Phi}} \\ &= \tilde{\mathbf{\Phi}}^\dagger \left(\partial^2 - \hat{\alpha}^2 - 2i\sigma_2\hat{\alpha}\partial_5 + \partial_5^2\right) \tilde{\mathbf{\Phi}} \\ &= \tilde{\mathbf{\Phi}}^\dagger \begin{pmatrix} \partial^2 - \hat{\alpha}^2 + \partial_5^2 & -2\hat{\alpha}\partial_5 \\ +2\hat{\alpha}\partial_5 & \partial^2 - \hat{\alpha}^2 + \partial_5^2 \end{pmatrix} \tilde{\mathbf{\Phi}}. \end{aligned} \tag{3.34}$$

The 4D Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{\rm free}_{4,\phi} &= \int_0^{2\pi R} dy\ \mathcal{L}^{\rm free}_{5,\phi} \\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\int_0^{2\pi R} dy \\
&\quad \left(\tilde{\phi}^\dagger_{1,n}\frac{\eta_n\cos\hat{n}y}{\sqrt{\pi R}},\ \tilde{\phi}_{2,n}\frac{\sin\hat{n}y}{\sqrt{\pi R}}\right)\begin{pmatrix}\partial^2-\hat{\alpha}^2+\partial_5^2 & -2\hat{\alpha}\partial_5 \\ +2\hat{\alpha}\partial_5 & \partial^2-\hat{\alpha}^2+\partial_5^2\end{pmatrix}\begin{pmatrix}\tilde{\phi}_{1,m}\frac{\eta_m\cos\hat{m}y}{\sqrt{\pi R}} \\ \tilde{\phi}^\dagger_{2,m}\frac{\sin\hat{m}y}{\sqrt{\pi R}}\end{pmatrix} \\
&= \sum_{n=0}^{\infty}\left(\tilde{\phi}^\dagger_{1,n},\tilde{\phi}_{2,n}\right)\begin{pmatrix}\partial^2-\hat{\alpha}^2-\hat{n}^2 & -2\hat{\alpha}\hat{n} \\ -2\hat{\alpha}\hat{n} & \partial^2-\hat{\alpha}^2-\hat{n}^2\end{pmatrix}\begin{pmatrix}\tilde{\phi}_{1,n} \\ \tilde{\phi}^\dagger_{2,n}\end{pmatrix} \\
&= \phi_0^\dagger\left[\partial^2-\hat{\alpha}^2\right]\phi_0 \\
&\quad +\sum_{n=1}^{\infty}\left(\phi^\dagger_{+,n},\phi^\dagger_{-,n}\right)\begin{pmatrix}\partial^2-(\hat{\alpha}+\hat{n})^2 & 0 \\ 0 & \partial^2-(\hat{\alpha}-\hat{n})^2\end{pmatrix}\begin{pmatrix}\phi_{+,n} \\ \phi_{-,n}\end{pmatrix}.
\end{aligned}
\tag{3.35}
$$

Here we define the mass eigen states as follows;

$$
\begin{pmatrix}\phi_{+,n} \\ \phi_{-,n}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\tilde{\phi}_{1,n} \\ \tilde{\phi}^\dagger_{2,n}\end{pmatrix}\quad (n\geq 1),\qquad \phi_0=\tilde{\phi}_{1,0},
\tag{3.36}
$$

$$
\Longleftrightarrow\quad \begin{pmatrix}\tilde{\phi}_{1,n} \\ \tilde{\phi}^\dagger_{2,n}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\phi_{+,n} \\ \phi_{-,n}\end{pmatrix}.
\tag{3.37}
$$

For more convenience, we define

$$
\phi_n = \begin{cases}\phi_{+,n} & (n\geq 1), \\ \phi_0 & (n=0), \\ \phi_{-,-n} & (n\leq 1),\end{cases}
\tag{3.38}
$$

then the Lagrangian is becomes

$$
\begin{aligned}
\mathcal{L}^{\rm free}_{4,\phi} &= \phi_0^\dagger\left[\partial^2-\hat{\alpha}^2\right]\phi_0 \\
&\quad +\sum_{n=1}^{\infty}\phi^\dagger_{+,n}\left[\partial^2-(\hat{\alpha}+\hat{n})^2\right]\phi_{+,n}+\sum_{n=1}^{\infty}\phi^\dagger_{-,n}\left[\partial^2-(\hat{\alpha}-\hat{n})^2\right]\phi_{-,n} \\
&= \sum_{n=-\infty}^{\infty}\phi_n^\dagger\left[\partial^2-(\hat{\alpha}+\hat{n})^2\right]\phi_n.
\end{aligned}
\tag{3.39}
$$

We have here got the KK-modes Lagrangian but the zero-mode has mass α/R. Contexts enable us to distinguish the original ϕ_1,ϕ_2 with the mass eigen states ϕ_n when $n=1,2$; most importantly, the formers are 5D fields and the latters are 4D fields.

It is important to have the direct relation between ϕ_1, ϕ_2 and ϕ_n, when we calculate the interactions among 4D mass eigen modes;

$$
\begin{aligned}
\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} &= e^{-i\sigma_2 \hat{\alpha} y} \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \cos\hat{\alpha}y & -\sin\hat{\alpha}y \\ \sin\hat{\alpha}y & \cos\hat{\alpha}y \end{pmatrix} \begin{pmatrix} \tilde{\phi}_{1,n} \frac{\eta_n \cos\hat{n}y}{\sqrt{\pi R}} \\ \tilde{\phi}_{2,n}^\dagger \frac{\sin\hat{n}y}{\sqrt{\pi R}} \end{pmatrix} \\
&= \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \begin{pmatrix} \tilde{\phi}_{1,n} \cos\cos - \tilde{\phi}_{2,n}^\dagger \sin\sin \\ \tilde{\phi}_{1,n} \sin\cos + \tilde{\phi}_{2,n}^\dagger \cos\sin \end{pmatrix} \\
&= \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \begin{pmatrix} \phi_{+,n} (\cos\cos - \sin\sin) + \phi_{-,n} (\cos\cos + \sin\sin) \\ \phi_{+,n} (\sin\cos + \cos\sin) + \phi_{-,n} (\sin\cos - \cos\sin) \end{pmatrix} \\
&\quad + \frac{1}{\sqrt{2\pi R}} \begin{pmatrix} \phi_0 \cos\hat{\alpha}y \\ \phi_0 \sin\hat{\alpha}y \end{pmatrix} \\
&= \frac{1}{\sqrt{2\pi R}} \left[\sum_{n=1}^{\infty} \begin{pmatrix} \phi_n \cos(\hat{\alpha}+\hat{n})y + \phi_{-n} \cos(\hat{\alpha}-\hat{n})y \\ \phi_n \sin(\hat{\alpha}+\hat{n})y + \phi_{-n} \sin(\hat{\alpha}-\hat{n})y \end{pmatrix} + \begin{pmatrix} \phi_0 \cos\hat{\alpha}y \\ \phi_0 \sin\hat{\alpha}y \end{pmatrix} \right] \\
&= \sum_{n=-\infty}^{\infty} \frac{\phi_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos(\hat{\alpha}+\hat{n})y \\ \sin(\hat{\alpha}+\hat{n})y \end{pmatrix}. \qquad (3.40)
\end{aligned}
$$

We can see that this satisfies the original boundary conditions and leads to the 4D Lagrangian in terms of ϕ_n.

3.2.4 ψ

Defining the Dirac field

$$\Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix}, \qquad (3.41)$$

we can impose the following boundary condition;

$$\mathcal{T}: \quad \Psi(y - 2\pi R) = \Psi(y), \qquad (3.42)$$

$$\mathcal{P}: \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Psi(-y) = \gamma^5 \Psi(-y) = \Psi(y), \qquad (3.43)$$

where we have assigned the parity so that it is consistent with the representation of $\mathbf{\Phi}$ (ψ_1 and ψ_2 must have the same parity with ϕ_1 and ϕ_2 respectively due to the SUSY). Therefore the KK expansion is

$$\Psi(y) = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix}(y) = \sum_{n=0}^{\infty} \begin{pmatrix} \psi_{1,n} \frac{\eta_n}{\sqrt{\pi R}} \cos \hat{n}y \\ \bar{\psi}_{2,n} \frac{1}{\sqrt{\pi R}} \sin \hat{n}y \end{pmatrix} \tag{3.44}$$

$$= \frac{P_L \Psi_0}{\sqrt{2\pi R}} + \sum_{n=1}^{\infty} \left[P_L \Psi_n \frac{\cos \hat{n}y}{\sqrt{\pi R}} + P_R \Psi_n \frac{\sin \hat{n}y}{\sqrt{\pi R}} \right]. \tag{3.45}$$

Here we have defined following Dirac fields

$$\Psi_0 = \begin{pmatrix} \psi_{1,0} \\ \bar{\psi}_{1,0} \end{pmatrix}, \quad \Psi_n = \begin{pmatrix} \psi_{1,n} \\ \bar{\psi}_{2,n} \end{pmatrix}, \quad (n \geq 1). \tag{3.46}$$

Note that Ψ_0 is a Majorana field in 4D sense.

Since the 5D Lagrangian is

$$\begin{aligned} \mathcal{L}_{5,\psi}^{\text{free}} &= -\left(\psi_2, \bar{\psi}_1\right) i\Gamma^M \partial_M \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} = -\overline{\Psi} i\Gamma^M \partial_M \Psi \\ &= -\left\{ \frac{\overline{\Psi}_0 P_R}{\sqrt{2\pi R}} + \sum_{m=1}^{\infty} \left[\frac{\cos \hat{m}y}{\sqrt{\pi R}} \overline{\Psi}_m P_R + \frac{\sin \hat{m}y}{\sqrt{\pi R}} \overline{\Psi}_m P_L \right] \right\} \\ &\quad \times \left\{ i\partial\!\!\!/ \Psi + \gamma^5 \sum_{n=1}^{\infty} \hat{n} \left[-P_L \Psi_n \frac{\sin \hat{n}y}{\sqrt{\pi R}} + P_R \Psi_n \frac{\cos \hat{n}y}{\sqrt{\pi R}} \right] \right\}, \end{aligned} \tag{3.47}$$

the 4D Lagrangian is

$$\begin{aligned} \mathcal{L}_{4,\psi}^{\text{free}} &= \int_0^{2\pi R} dy\ \mathcal{L}_{5,\psi}^{\text{free}} \\ &= -\overline{\Psi}_0 P_R i\partial\!\!\!/ P_L \Psi_0 - \sum_{n=1}^{\infty} \left\{ \overline{\Psi}_n i\partial\!\!\!/ \Psi_n + \hat{n} \overline{\Psi}_n (P_R \gamma^5 P_R - P_L \gamma^5 P_L) \Psi_n \right\} \\ &= -\frac{1}{2} \overline{\Psi}_0 i\partial\!\!\!/ \Psi_0 - \sum_{n=1}^{\infty} \overline{\Psi}_n \left[i\partial\!\!\!/ - \hat{n} \right] \Psi_n. \end{aligned} \tag{3.48}$$

If we define

$$\Psi_n' = \begin{pmatrix} \psi_{1,n} \\ -\bar{\psi}_2 \end{pmatrix}, \tag{3.49}$$

the Lagrangian can be writen

$$\mathcal{L}_{4,\psi}^{\text{free}} = -\frac{1}{2} \overline{\Psi}_0 i\partial\!\!\!/ \Psi_0 - \sum_{n=1}^{\infty} \overline{\Psi}_n' \left[i\partial\!\!\!/ + \hat{n} \right] \Psi_n'. \tag{3.50}$$

Note that the difference is the sign of mass term. This Ψ', as well as Ψ, is used in the later calculation of Feynman diagram.

3.2.5 λ^a

Let us next consider the gaugino. The 5D Lagrangian is

$$\mathcal{L}^{\rm free}_{5,\lambda} = -\left(-\lambda_2, \bar{\lambda}_1\right)\begin{pmatrix} \partial_5 & i\sigma^\mu\partial_\mu \\ i\bar{\sigma}^\mu\partial_\mu & -\partial_5 \end{pmatrix}\left(\begin{array}{c} \lambda_1 \\ -\bar{\lambda}_2 \end{array}\right) = -\left(-\lambda_2, \bar{\lambda}_1\right) i\Gamma^M\partial_M \left(\begin{array}{c} \lambda_1 \\ -\bar{\lambda}_2 \end{array}\right). \tag{3.51}$$

The boundary condition is

$$\mathcal{P}: \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\left(\begin{array}{c} \lambda_1 \\ -\lambda_2 \end{array}\right)(-y) = \left(\begin{array}{c} \lambda_1 \\ -\lambda_2 \end{array}\right)(y), \tag{3.52}$$

$$\mathcal{T}: \quad e^{-i\sigma_2 2\pi\alpha}\left(\begin{array}{c} \lambda_1 \\ -\lambda_2 \end{array}\right)(y-2\pi R) = \left(\begin{array}{c} \lambda_1 \\ -\lambda_2 \end{array}\right)(y). \tag{3.53}$$

The discussion of KK expansion is quite similar to the ϕ case. Therefore we jump to the final expansion form;

$$\left(\begin{array}{c} \lambda_1 \\ -\lambda_2 \end{array}\right) = \sum_{n=-\infty}^{\infty}\frac{\lambda_n}{\sqrt{2\pi R}}\left(\begin{array}{c} \cos(\hat{\alpha}+\hat{n})y \\ \sin(\hat{\alpha}+\hat{n})y \end{array}\right). \tag{3.54}$$

We can see that this satisfies the boundary conditions. The 4D Lagrangian is

$$\begin{aligned} \mathcal{L}^{\rm free}_{4,\lambda} &= \int_0^{2\pi R} dy\ \mathcal{L}^{\rm free}_{5,\lambda} \\ &= -\sum_{n,m}\int_0^{2\pi R}\frac{dy}{2\pi R}\left(\lambda_n \sin(\hat{\alpha}+\hat{n})y,\ \bar{\lambda}_n\cos(\hat{\alpha}+\hat{n})y\right) \\ &\qquad\times \begin{pmatrix} \partial_5 & i\sigma^\mu\partial_\mu \\ i\bar{\sigma}^\mu\partial_\mu & -\partial_5 \end{pmatrix}\left(\begin{array}{c} \lambda_m\cos(\hat{\alpha}+\hat{m})y \\ \bar{\lambda}_m\sin(\hat{\alpha}+\hat{m})y \end{array}\right) \\ &= -\frac{1}{2}\sum_{n=-\infty}^{\infty}\left[\bar{\lambda}_n i\bar{\sigma}^\mu\partial_\mu\lambda_n + \lambda_n i\sigma^\mu\partial_\mu\bar{\lambda}_n - (\hat{\alpha}+\hat{n})\left(\lambda_n\lambda_n + \bar{\lambda}_n\bar{\lambda}_n\right)\right] \\ &= -\sum_{n=-\infty}^{\infty}\frac{1}{2}\overline{\Upsilon}_n\left[i\partial\!\!\!/ - (\hat{\alpha}+\hat{n})\right]\Upsilon_n, \end{aligned} \tag{3.55}$$

where

$$\Upsilon_n = \left(\begin{array}{c} \lambda_n \\ \bar{\lambda}_n \end{array}\right) \tag{3.56}$$

is Majorana field.

3.2.6 summary

Here's the summary of the KK expansion of each field and its 4D Lagrangian.

$$\begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\phi_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos(\hat{\alpha}+\hat{n})y \\ \sin(\hat{\alpha}+\hat{n})y \end{pmatrix}, \tag{3.57}$$

$$\mathcal{L}_{4,\phi}^{\text{free}} = \sum_{n=-\infty}^{\infty} \phi_n^\dagger \left[\partial^2 - (\hat{\alpha}+\hat{n})^2\right] \phi_n, \tag{3.58}$$

$$\begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} = \sum_{n=0}^{\infty} \begin{pmatrix} \psi_{1,n} \frac{\eta_n}{\sqrt{\pi R}} \cos \hat{n}y \\ \bar{\psi}_{2,n} \frac{1}{\sqrt{\pi R}} \sin \hat{n}y \end{pmatrix}, \tag{3.59}$$

$$\mathcal{L}_{4,\psi}^{\text{free}} = -\frac{1}{2}\overline{\Psi}_0 i\partial\!\!\!/ \Psi_0 - \sum_{n=1}^{\infty} \overline{\Psi}_n \left[i\partial\!\!\!/ - \hat{n}\right] \Psi_n, \tag{3.60}$$

$$= -\frac{1}{2}\overline{\Psi}_0 i\partial\!\!\!/ \Psi_0 - \sum_{n=1}^{\infty} \overline{\Psi}'_n \left[i\partial\!\!\!/ + \hat{n}\right] \Psi'_n, \tag{3.61}$$

$$A_\mu^a(y) = \sum_{n=1}^{\infty} A_{\mu,n}^a \frac{\eta_n \cos \hat{n}y}{\sqrt{\pi R}}, \quad A_5^a(y) = \sum_{n=1}^{\infty} A_{5,n}^a \frac{\sin \hat{n}y}{\sqrt{\pi R}}, \tag{3.62}$$

$$\mathcal{L}_{4,A}^{\text{free}} = \sum_{n=0}^{\infty} \frac{1}{2} A_{\mu,n} \left[g^{\mu\nu}(\partial^2 - \hat{n}^2) - \partial^\mu \partial^\nu (1-\xi^{-1})\right] A_{\nu,n} \tag{3.63}$$

$$+ \sum_{n=1}^{\infty} \frac{1}{2} A_{5,n} \left[\partial^2 - \xi \hat{n}^2\right] A_{5,n}, \tag{3.64}$$

$$\Sigma^a(y) = \sum_{n=1}^{\infty} \Sigma_n^a \frac{\sin \hat{n}y}{\sqrt{\pi R}}, \tag{3.65}$$

$$\mathcal{L}_{4,\Sigma}^{\text{free}} = \sum_{n=1}^{\infty} \frac{1}{2} \Sigma_n^a \left[\partial^2 - \hat{n}^2\right] \Sigma_n^a, \tag{3.66}$$

$$\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\lambda_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos(\hat{\alpha}+\hat{n})y \\ \sin(\hat{\alpha}+\hat{n})y \end{pmatrix}, \tag{3.67}$$

$$\mathcal{L}_{4,\lambda}^{\text{free}} = -\sum_{n=-\infty}^{\infty} \frac{1}{2} \overline{\Upsilon}_n \left[i\partial\!\!\!/ - (\hat{\alpha}-\hat{n})\right] \Upsilon_n. \tag{3.68}$$

3.3 Interaction terms

In order to see the interaction terms, just substitute the KK expansion summarized above and calculate the y integral. The whole interaction terms are too intricate to write down and thus we derive the terms which are required for our purpose only when it's necessary (like in Appendix C).

Note that the gauge coupling is renormalized by the factor $\sqrt{2\pi R}$;

$$g|_{4D} = \frac{g|_{5D}}{\sqrt{2\pi R}}. \tag{3.69}$$

Practically, before substituting the KK expansion, it is a good idea to transform ϕ and λ into $\tilde{\phi}$ and $\tilde{\lambda}$,

$$\begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} = e^{-i\sigma_2 \hat{\alpha} y} \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix}, \quad \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = e^{-i\sigma_2 \hat{\alpha} y} \begin{pmatrix} \tilde{\lambda}_1 \\ -\tilde{\lambda}_2 \end{pmatrix} \tag{3.70}$$

and then perform the KK expansion as follows

$$\begin{aligned}
\begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} &= e^{+i\sigma_2 \hat{\alpha} y} \begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} \\
&= \begin{pmatrix} \cos\hat{\alpha} y & \sin\hat{\alpha} y \\ -\sin\hat{\alpha} y & \cos\hat{\alpha} y \end{pmatrix} \sum_{n=-\infty}^{\infty} \frac{\phi_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos(\hat{\alpha}+\hat{n}) y \\ \sin(\hat{\alpha}+\hat{n}) y \end{pmatrix} \\
&= \sum_{n=-\infty}^{\infty} \frac{\phi_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos\hat{n} y \\ \sin\hat{n} y \end{pmatrix}, \\
\begin{pmatrix} \tilde{\lambda}_1 \\ -\tilde{\lambda}_2 \end{pmatrix} &= e^{+i\sigma_2 \hat{\alpha} y} \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\lambda_n}{\sqrt{2\pi R}} \begin{pmatrix} \cos\hat{n} y \\ \sin\hat{n} y \end{pmatrix}.
\end{aligned} \tag{3.71}$$

This transformation doesn't change the Lagrangian except for the terms which have ∂_5 because it's a $SU(2)_R$ transformation if we assume y is constant.

3.4 Lagrangian on the brane

So far we have seen the Lagrangian for the bulk fields (the Vector multiplet and the Hyper multiplets) in the 5D space-time. We can also consider fields stuck on the branes (fixed points of Z_2, $y = 0, \pi R$). Actually in the compact SUSY model the Higgs fields live on the brane, as we will see in section 5.

In this subsection, we suppose we have a brane at $y = 0$ and then seek for the Lagrangian for a chiral superfield $H(x)$ living on the brane. We use the following notation for the component fields of H;

$$H = h + \sqrt{2}\theta\, \tilde{h} + \theta^2 F_h \tag{3.72}$$

3.4.1 Kinetic term

The kinetic term for H should be the usual 4D SUSY kinetic term. The 5D Lagrangian can be expressed using the Dirac delta function;

$$\mathcal{L}^{\rm brane}_{5,\rm kin} = \delta(y) \int d^4\theta \, H^\dagger e^{-2gV} H, \tag{3.73}$$

and we obtain

$$\mathcal{L}^{\rm brane}_{4,\rm kin} = \int_0^{2\pi R} dy \, \mathcal{L}^{\rm brane}_{5,\rm kin} = \int d^4\theta \, H^\dagger e^{-2gV(y=0)} H. \tag{3.74}$$

Since $g_{5D} V(x, y = 0) = g_{4D} \sum_{n=0}^{\infty} V_n(x)$, S has the usual gauge interactions including the non-zero gauge bosons and gauginos.

3.4.2 Interaction term

Let us derive the 4D interaction terms of the following Yukawa like coupling on the brane using the components fields;

$$\mathcal{L}^{\rm brane}_{5,\rm int} = \delta(y) \, \lambda \int d^2\theta \, \Phi_A \Phi_B H + \text{h.c.}, \tag{3.75}$$

where Φ_A and Φ_B are chiral superfields in the bulk with even parity, and H is a superfield on the brane. Then the component expression is given by

$$\mathcal{L}^{\rm brane}_{5,\rm int} = \delta(y) \, \lambda \left[F_A \phi_B h + \phi_A F_B h + \phi_A \phi_B F_h - \phi_A \psi_B \tilde{h} - \psi_A \phi_B \tilde{h} - \psi_A \psi_B h \right] + \text{h.c.}. \tag{3.76}$$

We want to eliminate the auxiliary fields F_A, F_B and F_h and perform the KK expansion so that we obtain the 4D Lagrangian $\mathcal{L}_4 = \int dy \mathcal{L}_5$. To this end, let us next recall the 5D Lagrangian containing the auxiliary fields in the bulk;

$$\begin{aligned} \mathcal{L}^{\rm bulk}_5 &\supset \sum_{*=A,B} \left\{ |F_*|^2 - F_*^\dagger \left[\partial_5 + ig(A_5 + i\Sigma) \right] \phi_*^{C\dagger} + \text{h.c.} \right\} \\ &= \sum_{*=A,B} \left\{ \left| F_* - \left[\partial_5 + ig(A_5 + i\Sigma) \right] \phi_*^{C\dagger} \right|^2 - \left| \left[\partial_5 - ig(A_5 - i\Sigma) \right] \phi_*^{C} \right|^2 \right\}. \end{aligned} \tag{3.77}$$

The second term in the second line contributes to the 5D part of the kinetic term of ϕ_*. From the first term, we have defined F'_* as

$$F'_* = F_* - \left[\partial_5 + ig(A_5 + i\Sigma) \right] \phi_*^{C\dagger}. \tag{3.78}$$

Therefore the total 5D Lagrangian relevant to the auxiliary fields is

$$\begin{aligned}
\mathcal{L}_5^{\text{total}} &= \mathcal{L}_5^{\text{brane}} + \mathcal{L}_5^{\text{bulk}} \\
&= |F'_A|^2 + |F'_B|^2 + \delta(y)\,|F_h|^2 + \delta(y)\lambda\left(F_A\phi_B h + \phi_A F_B h + \phi_A\phi_B F_h\right) + \text{h.c.} \\
&= |F'_A|^2 + |F'_B|^2 + \delta(y)\Big\{|F_h|^2 + \lambda\,\phi_A\phi_B F_h + \text{h.c.}\Big\} \\
&\quad + \delta(y)\lambda\left\{\left(F'_A + \partial_5\phi_A^{C\dagger}\right)\phi_B h + \phi_A\left(F'_B + \partial_5\phi_B^{C\dagger}\right)h\right\} + \text{h.c.} \\
&= \left|F'_A + \delta(y)\lambda^*\phi_B^\dagger h^\dagger\right|^2 + \left|F'_B + \delta(y)\lambda^*\phi_A^\dagger h^\dagger\right|^2 + \delta(y)\Big\{\left|F_h + \lambda^*\phi_A^\dagger\phi_B^\dagger\right|^2 - |\lambda\phi_A\phi_B|^2\Big\} \\
&\quad - |\delta(y)\lambda\,\phi_B h|^2 - |\delta(y)\lambda\,\phi_A h|^2 + \delta(y)\left\{\lambda\,\phi_B h\partial_5\phi_A^{C\dagger} + \lambda\phi_A\partial_5\phi_B^{C\dagger}h + \text{h.c.}\right\},
\end{aligned} \tag{3.79}$$

This looks disastrous at first sight since we have got divergent pieces $\delta(y)^2$ in the Lagrangian. However, it is actually not the case. When we have a brane at $y = 0$, the solution of the equation of motion for $\phi_A^C(y)$ becomes non-continuous at $y = 0$, and thus $\partial_5\phi_A^C$ and $\delta(y)\phi_B h$ cancel out each other at $y = 0$ and thus the divergence does not appear in $\mathcal{L}_5^{\text{total}}$. Similar discussion is found at section 4 of the [51].

However it is still difficult to derive the on-shell 4D Lagrangian of the components fields. To solve this, we change our strategy. Before eliminating the auxiliary fields, we first KK expand the fields including the auxiliary fields and integrate $\mathcal{L}_5$ with respect to y, and then remove all the modes of the auxiliary fields.

To KK-expand F_*, we first need to know its boundary condition. From the equation of motion, F_* has to have the same boundary condition as $\partial_5\phi_*^{C\dagger}$ and thus can be expanded as

$$F_* = \sum_{n=-\infty}^{\infty} F_{*,n}\frac{\cos(\hat{\alpha}+\hat{n})y}{\sqrt{2\pi R}} \qquad \left(F'_* = \sum_{n=-\infty}^{\infty} F'_{*,n}\frac{\cos(\hat{\alpha}+\hat{n})y}{\sqrt{2\pi R}}\right). \tag{3.80}$$

Substituting the KK expanded form of F'_* and

$$\begin{pmatrix} \phi_* \\ \phi_*^{C\dagger} \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\phi_{*,n}}{\sqrt{2\pi R}} \begin{pmatrix} \cos(\hat{\alpha}+\hat{n})y \\ \sin(\hat{\alpha}+\hat{n})y \end{pmatrix}. \tag{3.81}$$

into the $\mathcal{L}_5^{\text{total}}$ and integrate it with respect to y, we obtain the 4D Lagrangian. Let us first

KK-expand the terms which don't have F'_A and F'_B;

$$\begin{aligned}
\mathcal{L}_4^{\text{total}} &= \int_0^{2\pi R} dy\, \mathcal{L}_5^{\text{total}} \\
&\supset \int_0^{2\pi R} dy \left\{ \delta(y)\Big(|F_h|^2 + \lambda\, \phi_A\phi_B F_h + \text{h.c.}\Big) + \delta(y)\lambda\left(\partial_5\phi_A^{C\dagger}\phi_B + \phi_A\partial_5\phi_B^{C\dagger}\right)h + \text{h.c.}\right\} \\
&= \left|F_h + \lambda^*\, \phi_A(0)^\dagger\phi_B(0)^\dagger\right|^2 - |\lambda|^2\, |\phi_A(0)|^2\, |\phi_B(0)|^2 + \frac{\lambda}{2\pi R}\sum_{n,m}(2\hat{\alpha} + \hat{n} + \hat{m})\phi_{A,n}\phi_{B,m}h \\
&= \left|F_h + \lambda^*\, \phi_A(0)^\dagger\phi_B(0)^\dagger\right|^2 - |\lambda_{4D}|^2 \left|\sum_{n=-\infty}^{\infty}\phi_{A,n}\right|^2 \left|\sum_{m=-\infty}^{\infty}\phi_{B,m}\right|^2 \\
&\qquad + \lambda_{4D}\sum_{n,m}(2\hat{\alpha} + \hat{n} + \hat{m})\phi_{A,n}\phi_{B,m}h. \qquad (3.82)
\end{aligned}$$

Here we have defined λ_{4D} as $\lambda/2\pi R$. Notice that we obtain the soft SUSY breaking term

$$\lambda_{4D} 2\hat{\alpha}\phi_{A,0}\phi_{B,0}h. \qquad (3.83)$$

This plays the role as the soft A-term in the compact SUSY model.

When dealing with the terms containing F'_A and F'_B,

$$\begin{aligned}
\mathcal{L}_4^{\text{total}} &= \int_0^{2\pi R} dy\, \mathcal{L}_5^{\text{total}} \\
&\supset \int_0^{2\pi R} dy \left\{ |F'_A|^2 + |F'_B|^2 + \delta(y)\lambda\left(F'_A\phi_B + \phi_A F'_B\right)h + \text{h.c.}\right\}, \qquad (3.84)
\end{aligned}$$

it is better to use the tilde basis defined in the previous subsection. If we define $\tilde{F}_*$ and add the corresponding terms properly, KK expansions can be written as

$$\begin{pmatrix} \tilde{\phi}_* \\ \tilde{\phi}_*^{C\dagger} \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\phi_{*,n}}{\sqrt{2\pi R}} \begin{pmatrix} \cos\hat{n}y \\ \sin\hat{n}y \end{pmatrix}, \quad \tilde{F}'_* = \sum_{n=-\infty}^{\infty} F'_{*,n}\frac{\cos\hat{n}y}{\sqrt{2\pi R}}, \qquad (3.85)$$

and the Lagrangian becomes

$$\mathcal{L}_4^{\text{total}} \supset \sum_{n=-\infty}^{\infty}\left|F'_{A,n}\right|^2 + \sum_{n=-\infty}^{\infty}\left|F'_{B,n}\right|^2 + \frac{\lambda}{2\pi R}\sum_{n,m}\left(F'_{A,n}\phi_{B,m} + F'_{B,n}\phi_{A,m}\right)h + \text{h.c.}. \qquad (3.86)$$

Therefore the equations of motion for $F'_{A,n}$ and $F'_{B,n}$ are

$$F'_{A,n}{}^\dagger + \frac{\lambda}{2\pi R}\sum_{m=-\infty}^{\infty}\phi_{B,m}h = 0, \quad (A \longleftrightarrow B), \qquad (3.87)$$

and we obtain

$$\mathcal{L}_4^{\text{total}} \supset -\left|\lambda_{4D}\right|^2 \sum_{n=-\infty}^{\infty} \left\{ \left| \sum_{m=-\infty}^{\infty} \phi_{A,m} \right|^2 + \left| \sum_{m=-\infty}^{\infty} \phi_{B,m} \right|^2 \right\} |h|^2 \,. \tag{3.88}$$

Note that the coupling of the 4-point interaction term $|\phi_{B,m}|^2|h|^2$ is $|\lambda|^2 \sum_n$, which is obviously divergent since it's a infinite summation of a constant. This may look disastrous again at first sight, but its divergence is actually canceled out by the last term in Eq.(3.82), because the KK modes of the $\phi_{A,n}$ generate the divergent 4-point effective interaction of $\phi_{B,m}$ and h. We will see this cancellation concretely in subsection 5.2.

In summary for the later reference, if we have the brane interaction term of the form

$$\mathcal{L}_5^{\text{brane}} = \delta(y)\,\lambda \int d^2\theta\, \Phi_A \Phi_B H + \text{h.c.}, \tag{3.89}$$

the 4D Lagrangian after removing the auxiliary fields is given by

$$\begin{aligned}
\mathcal{L}_4^{\text{brane}} = & -|\lambda_{4D}|^2 \left| \sum_{n=-\infty}^{\infty} \phi_{A,n} \right|^2 \left| \sum_{m=-\infty}^{\infty} \phi_{B,m} \right|^2 \\
& - |\lambda_{4D}|^2 \sum_{n=-\infty}^{\infty} \left\{ \left| \sum_{m=-\infty}^{\infty} \phi_{A,m} \right|^2 + \left| \sum_{m=-\infty}^{\infty} \phi_{B,m} \right|^2 \right\} |h|^2 \\
& + \lambda_{4D} \sum_{n,m=-\infty}^{\infty} (2\hat{\alpha} + \hat{n} + \hat{m}) \phi_{A,n}\phi_{B,m} h + \text{h.c.} \\
& - \lambda_{4D} \sum_{n,m=-\infty}^{\infty} \left[\phi_{A,n}\psi_{B,m}\tilde{h} + \psi_{A,n}\phi_{B,m}\tilde{h} + \psi_{A,n}\psi_{B,m} h \right] + \text{h.c.}.
\end{aligned} \tag{3.90}$$

3.5 Radion Mediation

So far, we have seen the SUSY breaking and the soft terms out of the Scherk Schwarz mechanism, by

- expressing the 5D Lagrangian with component fields
- setting the boundary conditions (twisted by α) for the each component fields
- performing the KK expansion that respects the boundary conditions

However there is another way to see the SUSY breaking effect by Scherk Schwarz mechanism, which is called Radion Mediation [53, 54], and we see this idea in this subsection.

First, recall the full 5D Lagrangian expressed by the 4D superspace notation;

$$
\begin{aligned}
\mathcal{L}_5 = \mathcal{L}_5^{\text{Vector},1} &+ \mathcal{L}_5^{\text{Vector},2} + \mathcal{L}_5^{\text{Hyper},1} + \mathcal{L}_5^{\text{Hyper},2}, \\
\mathcal{L}_5^{\text{Vector},1} &= \frac{1}{16kg^2}\mathrm{Tr}\left\{\int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}\right\}, \qquad (3.91)\\
\mathcal{L}_5^{\text{Vector},2} &= -\frac{1}{2kg^2}\int d^4\theta\ \mathrm{Tr}\Big\{(\partial_5 - \sqrt{2}ig\chi^\dagger)e^{-2gV}(\partial_5 - \sqrt{2}ig\chi)e^{2gV} \\
&\qquad - \frac{1}{2}\partial_5 e^{-2gV}\partial_5 e^{2gV} + g^2(\chi\chi + \chi^\dagger\chi^\dagger)\Big\}, \\
\mathcal{L}_5^{\text{Hyper},1} &= \int d^4\theta\left[\Phi^\dagger e^{-2gV}\Phi + \Phi^C e^{2gV}\Phi^{C\dagger}\right], \qquad (3.92)\\
\mathcal{L}_5^{\text{Hyper},2} &= \int d^2\theta\, \Phi^C\left(\partial_5 - \sqrt{2}ig\chi\right)\Phi + \text{h.c.}. \qquad (3.93)
\end{aligned}
$$

We first define the dimension less coordinate φ and dimension less superfield $\underline{\chi}$ as follows;

$$
y = R\varphi, \quad \partial_5 = \frac{1}{R}\partial_\varphi, \qquad (3.94)
$$

$$
\underline{\chi} = R\chi, \qquad (3.95)
$$

and then rewrite the 4D Lagrangian

$$
\begin{aligned}
\mathcal{L}_4 &= \int_0^{2\pi R} dy\,\mathcal{L}_5 = \int_0^{2\pi} d\varphi\, R\,\mathcal{L}_5 \\
&= \int_0^{2\pi} d\varphi\Bigg[\frac{1}{16kg^2}\mathrm{Tr}\left\{\int d^2\theta\, R\mathcal{W}^\alpha\mathcal{W}_\alpha + \text{h.c.}\right\}, \qquad (3.96)\\
&\quad - \frac{1}{2kg^2}\int d^4\theta\,\frac{1}{R}\,\mathrm{Tr}\Big\{(\partial_\varphi - \sqrt{2}ig\underline{\chi}^\dagger)e^{-2gV}(\partial_\varphi - \sqrt{2}ig\underline{\chi})e^{2gV} \\
&\qquad - \frac{1}{2}\partial_\varphi e^{-2gV}\partial_\varphi e^{2gV} + g^2(\underline{\chi}\underline{\chi} + \underline{\chi}^\dagger\underline{\chi}^\dagger)\Big\}, \\
&\quad + \int d^4\theta\, R\left[\Phi^\dagger e^{-2gV}\Phi + \Phi^C e^{2gV}\Phi^{C\dagger}\right], \qquad (3.97)\\
&\quad + \int d^2\theta\,\Phi^C\left(\partial_\varphi - \sqrt{2}ig\underline{\chi}\right)\Phi + \text{h.c.}\Bigg]. \qquad (3.98)
\end{aligned}
$$

Now, suppose that R comes from vev of a sprion field T, and perform the following substitution;

$$
\int d^2\theta\, R^n \to \int d^2\theta\, T^n, \qquad (3.99)
$$

$$
\int d^4\theta\, R^n \to \int d^4\theta\left(\frac{T+T^\dagger}{2}\right)^n. \qquad (3.100)
$$

Therefore the 4D Lagrangian becomes

$$
\begin{aligned}
\mathcal{L}_4 = \int_0^{2\pi} d\varphi \Bigg[& \frac{1}{16kg^2} \mathrm{Tr} \left\{ \int d^2\theta \, T \, \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right\}, && (3.101) \\
& - \frac{1}{2kg^2} \int d^4\theta \, \frac{2}{T+T^\dagger} \, \mathrm{Tr} \Big\{ (\partial_\varphi - \sqrt{2} i g \underline{\chi}^\dagger) e^{-2gV} (\partial_\varphi - \sqrt{2} i g \underline{\chi}) e^{2gV} \\
& \qquad - \frac{1}{2} \partial_\varphi e^{-2gV} \partial_\varphi e^{2gV} + g^2 (\underline{\chi}\underline{\chi} + \underline{\chi}^\dagger \underline{\chi}^\dagger) \Big\}, \\
& + \int d^4\theta \, \frac{T+T^\dagger}{2} \, \left[\Phi^\dagger e^{-2gV} \Phi + \Phi^C e^{2gV} \Phi^{C\dagger} \right], && (3.102) \\
& + \int d^2\theta \, \Phi^C \left(\partial_\varphi - \sqrt{2} i g \underline{\chi} \right) \Phi + \text{h.c.} \Bigg]. && (3.103)
\end{aligned}
$$

If we assume $\langle T \rangle = R$, this Lagrangian is same as the previous one at the vacuum of T. Now, instead of having twisted boundary conditions, suppose that the spurion T has the SUSY breaking effect through the following F-term;

$$
\langle T \rangle = R + \theta^2 f. \tag{3.104}
$$

Then, we can show that this theory at the vacuum of T is same as the previous theory if we take

$$
f = 2\alpha. \tag{3.105}
$$

We can check this statement just by redefining the chiral superfield so that they become canonical, as follows.

gaugino mass term by Radion mediation

First, when T has F term, the Lagrangian becomes

$$
\begin{aligned}
\mathcal{L}_4^{\text{Vector},1} &= \int_0^{2\pi} d\varphi \, \frac{1}{16kg^2} \mathrm{Tr} \left\{ \int d^2\theta (R + \theta^2 f) \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right\}, \\
&= \int_0^{2\pi R} dy \left[\frac{1}{16kg^2} \mathrm{Tr} \left\{ \int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right\} + \frac{f}{4R} \lambda_1 \lambda_1 + \frac{f^\dagger}{4R} \bar{\lambda}_1 \bar{\lambda}_1 \right]. && (3.106)
\end{aligned}
$$

In addition to the SUSY preserving part, we have the SUSY breaking soft mass term for λ_1. Secondly,

$$
\begin{aligned}
\mathcal{L}_4^{\text{Vector},2} = - \frac{1}{2kg^2} \int_0^{2\pi} d\varphi \int d^4\theta \, \frac{2}{T+T^\dagger} \, \mathrm{Tr} \Big\{ & (\partial_\varphi - \sqrt{2} i g \underline{\chi}^\dagger) e^{-2gV} (\partial_\varphi - \sqrt{2} i g \underline{\chi}) e^{2gV} \\
& - \frac{1}{2} \partial_\varphi e^{-2gV} \partial_\varphi e^{2gV} + g^2 (\underline{\chi}\underline{\chi} + \underline{\chi}^\dagger \underline{\chi}^\dagger) \Big\}, && (3.107)
\end{aligned}
$$

and now if we substitute

$$\frac{2}{T+T^{\dagger}}=\frac{1}{2R+\theta^2 f+\bar{\theta}^2 f^{\dagger}}=\frac{1}{R}\left(1-\theta^2\frac{f}{2R}-\bar{\theta}^2\frac{f^{\dagger}}{2R}+\theta^4\frac{|f|^2}{2R^2}\right) \tag{3.108}$$

the chiral superfield χ is not canonical if we see

$$\mathcal{L}_5^{\mathrm{Vector},2}\supset\int_0^{2\pi R}dy\int d^4\theta\,\left(1-\theta^2\frac{f}{2R}-\bar{\theta}^2\frac{f^{\dagger}}{2R}+\theta^4\frac{|f|^2}{2R^2}\right)\chi^{\dagger}\chi, \tag{3.109}$$

and thus we define a canonical chiral superfiled $\tilde{\chi}$ as

$$\chi=\left(1+\frac{f}{2R}\theta^2\right)\tilde{\chi}. \tag{3.110}$$

so that we obtain

$$\begin{aligned}\mathcal{L}_4^{\mathrm{Vector},2}&\supset\int_0^{2\pi R}dy\int d^4\theta\,\left(1-\theta^2\frac{f}{2R}-\bar{\theta}^2\frac{f^{\dagger}}{2R}+\theta^4\frac{|f|^2}{2R^2}\right)\chi^{\dagger}\chi\\&=\int_0^{2\pi R}dy\left[\int d^4\theta\,\tilde{\chi}^{\dagger}\tilde{\chi}+\frac{|f|^2}{4R^2}|\phi_{\tilde{\chi}}|^2\right].\end{aligned} \tag{3.111}$$

This looks like at first sight, we have soft mass $(f/2R)^2$ for $\phi_{\tilde{\chi}}$. However from the other term, we obtain

$$\begin{aligned}\mathcal{L}_4^{\mathrm{Vector},2}&\supset-\frac{1}{2kg^2}\int_0^{2\pi R}dy\int d^4\theta\,\left(1-\theta^2\frac{f}{2R}-\bar{\theta}^2\frac{f^{\dagger}}{2R}+\theta^4\frac{|f|^2}{2R^2}\right)\mathrm{Tr}\left(\chi\chi+\chi^{\dagger}\chi^{\dagger}\right)\\&=-\frac{1}{2}\int_0^{2\pi R}dy\left[\int d\theta^4\,\left(\tilde{\chi}\tilde{\chi}+\tilde{\chi}^{\dagger}\tilde{\chi}^{\dagger}\right)-\frac{f^{\dagger}}{2R}\left(2F_{\tilde{\chi}}\phi_{\tilde{\chi}}-\lambda_2\lambda_2+\mathrm{h.c.}\right)\right].\end{aligned} \tag{3.112}$$

If we eliminate the auxiliary field F_χ the $\frac{|f|^2}{4R^2}|\phi_{\tilde{\chi}}|^2$ in $\mathcal{L}_4^{\mathrm{Vector},2}$ is canceled out. Furthermore we have now the other gaugino λ_2 soft mass term. We can identify the $\tilde{\chi}$ defined here with the $\tilde{\chi}$ defined in the previous subsection since they have the same Lagrangian.

sfermion mass term by Radion mediation

Let us next focus on the Lagrangian of the Hyper multiplets,

$$\begin{aligned}
\mathcal{L}_4^{\text{Hyper},1} &= \int d\varphi \int d^4\theta \, \frac{T+T^\dagger}{2} \, \left[\Phi^\dagger e^{-2gV}\Phi + \Phi^C e^{2gV}\Phi^{C\dagger}\right], \\
&= \int dy \int d^4\theta \left(1+\theta^2\frac{f}{2R}+\bar\theta^2\frac{f^\dagger}{2R}\right) \left[\Phi^\dagger e^{-2gV}\Phi + \Phi^C e^{2gV}\Phi^{C\dagger}\right], \\
&= \int dy \int d^4\theta \left(1+\theta^2\frac{f}{2R}+\bar\theta^2\frac{f^\dagger}{2R}\right) \\
&\qquad\qquad \times \left(1-\frac{f^\dagger}{2R}\bar\theta^2\right)\left(1-\frac{f}{2R}\theta^2\right)\left[\tilde\Phi^\dagger e^{-2gV}\tilde\Phi + \tilde\Phi^C e^{2gV}\tilde\Phi^{C\dagger}\right] \qquad (3.113) \\
&= \int dy \left[\int d^4\theta \left[\tilde\Phi^\dagger e^{-2gV}\tilde\Phi + \tilde\Phi^C e^{2gV}\tilde\Phi^{C\dagger}\right] - \frac{|f|^2}{4R^2}\tilde\phi_1^\dagger\tilde\phi_1 - \frac{|f|^2}{4R^2}\tilde\phi_2^\dagger\tilde\phi_2\right]. \qquad (3.114)
\end{aligned}$$

In the third equality, we have defined the canonical chiral superfields $\tilde\Phi$ and $\tilde\Phi^C$ as

$$\Phi = \left(1-\frac{f}{2R}\theta^2\right)\tilde\Phi, \quad \Phi^C = \left(1-\frac{f}{2R}\theta^2\right)\tilde\Phi^C, \qquad (3.115)$$

so that we have canonical kinetic term for $\tilde\Phi$ and $\tilde\Phi^C$.

Finally,

$$\begin{aligned}
\mathcal{L}_5^{\text{Hyper},2} &\supset \int dy \int d^2\theta \, \Phi^C\left(\partial_5 - \sqrt{2}ig\chi\right)\Phi + \text{h.c.} \\
&= \int dy \int d^2\theta \left(1-\frac{f}{2R}\theta^2\right)^2 \tilde\Phi^C\left(\partial_5 - \sqrt{2}ig(1+\frac{f}{2R}\theta^2)\tilde\chi\right)\tilde\Phi + \text{h.c.} \\
&= \int dy \left[\int d^2\theta \, \tilde\Phi^C\left(\partial_5 - \sqrt{2}ig\tilde\chi\right)\tilde\Phi - \frac{f}{2R}\tilde\phi_2\left(\partial_5 - \sqrt{2}ig\phi_{\tilde\chi}\right)\tilde\phi_1\right] + \text{h.c.} \qquad (3.116)
\end{aligned}$$

Notice that fields with tilde $\tilde\Phi$ and $\tilde\Phi^C$ defined in this subsection correspond to the fields with tildes that are defined in the previous subsection, if we take $f = 2\alpha$, in the sense that they produce the same Lagrangian.

soft A-term by Radion mediation

We can also see the soft SUSY breaking term from the interaction on the brane by the Radion mediation method. Let us consider the same interaction term as that was introduced in subsubsection 3.4.2;

$$\mathcal{L}_{5,\text{int}}^{\text{brane}} = \delta(y)\,\lambda\int d^2\theta \, \Phi_A\Phi_B H + \text{h.c.}, \qquad (3.117)$$

where Φ_A and Φ_B are chiral superfields in the bulk with even parity, and H is a superfield on the brane. If we rewrite Φ_A and Φ_B with their canonical counterparts, $\tilde{\Phi}_A$ and $\tilde{\Phi}_B$, the Lagrangian becomes

$$\begin{aligned}
\mathcal{L}_{5,\mathrm{int}}^{\mathrm{brane}} &= \delta(y)\,\lambda \int d^2\theta\,\Phi_A\Phi_B H + \mathrm{h.c.} \\
&= \delta(y)\,\lambda \int d^2\theta \left(1 - \frac{f}{2R}\theta^2\right)^2 \tilde{\Phi}_A\tilde{\Phi}_B H + \mathrm{h.c.} \\
&= \delta(y)\,\lambda \left\{ \int d^2\theta\,\tilde{\Phi}_A\tilde{\Phi}_B H - \frac{f}{R}\tilde{\phi}_A\tilde{\phi}_B h \right\} + \mathrm{h.c.},
\end{aligned} \tag{3.118}$$

which produce the same soft term in Eq.(3.83) if we take $f = 2\alpha$.

4 The Radiative Correction to the Gaugino and Sfermion Mass

In the previous section we have described the Scherk-Schwarz mechanism and derived the 4D Lagrangian compactified over S^1/Z_2. Before we move on to the compact SUSY model setup, we calculate the radiative corrections to the gaugino and the sfermion masses within the simple model described in the previous section.

Now that we have infinite KK-modes in the 4D effective Lagrangian, the loop computation is rather intricate than the usual loop calculation of 4D quantum field theory. We can perform the KK-modes loop calculation using the techniques summarized in the Appendix B which is often used in the thermal field theory since the time direction can be treated as S^1 with a finite radius. Note that since the 5D gauge theory is not renormalizable, the loop correction has the cutoff Λ dependence.

4.1 Abelian Gaugino Mass Correction

We first calculate the radiative correction to the Abelian gaugino mass, to focus on the difference in the regularization scheme in the simpler case. The relevant interaction terms in 5D Lagrangian are gaugino-fermion-sfermion 3-point interactions and we are interested only in the zero-mode gaugino;

$$\begin{aligned}\mathcal{L}_{5,\phi\lambda\psi} &= -\sqrt{2}ig\left(\phi_1^\dagger, \phi_2\right)\left[\begin{pmatrix}\lambda_1 \\ -\lambda_2\end{pmatrix}\psi_1 + \begin{pmatrix}\bar{\lambda}_2 \\ \bar{\lambda}_1\end{pmatrix}\bar{\psi}_2\right] + \text{h.c.} \\ &= -\sqrt{2}ig\left(\tilde{\phi}_1^\dagger, \tilde{\phi}_2\right)\left[\begin{pmatrix}\tilde{\lambda}_1 \\ -\tilde{\lambda}_2\end{pmatrix}\psi_1 + \begin{pmatrix}\bar{\tilde{\lambda}}_2 \\ \bar{\tilde{\lambda}}_1\end{pmatrix}\bar{\psi}_2\right] + \text{h.c.} \\ &\supset -\frac{\sqrt{2}ig}{\sqrt{2\pi R}}\left(\tilde{\phi}_1^\dagger\lambda_0\psi_1 + \tilde{\phi}_2\bar{\lambda}_0\bar{\psi}_2\right) + \text{h.c.}. \end{aligned} \tag{4.1}$$

In the second equality, we rotate ϕ and λ so they are in tilde basis, and in the next line we only care the zero mode of gaugino - recall the relation

$$\begin{pmatrix}\tilde{\lambda}_1 \\ -\tilde{\lambda}_2\end{pmatrix} = \begin{pmatrix}\frac{\lambda_0}{\sqrt{2\pi R}} \\ 0\end{pmatrix} + (\text{non-zero modes}). \tag{4.2}$$

Replacing $g/\sqrt{2\pi R}$ with g and integrating them with respect to y, we obtain the corresponding 4D Lagrangian;

$$
\begin{aligned}
\mathcal{L}_{4,\phi\lambda_0\psi} &= -\sqrt{2}ig\int_0^{2\pi R} dy\left(\tilde{\phi}_1^\dagger\lambda_0\psi_1 + \tilde{\phi}_2\bar{\lambda}_0\bar{\psi}_2\right) + \text{h.c.} \\
&= -\sqrt{2}ig\int_0^{2\pi R} dy\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} \\
&\qquad\left(\phi_m^\dagger\lambda_0\psi_{1,n}\frac{\eta_n}{\sqrt{2\pi R}}\cos\hat{m}y\cos\hat{n}y + \phi_m^\dagger\bar{\lambda}_0\bar{\psi}_{2,n}\frac{1}{\sqrt{2\pi R}}\sin\hat{m}y\sin\hat{n}y\right) + \text{h.c.} \\
&= -\sqrt{2}ig\phi_0^\dagger\lambda_0\psi_{1,0} - ig\sum_{n=1}^{\infty}\left[\phi_n^\dagger\left(\lambda_0,\bar{\lambda}_0\right)\begin{pmatrix}\psi_{1,n}\\ \bar{\psi}_{2,n}\end{pmatrix} + \phi_{-n}^\dagger\left(\lambda_0,\bar{\lambda}_0\right)\begin{pmatrix}\psi_{1,n}\\ -\bar{\psi}_{2,n}\end{pmatrix}\right] + \text{h.c.} \\
&= -\sqrt{2}ig\phi_0^\dagger\overline{\Upsilon}_0 P_L\Psi_0 - ig\sum_{n=1}^{\infty}\left[\phi_n^\dagger\overline{\Upsilon}_0\Psi_n + \phi_{-n}^\dagger\overline{\Upsilon}_0\Psi'_n\right] + \text{h.c..} \qquad (4.3)
\end{aligned}
$$

Therefore 1PI graph at 1-loop is given by Fig.2.

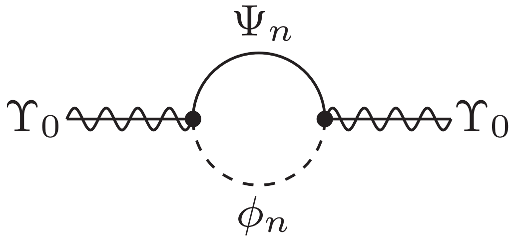


Figure 2: The loop diagram of radiative corrections to the gaugino mass by the 3-point interaction between gaugino, fermion and sfermion.

Taking the external momentum to be p and the loop momentum to be k, the Feynman diagram is expressed as follows;

$$
\begin{aligned}
\Sigma(p) &= \text{Fig.2} = (\text{zero mode}) + (\text{positive modes}) + (\text{negative modes}) \\
&= \sum_{ij} \int \frac{d^4k}{i(2\pi)^4} \Bigg\{ (-\sqrt{2} ig T^a_{ij} P_L) \frac{\not{k}}{k^2} (\sqrt{2} ig T^a_{ji} P_R) \frac{1}{(p-k)^2 + \hat{\alpha}^2} \\
&\qquad + (\sqrt{2} ig T^a_{ij} P_R) \frac{\not{k}}{k^2} (-\sqrt{2} ig T^a_{ji} P_L) \frac{1}{(p-k)^2 + \hat{\alpha}^2} \Bigg\} \\
&+ \sum_{n=1}^{\infty} \sum_{ij} \int \frac{d^4k}{i(2\pi)^4} (-igT^a_{ij}) \frac{\not{k} - \hat{n}}{k^2 + \hat{n}^2} (igT^a_{ji}) \frac{1}{(p-k)^2 + (\hat{\alpha} + \hat{n})^2} \times 2 \\
&+ \sum_{n=1}^{\infty} \sum_{ij} \int \frac{d^4k}{i(2\pi)^4} (-igT^a_{ij}) \frac{\not{k} + \hat{n}}{k^2 + \hat{n}^2} (igT^a_{ji}) \frac{1}{(p-k)^2 + (\hat{\alpha} - \hat{n})^2} \times 2 \\
&= 2g^2 T(F) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{\not{k} - \hat{n}}{[k^2 + \hat{n}^2][(k-p)^2 + (\hat{\alpha} + \hat{n})^2]}.
\end{aligned} \tag{4.4}
$$

The factor $\times 1/2$ in the zero mode loop comes from the Majorana property of Ψ_0. In the last line, $T(F)$ is defined as the number of the flavor times the trace of the fundamental representation, i.e. $T(F) = F \times T(N)$. $T(N)$ is $\frac{1}{2}$ in the non-Abelian case, and is 1 in the Abelian case. By the Feynman technique $\frac{1}{ab} = \int_0^1 dx \, \frac{1}{[ax+b(1-x)]^2}$, we obtain the following

$$
\Sigma(p) = 2g^2 F \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \int_0^1 dx \, \frac{\not{k} - \hat{n}}{[k^2 - 2xpk + xp^2 + \hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2]^2}. \tag{4.5}
$$

We evaluate this in three different ways.

Method 1: DR bar scheme

Here's the procedure of the first method;

- Step1: integrate with respect to k, with $\overline{\text{DR}}$ method to regularize the infinity
- Step2: expand around $\alpha = 0$, and then perform the x integration
- Step3: perform the summation by n, if possible

For notational simplicity, we introduce $\hat{p} = pR$, which is dimensionless and thought to be small.

$$
\begin{aligned}
\Sigma(p) &= 2g^2 F \sum_{n=-\infty}^{\infty} \int_0^1 dx \int \frac{d^4 k}{i(2\pi)^4} \frac{\not{k} - \hat{n}}{[k^2 - 2xpk + xp^2 + \hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2]^2} \\
&= 2g^2 F \sum_{n=-\infty}^{\infty} \int_0^1 dx \frac{\Gamma(2 - D/2)}{(4\pi)^{D/2}} \frac{x\not{p} - \hat{n}}{[x(1-x)\hat{p}^2 + n^2 + 2x\alpha n + x\alpha^2]^{2-D/2}} \\
&= 2g^2 F \sum_{n=-\infty}^{\infty} \int_0^1 dx \frac{x\not{p} - \hat{n}}{16\pi^2} \Big\{ \bar{\epsilon}^{-1} - \ln\left[x(1-x)\hat{p}^2 + n^2 + 2x\alpha n + x\alpha^2\right] \Big\} + \mathcal{O}(\epsilon) \\
&= \frac{g^2 F}{8\pi^2} \sum_{n=-\infty}^{\infty} \int_0^1 dx (x\not{p} - \hat{n}) \Big\{ \bar{\epsilon}^{-1} - \ln\left[n^2 + 2x\alpha n + x\alpha^2\right] \Big\} + \mathcal{O}(\hat{p}^2) \\
&= \frac{g^2 F}{8\pi^2} \sum_{n=-\infty}^{\infty} \left\{ \not{p} \frac{1}{2} \bar{\epsilon}^{-1} - \int_0^1 dx (x\not{p} - \hat{n}) \ln\left[n^2 + 2x\alpha n + x\alpha^2\right] \right\} + \mathcal{O}(\hat{p}^2). \qquad (4.6)
\end{aligned}
$$

After the fourth line, we have omitted the sum of $\mathcal{O}(\epsilon)$. The last line can be evaluated as follows.

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty} \int_0^1 dx (x\not{p} - \hat{n}) \ln\left[n^2 + 2x\alpha n + x\alpha^2\right] \\
&= \not{p} \int_0^1 dx\ x \ln[x\alpha^2] + \not{p} \sum_{n=1}^{\infty} \int_0^1 dx x \ln\left[n^2 + 2x\alpha n + x\alpha^2\right]\left[n^2 - 2x\alpha n + x\alpha^2\right] \\
&\quad - R^{-1} \sum_{n=1}^{\infty} n \int_0^1 dx \ln\left[\frac{n^2 + 2x\alpha n + x\alpha^2}{n^2 - 2x\alpha n + x\alpha^2}\right] \\
&= \not{p} \left(-\frac{1}{4} + \frac{1}{2} \ln\alpha^2 + \sum_{n=1}^{\infty} \int_0^1 dx x \left[\ln n^4 - \frac{2x(2x-1)}{n^2} \alpha^2 + \mathcal{O}(\alpha^4) \right] \right) \\
&\quad - R^{-1} \sum_{n=1}^{\infty} \int_0^1 dx\ \left[4x\alpha + \frac{4x^2(4x-3)}{3n^2} \alpha^3 + \mathcal{O}(\alpha^5) \right] \\
&= \not{p} \left(-\frac{1}{4} + \frac{1}{2} \ln\alpha^2 + \sum_{n=1}^{\infty} \left[\frac{1}{2} \ln n^4 - \frac{1}{3n^2} \alpha^2 \right] \right) - \hat{\alpha} \sum_{n=1}^{\infty} 2 + \mathcal{O}(\alpha^4). \qquad (4.7)
\end{aligned}
$$

Since $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, the result is

$$
\Sigma(p) = \frac{g^2 F}{8\pi^2} \left\{ \not{p} \left(\frac{1}{2} \sum_{n=-\infty}^{\infty} \bar{\epsilon}^{-1} + \frac{1}{4} - \frac{1}{2} \ln\alpha^2 - \frac{1}{2} \sum_{n=1}^{\infty} \ln n^4 + \frac{\pi^2}{18} \alpha^2 \right) + \hat{\alpha} \left(2 \sum_{n=1}^{\infty} 1 \right) \right\} + \mathcal{O}(\hat{p}^2, \alpha^4). \qquad (4.8)
$$

As you can see, there are three kinds of divergences, two in the wave function renormalization and one in the mass term. We cannot perform the summation by n any further which means the regularization is not complete. Therefore it is difficult to discuss physics in this method.

Method 2: KK-regularization

The second method is as follows;

- Step1: take the infinite summation by n first
- Step2: evaluate the momentum integral with the cutoff Λ
- Step3: perform the x integration and expand around $\alpha = 0$ if necessary

This scheme is called KK-regularization in the literatures [55–57]. In this way, we can evaluate the divergence with only one parameter Λ, which is one of the upsides of this method. As the downside, the introduction of the momentum cutoff breaks the gauge symmetry and SUSY.

Let us go back to Eq.(4.5) and see how this method works. We first rearrange the infinite sum and integrations and rewrite the integrand as follows;

$$
\begin{aligned}
\Sigma(p) &= 2g^2 F \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \; \frac{\not{k} - \hat{n}}{[\hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2 + k^2 - 2xpk + xp^2]^2} \\
&= -2g^2 F R^{-1} \int_0^1 dx \int \frac{d^4\hat{k}'}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{n - x\hat{\not{p}}}{(n + x\alpha + iq)^2 (n + x\alpha - iq)^2},
\end{aligned} \tag{4.9}
$$

where we have defined $\hat{k} = kR$, $\hat{k}' = \hat{k} - x\hat{p}$ and $q^2 = \hat{k}'^2 + x(1-x)(\alpha^2 + \hat{p}^2) = \hat{k}'^2 + c^2$. Note that we have made it Euclidean by the Wick rotation from the beginning. Note also that there is a mathematical subtlety when exchanging the infinite sum and the integral, which may result in a different result. The technique to evaluate the infinite sum $\sum_{n=-\infty}^{\infty}$ and perform the momentum integral $\int d^4k$ with cutoff Λ is summarized in the Appendix B. Using Eq.(B.40) and Eq.(B.39), we obtain

$$
\begin{aligned}
\Sigma(p) &= -2g^2 F R^{-1} \int_0^1 dx \; [\mathcal{I}_2(x\alpha, c) - x\hat{\not{p}}\, \mathcal{I}_1(x\alpha, c)] \\
&= \frac{g^2 F}{8\pi^2} \left\{ \not{p} \left(\frac{\pi}{2}\hat{\Lambda} - 2\int_0^1 dx\, x \ln[2\pi\sqrt{x\alpha^2 + x(1-x)\hat{p}^2}] \right) + \hat{\alpha} \left(\frac{\pi}{2}\hat{\Lambda} - 1 \right) \right\}.
\end{aligned} \tag{4.10}
$$

Here we have used $\hat{\Lambda} = \Lambda R$, a dimensionless cutoff regularized by R. We have divergences both in the wave function renormalization and in the mass renormalization term. Both are expressed by the cutoff Λ and we can see the cancellation as follows.

We define the wave function renormalization δZ and δm as

$$\Sigma(p) = \delta Z \not{p} + \delta m. \tag{4.11}$$

Then the 1PI 2-point vertex function is

$$\Gamma_{\lambda_0}(p) = -\not{p} - m - \Sigma(p) = -(1+\delta Z)\left(\not{p} + \frac{m+\delta m}{1+\delta Z}\right). \tag{4.12}$$

Now, the tree-level mass m is $\hat{\alpha}$. Therefore, by looking at the pole mass, the mass correction is

$$\begin{aligned}
\delta M_\lambda &= [\delta m - \hat{\alpha}\delta Z]_{p^2=-\hat{\alpha}^2} \\
&= \frac{g^2 F}{8\pi^2}\hat{\alpha}\left\{\left(\frac{\pi}{2}\hat{\Lambda} - 1\right) - \left(\frac{\pi}{2}\hat{\Lambda} - 2\int_0^1 dx\, x \ln[2\pi\sqrt{x\alpha^2 - x(1-x)\alpha^2}]\right)\right\} \\
&= \frac{g^2 F}{8\pi^2}\hat{\alpha}\left(\ln[2\pi\alpha] - \frac{3}{2}\right).
\end{aligned} \tag{4.13}$$

Here are comments regarding this result;

- It is proportionate to α, and thus it vanishes when $\alpha = 0$. This is consistent with the fact that when $\alpha = 0$, the supersymmetry preserves even after the compactification, and thus the gaugino should be massless.
- The linear divergence in the wave function renormalization is consistent with the fact that 5D gauge coupling is expected to have linear divergence from the dimensional analysis [58, 59].
- Still this divergence is cancelled out in the overall mass correction. This is reasonable because the UV divergence is a local effect and there is locally the 5D Lorentz invariance from the construction of this theory. If there is a 5D Lorentz invariance, there must be no mass correction to the zero-mode gaugino mass. Similar discussion is found in [60].

One may wonder if the cancellation of the linear divergence is really scheme independent since the introduction of the momentum cutoff Λ breaks the SUSY and gauge symmetry. We take one more different method and show that there is no divergence in the mass correction.

Method 3: The winding method

We can evaluate the Feynman diagrams that have KK-modes loop without introducing the momentum cutoff Λ following the method in [60]. We show that the linear divergence does not appear in the mass correction in this method as well as in the KK-regularization method.

The key equality in this method is the Poisson resummation formula;

$$\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty} F(n/R) = \sum_{m=-\infty}^{\infty} f(2\pi R m), \tag{4.14}$$

where F and f are continuous functions and f is the Fourier transformation of F;

$$f(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, F(k)\, e^{-ikx}. \tag{4.15}$$

Using this formula, we rewrite the KK-modes summation into the winding number summation, and this is why we call this scheme the winding method. The calculation is written down below followed by the explanation in each step.

$$\begin{aligned}
\Sigma(p) &= 2g^2 F \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx\, \frac{x\not{p} - \hat{n}}{[k^2 + x(1-x)p^2 + \hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2]^2} && (4.16)\\
&= 2g^2 F \sum_{n=-\infty}^{\infty} \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \left(x\not{p} - \hat{n}\right) \int_0^{\infty} dl\, l\, e^{-l[k^2 + x(1-x)p^2 + \hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2]} && (4.17)\\
&= \frac{g^2}{8\pi^2} F \sum_{n=-\infty}^{\infty} \int_0^1 dx\, \left(x\not{p} - \hat{n}\right) \int_0^{\infty} \frac{dl}{l}\, e^{-l[x(1-x)p^2 + \hat{n}^2 + 2x\hat{\alpha}\hat{n} + x\hat{\alpha}^2]} && (4.18)\\
&= \frac{g^2}{8\pi^2} F R \sum_{m=-\infty}^{\infty} \int_0^1 dx \int_0^{\infty} \frac{dl}{l} \left(-i\frac{2\pi R m}{2l}\right) \sqrt{\frac{\pi}{l}}\, e^{-2\pi i m x \alpha - \frac{\pi^2 R^2 m^2}{l}} && (4.19)\\
&= \frac{g^2}{8\pi^2} F \frac{1}{R} \sum_{m=-\infty}^{\infty} \left(-\frac{1}{2\pi}\right) \frac{m}{|m|^3} \frac{1 - e^{-2\pi i m x \alpha}}{2\pi m \alpha} && (4.20)\\
&= \frac{g^2}{8\pi^2} F \frac{\alpha}{R} \left(\ln[2\pi\alpha] - \frac{3}{2}\right) + \mathcal{O}(\alpha^3). && (4.21)
\end{aligned}$$

In the second equality, we have used the following identity

$$\frac{1}{A^2} = \int_0^{\infty} dl\, l\, e^{-lA}, \tag{4.22}$$

for $A > 0$ and in the forth equality we have performed the Gaussian integration with respect to momentum k. In the fourth equality, we have used the Poisson resummation formula and rewritten the integrand with its Fourier transform, using the on-shell condition ($\not{p} = -\hat{\alpha}$). The fifth equality is just a integration by l and then x. In the last equality, we have expanded around $\alpha = 0$ and dropped $m = 0$ contribution to perform the summation by m. The $m = 0$ dropping looks valid since the integrand of the fifth line vanish when $m = 0$. However this reasoning is mathematically subtle since we have exchanged the summation and integral which does not converge. Instead we rely on the physical reasoning to drop the $m = 0$ divergence (same as in [60]); $m = 0$ corresponds to no-winding and thus should be vanish by local 5D Lorentz invariance. This $m = 0$ corresponds to the linear divergence in KK-regularization and the resulting mass correction from the rest is exactly same as Eq.(4.13);

$$\delta M_\lambda = \frac{g^2}{8\pi^2}\hat{\alpha}F\left(\ln[2\pi\alpha] - \frac{3}{2}\right). \tag{4.23}$$

This implies that the cancellation of linear and higher divergences is not scheme dependent.

In the following calculation, we use the KK-regularization method since the cutoff Λ, which was introduced to renormalize the divergence, can be understood as the scale where the 5D effective field theory picture breaks.

4.2 Non-Abelian Gaugino Mass Correction

In this section let us discuss the radiative correction to the Non-Abelian gaugino. The relevant terms are $\mathcal{L}_{\phi\lambda\psi}$, $\mathcal{L}_{\lambda\Sigma\lambda}$ and $\mathcal{L}_\lambda^{\text{gauge-int}}$. Therefore, the 1-loop 1PI graph falls into following components;

$$\begin{aligned}\Sigma(p) &= \Sigma_{\phi\lambda\psi} + \Sigma_{\lambda\Sigma\lambda} + \Sigma_\lambda^{\text{gauge-int}},\\ \Sigma_\lambda^{\text{gauge-int}} &= \Sigma_{\lambda A_\mu\lambda} + \Sigma_{\lambda A_5\lambda}.\end{aligned} \tag{4.24}$$

Each term represents the Feynman diagram in Fig.3.

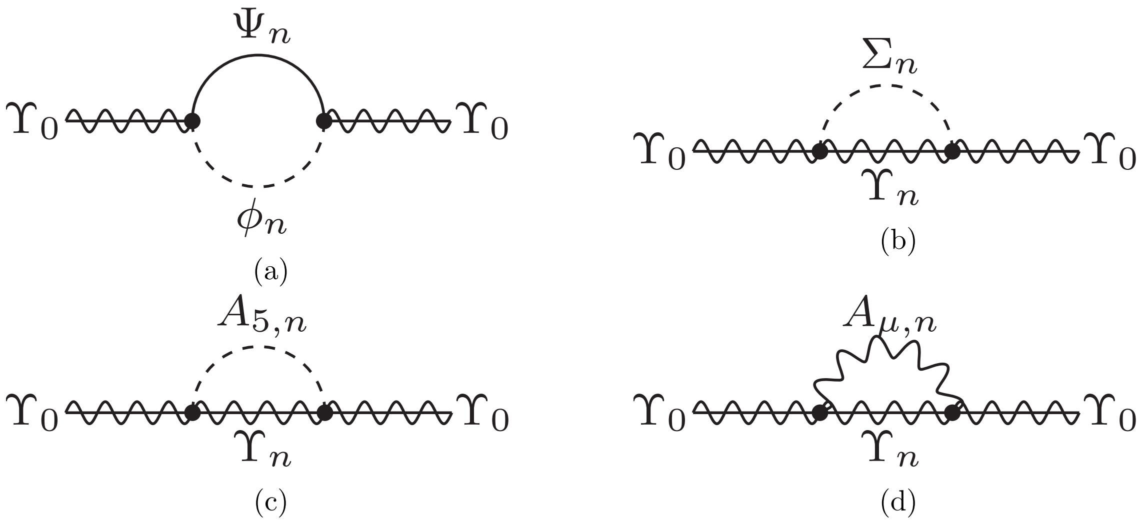


Figure 3: The loop diagram of radiative corrections to the gaugino mass by the 3-point interaction between gaugino, fermion and sfermion.

Taking the external momentum to be p and the loop momentum to be k, each diagram is expressed as

$$\Sigma_{\phi\lambda\psi} = \text{Fig. 3 (a)} = 2g^2T(F)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}-\hat{n}}{[k^2+\hat{n}^2][(k-p)^2+(\hat{\alpha}+\hat{n})^2]}, \tag{4.25}$$

$$\Sigma_{\lambda\Sigma\lambda} = \text{Fig. 3 (b)} = \frac{g^2C(A)}{2}\sum_{n\neq 0}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+(\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}, \tag{4.26}$$

$$\Sigma_{\lambda A_5\lambda} = \text{Fig. 3 (c)} = \frac{g^2C(A)}{2}\sum_{n\neq 0}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}-(\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}, \tag{4.27}$$

$$\Sigma_{\lambda A_\mu\lambda} = \text{Fig. 3 (d)} = g^2C(A)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+2(\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}, \tag{4.28}$$

$$+\,g^2C(A)\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+2\hat{\alpha}}{[k^2+\hat{\alpha}^2][(k-p)^2]}. \tag{4.29}$$

The detailed derivation of these equations are written in the Appendix C.1. Combining these results give us

$$\begin{aligned}\Sigma(p) = {} & 2g^2T(F)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}-\hat{n}}{[k^2+\hat{n}^2][(k-p)^2+(\hat{\alpha}+\hat{n})^2]}\\ & +2g^2C(A)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+\hat{\alpha}+\hat{n}}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}\\ & +2g^2C(A)\int\frac{d^4k}{(2\pi)^4}\frac{\hat{\alpha}}{[k^2+\hat{\alpha}^2][(k-p)^2]}.\end{aligned} \tag{4.30}$$

Evaluation of Σ by KK-regularization

We evaluate the Eq.(4.30) in the KK-regularization scheme as in the Abelian gaugino case. Using Eq.(B.39) and Eq.(B.40), the first term in Eq.(4.30) is evaluated as follows;

$$
\begin{aligned}
&2g^2T(F)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}-\hat{n}}{[k^2+\hat{n}^2][(k-p)^2+(\hat{\alpha}+\hat{n})^2]}\\
&=2g^2T(F)\int_0^1 dx\sum_{n=-\infty}^{\infty}\int\frac{d^4k'}{(2\pi)^4}\frac{x\not{p}-\hat{n}}{[(\hat{n}+x\hat{\alpha})^2+k'^2+c^2]^2}\qquad\left(c^2=x(1-x)(p^2+\hat{\alpha}^2)\right)\\
&=2g^2T(F)R^{-1}\int_0^1 dx\,\left[x\not{p}\mathcal{I}_1(x\alpha,c)-\mathcal{I}_2(x\alpha,c)\right]\\
&=\frac{2g^2T(F)}{8\pi^2}\left\{\not{p}\left(\frac{\pi}{4}\hat{\Lambda}-\int_0^1 dx\,x\ln[2\pi\sqrt{x\alpha^2+x(1-x)\hat{p}^2}]\right)+\hat{\alpha}\left(\frac{\pi}{4}\hat{\Lambda}-\frac{1}{2}\right)\right\}.
\end{aligned}\tag{4.31}
$$

Similarly the second term in Eq.(4.30) is;

$$
\begin{aligned}
&2g^2C(A)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+\hat{\alpha}+\hat{n}}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}\\
&=2g^2C(A)\int_0^1 dx\sum_{n=-\infty}^{\infty}\int\frac{d^4k'}{(2\pi)^4}\frac{(1-x)\not{p}+\hat{\alpha}+\hat{n}}{[(\hat{n}+x\hat{\alpha})^2+k'^2+c^2]^2}\qquad\left(c^2=x(1-x)(p^2+\hat{\alpha}^2)\right)\\
&=2g^2C(A)R^{-1}\int_0^1 dx\,[((1-x)\not{p}+\alpha)\mathcal{I}_1(x\alpha,c)+\mathcal{I}_2(x\alpha,c)]\\
&=\frac{2g^2C(A)}{8\pi^2}\left\{\not{p}\left(\frac{\pi}{4}\hat{\Lambda}-\int_0^1 dx\,(1-x)\ln[2\pi\sqrt{x\alpha^2+x(1-x)\hat{p}^2}]\right)\right.\\
&\qquad\left.+\hat{\alpha}\left(\frac{\pi}{4}\hat{\Lambda}+\frac{1}{2}-\int_0^1 dx\,\ln[2\pi\sqrt{x\alpha^2+x(1-x)\hat{p}^2}]\right)\right\}.
\end{aligned}\tag{4.32}
$$

The third term doesn't have the KK summation and this is same as the usual 4D loop diagram. We evaluate this by introducing a naive momentum cutoff (the last equation in Eq.(A.47)) and we obtain

$$
\begin{aligned}
&2g^2C(A)\int\frac{d^4k}{(2\pi)^4}\frac{\hat{\alpha}}{[k^2+\hat{\alpha}^2][(k-p)^2]}\\
&=2g^2C(A)\hat{\alpha}\int_0^1 dx\int\frac{d^4k'}{(2\pi)^4}\frac{1}{[k'^2+x(1-x)p^2+(1-x)\hat{\alpha}^2]^2}\\
&=\frac{2g^2C(A)}{8\pi^2}\hat{\alpha}\left(\ln\hat{\Lambda}-\int_0^1 dx\,\ln[\sqrt{x\alpha^2+x(1-x)\hat{p}^2}]-\frac{1}{2}\right).
\end{aligned}\tag{4.33}
$$

In total, we obtain

$$\Sigma(p) = \frac{g^2}{4\pi^2}\Bigg\{\not{p}\Bigg(\frac{T(F)+C(A)}{4}\pi\hat{\Lambda} - \int_0^1 dx\ (xT(F)+(1-x)C(A))\ln[2\pi\sqrt{x\alpha^2+x(1-x)\hat{p}^2}]\Bigg)$$
$$+\hat{\alpha}\Bigg(\frac{T(F)+C(A)}{4}\pi\hat{\Lambda} + \frac{C(A)}{2}\ln\hat{\Lambda}^2 - \frac{T(F)}{2}$$
$$-2C(A)\int_0^1 dx\ \ln[\sqrt{x\alpha^2+x(1-x)\hat{p}^2}] - C(A)\ln[2\pi]\Bigg)\Bigg\} \tag{4.34}$$

Defining $\Sigma(p) = \delta Z\not{p} + \delta m$, the 1PI 2-point vertex function is

$$\Gamma_{\lambda_0}(p) = -\not{p} - m - \Sigma(p) = -(1+\delta Z)\left(\not{p} + \frac{m+\delta m}{1+\delta Z}\right). \tag{4.35}$$

Since the tree-level mass m is $\hat{\alpha}$, the mass correction is

$$\begin{aligned}\delta M_\lambda &= [\delta m - \hat{\alpha}\delta Z]_{p^2=-\hat{\alpha}^2} \\ &= \frac{g^2}{16\pi^2}\hat{\alpha}\Bigg(4C(A)\ln[2\pi\hat{\Lambda}] + (2T(F)-6C(A))\ln[2\pi\alpha] + 5C(A) - 3T(F)\Bigg)\end{aligned} \tag{4.36}$$

Here are comments on this result;

- First of all, if we put $C(A) = 0$ and $T(F) = 1$, this value is of course same as the Abelian case result, i.e. Eq.(4.13), and the comments on the proportionality to α and the cancellation of linear divergence still stand.

- The coefficient of $\ln[2\pi\alpha]$ should be same as the MSSM gaugino beta function;

$$\frac{dM_\lambda}{d\ln\mu} = \frac{g^2}{16\pi^2}(2T(F)-6C(A))\hat{\alpha}. \tag{4.37}$$

  This is because the IR divergent $\ln[\alpha]$ term comes only from the zero mode loop diagrams, and the zero mode theory is same as the MSSM with the soft parameter $\hat{\alpha}$.

- The main difference from the Abelian case is that there is a logarithmic divergent term. This comes from the "mis-match" in the zero-mode (the last line in the Eq.(4.30)) and can be understood as the brane localized correction. We discuss on this further in the Appendix E.

4.3 Sfermion Mass Correction

In this section let us evaluate the radiative correction to the sfermion mass. The procedure is almost same as the gaugino case.

The relevant interaction terms are $\mathcal{L}_{\phi\lambda\psi}$, $\mathcal{L}_{\phi\Sigma\Sigma\phi}$, $\mathcal{L}_{\phi\phi\phi\phi}$ and $\mathcal{L}^{\text{gauge-int}}$. Therefore, the 2-point 1PI Green function falls into following parts;

$$\begin{aligned} \Pi(p) &= \Pi_{\phi\lambda\psi}(p) + \Pi_{\phi\Sigma\Sigma\phi}(p) + \Pi_{\phi\phi\phi\phi}(p) + \Pi^{\text{gauge-int}}(p), \\ \text{with} \qquad \Pi^{\text{gauge-int}}(p) &= \Pi_{\phi A_\mu \phi}(p) + \Pi_{\phi A_5 \phi}(p) + \Pi_{\phi A^\mu A_\mu \phi}(p) + \Pi_{\phi A_5 A_5 \phi}(p). \end{aligned} \tag{4.38}$$

Each term represents the Feynman diagrams in Fig.4.

Taking the external momentum to be p and the loop momentum to be k, each diagram is expressed as follows (see the Appendix C.2 for the detailed calculation);

$$\Pi_{\phi\lambda\psi} = \text{Fig.4 (a)} = 4g^2 C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{k\cdot(k-p) + (\hat{\alpha}+\hat{n})\hat{n}}{[k^2 + (\hat{\alpha}+\hat{n})^2][(k-p)^2 + \hat{n}^2]}, \tag{4.39}$$

$$\begin{aligned} \Pi_{\phi A_5 \phi} = \text{Fig.4 (b)} &= \frac{g^2}{2} C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{(2\hat{\alpha}+\hat{n})^2}{[k^2 + (\hat{\alpha}+\hat{n})^2][(k-p)^2 + \hat{n}^2]} \\ &\quad - 2g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{\hat{\alpha}^2}{[k^2 + \hat{\alpha}^2][(k-p)^2]}, \end{aligned} \tag{4.40}$$

$$\begin{aligned} \Pi_{\phi A_\mu \phi} = \text{Fig.4 (c)} &= \frac{g^2}{2} C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{(k+p)^2}{[k^2 + (\hat{\alpha}+\hat{n})^2][(k-p)^2 + \hat{n}^2]} \\ &\quad + \frac{g^2}{2} C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{(k+p)^2}{[k^2 + \hat{\alpha}^2][(k-p)^2]}. \end{aligned} \tag{4.41}$$

for loops from 3-point couplings, and

$$\Pi_{\phi\Sigma\Sigma\phi} = \text{Fig.4 (d)} = -g^2 C(N) \sum_{n=1}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{n}^2}, \tag{4.42}$$

$$\Pi_{\phi\phi\phi\phi} = \text{Fig.4 (e)} = -\frac{3}{2} g^2 C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + (\hat{\alpha}+\hat{n})^2} \tag{4.43}$$

$$+ \frac{1}{2} g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{\alpha}^2}, \tag{4.44}$$

$$\Pi_{\phi A_5 A_5 \phi} = \text{Fig.4 (f)} = -g^2 C(N) \sum_{n=1}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{n}^2}, \tag{4.45}$$

$$\Pi_{\phi A_\mu A_\mu \phi} = \text{Fig.4 (g)} = -4g^2 C(N) \sum_{n=0}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{n}^2}, \tag{4.46}$$

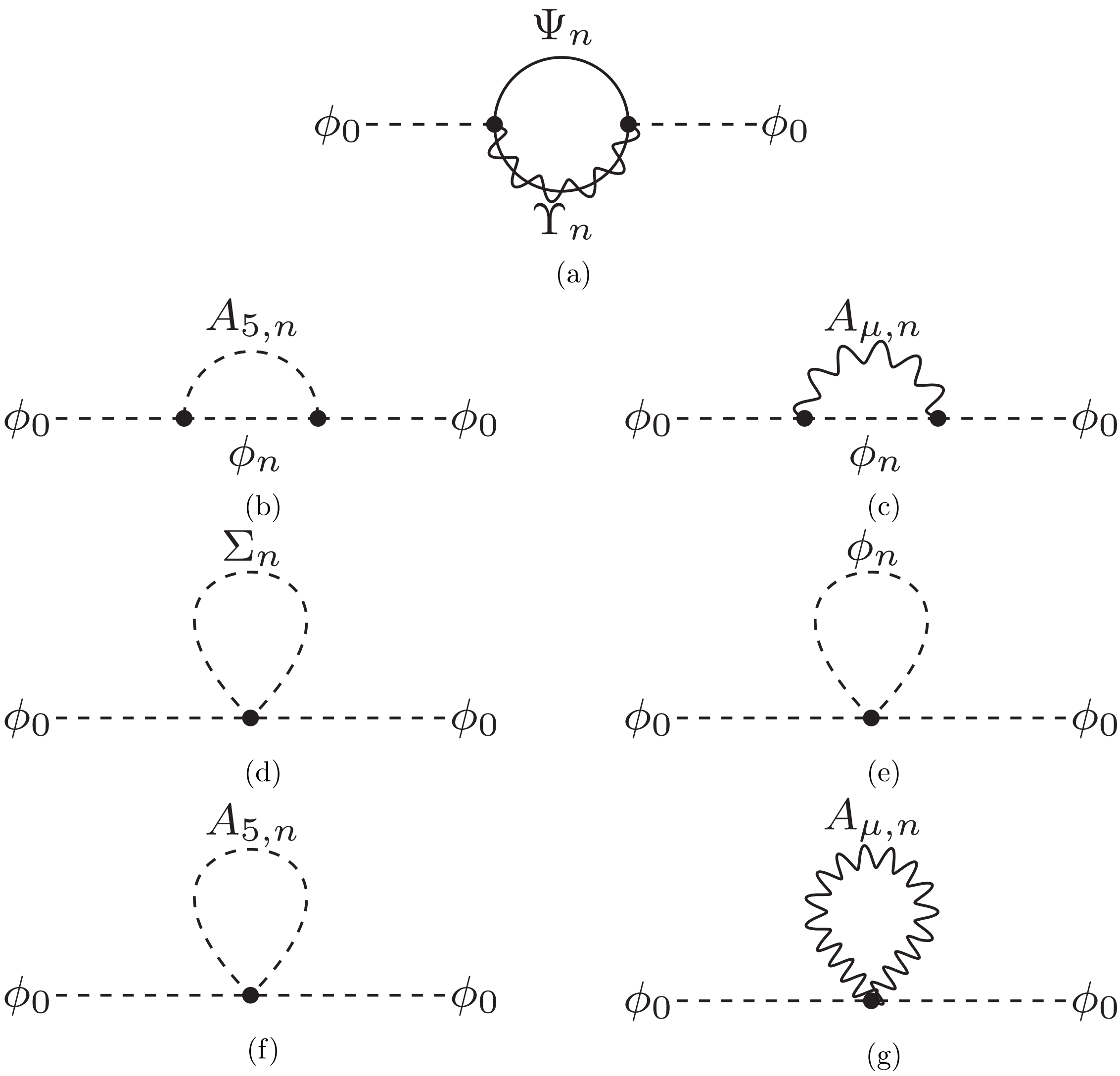


Figure 4: The 1-loop Feynman diagrams of radiative corrections to the sfermion mass.

for loops from 4-point couplings.

In total,

$$\Pi(p) = -g^2 C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{p^2+\hat{\alpha}^2}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]} \\ + g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{p^2-3\hat{\alpha}^2}{[k^2+\hat{\alpha}^2][(k-p)^2]}. \tag{4.47}$$

The quadratic divergent terms are canceled out, and of course this is zero when $\alpha = 0, p = 0$.

Evaluation of Π by KK-regularization

First term is

$$\begin{aligned} &- g^2 C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{p^2+\hat{\alpha}^2}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]} \\ &= -g^2 C(N)(p^2+\hat{\alpha}^2) \int_0^1 dx \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{[k^2+(\hat{n}+x\hat{\alpha})^2+c^2]^2} \\ &= -g^2 C(N)(p^2+\hat{\alpha}^2) \int_0^1 dx\, \mathcal{I}_1(x\alpha, c) \\ &= -\frac{g^2}{8\pi^2} C(N)(p^2+\hat{\alpha}^2) \left(\frac{\pi}{2}\hat{\Lambda} - \int_0^1 dx\, \ln[2\pi\sqrt{x\alpha^2+x(1-x)\hat{p}^2}] \right). \end{aligned} \tag{4.48}$$

Second term is

$$\begin{aligned} &g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{p^2-3\hat{\alpha}^2}{[k^2+\hat{\alpha}^2][(k-p)^2]} \\ &= g^2 C(N)(p^2-3\hat{\alpha}^2) \int_0^1 dx \int \frac{d^4k}{i(2\pi)^4} \frac{1}{[k^2+x\hat{\alpha}^2+x(1-x)p^2]^2} \\ &= \frac{g^2}{8\pi^2} C(N)(p^2-3\hat{\alpha}^2) \left(\ln\hat{\Lambda} - \int_0^1 dx\, \ln\sqrt{x\alpha^2+x(1-x)p^2} - \frac{1}{2} \right). \end{aligned} \tag{4.49}$$

The total $\Pi(p)$ is just a sum of above two.

If we define $\Pi(p) = -\delta Z p^2 - \delta m^2$, the 2-point vertex function becomes

$$\Gamma(p) = p^2 + m^2 - \Pi(p) = (1+\delta Z)\left(p^2 + \frac{m^2+\delta m^2}{1+\delta Z} \right), \tag{4.50}$$

and thus the mass correction at 1-loop level is

$$\begin{aligned} \delta M_\phi^2 &= \delta m^2 - \hat{\alpha}^2 \delta Z|_{p^2=-\hat{\alpha}^2} \\ &= \frac{g^2 C(N)}{8\pi^2} 4\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \ln\alpha + \frac{1}{2} \right). \end{aligned} \tag{4.51}$$

Here are comments on this result;

- The proportionality to $\hat{\alpha}^2$ and the cancellation of the quadratic (cubic, if we take the infinite sum into account) has the same physical reasoning as the gaugino case.

- For the same reason as the gaugino case, the coefficient of $\ln \alpha$ corresponds to the beta function in the case of MSSM;

$$\frac{dM_\phi^2}{d\ln\mu} = -\frac{g^2}{8\pi^2}C(N)4\hat{\alpha}^2 = \begin{cases} -\frac{g^2}{16\pi^2}\frac{32}{3}\hat{\alpha}^2 & (G = SU(3)) \\ -\frac{g^2}{16\pi^2}6\hat{\alpha}^2 & (G = SU(2)) \\ -\frac{g^2}{16\pi^2}\frac{24}{5}Y^2\hat{\alpha}^2 & (G = U(1)_Y) \end{cases} \tag{4.52}$$

  Here we have used $C(N) = \frac{N^2-1}{2N}$ when $G = SU(N)$, and $C(N) = 3Y^2/5$ when $G = U(1)_Y$.

- There is again logarithmic divergence.

5 compact SUSY model

So far we have seen the 4D Lagrangian, which was from a compactification of 5D SUSY gauge theory over S^1/Z_2, as a model case of the Scherk-Schwarz mechanism, and calculated the 1-loop radiative corrections to the gaugino and sfermion masses in the previous section.

In this section, we finally discuss the compact SUSY model [43–45, 47–49], a realistic application of the Scherk-Schwarz mechanism.

5.1 The model setup

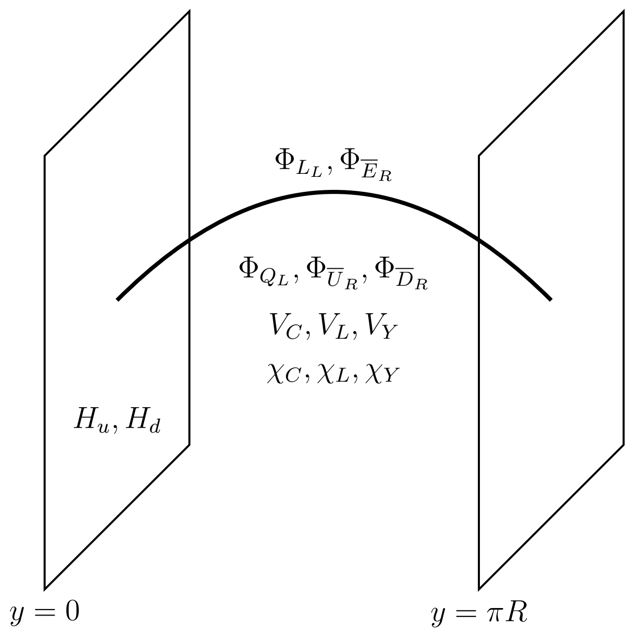


Figure 5: The schematic picture of the compact SUSY model.

As in the schematic Figure is Fig.5, we consider the 5D SUSY gauge theory compactified over S^1/Z_2 with the twisted boundary condition parametrized by α, and take the gauge group to be same as the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Therefore there are three gauge multiplets in the bulk, (V_C, χ_C), (V_L, χ_L) and (V_Y, χ_Y), which are adjoint representation of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, respectively.

We next put the Hypermultiplets in the bulk as the matter fields, so that the all the zero modes is same as the matter content of the MSSM except for Higgs fields (we introduce them later). We write them as (Φ_X, Φ_X^C), where $X = Q_L(U_L, D_L), \overline{U}_R, \overline{D}_R, L_L(N_L, E_L), \overline{E}_R$. The notation for the component fields is

$$\Phi_X = \phi_X + \sqrt{2}\,\theta\psi_X + \theta^2 F_X, \tag{5.1}$$
$$\Phi_X^C = \phi_X^C + \sqrt{2}\,\theta\psi_X^C + \theta^2 F_X^C. \tag{5.2}$$

They are summarized in the Table 1.

chiral superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(3)_F$
$\Phi_{Q_L} = \begin{pmatrix} \Phi_{U_L} \\ \Phi_{D_L} \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$\mathbf{3}$
$\Phi_{\overline{U}_R}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$1/3$	$\overline{\mathbf{3}}$
$\Phi_{\overline{D}_R}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$\overline{\mathbf{3}}$
$\Phi_{L_L} = \begin{pmatrix} \Phi_{N_L} \\ \Phi_{E_L} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{3}$
$\Phi_{\overline{E}_R}$	$\mathbf{1}$	$\mathbf{1}$	$+1$	$\overline{\mathbf{3}}$
$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$\mathbf{1}$
$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{1}$

Table 1: The table of the representation of chiral superfield. $SU(2)_L$ doublets' components are explicitly written on the right hand side in the chiral superfield row. Each Φ_X^C has opposite transformation against Φ_X.

Among the matter fields in MSSM, the rest is Higgs fields. We use the following conventional notation for them;

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \tag{5.3}$$

where the right hand sides are the doublets of $SU(2)_L$. We assume that Higgs fields live on the brane, not in the bulk. This is just a choice for the following phenomenological reason. If we put the Higgs fields in the bulk, they too have soft mass terms, which is not desirable since we want to realize electro-weak breaking through the negative mass term coming from the radiative correction. Therefore we incorporate the Higgs fields on the brane in the compact SUSY model. The notation for the component fields of the Higgs superfield is

$$H_*^{\#} = h_*^{\#} + \sqrt{2}\,\theta\tilde{h}_*^{\#} + \theta^2 F_{h_*^{\#}}, \tag{5.4}$$

where $* = u, d$ and $\# = +, 0, -$.

The Lagrangian on the brane is given by

$$\begin{aligned}\mathcal{L}_5^{\mathrm{brane}} &= \delta(y)\int d^4\theta\,\left[H_u^\dagger e^{-2gV}H_u + H_d^\dagger e^{-2gV}H_d\right] + \delta(y)\left[\int d^2\theta\,W^{\mathrm{brane}} + \mathrm{h.c.}\right]\\ W^{\mathrm{brane}} &= Y_u\Phi_{\overline{U}_R}\Phi_{Q_L}H_u - Y_d\Phi_{\overline{D}_R}\Phi_{Q_L}H_d - Y_e\Phi_{\overline{E}_R}\Phi_{L_L}H_d + \mu H_u H_d.\end{aligned} \tag{5.5}$$

Here Y_u, Y_d and Y_e are Yukawa couplings that are constant 3 by 3 matrices of the order of 1. μ is another dimension 1 supersymmetric parameter of the model, whose origin has to be discussed later. We assume μ has the same order as $\hat{\alpha}$ in the later loop calculation.

Soft parameters at tree-level

Combining all the zero-modes in the bulk and the Higgs fields on the brane, the compact SUSY model contains the MSSM matter contents. All the other fields are KK modes with n/R mass, and thus we assume KK scale is higher than TeV so that LHC cannot detect them. The SUSY is broken by the Scherk-Schwarz mechanism. Using the conventional notation of MSSM, the soft breaking parameters are

$$M_{1/2} = \frac{\alpha}{R}, \quad m^2_{\tilde{Q},\tilde{U},\tilde{D},\tilde{L},\tilde{E}} = \left(\frac{\alpha}{R}\right)^2, \quad A_0 = -\frac{2\alpha}{R}, \quad m^2_{H_u,H_d} = 0, \quad b = 0, \tag{5.6}$$

at tree-level. The sparticles except for the higgsino are completely degenerate, and this degeneracy is broken by the quantum corrections and thus the nearly degenerate spectrum is expected in the compact SUSY model. If that is the case, this model can be a solution to the tension between LHC result and low scale SUSY model, as we have explained in section 1. Further more this model has only four parameters,

$$\hat{\alpha} = \frac{\alpha}{R}, \quad \frac{1}{R}, \quad \hat{\Lambda} = \Lambda R, \quad \mu, \tag{5.7}$$

and we can show that this model still has the viable phenomenology.

First things first, it is not trivial that we can maintain the compressed spectrum after we take the correction by KK modes into account. At least, there must be a constraint to the parameters of the model for the successful phenomenology. Therefore we calculate the radiative corrections to the gaugino and squark masses.

5.2 The 1-loop radiative corrections to the soft parameters

In this subsection, we summarize the soft parameters at 1-loop level. To that end, we need the following results;

- corrections from the gauge interaction in the bulk in section 4
- corrections from Yukawa interactions on the brane which we calculate later
- the 1-loop analysis on the Higgs mass parameters in [47]

For the phenomenological analysis in the next subsection, we incorporate the 1-loop results into the MSSM parameter with DR-bar scheme and choose the renormalization scale to be $1/(2\pi R)$.

5.2.1 The mass correction from the gauge interaction in the bulk

We first summarize the correction to the gaugino and sfermion masses from the gauge interaction in the bulk, using the results derived in section 4.

The gaugino mass correction from the gauge coupling

As we have derived in Eq.(4.36), the gaugino mass correction with KK-regularization is evaluated as

$$\delta M_\lambda^{(\mathrm{KK})} = \frac{g^2}{16\pi^2}\hat{\alpha}\left(4C(A)\ln[2\pi\hat{\Lambda}] + (2T(F) - 6C(A))\ln[2\pi\alpha] + 5C(A) - 3T(F)\right), \quad (5.8)$$

whereas the correction of MSSM gaugino with DR-bar scheme is given by [2]

$$\delta M_\lambda^{(\overline{\mathrm{DR}})} = \frac{g^2}{16\pi^2}\hat{\alpha}\left(-C(A)\left[3\ln\hat{\alpha}^2/Q^2 - 5\right] + T(F)\left[\ln\hat{\alpha}^2/Q^2 - 1\right]\right). \quad (5.9)$$

Therefore we find

$$\begin{aligned}\delta M_\lambda &= \delta M_\lambda^{(\mathrm{KK})} - \delta M_\lambda^{(\overline{\mathrm{DR}})}(Q = 1/2\pi R) \\ &= \frac{g^2}{16\pi^2}\hat{\alpha}\left(4C(A)\ln[2\pi\hat{\Lambda}] - 2T(F)\right).\end{aligned} \quad (5.10)$$

[2]To evaluate 4D loop integral with DR-bar scheme, we use the formulae in Appendix A.9.

By substituting the group factor $C(A)$ and $T(F)$ in the MSSM setup, we obtain the mass corrections of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gaugino

$$\delta M_1 = \frac{g'^2}{16\pi^2}\hat{\alpha}\,(-10)\,, \tag{5.11}$$
$$\delta M_2 = \frac{g_2^2}{16\pi^2}\hat{\alpha}\left(8\ln[2\pi\hat{\Lambda}] - 12\right), \tag{5.12}$$
$$\delta M_3 = \frac{g_3^2}{16\pi^2}\hat{\alpha}\left(12\ln[2\pi\hat{\Lambda}] - 12\right), \tag{5.13}$$

respectively. Note that Higgs fields are not taken into account here since we need to treat them differently.

The stop mass correction from the gauge coupling

Among the sfermion, the most interesting particle is stop from the view point naturalness, Higgs boson mass analysis, and electro-weak symmetry breaking. Therefore we first seek for the mass correction to the stop.

The correction from gauge interaction is

$$\begin{aligned}\delta_{\text{gauge}}M_\phi^{(\text{KK})2} &= \frac{g^2}{16\pi^2}C(N)8\hat{\alpha}^2\left(\ln\hat{\Lambda} - \ln\alpha + \frac{1}{2}\right),\\ \delta_{\text{gauge}}M_\phi^{(\overline{\text{DR}})\,2} &= -\frac{g^2}{16\pi^2}C(N)8\hat{\alpha}^2\left(\ln\hat{\alpha}/Q - 1\right),\end{aligned} \tag{5.14}$$

in KK-regularization scheme and DR-bar scheme respectively. Therefore the correction as the MSSM soft parameter at scale $1/(2\pi R)$ is

$$\begin{aligned}\delta_{\text{gauge}}M_{\tilde{t}_L}^2 &= \delta_{\text{gauge}}M_{\tilde{t}_L}^{(\text{KK})2} - \delta_{\text{gauge}}M_{\tilde{t}_L}^{(\overline{\text{DR}})\,2}(Q = 1/2\pi R)\\ &= \frac{g^2}{16\pi^2}C(N)8\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right),\\ &= \frac{g'^2}{16\pi^2}\frac{8}{36}\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_2^2}{16\pi^2}6\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_3^2}{16\pi^2}\frac{32}{3}\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right).\end{aligned} \tag{5.15}$$

Similarly, the correction to the right-handed stop mass is

$$\delta M_{\tilde{t}_R}^2 = \frac{g'^2}{16\pi^2}\frac{8}{9}\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_3^2}{16\pi^2}\frac{32}{3}\hat{\alpha}^2\left(\ln\hat{\Lambda} - \frac{1}{2}\right). \tag{5.16}$$

The slepton mass correction from the gauge coupling

We next write down the slepton mass correction from the gauge interaction. For the Left-handed stau, the correction is

$$\begin{aligned}\delta_{\text{gauge}} M_{\tilde{\tau}_L}^2 &= \frac{g^2}{16\pi^2} C(N) 8\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \frac{1}{2}\right), \\ &= \frac{g'^2}{16\pi^2}\frac{8}{4}\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_2^2}{16\pi^2} 6\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \frac{1}{2}\right),\end{aligned} \tag{5.17}$$

and for the Right-handed,

$$\begin{aligned}\delta_{\text{gauge}} M_{\tilde{\tau}_R}^2 &= \frac{g^2}{16\pi^2} C(N) 8\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \frac{1}{2}\right), \\ &= \frac{g'^2}{16\pi^2} 8\hat{\alpha}^2 \left(\ln\hat{\Lambda} - \frac{1}{2}\right).\end{aligned} \tag{5.18}$$

5.2.2 The radiative corrections from the interactions on the brane

We next derive the KK expanded 4D Yukawa interaction terms on the brane and calculate the radiative corrections from them. We assume

$$Y_u \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad Y_d \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e \simeq 0, \tag{5.19}$$

since we are interested mainly in the interaction of stop, and we know from the experiment that the Yukawa coupling of the third generation is larger enough than the first and second ones. We just don't care the Yukawa coupling of the lepton fields because it's irrelevant for stop fields. The superpotential of the brane becomes

$$\begin{aligned}W^{\text{brane}} &\simeq y_t \Phi_{\bar{t}_R} \Phi_{Q_{3,L}} H_u - y_b \Phi_{\bar{b}_R} \Phi_{Q_{3,L}} H_d + \mu H_u H_d \\ &= y_t \Phi_{\bar{t}_R}\left(\Phi_{t_L} H_u^0 - \Phi_{b_L} H_u^+\right) - y_b \Phi_{\bar{b}_R}\left(\Phi_{t_L} H_d^- - \Phi_{b_L} H_d^0\right) + \mu\left(H_u^+ H_d^- - H_u^0 H_d^0\right).\end{aligned} \tag{5.20}$$

In the second line, we have expressed the terms using the components of the $SU(2)_L$ doublets.

Even though we have seen the 4D interaction terms on the brane in subsection 3.4 (the result is summarized in Eq.(3.4.2)), we cannot simply use the result there because each field has multiple couplings and Higgs fields have μ-term now (The discussion for the tree-level soft term is the same). Therefore we first derive the full interaction on the brane in the current setup. The procedure for the KK-expansion and auxiliary-fields-removal is the same. After we get the 4D Lagrangian, the procedure of the 1-loop calculation with KK-regularization scheme is pretty much the same as the gauge interaction case. We put the detailed calculation in Appendix D. The result is given by

$$\delta_{y_t} M_{\tilde{t}_L}^2 = \frac{|y_t|^2}{16\pi^2}\hat{\alpha}^2\left(-12\ln[2\pi\hat{\Lambda}]+6\right). \tag{5.21}$$

The mass correction to the right-handed stop $\tilde{t}_R$ is twice as large as that of left-handed stop.

The correction to the gaugino mass from the interaction with Higgs on the brane

There are also interactions on the brane between the Higgs, the Higgsino and the gaugino, which contributes to the gaugino mass correction. It is usual 4D 1-loop calculation and we evaluate it using KK-regularization scheme;

$$\begin{aligned}\Sigma(p) &= 2g^2T(F)\int\frac{d^4k}{i(2\pi)^4}\frac{\not{k}}{[k^2+\mu^2][(k-p)^2+\mu^2]}\\ &= 2g^2T(F)\not{p}\int_0^1 dx\left\{\ln\frac{\hat{\Lambda}^2}{D^2}-1\right\}\\ &= 2g^2T(F)\not{p}\int_0^1 dx\left\{-\ln D^2\right\},\end{aligned} \tag{5.22}$$

where $D^2 = x(1-x)p^2+\mu^2$, and the second line is evaluated with KK-regularization and the third is with DR-bar scheme. Therefore the mass correction is

$$\delta M_\lambda = -\frac{g^2}{8\pi^2}\hat{\alpha}\,T(F)\left\{2\ln\hat{\Lambda}-1\right\}, \tag{5.23}$$

in a general expression. By substituting the group factor in the MSSM setup, we obtain the mass corrections of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gaugino

$$\delta M_1 = -\frac{g'^2}{8\pi^2}\hat{\alpha}\left\{2\ln\hat{\Lambda}-1\right\},\quad \delta M_2 = -\frac{g_2^2}{8\pi^2}\hat{\alpha}\left\{2\ln\hat{\Lambda}-1\right\},\quad \delta M_3 = 0, \tag{5.24}$$

respectively.

5.2.3 Summary

Combining the above results and the results regarding the Higgs sector in [47], the summary is given as follows;

$$\frac{\delta M_1}{\hat{\alpha}} = \frac{g'^2}{8\pi^2}\left(-2\ln\hat{\Lambda} - 4\right), \tag{5.25}$$

$$\frac{\delta M_2}{\hat{\alpha}} = \frac{g_2^2}{8\pi^2}\left(2\ln[\hat{\Lambda}] + 4\ln[2\pi] - 5\right), \tag{5.26}$$

$$\frac{\delta M_3}{\hat{\alpha}} = \frac{g_3^2}{8\pi^2}\left(6\ln[2\pi\hat{\Lambda}] - 6\right), \tag{5.27}$$

$$\begin{aligned}\frac{\delta m^2_{\tilde{t}_L}}{\hat{\alpha}^2} &= \frac{g'^2}{16\pi^2}\frac{8}{36}\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_2^2}{16\pi^2}6\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_3^2}{16\pi^2}\frac{32}{3}\left(\ln\hat{\Lambda} - \frac{1}{2}\right)\\ &\quad + \frac{|y_t|^2}{16\pi^2}\left(-12\ln[2\pi\hat{\Lambda}] + 6\right),\end{aligned} \tag{5.28}$$

$$\frac{\delta m^2_{\tilde{t}_R}}{\hat{\alpha}^2} = \frac{g'^2}{16\pi^2}\frac{8}{9}\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{g_3^2}{16\pi^2}\frac{32}{3}\left(\ln\hat{\Lambda} - \frac{1}{2}\right) + \frac{|y_t|^2}{16\pi^2}\left(-24\ln[2\pi\hat{\Lambda}] + 12\right), \tag{5.29}$$

$$\frac{\delta m^2_{H_u}}{\hat{\alpha}^2} = -\frac{33}{8\pi^2}y_t^2 + \frac{9(g_2^2 + g_1^2/5)}{16\pi^2}, \tag{5.30}$$

$$\frac{\delta m^2_{H_d}}{\hat{\alpha}^2} = \frac{9(g_2^2 + g_1^2/5)}{16\pi^2}, \tag{5.31}$$

$$\frac{\delta b}{\hat{\alpha}} = \frac{9}{8\pi^2}y_t^2 - \frac{3(g_2^2 + g_1^2/5)}{8\pi^2}. \tag{5.32}$$

5.3 Phenomenological analysis

In this subsection, we discuss the phenomenology of the compact SUSY model using the 1-loop result derived in the previous subsection.

5.3.1 MSSM mass spectrum

We first show some selected mass spectrum by using the program `SOFTSUSY3` [61] to obtain the physical parameters. We need to set the three parameters $1/R, \alpha/R$ and ΛR in the model to decide the physical parameters.

In Fig. 6, we first plot the masses of the lightest neutralino $\tilde{\chi}_0^1$, the lightest stop $\tilde{t}_1$ and the gluino $\tilde{g}$ by choosing $1/R$ as the horizontal axis and setting $\alpha/R = 1.5$ TeV. We plot the three lines for each sparticles that corresponds to $\Lambda R = 1, 5, 20$ so that we can see the dependence of the phenomenology on the cutoff scale. Here are comments regarding Fig. 6;

- If $\Lambda R \lesssim 20$ and $1/R$ is around 100 TeV, the nearly degenerate spectrum is still realized.
- If the ΛR becomes larger than 20, the mass difference gets larger due to the difference in $\ln \Lambda$ term.
- In the region where $1/R \lesssim 10^6$ GeV the main component of the neutralino is higgsino, while in the other region, the main component is the bino. This is why the slope of the neutralino curve is changing.
- The mass of the higgsino and thus the mass of the neutralino depend on the electroweak breaking condition, which is a physics of a few hundred GeV scale. Therefore the compressed spectrum realized near $1/R \sim 10^6$ GeV region is not trivial.

In Fig. 7, we next plot the lines for the sparticles (the lightest stop $\tilde{t}_1$, the gluino $\tilde{g}$ and the neutralino $\tilde{\chi}_1^0$) to have 1.5 TeV mass, in $1/R$ versus α/R plane. We found that in some region where α/R is small, the lightest stop becomes the LSP. In that case, a stable stop hadron should be observed at LHC experiment and it's inconsistent with the current constraint [62].

In Fig. 8, we have plotted the full sparticle spectra by setting $1/R = 100$ TeV and $\alpha/R = 1.5$ TeV. In the fist figure (a) we have set $\Lambda R = 20$, and the second figure (b) is when $\Lambda R = 1$, which corresponds to the case where $\ln \Lambda R$ terms are absent. From the figures we can confirm that the compressed spectrum is realized.

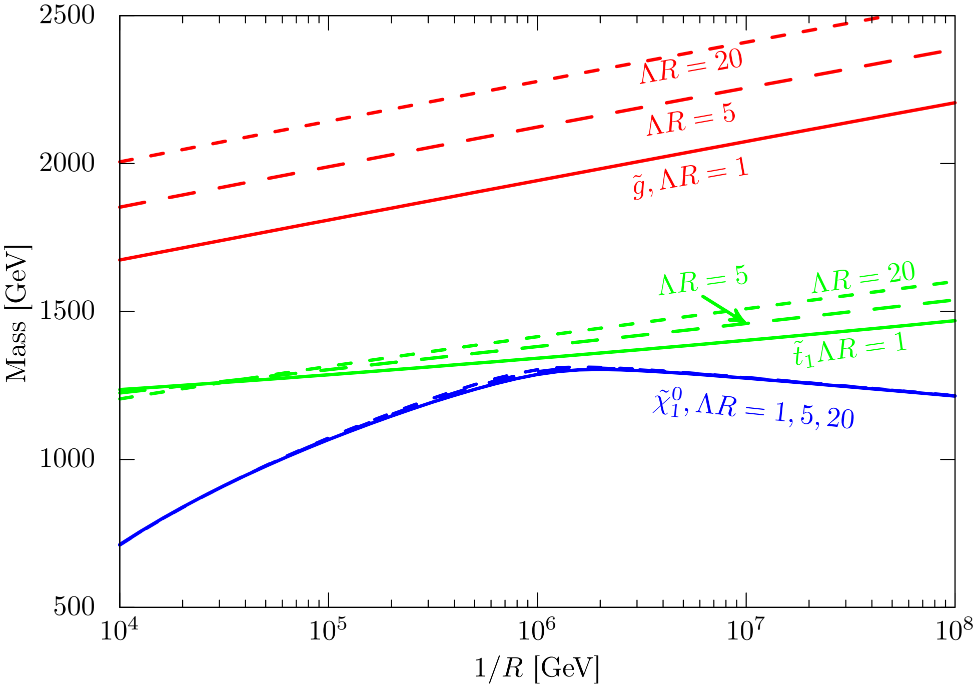


Figure 6: The mass dependence on $1/R$ for $\alpha/R = 1500$ GeV. The lines represent the lightest neutralino (red), the lightest stop (green) and the gluino (blue). The solid, long-dashed and short-dashed lines corresponds to $\Lambda R = 1$, 5 and 20, respectively.

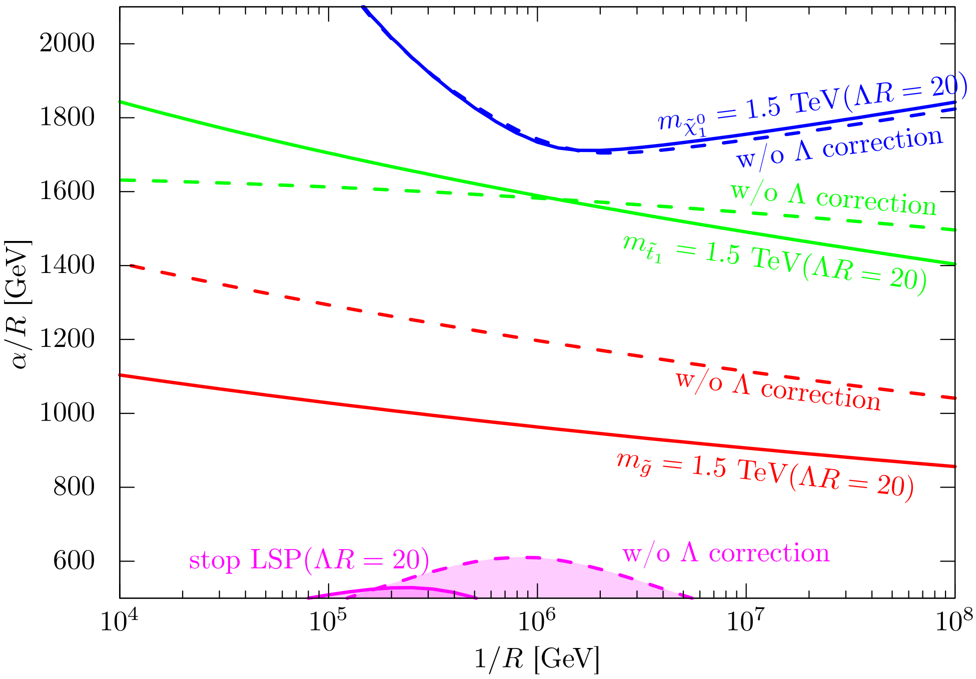
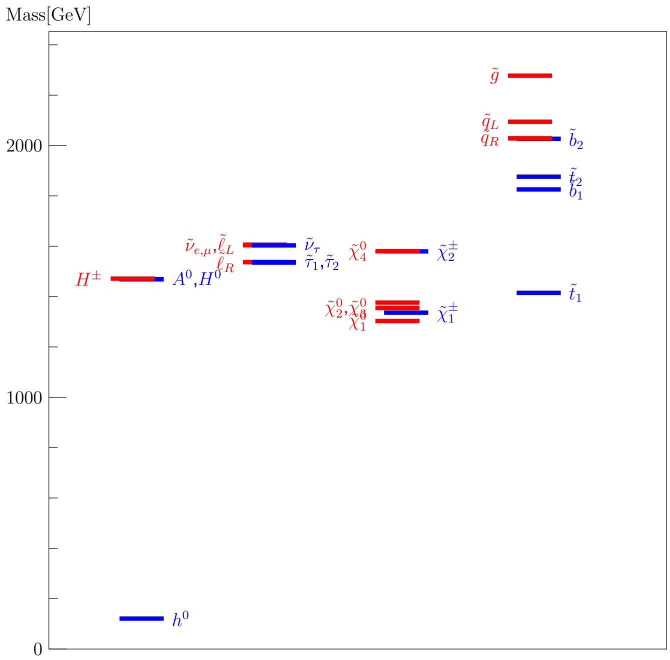
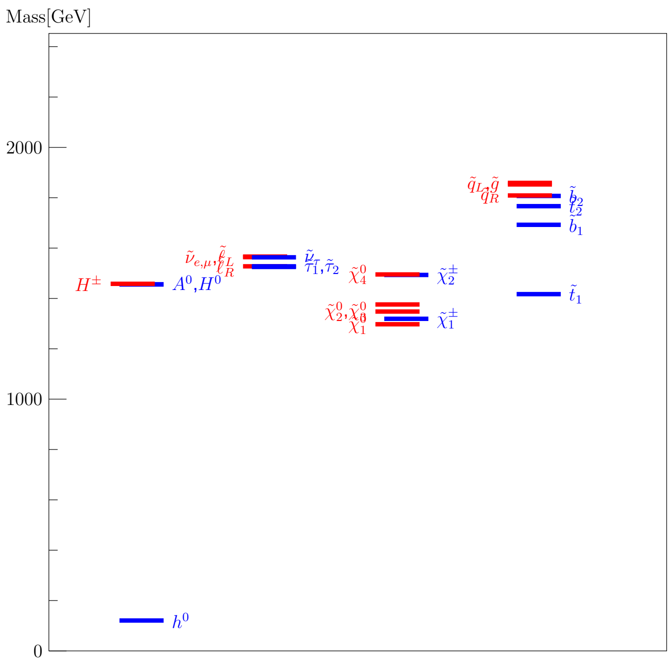


Figure 7: Mass contour lines of 1.5 TeV: Mass contour lines of 1.5 TeV for the lightest neutralino (blue), the lightest stop (green) and the gluino (red). In solid (dashed) lines we represent the cases of with (without) correction of $\Lambda R = 20$. The purple region is where the lightest stop is the LSP.



(a) $\Lambda R = 20$



(b) Without Λ corrections

Figure 8: Examples of mass spectra. We set $1/R = 10^6$ GeV and $\alpha/R = 1500$ GeV. We can confirm that the nearly degenerate spectrum is realized in both cases.

5.3.2 Collider signature

In the previous subsubsection, we have seen that the compressed spectrum seems to be realized at least in some parameter region ($\alpha/R \sim 1.5$ TeV, $1/R \sim 10^6$ GeV, $\Lambda R \lesssim 20$). Whether the compression is enough or not depends on the LHC experiment. We therefore discuss the LHC signatures of the compact SUSY model in this subsubsection. To that end, we make the decay table by using the program `SDECAY` [63]. For the Monte-Carlo simulation, we have used `MG5aMC@NLO` [64,65], `Pythia6` [66] and `Delphes3` [67,68] with `FastJet` [69,70]

In Fig. 9, we show the current constraint from multi-jets and missing energy by ATLAS [71] with $\Lambda R = 20$ (red region) and without the KK-correction (blue region) for comparison. The region below the line is excluded since there the sparticles are too light to avoid LHC constraint. We can see that the constraint is relatively loose around the $1/R \sim 10^6$ GeV region, thanks to the compressed spectrum we have confirmed in the previous subsubsection 5.3.1. The spectrum is more compressed in the case of no Λ correction than in the $\Lambda R = 20$ case. However in some region, the constraint is more strong in the without-Λ-correction case. This may be because the more compressed the spectrum is the more sparticles are produced by the collider, and therefore the more compression doesn't necessarily mean the looser constraint.

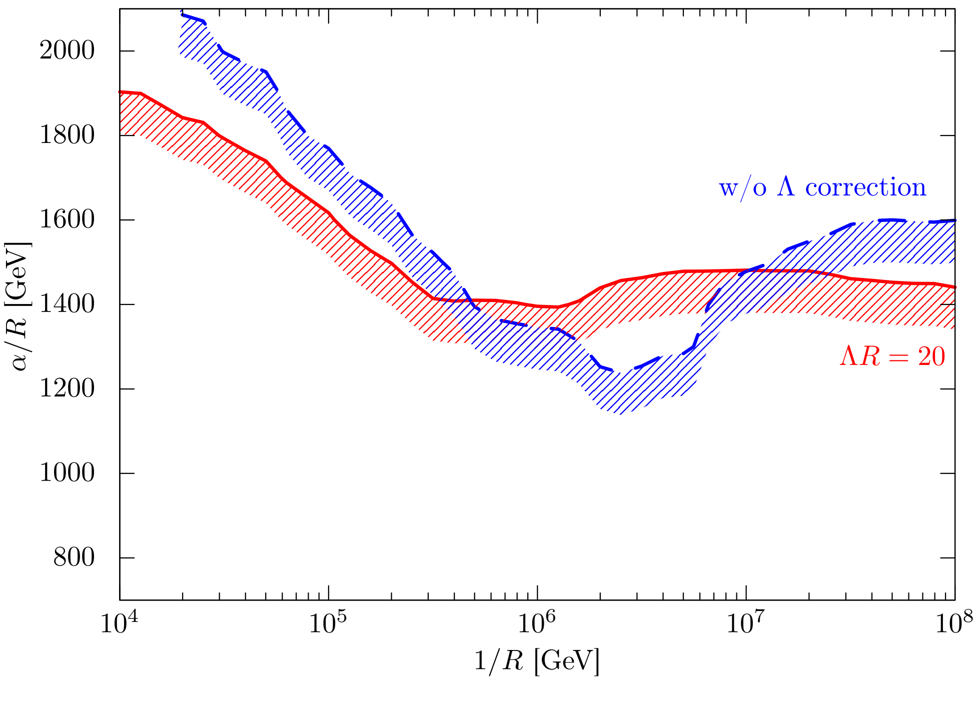


Figure 9: Collider constrains on the $1/R$–α/R plane.

6 Conclusion and outlook

In this thesis, we have first investigated the Scherk-Schwarz mechanism in detail, in the simplest S^1/Z_2 extra-dimension case. We have written down the 4D effective Lagrangian of the bulk and from the fields on the brane. After that we have calculated the radiative corrections to the gaugino and sfermion masses. We have next presented the compact SUSY model setup as a realistic application of the Scherk-Schwarz mechanism and calculated the 1-loop radiative corrections to the gaugino and the sfermions masses from the gauge and Yukawa interactions. We have found that the compact SUSY model still has the compressed spectrum at 1-loop level, which is the essence for the model to be an solution to the tension between the hierarchy problem and LHC result without anthropic principle nor R-parity violation. In the corrections to the gaugino and sfermion masses, we have seen the cancellation of the divergences, which can be expected since the SUSY breaking is caused by the Scherk-Schwarz mechanism that is a global phenomenon while the UV divergence is the local effect. We have also seen the logarithmic divergences both from gauge interactions and Yukawa interactions.

There are more phenomenological issues worth discussing like the dark matter, gravitino and the 1-loop correction to the A-term, but we leave them to the future work since the discovery of the SUSY signal has the highest priority. It might be rather worthwhile to improve the way of the search for the missing transverse energy signal at the Collider analysis.

Even if we have the fully analyze the compact SUSY model, and if the all results agrees with the experiment, it cannot be the full understanding of the physics since the compact SUSY model that is based on the 5D gauge theory is not UV complete. To begin with, the quantum gravity is not well described in the quantum field theory. Therefore we eventually need to start searching for more fundamental model, and the superstring theory is one of the primary candidate for the final answer. Actually, there has been several works on the realization of the Scherk-Schwarz mechanism in the framework of the superstring thoery (for general discussion, see [72, 73] and for phenomenological discussion see [55, 56, 74, 75]). It might be interesting to discuss how to embed the compact SUSY model onto the superstring theory to find the more phenomenological implication, for example, we might have a reasoning that only Higgs fields live on the brane, the origin of the μ term and so on.

Furthermore, the cosmological history including the inflation has to be discussed if one takes the model seriously. If the inflation scale and the reheating temperature is small enough the discussion is same as the usual one. However once the scale goes above the KK scale, the usual 4D discussion cannot be applied and the higher dimensional cosmology is needed. This is an universal issue for any models with extra dimensions.

A Notation and formulae

Here we show the notations that are needed in this thesis to follow all the details of the calculation.

A.1 The metric

First things first, our metric is mostly plus

$$g_{MN} = \mathrm{diag}(-,+,+,+,+), \tag{A.1}$$

this choice affects the signs in the Lagrangian and the Feynman diagram.

A.2 2-spinors

The Pauli matrices

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.2}$$

and its indices notation;

$$\sigma^\mu = (-1, \sigma_i), \quad \sigma_\mu = (1, \sigma_i), \quad \bar{\sigma}^\mu = (-1, -\sigma_i), \quad \bar{\sigma}_\mu = (1, -\sigma_i). \tag{A.3}$$

In other literatures, although the metric is the same, the sign of σ^0 is opposite and thus the kinetic term of fermion looks different.

With

$$\sigma^{\mu\nu} = \frac{i}{2}\sigma^{[\mu}\bar{\sigma}^{\nu]}, \quad \bar{\sigma}^{\mu\nu} = \frac{i}{2}\bar{\sigma}^{[\mu}\sigma^{\nu]}, \tag{A.4}$$

the 2 spinors are defined

$$\text{Left-Handed:} \quad \xi_\alpha \to \exp\left[\frac{i}{2}\theta_{\mu\nu}\sigma^{\mu\nu}\right]_\alpha^{\ \beta} \xi_\beta \tag{A.5}$$

$$\text{Right-Handed:} \quad \chi^{\dot{\alpha}} \to \exp\left[\frac{i}{2}\theta_{\mu\nu}\bar{\sigma}^{\mu\nu}\right]^{\dot{\alpha}}_{\ \dot{\beta}} \chi^{\dot{\beta}}. \tag{A.6}$$

$\bar{\chi}_\alpha$ and $\bar{\xi}^{\dot{\alpha}}$ are Left and Right-Handed spinors.

We use the following antisymmetric tensor to raise and down the spinor indices

$$\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \epsilon_{\dot{\alpha}\dot{\beta}} = i\sigma_2. \tag{A.7}$$

In order not to get the wrong sign when contracting the spinor indices, we stick to "the left-top and right-down rule".

Several useful formulae regarding the spinors are listed below;

$$\begin{aligned}
&\xi_1\xi_2 = \xi_1^{\alpha}\xi_{2\alpha} = \epsilon^{\alpha\beta}\xi_{1\beta}\xi_{2\alpha} = \xi_2\xi_1, \\
&\chi_1\chi_2 = \chi_1^{\dot{\alpha}}\chi_{2\dot{\alpha}} = \chi_1^{\dot{\alpha}}\chi_2^{\dot{\beta}}\epsilon_{\dot{\beta}\dot{\alpha}} = \chi_2\chi_1, \\
&\bar{\xi}_1\bar{\sigma}^{\mu}\partial_{\mu}\xi_2 = \xi_2\sigma^{\mu}\partial_{\mu}\bar{\xi}_1 + \partial^{\mu}(\cdots)
\end{aligned} \tag{A.8}$$

A.3 Gamma matrices and fermion representation

We consider general D dimensional spacetime. Gamma matrices is a set of D matrices which satisfies the Clifford algebra;

$$\{\gamma^{\mu}, \gamma^{\nu}\} = -2g^{\mu\nu}. \tag{A.9}$$

$2m \times 2m$ matrices are the irreducible representation when $D = 2m$ or $D = 2m + 1$. We consider only hermitian representations;

$$\gamma^{\mu\dagger} = \gamma^0\gamma^{\mu}\gamma^0. \tag{A.10}$$

We can difine Dirac fermion Ψ which has the following Lorentz transformation property;

$$\delta\Psi = i\theta_{\mu\nu}\frac{i}{4}\gamma^{[\mu}\gamma^{\nu]}\Psi. \tag{A.11}$$

This is a $2m$ dimensinal representation of the Lorentz group (we can ckeck the generator $\frac{i}{4}\gamma^{[\mu}\gamma^{\nu]}$ satisfies the commutation relation of the Lorentz group).

When we calculate, we take the chiral representation in 4D

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.12}$$

γ^5 satisfies $\{\gamma^{\mu}, \gamma^5\} = 0$ and $(\gamma^5)^2 = 1$.

Trace techniques

Here are the useful formula used in the calculation of Feynman diagrams in this thesis;

$$\begin{aligned}
&\gamma^{\mu}\gamma_{\mu} = -4, \quad \gamma^{\mu}\not{a}\gamma_{\mu} = 2\not{a}, \quad \gamma^{\mu}\not{a}\not{b}\gamma_{\mu} = 4ab, \\
&\mathrm{Tr}[\text{ odd \# of } \gamma^{\mu}] = 0, \quad \mathrm{Tr}[\gamma^{\mu}\gamma^{\nu}] = \frac{1}{2}\mathrm{Tr}[\{\gamma^{\mu}, \gamma^{\nu}\}] = -4g^{\mu\nu}, \\
&\mathrm{Tr}[\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}), \\
&\mathrm{Tr}[\gamma^5 \text{ odd \# of } \gamma^{\mu}] = 0, \quad \mathrm{Tr}[\gamma^5\gamma^{\mu}\gamma^{\nu}] = 0.
\end{aligned} \tag{A.13}$$

Gamma matrices notation in 5D

In 5D, we use $\Gamma^{M=\mu,5}$ to clarify the difference;

$$\Gamma^\mu = \gamma^\mu, \quad \Gamma^5 = \gamma^4 = -i\gamma^5. \tag{A.14}$$

γ^4 can be either $\pm i\gamma^5$, and we take $-$ as a choice.

A.4 Complex conjugate of Dirac field

It is known there exist some $2m \times 2m$ matrix C such that

$$\left(C\Gamma^{(r)}\right)^T = -t_r C\Gamma^{(r)}, \quad t_r = \pm 1. \tag{A.15}$$

where $\Gamma^{(r)}$ is any rank r clifford matrix ($\Gamma^{(r)} \sim \gamma^{\mu_1} \cdots \gamma^{\mu_r}$). Especially

$$C^T = -t_0 C, \quad [C\gamma^\mu]^T = -t_1 C\gamma^\mu, \quad \left(\gamma^{\mu T} = t_0 t_1 C\gamma^\mu C^{-1}\right). \tag{A.16}$$

Since we can show

$$t_2 = -t_0, \quad t_3 = -t_1, \quad t_{r+4} = t_r, \tag{A.17}$$

the sign of t_0 and t_1 determines the symmetric property.

To define complex conjugate, we first define

$$B = it_0 C\gamma^0, \tag{A.18}$$

and note that this satisfies

$$(\gamma^\mu)^* = -t_0 t_1 B\gamma^\mu B^{-1}, \quad B^* B = -t_1. \tag{A.19}$$

Then we can see

$$\Psi^C = B^{-1}\Psi^* \tag{A.20}$$

has the same Lorentz transformation property as the Dirac field since

$$\delta\Psi^* = -\frac{1}{4}\theta_{\mu\nu}\gamma^{*[\mu}\gamma^{*\nu]}\Psi^* = -\frac{1}{4}\theta_{\mu\nu}B\gamma^{[\mu}\gamma^{\nu]}B^{-1}\Psi^*. \tag{A.21}$$

Ψ^C is the complex conjugate for the Dirac field.

In 4D, we have two choices for C;

$$\begin{cases} C_+ = i\gamma^3\gamma^4 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, & (t_0 = +1, t_1 = +1) \\ C_- = i\gamma^2\gamma^0 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, & (t_0 = +1, t_1 = -1) \end{cases} \tag{A.22}$$

here the explicit forms are in the chiral representation. For each choices, the complex conjugates of a Dirac Field

$$\Psi = \begin{pmatrix} \psi_{1\alpha} \\ \bar{\psi}_2^{\dot{\alpha}} \end{pmatrix} \tag{A.23}$$

are

$$\Psi^{C_+} = \begin{pmatrix} \psi_{2\alpha} \\ -\bar{\psi}_1^{\dot{\alpha}} \end{pmatrix}, \quad \Psi^{C_-} = \begin{pmatrix} \psi_{2\alpha} \\ \bar{\psi}_1^{\dot{\alpha}} \end{pmatrix}. \tag{A.24}$$

Note that C_+/C_- can/cannot be extended to 5D since

$$\left[C_+\gamma^4\right]^T = -C_+\gamma^4, \quad \left[C_-\gamma^4\right]^T \neq +C_-\gamma^4. \tag{A.25}$$

A.5 Majorana field

Majorana field can be defined from a Dirac field Ψ with a constraint

$$\Psi = \Psi^C. \tag{A.26}$$

Note that this condition is consistent only when

$$B^*B = -t_1 = 1 \quad \Longleftrightarrow \quad t_1 = -1, \tag{A.27}$$

since

$$\Psi = \Psi^C = (\Psi^C)^C = B^{-1}(B^{-1}\Psi^*)^* = B^{-1}(B^{-1})^*\Psi. \tag{A.28}$$

In 4D, we have to take C_- to define Majorana field, and in the case of the chiral representation,

$$\begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} = \Psi = \Psi^C = \begin{pmatrix} \psi_2 \\ \bar{\psi}_1 \end{pmatrix}, \tag{A.29}$$

so we can make a Majorana field out of a Left-Handed weyl spinor ψ_1;

$$\Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_1 \end{pmatrix}. \tag{A.30}$$

Note that since there is no $t_1 = -1$ matrix C in 5D, we cannot define Majorana field.

A.6 Symplectic Majorana field

When $t_1 = +1$, like in 5D, we cannot define Majorana fields but we can define symplectic Majorana fields as follows. When we have even number (say, $2k$) of Dirac fields $\Psi_{i=1\sim 2k}$, and these satisfy the condition;

$$\Psi_i^C = \epsilon_{ij}\Psi_j, \tag{A.31}$$

where ϵ_{ij} is a non-singlar antisymmetric matrix, we call Ψ_i symplectic Majorana fields.

In 5D case, suppose we have two Dirac fields

$$\Upsilon_1 = \begin{pmatrix} \lambda_1 \\ \bar{\lambda}_2 \end{pmatrix}, \Upsilon_2 = \begin{pmatrix} \lambda_3 \\ \bar{\lambda}_4 \end{pmatrix}. \tag{A.32}$$

If we set a following condition, these fields are symplectic Majorana fields;

$$\begin{cases} \begin{pmatrix} \lambda_1 \\ \bar{\lambda}_2 \end{pmatrix} = \Upsilon_1 = \Upsilon_2^C = \begin{pmatrix} \lambda_4 \\ -\bar{\lambda}_3 \end{pmatrix} \\ \begin{pmatrix} \lambda_3 \\ \bar{\lambda}_4 \end{pmatrix} = \Upsilon_2 = -\Upsilon_1^C = -\begin{pmatrix} \lambda_2 \\ -\bar{\lambda}_1 \end{pmatrix} \end{cases} \iff \begin{cases} \lambda_3 = -\lambda_2 \\ \lambda_4 = \lambda_1 \end{cases} \tag{A.33}$$

In other words, we can form a set of symplectic Majorana fields out of two Left-Handed weyl spinor λ_1, λ_2;

$$\Upsilon_1 = \begin{pmatrix} \lambda_1 \\ \bar{\lambda}_2 \end{pmatrix}, \quad \Upsilon_2 = \begin{pmatrix} -\lambda_2 \\ \bar{\lambda}_1 \end{pmatrix}. \tag{A.34}$$

A.7 Lie Group and its generator and structure constants

This subsection is the summary for features that comes from group theory. A representation of a group G is written by T_R^a $(a = 1, 2, \cdots, D(A))$, and they satisfy the commutation relation

$$[T_R^a, T_R^b] = if^{abc}T_R^c, \tag{A.35}$$

where f^{abc} is a structure constant. We define $T(R)$, trace of a representation R, as

$$\mathrm{Tr}(T_R^a T_R^b) = T(R)\delta^{ab}. \tag{A.36}$$

We have written $T(R) = k$ in the Lagrangian section. The quadratic Casimir $C(R)$ is defined as

$$\sum_{a,j} T^a_{R\ ij} T^a_{R\ jk} = C(R)\delta_{ik} \tag{A.37}$$

By taking the sum of Eq.(A.37) with respect to $i = k$, we obtain the relation $T(R)D(A) = C(R)D(R)$, where $D(R)$ is dimension of the representation R.

For fundamental representation of $G = SU(N)$, we write $T^a_R = T^a$ and take $T(R) = T(N) = 1/2$, and we can show $C(N) = T(N)D(A)/D(N) = \frac{N^2-1}{2N}$.

For Adjoint representation, $T^a_{R\ ij} = T^a_{A\ ij} = -if^{aij}$, $C(A) = T(A) = N$, i.e.

$$\mathrm{Tr}(T^a_A T^b_A) = \sum_{b,c} f^{abc} f^{dbc} = T(A)\delta^{ad} = C(A)\delta^{ad} = N\delta^{ad}. \tag{A.38}$$

A.8 Feynman rules

The Feynman rules depends on the choice of the metric, gamma matrices (4-sigma matrices) and the overall factor for propagator, vertex and loop integral. In order for the reader not to get confused, we summarize the Feynman rules that are used in this paper.

- Internal propagators

$$(\text{Boson}) = \frac{1}{k^2 + m^2}, \tag{A.39}$$

$$(\text{Fermion}) = \frac{\not{k} - m}{k^2 + m^2} = [-\not{k} - m]^{-1}, \quad (\text{for } \mathcal{L} = -\overline{\Psi}(i\not{\partial} - m)\Psi) \tag{A.40}$$

$$(\text{Gauge boson}) = \frac{g_{\mu\nu}}{k^2 + m^2}, \quad (\text{Feynman gauge}). \tag{A.41}$$

- Vertex is written as it is. No factor such as $\times i$.

- Loop momentum integral

$$\int \frac{d^4 k}{i(2\pi)^4} \tag{A.42}$$

In some of the other literatures, the internal propagator(P), the vertex(V) and the loop integral(L) have factors $\times(-i)$, $\times i$ and $\times i$ respectively, which means we have different factor i^{-P+V+L} for the scattering amplitude. However, thanks to the relation $P - V + 1 = L$ for any diagram, the difference is always i and thus this difference doesn't affect the relative factors. Threfore the physical quantities, that must be defined by the absolute value of the scattering amplitude, are the same in both notation.

Under the notation above, two point Green function for a boson is given by

$$\begin{aligned} iG_b(P) &= \frac{1}{p^2+m^2} + \frac{1}{p^2+m^2}\Pi(p)\frac{1}{p^2+m^2} + \cdots \\ &= \left[p^2+m^2-\Pi(p)\right]^{-1} = \Gamma(p)^{-1}, \end{aligned} \tag{A.43}$$

where $\Gamma(p)$ is the 1PI 2-point vertex function. Two point Green function for a fermion is given by

$$\begin{aligned} iG_f(P) &= \frac{1}{-\not{p}-m} + \frac{1}{-\not{p}-m}\Sigma(p)\frac{1}{-\not{p}-m} + \cdots \\ &= \left[-\not{p}-m-\Sigma(p)\right]^{-1} = \Gamma(p)^{-1}, \end{aligned} \tag{A.44}$$

where $\Gamma(p)$ is the 1PI 2-point vertex function.

A.9 The 4D momentum integration formulae

After the Feynman parameter integral

$$\frac{1}{ab} = \int_0^1 dx\, \frac{1}{[ax+b(1-x)]^2}, \tag{A.45}$$

all we need to calculate the loop integrals in this thesis is following formulae;

$$\int \frac{d^n k}{i(2\pi)^n}\frac{1}{[k^2+D^2]} = -\frac{D^2}{16\pi^2}\left\{\bar{\epsilon}^{-1}+1-\ln D^2\right\} = \frac{1}{16\pi^2}\left\{\Lambda^2 - D^2\ln\Lambda^2/D^2\right\}, \tag{A.46}$$

$$\int \frac{d^n k}{i(2\pi)^n}\frac{1}{[k^2+D^2]^2} = \frac{1}{16\pi^2}\left\{\bar{\epsilon}^{-1}-\ln D^2\right\} = \frac{1}{16\pi^2}\left\{\ln\Lambda^2/D^2 - 1\right\}, \tag{A.47}$$

$$\int \frac{d^n k}{i(2\pi)^n}\frac{k^2}{[k^2+D^2]^2} = -\frac{2D^2}{16\pi^2}\left\{\bar{\epsilon}^{-1}+1-\ln D^2\right\} = \frac{1}{16\pi^2}\left\{\Lambda^2 - 2D^2\ln\Lambda^2/D^2\right\}. \tag{A.48}$$

where $n = 4-2\epsilon, \bar{\epsilon}^{-1} = \epsilon^{-1} - \gamma + \ln[4\pi]$. Here the first equations are regularized by $\overline{\mathrm{DR}}$ scheme, while the second equations are regularized by naive cutoff which corresponds to KK-regularization scheme.

B The evaluation of the loop integrals by the KK regularization scheme

Our goal in this section is to evaluate the following 4-momenta integrals of infinite sum;

$$\begin{aligned}\mathcal{I}_1(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{[(n+a)^2+k^2+c^2]^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{[n+a+iq]^2[n+a-iq]^2} \qquad \text{(B.1)} \\ \mathcal{I}_2(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{n}{[(n+a)^2+k^2+c^2]^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{n}{[n+a+iq]^2[n+a-iq]^2}, \qquad \text{(B.2)} \\ \mathcal{I}_3(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{k^2+c^2}{[(n+a)^2+k^2+c^2]^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{q^2}{[n+a+iq]^2[n+a-iq]^2}, \qquad \text{(B.3)}\end{aligned}$$

with momentum cutoff Λ. Here q^2 is defined as k^2+c^2. We assume $0 < a, c \ll 1$ and seek for the results up to $\mathcal{O}(a^2, c^2)$. We don't care the mass dimension in this section. In the actual calculation, $a = x\alpha$, $c^2 = x(1-x)(\alpha^2+\hat{p}^2)$, where x is a Feynman parameter which are to be integrated over $[0,1]$. Note that in this appendix, we assume all the variables are normalized to be dimensionless from the beginning.

B.1 Infinite sum

Since $\coth[i\pi z]/2$ has poles at $z \in \mathbf{Z}$ with its residues $1/(2\pi i)$, we can rewrite the infinite sum by $n \in \mathbf{Z}$ into the complex integral as follows;

$$\begin{aligned}\sum_{n=-\infty}^{\infty} f(n) &= -\oint_{C_1+C_3+C_2+C_4} dz \, f(z)\frac{\coth[i\pi z]}{2} \\ &= -\oint_{C_1+C_5+C_2+C_6} dz \, f(z)\frac{\coth[i\pi z]}{2} = -2\pi i \mathrm{Res}\left[f(z)\frac{\coth[i\pi z]}{2}\right], \qquad \text{(B.4)}\end{aligned}$$

where the contours $C_1 \sim C_6$ are given in Fig.10 and the Res in the last form means the residues in upper and lower regions (inside C_1+C_5 and C_2+C_6 respectively). Here we have

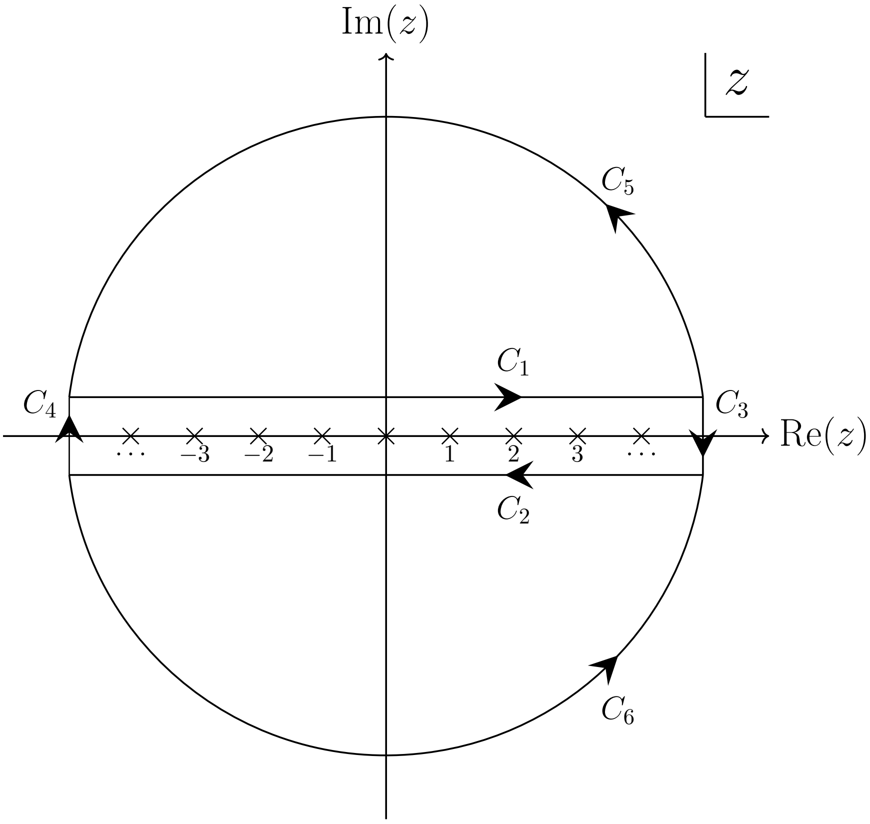


Figure 10: Contours and the poles.

assumed that $f(z)$ doesn't have poles along the real axis in the first line and that $z|f(z)| \to 0$ as $z \to \infty$ when exchanging the contour in the second line.

Then we can show following equations by carefully picking up the residues;

$$\sum_{n=-\infty}^{\infty} \frac{1}{[n+a+iq]^2[n+a-iq]^2} = \frac{\pi}{2q^3}\left[\coth_+(\pi q, \pi a) - \pi q \coth'_+(\pi q, \pi a)\right] \tag{B.5}$$

$$\sum_{n=-\infty}^{\infty} \frac{n}{[n+a+iq]^2[n+a-iq]^2}$$
$$= \frac{\pi}{2q^3}\left[-a\coth_+(\pi q, \pi a) + \pi a q \coth'_+(\pi q, \pi a) - i\pi q^2 \coth'_-(\pi q, \pi a)\right]. \tag{B.6}$$

Here we have defined

$$
\begin{aligned}
\coth_+(x,y) &= \frac{1}{2}\left(\coth[x+iy]+\coth[x-iy]\right) \\
&= 1+\frac{1}{e^{2(x+iy)}-1}+\frac{1}{e^{2(x-iy)}-1}=\frac{\sinh[2x]}{\cosh[2x]-\cos[2y]}, && \text{(B.7)} \\
\coth_-(x,y) &= \frac{1}{2}\left(\coth[x+iy]-\coth[x-iy]\right) \\
&= \frac{1}{e^{2(x+iy)}-1}-\frac{1}{e^{2(x-iy)}-1}=-i\frac{\sin[2y]}{\cosh[2x]-\cos[2y]}, && \text{(B.8)} \\
\coth'_+(x,y) &= \frac{1}{2}\left(\coth'[x+iy]+\coth'[x-iy]\right) \\
&= \frac{-2}{(e^{2(x+iy)}-1)^2}+\frac{-2}{(e^{2(x-iy)}-1)^2}+\frac{-2}{e^{2(x+iy)}-1}+\frac{-2}{e^{2(x-iy)}-1} \\
&= 4\frac{\cosh^2[x]\sin^2[y]-\sinh^2[x]\cos^2[y]}{(\cosh[2x]-\cos[2y])^2}, && \text{(B.9)} \\
\coth'_-(x,y) &= \frac{1}{2}\left(\coth'[x+iy]-\coth'[x-iy]\right) \\
&= \frac{-2}{(e^{2(x+iy)}-1)^2}+\frac{2}{(e^{2(x-iy)}-1)^2}+\frac{-2}{e^{2(x+iy)}-1}+\frac{2}{e^{2(x-iy)}-1} \\
&= 2i\frac{\sinh[2y]\sin[2y]}{(\cosh[2x]-\cos[2y])^2}. && \text{(B.10)}
\end{aligned}
$$

To obtain the exponential forms, we have used

$$
\coth[z]=1-\frac{2}{e^{2z}-1},\quad \coth'[z]=-\frac{4e^{2z}}{(e^{2z}-1)^2}=\frac{-4}{(e^{2z}-1)^2}+\frac{-4}{e^{2z}-1}, \tag{B.11}
$$

and we transformed them into hyperbolic functions in the next line. Note that $\coth_+(x,y)$, $\coth'_+(x,y)$ are real and $\coth_-(x,y)$, $\coth'_-(x,y)$ are pure imaginary, and thus we can make sure that all the four infinite sum are real. Note also that $\coth_-(x,y)$, $\coth'_+(x,y)$ and $\coth'_-(x,y)$ exponentially converge to zero as $x\to 0$, while $\coth_+(x,y)$ converges to 1. This means only $\coth_+(k,*)$ containing terms can diverge when integrating with respect to momentum k.

By substituting the above equations into the original equations, and using

$$
\begin{aligned}
\int\frac{d^4k}{(2\pi)^4} &= \frac{1}{16\pi^2}\int_0^{\Lambda^2}dk^2k^2=\frac{1}{8\pi^2}\int_0^{\Lambda}dk\,k^3 \\
&= \frac{1}{16\pi^2}\int_{c^2}^{\Lambda^2+c^2}dq^2(q^2-c^2)=\frac{1}{8\pi^2}\int_c^{\tilde{\Lambda}}dq\;(q^3-c^2q), && \text{(B.12)}
\end{aligned}
$$

where $\tilde{\Lambda} = \sqrt{\Lambda^2 + c^2} \simeq \Lambda$, we obtain

$$\begin{aligned}
\mathcal{I}_1(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{[n+a+iq]^2[n+a-iq]^2} \\
&= \frac{1}{8\pi^2} \int_c^{\Lambda} dq\ (q^3 - c^2 q) \frac{\pi}{2q^3} \left[\coth_+(\pi q, \pi a) - \pi q \coth'_+(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \int_c^{\Lambda} dq\ (1 - c^2 q^{-2}) \left[\coth_+(\pi q, \pi a) - \pi q \coth'_+(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \left\{ I_+(0) - c^2 I_+(-2) - \pi I'_+(1) + \pi c^2 I'_+(-1) \right\}, \qquad \text{(B.13)} \\
\mathcal{I}_3(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{q^2}{[n+a+iq]^2[n+a-iq]^2} \\
&= \frac{1}{16\pi} \left\{ I_+(2) - c^2 I_+(0) - \pi I'_+(3) + \pi c^2 I'_+(1) \right\}, \qquad \text{(B.14)}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{I}_2(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{n}{[n+a+iq]^2[n+a-iq]^2} \\
&= \frac{1}{8\pi^2} \int_c^{\Lambda} dq\ (q^3 - c^2 q) \frac{\pi}{2q^3} \left[-a \coth_+(\pi q, \pi a) + \pi a q \coth'_+(\pi q, \pi a) - i\pi q^2 \coth'_-(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \int_c^{\Lambda} dq\ (1 - c^2 q^{-2}) \left[-a \coth_+(\pi q, \pi a) + \pi a q \coth'_+(\pi q, \pi a) - i\pi q^2 \coth'_-(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \left\{ -a I_+(0) + a c^2 I_+(-2) + \pi a I'_+(1) - \pi a c^2 I'_+(-1) - i\pi I'_-(2) + i\pi c^2 I'_-(0) \right\}. \qquad \text{(B.15)}
\end{aligned}$$

In the last equation, we have defined following q integral;

$$I_+(n) = \int_c^{\Lambda} dq\, q^n \coth_+(\pi q, \pi a), \qquad \text{(B.16)}$$

$$I'_+(n) = \int_c^{\Lambda} dq\, q^n \coth'_+(\pi q, \pi a), \qquad \text{(B.17)}$$

$$I'_-(n) = \int_c^{\Lambda} dq\, q^n \coth'_-(\pi q, \pi a). \qquad \text{(B.18)}$$

Thus the rest of the work is the evaluation of these three integrals.

B.2 Polylogarithm

To evaluate the integrals, it is useful to use Polylogarithm. The definition is

$$\mathrm{Li}_s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^s}. \tag{B.19}$$

When $s = 1$, this is nothing but a Taylor expansion of $\ln(1 - z)$. Thus we can say $\mathrm{Li}_s(z)$ is generalized logarithmic function.

When $\mathrm{Re}(s) > 0$, the integral representations are known as follows

$$\mathrm{Li}_s(z) = \frac{1}{\Gamma(s)} \int_0^\infty dt \, \frac{t^{s-1}}{e^t/z - 1}. \tag{B.20}$$

Especially, when $n \geq 0$,

$$\int_0^\infty dx \, \frac{x^n}{e^{2\pi(x \pm iy)} - 1} = \frac{1}{(2\pi)^{n+1}} \int_0^\infty dt \, \frac{t^n}{e^t/e^{\mp 2\pi i y} - 1} = \frac{n!}{(2\pi)^{n+1}} \mathrm{Li}_{n+1}(e^{\mp 2\pi i y}), \tag{B.21}$$

$$\begin{aligned}
\int_0^\infty dx \, \frac{x^n}{(e^{2\pi(x \pm iy)} - 1)^2} &= \frac{1}{(2\pi)^{n+1}} \int_0^\infty dt \, \frac{t^n}{(e^t/e^{\mp 2\pi i y} - 1)^2} \\
&= \frac{1}{(2\pi)^{n+1}} \int_0^\infty dt \left[t^n \frac{d}{dt} \left(\frac{-1}{e^t/e^{\mp 2\pi i y} - 1} \right) - \frac{t^n}{e^t/e^{\mp 2\pi i y} - 1} \right] \\
&= \frac{1}{(2\pi)^{n+1}} \left[\frac{-t^n}{e^t/e^{\mp 2\pi i y} - 1} \right]_0^\infty + \frac{1}{(2\pi)^{n+1}} \int_0^\infty dt \left[\frac{n t^{n-1} - t^n}{e^t/e^{\mp 2\pi i y} - 1} \right] \\
&= \frac{n!}{(2\pi)^{n+1}} \left(\mathrm{Li}_n(e^{\mp 2\pi i y}) - \mathrm{Li}_{n+1}(e^{\mp 2\pi i y}) \right).
\end{aligned} \tag{B.22}$$

Here, when $n = 0$, we have used

$$\mathrm{Li}_0(z) = \sum_{n=1}^{\infty} z^n = \frac{z}{1 - z} = \frac{1}{z^{-1} - 1}. \tag{B.23}$$

Following Taylor expansion forms are used when Polylogarithms are evaluated in the result subsection.

$$\mathrm{Li}_1(e^{-2\pi i a}) + \mathrm{Li}_1(e^{+2\pi i a}) = -2\ln[2\pi a] + \mathcal{O}(a^2), \tag{B.24}$$

$$\mathrm{Li}_2(e^{-2\pi i a}) - \mathrm{Li}_2(e^{+2\pi i a}) = 4\pi i(\ln[2\pi a] - 1)a + \mathcal{O}(a^3). \tag{B.25}$$

B.3 Integral formulae

Our goal now is to evaluate $I_+(n)$ and $I'_\pm(n)$. Since c and a are assumed to be small, we seek for results only up to $\mathcal{O}(c^2, a^2)$. We also assume that Λ is large. When $n \geq 0$, we can use the integral representations of Polylogarithm as follows;

$$
\begin{aligned}
I_+(n) &= \int_c^\Lambda dq\, q^n \coth_+(\pi q, \pi a) \\
&= \int_c^\Lambda dq\, q^n + \int_c^\Lambda dq\, \frac{q^n}{e^{2\pi(q+ia)}-1} + \int_c^\Lambda dq\, \frac{q^n}{e^{2\pi(q-ia)}-1} \\
&\simeq \int_c^\Lambda dq\, q^n + \int_0^\Lambda dq\, \frac{q^n}{e^{2\pi(q+ia)}-1} + \int_0^\Lambda dq\, \frac{q^n}{e^{2\pi(q-ia)}-1} \\
&\qquad\qquad - \frac{1}{2\pi}\int_0^c dq\, \frac{q^n}{q+ia} - \frac{1}{2\pi}\int_0^c dq\, \frac{q^n}{q-ia} \\
&\simeq \frac{\Lambda^{n+1}-c^{n+1}}{n+1} + \frac{n!}{(2\pi)^{n+1}}\left[\mathrm{Li}_{n+1}(e^{-2\pi ia}) + \mathrm{Li}_{n+1}(e^{+2\pi ia})\right] + J_+(n).
\end{aligned}
\tag{B.26}
$$

In the third line, we have divided the interval of the integration $[c, \Lambda]$ into $[0, \Lambda]$ and $[0, c]$, and expand the integrand in $[0, c]$ with respect to q and a since c is small. In the fourth line, we have used the integral representations of Polylogarithms since Λ is large and the integrands converges to zero as $q \to \infty$. We consult the Taylor expansion form of Polylogarithms when we apply these equations with fixed $n = 1, 2, 3$. The last term in the last line is defined as

$$
J_+(n) = -\frac{1}{2\pi}\int_0^c dq\, \left[\frac{q^n}{q+ia} + \frac{q^n}{q-ia}\right]. \tag{B.27}
$$

This term matters only when n is small since $J_+(n) = \mathcal{O}([a|c]^n)$, so the following equations are sufficient;

$$
J_+(0) = -\frac{1}{2\pi}\ln[1+\frac{c^2}{a^2}], \quad J_+(1) = -\frac{c}{\pi} + \frac{a}{\pi}\tan^{-1}[c/a]. \tag{B.28}
$$

Similarly, we can evaluate $I'_+(n)$ and $I'_-(n)$ as follows.

$$
\begin{aligned}
I'_\pm(n) &= \int_c^\Lambda dq\, q^n \coth_\pm(\pi q, \pi a) \\
&= -2\Bigg[\int_c^\Lambda dq\, \frac{q^n}{(e^{2\pi(q+ia)}-1)^2} \pm \int_c^\Lambda dq\, \frac{q^n}{(e^{2\pi(q-ia)}-1)^2} \\
&\qquad\qquad + \int_c^\Lambda dq\, \frac{q^n}{e^{2\pi(q+ia)}-1} \pm \int_c^\Lambda dq\, \frac{q^n}{e^{2\pi(q-ia)}-1}\Bigg] \\
&\simeq -\frac{2\cdot n!}{(2\pi)^{n+1}}\left[\mathrm{Li}_n(e^{-2\pi ia}) \pm \mathrm{Li}_n(e^{+2\pi ia})\right] + J'_\pm(n),
\end{aligned}
\tag{B.29}
$$

where we have used Eq.(B.21) and Eq.(B.22) to get the Polylogarithms, and $J'_{\pm}(n)$ is defined as

$$J'_{\pm}(n) = \frac{2}{(2\pi)^2}\int_0^c dq\left[\frac{q^n}{(q+ia)^2} \pm \frac{q^n}{(q-ia)^2} + \frac{2\pi q^n}{q+ia} \pm \frac{2\pi q^n}{q-ia}\right], \tag{B.30}$$

which is of the order of $\mathcal{O}([a|c]^{n-1})$. Only the following terms are in our interest;

$$J'_{+}(0) = -\frac{1}{c\pi^2(1+a^2/c^2)} + \frac{1}{\pi}\ln[1+c^2/a^2], \tag{B.31}$$

$$J'_{+}(1) = -\frac{1}{\pi^2(1+a^2/c^2)} + \frac{2c}{\pi} - \frac{2a}{\pi}\tan^{-1}[c/a] + \frac{1}{2\pi^2}\ln[1+c^2/a^2], \tag{B.32}$$

$$J'_{+}(2) = \frac{2c}{\pi^2} - \frac{c}{\pi^2(1+a^2/c^2)} + \frac{c^2}{\pi} - \frac{2a}{\pi^2}\tan^{-1}[c/a] - \frac{a^2}{\pi}\ln[1+c^2/a^2]. \tag{B.33}$$

and

$$J'_{-}(0) = -\frac{i}{a\pi^2(1+a^2/c^2)} - \frac{2i}{\pi}\tan^{-1}[c/a], \tag{B.34}$$

$$J'_{-}(1) = \frac{iac}{\pi^2(a^2+c^2)} - \frac{i}{\pi^2}\tan^{-1}[c/a] - \frac{ia}{\pi}\ln[1+c^2/a^2], \tag{B.35}$$

$$J'_{-}(2) = -\frac{2ia}{\pi} + \frac{iac}{\pi^2(1+a^2/c^2)} + \frac{2ia^2}{\pi}\tan^{-1}[c/a] - \frac{ia}{\pi^2}\ln[1+c^2/a^2]. \tag{B.36}$$

Furthermore, there are following cases where we need to evaluate $I_{\pm}^{(\prime)}(c,\Lambda,n,a)$ when n is less than (or equal to) zero;

$$\begin{aligned}
I_{+}(-2) &= \int_c^{\Lambda} dq\, q^{-2}\coth_{+}(\pi q,\pi a)\\
&= \int_c^{\Lambda} dq\, q^{-2} + \int_c^{\Lambda} dq\,\frac{q^{-2}}{e^{2\pi(q+ia)}-1} + \int_c^{\Lambda} dq\,\frac{q^{-2}}{e^{2\pi(q-ia)}-1}\\
&= \int_c^{\Lambda} dq\, q^{-2} + \frac{1}{2\pi}\int_c^{*} dq\,\frac{1}{q^2(q+ia)} + \frac{1}{2\pi}\int_c^{*} dq\,\frac{1}{q^2(q-ia)} + \text{const.}\\
&= \frac{1}{c} + \frac{1}{2\pi a^2}\ln[1+\frac{a^2}{c^2}] + \text{const.}.
\end{aligned} \tag{B.37}$$

In the third equality, we used the fact that the integrand converges to zero as q goes to infinity, and it matters only when q is small, and thus we divided the integration interval and expanded the integrand with respect to q. Similarly we can derive

$$\begin{aligned}
I'_{+}(-1) &= \int_c^{\Lambda} dq\, q^{-1}\coth'_{+}(\pi q,\pi a)\\
&= -\frac{1}{\pi^2(a^2+c^2)} + \frac{2\tan^{-1}[c/a]}{\pi a} + \frac{1}{\pi^2 a^2}\ln[1+a^2/c^2].
\end{aligned} \tag{B.38}$$

B.4 Result

Using the equations we have derived so far in this section, we finally acquire the results.

$$\begin{aligned}
\mathcal{I}_1(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{[(n+a)^2+k^2+c^2]^2} \\
&= \frac{1}{16\pi} \int_c^{\Lambda} dq\, (1-c^2q^{-2}) \left[\coth_+(\pi q, \pi a) - \pi q \coth'_+(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \left\{ I_+(0) - c^2 I_+(-2) - \pi I'_+(1) + \pi c^2 I'_+(-1) \right\} \\
&= \frac{1}{8\pi^2} \left\{ \frac{\pi\Lambda}{2} - \ln\left[2\pi\sqrt{a^2+c^2}\right] - \frac{6\pi+1}{4}c + \frac{a^2+c^2}{a}\pi \tan^{-1}\left[\frac{c}{a}\right] \right\} + \mathcal{O}([a|c]^2).
\end{aligned} \tag{B.39}$$

There is linear divergent term, infra-red divergent term, and other constants.

The second one is given by

$$\begin{aligned}
\mathcal{I}_2(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{n}{[(n+a)^2+k^2+c^2]^2} \\
&= \frac{1}{16\pi} \int_c^{\Lambda} dq\, (1-c^2q^{-2}) \left[-a\coth_+(\pi q, \pi a) + \pi a q \coth'_+(\pi q, \pi a) - i\pi q^2 \coth'_-(\pi q, \pi a)\right] \\
&= \frac{1}{16\pi} \left\{ -aI_+(0) + ac^2 I_+(-2) + \pi a I'_+(1) - \pi a c^2 I'_+(-1) - i\pi I'_-(2) + i\pi c^2 I'_-(0) \right\} \\
&= \frac{a}{8\pi^2} \left\{ -\frac{\pi\Lambda}{2} + 1 + \frac{2\pi+1}{4}c \right\} + \mathcal{O}([a|c]^3).
\end{aligned} \tag{B.40}$$

There is linear divergent term and other constants, and every term is proportionate to a, which is consistent with the fact that when $a = 0$, $\mathcal{I}_2$ must be zero. There is no infra-red divergence because when $n = 0$ and $k = 0$ (this is when the infra-red divergence occurs), the original formula is equal to zero because it has factor n in the numerator.

The last formula is

$$\begin{aligned}
\mathcal{I}_3(a,c) &= \int \frac{d^4k}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{q^2}{[(n+a)^2+k^2+c^2]^2} \\
&= \frac{1}{16\pi} \left\{ I_+(2) - c^2 I_+(0) - \pi I'_+(3) + \pi c^2 I'_+(1) \right\} \\
&= \frac{1}{8\pi^2} \left\{ \frac{\pi}{6}\Lambda^3 - \frac{\pi c^2}{2}\Lambda + 2a^2 \left(\ln[2\pi\sqrt{a^2+c^2}] - \frac{3}{2} \right) \right. \\
&\qquad \left. + c^2 \left(\ln[2\pi\sqrt{a^2+c^2}] - 1 \right) + \frac{\zeta_3}{\pi^2} \right\} + \mathcal{O}([a|c]^2).
\end{aligned} \tag{B.41}$$

There is a cubic and a linear divergent term, and infra-red divergence.

C Detailed calculation of 4D interaction terms and Feynman diagrams

The derivation of the 4D interaction terms and the corresponding 1-loop graph equations are quite complicated. Therefore we give some of the intermediate equations in this section, for the reader interested in the detailed calculations.

C.1 Gaugino 1-loop graph

C.1.1 $\Sigma_{\phi\lambda\psi}$

This is almost same as the Abelian case. The relevant interaction terms in 5D Lagrangian are

$$\mathcal{L}_{5,\phi\lambda\psi} \supset -\frac{\sqrt{2}ig}{\sqrt{2\pi R}}\left(\tilde{\phi}_1^\dagger \lambda_0 \psi_1 + \tilde{\phi}_2 \bar{\lambda}_0 \bar{\psi}_2\right) + \text{h.c.}. \tag{C.1}$$

The corresponding 4D Lagrangian;

$$\mathcal{L}_{4,\phi\lambda_0\psi} = -\sqrt{2}ig\phi_0^\dagger \overline{\Upsilon}_0 P_L \Psi_0 - ig\sum_{n=1}^{\infty}\left[\phi_n^\dagger \overline{\Upsilon}_0 \Psi_n + \phi_{-n}^\dagger \overline{\Upsilon}_0 \Psi'_n\right] + \text{h.c.}. \tag{C.2}$$

1PI graph at 1-loop is given by

$$\begin{aligned}
\Sigma(p)\delta^{ab} &= (\text{zero mode}) + (\text{positive modes}) + (\text{negative modes}) \\
&= \sum_{ij}\int \frac{d^4k}{i(2\pi)^4}\Bigg\{(-\sqrt{2}igT^a_{ij}P_L)\frac{\not{k}}{k^2}(\sqrt{2}igT^b_{ji}P_R)\frac{1}{(p-k)^2+\hat{\alpha}^2} \\
&\qquad + (\sqrt{2}igT^a_{ij}P_R)\frac{\not{k}}{k^2}(-\sqrt{2}igT^b_{ji}P_L)\frac{1}{(p-k)^2+\hat{\alpha}^2}\Bigg\} \\
&\quad + \sum_{n=1}^{\infty}\sum_{ij}\int \frac{d^4k}{i(2\pi)^4}(-igT^a_{ij})\frac{\not{k}-\hat{n}}{k^2+\hat{n}^2}(igT^b_{ji})\frac{1}{(p-k)^2+(\hat{\alpha}+\hat{n})^2}\times 2 \\
&\quad + \sum_{n=1}^{\infty}\sum_{ij}\int \frac{d^4k}{i(2\pi)^4}(-igT^a_{ij})\frac{\not{k}+\hat{n}}{k^2+\hat{n}^2}(igT^b_{ji})\frac{1}{(p-k)^2+(\hat{\alpha}-\hat{n})^2}\times 2 \\
&= 2g^2T(F)\delta^{ab}\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{i(2\pi)^4}\frac{\not{k}-\hat{n}}{[k^2+\hat{n}^2][(k-p)^2+(\hat{\alpha}+\hat{n})^2]}.
\end{aligned} \tag{C.3}$$

The factors $\times 2$ in non-zero modes diagrams come from the Majorana property of Υ_0. In the last line, $T(F)$ is defined as $F\times T(N)$, where $T(N)$ the trace of the fundamental representation, which is $\frac{1}{2}$ in the non-Abelian case.

C.1.2 $\Sigma_{\lambda\Sigma\lambda}$

The 5D Lagrangian is

$$\begin{aligned}\mathcal{L}_{5,\lambda\Sigma\lambda} &= igf^{abc}\left(\lambda_1^a\Sigma^b\lambda_2^c - \bar{\lambda}_1^a\Sigma^b\bar{\lambda}_2^c\right) = igf^{abc}\Sigma^b\left(\tilde{\lambda}_1^a\tilde{\lambda}_2^c - \bar{\tilde{\lambda}}_1^a\bar{\tilde{\lambda}}_2^c\right)\\ &\supset \frac{ig}{\sqrt{2\pi R}}f^{abc}\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\Sigma_n^b\left(-\lambda_0^a\lambda_m^c + \bar{\lambda}_0^a\bar{\lambda}_m^c\right)\frac{\sin\hat{n}y}{\sqrt{\pi R}}\frac{\sin\hat{m}y}{\sqrt{2\pi R}},\end{aligned} \tag{C.4}$$

which leads to the following 4D Lagrangian

$$\begin{aligned}\mathcal{L}_{4,\lambda\Sigma\lambda} &= \int_0^{2\pi R} dy\ \mathcal{L}_{5,\lambda\Sigma\lambda}\\ &\supset \frac{ig}{\sqrt{2}}f^{abc}\sum_{n=1}^{\infty}\Sigma_n^b\left(-\lambda_0^a\lambda_n^c + \bar{\lambda}_0^a\bar{\lambda}_n^c + \lambda_0^a\lambda_{-n}^c - \bar{\lambda}_0^a\bar{\lambda}_{-n}^c\right)\\ &= -\frac{ig}{\sqrt{2}}f^{abc}\sum_{n=1}^{\infty}\Sigma_n^b\overline{\Upsilon_0^a}\ \gamma^5\left(\Upsilon_n^c - \Upsilon_{-n}^c\right).\end{aligned} \tag{C.5}$$

Therefore, the 1-loop 1PI graph is

$$\begin{aligned}\Sigma(p)\delta^{ad} &= \sum_{n=1}^{\infty}\sum_{b,c}\int\frac{d^4k}{i(2\pi)^4}\left[\left(-\frac{ig}{\sqrt{2}}f^{abc}\gamma^5\right)\frac{\not{k} - (\hat{\alpha}+\hat{n})}{k^2 + (\hat{\alpha}+\hat{n})^2}\left(+\frac{ig}{\sqrt{2}}f^{cbd}\gamma^5\right)\frac{1}{(k-p)^2+\hat{n}^2}\right]\\ &\quad + (n\to -n)\\ &= -\frac{g^2}{2}\left(\sum_{b,c}f^{abc}f^{dbc}\right)\sum_{n\neq 0}\int\frac{d^4k}{i(2\pi)^4}\frac{\gamma^5[\not{k} - (\hat{\alpha}+\hat{n})]\gamma^5}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}\\ &= \frac{g^2C(A)}{2}\delta^{ad}\sum_{n\neq 0}\int\frac{d^4k}{i(2\pi)^4}\frac{\not{k} + (\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}.\end{aligned} \tag{C.6}$$

C.1.3 $\Sigma^{\lambda A_5\lambda}$

The relevant 5D Lagrangian term is

$$\begin{aligned}\mathcal{L}_{5,\lambda A_5\lambda} &= -gf^{abc}\left(-\lambda_2^a, \bar{\lambda}_1^a\right)i\Gamma^5 A_5^b\begin{pmatrix}\lambda_1^c\\ -\bar{\lambda}_2^c\end{pmatrix} = -gf^{abc}\left(-\tilde{\lambda}_2^a, \bar{\tilde{\lambda}}_1^a\right)i\Gamma^5 A_5^b\begin{pmatrix}\tilde{\lambda}_1^c\\ -\bar{\tilde{\lambda}}_2^c\end{pmatrix}\\ &\supset -\frac{g}{2\pi R\sqrt{\pi R}}f^{abc}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\left(\lambda_m^a\lambda_0^c - \bar{\lambda}_0^a\bar{\lambda}_m^c\right)A_{5,n}^b\sin\hat{m}y\sin\hat{n}y\\ &= +\frac{g}{2\pi R\sqrt{\pi R}}f^{abc}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\left(\lambda_0^a\lambda_m^c + \bar{\lambda}_0^a\bar{\lambda}_m^c\right)A_{5,n}^b\sin\hat{m}y\sin\hat{n}y,\end{aligned} \tag{C.7}$$

and the corresponding 4D Lagrangian is

$$\mathcal{L}_{4,\lambda A_5 \lambda} = \int_0^{2\pi R} dy\ \mathcal{L}_{5,\lambda A_5 \lambda} = \frac{g}{\sqrt{2}} f^{abc} \sum_{n=1}^{\infty} \overline{\Upsilon}_0^a A_{5,n}^b \left(\Upsilon_n^c - \Upsilon_{-n}^c \right). \tag{C.8}$$

Therefore

$$\begin{aligned}
\Sigma_{\lambda A_5 \lambda} \delta^{ad} &= \sum_{n \neq 0} \int \frac{d^4 k}{i(2\pi)^4} \left(\frac{g}{\sqrt{2}} f^{abc} \right) \frac{\not{k} - (\hat{\alpha} + \hat{n})}{k^2 + (\hat{\alpha} + \hat{n})^2} \left(-\frac{g}{\sqrt{2}} f^{cbd} \right) \frac{1}{(k-p)^2 + \hat{n}^2} \\
&= \frac{g^2 C(A)}{2} \delta^{ad} \sum_{n \neq 0} \int \frac{d^4 k}{i(2\pi)^4} \frac{\not{k} - (\hat{\alpha} + \hat{n})}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]},
\end{aligned} \tag{C.9}$$

where the factor $\frac{1}{2}$ is a symmetry factor.

C.1.4 $\Sigma^{\lambda A_\mu \lambda}$

The relevant 5D Lagrangian term is

$$\begin{aligned}
\mathcal{L}_{5,\lambda A_\mu \lambda} &= -g f^{abc} \left(-\lambda_2^a, \bar{\lambda}_1^a \right) i\gamma^\mu A_\mu^b \begin{pmatrix} \lambda_1^c \\ -\bar{\lambda}_2^c \end{pmatrix} = -g f^{abc} \left(-\tilde{\lambda}_2^a, \bar{\tilde{\lambda}}_1^a \right) i\gamma^\mu A_\mu^b \begin{pmatrix} \tilde{\lambda}_1^c \\ -\bar{\tilde{\lambda}}_2^c \end{pmatrix} \\
&\supset -\frac{g f^{abc}}{2\pi R \sqrt{\pi R}} \sum_{m,l=-\infty}^{\infty} \sum_{n=0}^{\infty} \left(\lambda_m^a i\sigma^\mu \bar{\lambda}_l^c \sin \hat{m} y \sin \hat{l} y + \bar{\lambda}_m^a i\bar{\sigma}^\mu \lambda_l^c \cos \hat{m} y \cos \hat{l} y \right) A_{n,\mu}^b \eta_n \cos \hat{n} y
\end{aligned} \tag{C.10}$$

and the corresponding 4D Lagrangian is

$$\begin{aligned}
\mathcal{L}_{4,\lambda A_\mu \lambda} &= \int_0^{2\pi R} dy\ \mathcal{L}_{5,\lambda A_\mu \lambda} \\
&= -g f^{abc} \lambda_0^a i\sigma^\mu \bar{\lambda}_0^c - \frac{g}{\sqrt{2}} f^{abc} \sum_{n=1}^{\infty} A_{n,\mu}^b \left\{ \bar{\lambda}_0^a i\bar{\sigma}^\mu (\lambda_n^c + \lambda_{-n}^c) + (\bar{\lambda}_n^a + \bar{\lambda}_{-n}^a) i\bar{\sigma}^\mu \bar{\lambda}_0^c \right\} \\
&= -\frac{g f^{abc}}{2} \overline{\Upsilon}_0^a i\gamma^\mu A_{0,\mu}^b \Upsilon_0^c - \frac{g}{\sqrt{2}} f^{abc} \sum_{n=1}^{\infty} \overline{\Upsilon}_0^a\ i\gamma^\mu A_{n,\mu}^b \left(\Upsilon_n^c + \Upsilon_{-n}^c \right).
\end{aligned} \tag{C.11}$$

Therefore

$$
\begin{aligned}
\Sigma_{\lambda A_\mu \lambda}\delta^{ad} &= \int \frac{d^4k}{i(2\pi)^4}\left(-gf^{abc}i\gamma^\mu\right)\frac{\not{k}-\hat{\alpha}}{k^2+\hat{\alpha}^2}\left(-gf^{cbd}i\gamma^\nu\right)\frac{g_{\mu\nu}}{(k-p)^2} \\
&\quad + \sum_{n\neq 0}\int \frac{d^4k}{i(2\pi)^4}\left(-\frac{g}{\sqrt{2}}f^{abc}i\gamma^\mu\right)\frac{\not{k}-(\hat{\alpha}+\hat{n})}{k^2+(\hat{\alpha}+\hat{n})^2}\left(-\frac{g}{\sqrt{2}}f^{cbd}i\gamma^\nu\right)\frac{g_{\mu\nu}}{(k-p)^2+\hat{n}^2} \\
&= g^2C(A)\delta^{ad}\int \frac{d^4k}{i(2\pi)^4}\frac{2\not{k}+4\hat{\alpha}}{[k^2+\hat{\alpha}^2][(k-p)^2]} \\
&\quad + \frac{g^2}{2}C(A)\delta^{ad}\sum_{n\neq 0}\int \frac{d^4k}{i(2\pi)^4}\frac{2\not{k}+4(\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]} \\
&= g^2C(A)\delta^{ad}\int \frac{d^4k}{i(2\pi)^4}\frac{\not{k}+2\hat{\alpha}}{[k^2+\hat{\alpha}^2][(k-p)^2]} \\
&\quad + g^2C(A)\delta^{ad}\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{i(2\pi)^4}\frac{\not{k}+2(\hat{\alpha}+\hat{n})}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}.
\end{aligned}
\tag{C.12}
$$

C.2 Sfermion 1-loop graphs

C.2.1 $\Pi_{\phi\lambda\psi}$

The relevant terms are

$$
\begin{aligned}
\mathcal{L}_{5,\phi\lambda\psi} &= -\sqrt{2}ig\left(\tilde{\phi}_1^\dagger,\tilde{\phi}_2\right)\left[\begin{pmatrix}\tilde{\lambda}_1\\ -\tilde{\lambda}_2\end{pmatrix}\psi_1+\begin{pmatrix}\bar{\tilde{\lambda}}_2\\ \bar{\tilde{\lambda}}_1\end{pmatrix}\bar{\psi}_2\right]+\text{h.c.} \\
&\supset -\frac{\sqrt{2}ig}{\sqrt{2\pi R}}\,\phi_0^\dagger\left[\tilde{\lambda}_1\psi_1+\bar{\tilde{\lambda}}_2\bar{\psi}_2\right]+\text{h.c.} \\
&= -\frac{\sqrt{2}ig}{\sqrt{2\pi R}}\,\phi_0^\dagger\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\left[\lambda_m\psi_{1,n}\frac{\cos\hat{m}y}{\sqrt{2\pi R}}\frac{\eta_n\cos\hat{n}y}{\sqrt{\pi R}}-\bar{\lambda}_m\bar{\psi}_{2,n}\frac{\sin\hat{m}y}{\sqrt{2\pi R}}\frac{\sin\hat{n}y}{\sqrt{\pi R}}\right]+\text{h.c.}.
\end{aligned}
\tag{C.13}
$$

The corresponding 4D Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{4,\phi_0\lambda\psi} &= -\sqrt{2}ig\left(\phi_0^\dagger\lambda_0\psi_{1,0}-\bar{\psi}_0\bar{\lambda}_0\phi_0\right) \\
&\quad - ig\phi_0^\dagger\sum_{n=1}^{\infty}\left[\left(\lambda_n+\lambda_{-n}\right)\psi_{1,n}-\left(\bar{\lambda}_n-\bar{\lambda}_{-n}\right)\bar{\psi}_{2,n}\right]+\text{h.c.} \\
&= -\sqrt{2}ig\left(\phi_0^\dagger\overline{\Upsilon}_0P_L\Psi_0-\overline{\Psi}_0P_R\Upsilon_0\phi_0\right) \\
&\quad - ig\sum_{n=1}^{\infty}\left[\phi_0^\dagger\overline{\Upsilon}_n\Psi_n'+\phi_0^\dagger\overline{\Upsilon}_{-n}\Psi_n-\overline{\Psi}_n'\Upsilon_n\phi_0-\overline{\Psi}_n\Upsilon_{-n}\phi_0\right].
\end{aligned}
\tag{C.14}
$$

Therefore the zero mode loop is

$$
\begin{aligned}
\Pi_{\phi\lambda\psi} = & - \int \frac{d^4k}{i(2\pi)^4} \mathrm{Tr}\Bigg[\left(-\sqrt{2}igT^a P_L\right) \frac{\not{k} - \hat{\alpha}}{k^2 + \hat{\alpha}^2} \left(+\sqrt{2}igT^a P_R\right) \frac{\not{k} - \not{p}}{(k-p)^2} \\
& + \left(+\sqrt{2}igT^a P_R\right) \frac{\not{k} - \hat{\alpha}}{k^2 + \hat{\alpha}^2} \left(-\sqrt{2}igT^a P_L\right) \frac{\not{k} - \not{p}}{(k-p)^2} \Bigg] \times \frac{1}{2} \\
= & -g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{\mathrm{Tr}[\not{k}(\not{k} - \not{p})]}{[k^2 + \hat{\alpha}^2][(k-p)^2]} \\
= & +4g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{k(k-p)}{[k^2 + \hat{\alpha}^2][(k-p)^2]}.
\end{aligned} \tag{C.15}
$$

The non-zero modes contribution is

$$
\begin{aligned}
\Pi_{\phi\lambda\psi} = & - \sum_{n=1}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \mathrm{Tr}\Bigg[(-igT^a) \frac{\not{k} - (\hat{\alpha} + \hat{n})}{k^2 + (\hat{\alpha} + \hat{n})^2} (+igT^a) \frac{\not{k} - \not{p} + \hat{n}}{(k-p)^2 + \hat{n}^2} \\
& + (-igT^a) \frac{\not{k} - (\hat{\alpha} - \hat{n})}{k^2 + (\hat{\alpha} - \hat{n})^2} (+igT^a) \frac{\not{k} - \not{p} - \hat{n}}{(k-p)^2 + \hat{n}^2} \Bigg] \\
= & -g^2 C(N) \sum_{n \neq 0} \int \frac{d^4k}{i(2\pi)^4} \frac{\mathrm{Tr}[(\not{k} - \hat{\alpha} - \hat{n})(\not{k} - \not{p} + \hat{n})]}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]} \\
= & +4g^2 C(N) \sum_{n \neq 0} \int \frac{d^4k}{i(2\pi)^4} \frac{k(k-p) + \hat{n}(\hat{\alpha} + \hat{n})}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]}.
\end{aligned} \tag{C.16}
$$

In total,

$$
\Pi_{\phi\lambda\psi} = +4g^2 C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{k(k-p) + \hat{n}(\hat{\alpha} + \hat{n})}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]}. \tag{C.17}
$$

C.2.2 $\Pi_{\phi A_5 \phi}$

Since

$$
\begin{aligned}
\mathcal{L}_{5,\phi} = & - \left(\mathcal{D}^M \phi_1\right)^\dagger \left(\mathcal{D}_M \phi_1\right) - \left(\mathcal{D}^M \phi_2^\dagger\right)^\dagger \left(\mathcal{D}_M \phi_2^\dagger\right) \\
\supset & - \left(\mathcal{D}_5 \phi_1\right)^\dagger \left(\mathcal{D}_5 \phi_1\right) - \left(\mathcal{D}_5 \phi_2^\dagger\right)^\dagger \left(\mathcal{D}_5 \phi_2^\dagger\right) \\
= & - \left(\partial_5 \phi_1^\dagger + ig\phi_1^\dagger A_5\right) \left(\partial_5 \phi_1 - igA_5 \phi_1\right) - \left(\partial_5 \phi_2 + ig\phi_2 A_5\right) \left(\partial_5 \phi_2^\dagger - igA_5 \phi_2^\dagger\right),
\end{aligned} \tag{C.18}
$$

the relevant terms are

$$
\begin{aligned}
\mathcal{L}_{5,\phi A_5 \phi} &= -ig\left(\phi_1^\dagger, \phi_2\right) A_5 \partial_5 \begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} + \text{h.c.} \\
&= -ig\left(\tilde{\phi}_1^\dagger, \tilde{\phi}_2\right) U A_5 \partial_5 U^\dagger \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} + \text{h.c.} \\
&= -ig\left(\tilde{\phi}_1^\dagger, \tilde{\phi}_2\right) A_5 \begin{pmatrix} \partial_5 & -\hat{\alpha} \\ +\hat{\alpha} & \partial_5 \end{pmatrix} \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} + \text{h.c.} \\
&= -\frac{ig}{2\pi R\sqrt{\pi R}} \sum_{n=1}^{\infty} \sum_{m,l=-\infty}^{\infty} \phi_m^\dagger \left(\cos \hat{m}y, \sin \hat{m}y\right) A_{5,n} \sin \hat{n}y \\
&\qquad \times \begin{pmatrix} \partial_5 & -\hat{\alpha} \\ +\hat{\alpha} & \partial_5 \end{pmatrix} \begin{pmatrix} \cos \hat{l}y \\ \sin \hat{l}y \end{pmatrix} \phi_l + \text{h.c.}.
\end{aligned} \tag{C.19}
$$

The corresponding 4D Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{4,\phi A_5 \phi} &= -\frac{ig}{\sqrt{2}\pi R} \sum_{n=1}^{\infty} \sum_{m,l=-\infty}^{\infty} (\hat{\alpha} + \hat{l}) \phi_m^\dagger A_{5,n} \phi_l \\
&\qquad \times \int dy \;\; \sin \hat{n}y \left(-\cos \hat{m}y \sin \hat{l}y + \sin \hat{m}y \cos \hat{l}y\right) + \text{h.c.} \\
&= -\frac{ig}{\sqrt{2}\pi R} \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \int dy \;\; \sin \hat{n}y \sin \hat{m}y \\
&\quad \left(-(\hat{\alpha} + \hat{m}) \phi_0^\dagger A_{5,n} \phi_m + \hat{\alpha} \phi_m^\dagger A_{5,n} \phi_0\right) + \text{h.c.} \\
&= -\frac{ig}{\sqrt{2}\pi R} \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \int dy \;\; \sin \hat{n}y \sin \hat{m}y (2\hat{\alpha} + \hat{m}) \left(-\phi_0^\dagger A_{5,n} \phi_m + \phi_m^\dagger A_{5,n} \phi_0\right) \\
&= -\frac{ig}{\sqrt{2}} \sum_{n=1}^{\infty} \Big(-(2\hat{\alpha} + \hat{n}) \phi_0^\dagger A_{5,n} \phi_n + (2\hat{\alpha} + \hat{n}) \phi_n^\dagger A_{5,n} \phi_0 \\
&\qquad\qquad + (2\hat{\alpha} - \hat{n}) \phi_0^\dagger A_{5,n} \phi_{-n} - (2\hat{\alpha} - \hat{n}) \phi_{-n}^\dagger A_{5,n} \phi_0\Big).
\end{aligned} \tag{C.20}
$$

The 1PI graph is

$$
\begin{aligned}
\Pi_{\phi A_5 \phi} = & \sum_{n=1}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \Bigg\{ \left(+ \frac{ig}{\sqrt{2}} (2\hat{\alpha} + \hat{n}) T^a \right) \frac{1}{k^2 + (\hat{\alpha} + \hat{n})^2} \left(- \frac{ig}{\sqrt{2}} (2\hat{\alpha} + \hat{n}) T^a \right) \frac{1}{(k-p)^2 + \hat{n}^2} \\
& + \left(- \frac{ig}{\sqrt{2}} (2\hat{\alpha} - \hat{n}) T^a \right) \frac{1}{k^2 + (\hat{\alpha} - \hat{n})^2} \left(+ \frac{ig}{\sqrt{2}} (2\hat{\alpha} - \hat{n}) T^a \right) \frac{1}{(k-p)^2 + \hat{n}^2} \Bigg\} \\
= & \frac{g^2}{2} C(N) \sum_{n \neq 0}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{(2\hat{\alpha} + \hat{n})^2}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]} \\
= & \frac{g^2}{2} C(N) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{(2\hat{\alpha} + \hat{n})^2}{[k^2 + (\hat{\alpha} + \hat{n})^2][(k-p)^2 + \hat{n}^2]} \\
& - 2g^2 C(N) \int \frac{d^4k}{i(2\pi)^4} \frac{\hat{\alpha}^2}{[k^2 + \hat{\alpha}^2][(k-p)^2]}
\end{aligned}
\tag{C.21}
$$

C.2.3 $\Pi_{\phi A_\mu \phi}$

Since

$$
\begin{aligned}
\mathcal{L}_{5,\phi} \supset & - (\mathcal{D}^\mu \phi_1)^\dagger (\mathcal{D}_\mu \phi_1) - \left(\mathcal{D}^\mu \phi_2^\dagger \right)^\dagger \left(\mathcal{D}_\mu \phi_2^\dagger \right) \\
= & - \left(\partial^\mu \phi_1^\dagger + ig \phi_1^\dagger A^\mu \right) (\partial_\mu \phi_1 - ig A_\mu \phi_1) - (\partial^\mu \phi_2 + ig \phi_2 A_\mu) \left(\partial_\mu \phi_2^\dagger - ig A_\mu \phi_2^\dagger \right),
\end{aligned}
\tag{C.22}
$$

the relevant terms are

$$
\begin{aligned}
\mathcal{L}_{5,\phi A_\mu \phi} = & -ig \left(\phi_1^\dagger, \phi_2 \right) A^\mu \partial_\mu \begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix} + \text{h.c.} = -ig \left(\tilde{\phi}_1^\dagger, \tilde{\phi}_2 \right) A^\mu \partial_\mu \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger \end{pmatrix} + \text{h.c.} \\
= & - \frac{ig}{2\pi R \sqrt{\pi R}} \sum_{n=0}^{\infty} \sum_{m,l=-\infty}^{\infty} \phi_m^\dagger \left(\cos \hat{m} y, \sin \hat{m} y \right) A_n^\mu \eta_n \cos \hat{n} y \partial_\mu \phi_l \begin{pmatrix} \cos \hat{l} y \\ \sin \hat{l} y \end{pmatrix} + \text{h.c.} \\
\supset & - \frac{ig}{2\pi R \sqrt{\pi R}} \sum_{n=0}^{\infty} \sum_{m,l=-\infty}^{\infty} \phi_m^\dagger A_n^\mu \partial_\mu \phi_l \eta_n \cos \hat{n} y \cos \hat{m} y \cos \hat{l} y + \text{h.c.} \\
\supset & - \frac{ig}{2\pi R \sqrt{\pi R}} \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \left(\phi_m^\dagger A_n^\mu \partial_\mu \phi_0 + \phi_0^\dagger A_n^\mu \partial_\mu \phi_m \right) \eta_n \cos \hat{n} y \cos \hat{m} y \\
& + \frac{ig}{2\pi R \sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_0^\dagger A_n^\mu \partial_\mu \phi_0 \eta_n \cos \hat{n} y + \text{h.c.}
\end{aligned}
\tag{C.23}
$$

The corresponding 4D Lagrangian is

$$\mathcal{L}_{4,\phi A_\mu \phi} \supset -ig\left[\phi_0^\dagger A_0^\mu \partial_\mu \phi_0 - \partial_\mu \phi_0^\dagger A_0^\mu \phi_0\right]$$
$$- \frac{ig}{\sqrt{2}}\sum_{n=1}^{\infty}\left\{(\phi_n^\dagger + \phi_{-n}^\dagger)A_n^\mu \partial_\mu \phi_0 + \phi_0^\dagger A_n^\mu \partial_\mu(\phi_n + \phi_{-n})\right.$$
$$\left. - \partial_\mu \phi_0^\dagger A_n^\mu(\phi_n + \phi_{-n}) - \partial_\mu(\phi_n^\dagger + \phi_{-n}^\dagger)A_n^\mu \phi_0\right\}. \tag{C.24}$$

The 1PI diagram is

$$\Pi_{\phi A_\mu \phi} = g^2 C(N)\int \frac{d^4k}{i(2\pi)^4}\frac{(k+p)^2}{[k^2+\hat{\alpha}^2][(k-p)^2]}$$
$$+ \frac{g^2}{2}C(N)\sum_{n\neq 0}^{\infty}\int \frac{d^4k}{i(2\pi)^4}\frac{(k+p)^2}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}$$
$$= \frac{g^2}{2}C(N)\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{i(2\pi)^4}\frac{(k+p)^2}{[k^2+(\hat{\alpha}+\hat{n})^2][(k-p)^2+\hat{n}^2]}$$
$$+ \frac{g^2}{2}C(N)\int \frac{d^4k}{i(2\pi)^4}\frac{(k+p)^2}{[k^2+\hat{\alpha}^2][(k-p)^2]}. \tag{C.25}$$

C.2.4 $\Pi_{\Sigma\phi\Sigma\phi}$

Since

$$\mathcal{L}_{5,\phi\Sigma\Sigma\phi} = -g^2\left(\tilde{\phi}_1^\dagger, \tilde{\phi}_2\right)\Sigma^2\begin{pmatrix}\tilde{\phi}_1 \\ \tilde{\phi}_2^\dagger\end{pmatrix}$$
$$\supset -\frac{g^2}{2\pi^2 R^2}\sum_{n,m=1}^{\infty}\phi_0^\dagger \Sigma_n \Sigma_m \phi_0 \sin\hat{n}y\sin\hat{m}y, \tag{C.26}$$

then

$$\mathcal{L}_{4,\phi_0\Sigma\Sigma\phi_0} = -g^2\sum_{n=1}^{\infty}\phi_0^\dagger \Sigma_n^2 \phi_0. \tag{C.27}$$

Therefore

$$\Pi_{\phi\Sigma\Sigma\phi} = -g^2 C(N)\sum_{n=1}^{\infty}\int \frac{d^4k}{i(2\pi)^4}\frac{1}{k^2+\hat{n}^2}. \tag{C.28}$$

C.2.5 $\Pi_{\phi\phi\phi\phi}$

Since

$$\begin{aligned}
\mathcal{L}_{5,\phi\phi\phi\phi} &= -\frac{g^2}{2}\left(\phi_1^\dagger T^a\phi_1 - \phi_2 T^a\phi_2^\dagger\right)^2 - 2g^2\left(\phi_1^\dagger T^a\phi_2^\dagger\right)\left(\phi_2 T^a\phi_1\right) \\
&\supset -\frac{g^2}{2(2\pi R)^2}\left[(\phi_0^\dagger T^a\phi_0)^2 + \sum_{n,m\neq 0}\left(\phi_0^\dagger T^a\phi_n\right)\left(\phi_m^\dagger T^a\phi_0\right)\cos\hat{n}y\cos\hat{m}y\times 2\right] \\
&\quad -\frac{2g^2}{(2\pi R)^2}\sum_{n,m=-\infty}^{\infty}\left(\phi_0^\dagger T^a\phi_n\right)\left(\phi_m^\dagger T^a\phi_0\right)\sin\hat{n}y\sin\hat{m}y, \qquad \text{(C.29)}
\end{aligned}$$

then

$$\mathcal{L}_{4,\phi_0\phi\phi\phi_0} = -\frac{g^2}{2}(\phi_0^\dagger T^a\phi_0)^2 - \frac{3}{2}g^2\sum_{n\neq 0}\left|\phi_0^\dagger T^a\phi_n\right|^2. \qquad \text{(C.30)}$$

Therefore

$$\begin{aligned}
\Pi_{\phi\phi\phi\phi} &= -\frac{g^2}{2}C(N)\int\frac{d^4k}{i(2\pi)^4}\frac{1}{k^2+\hat{\alpha}^2}\times 2 - \frac{3}{2}g^2C(N)\sum_{n\neq 0}\int\frac{d^4k}{i(2\pi)^4}\frac{1}{k^2+(\hat{\alpha}+\hat{n})^2} \\
&= -\frac{3}{2}g^2C(N)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{i(2\pi)^4}\frac{1}{k^2+(\hat{\alpha}+\hat{n})^2} + \frac{1}{2}g^2C(N)\int\frac{d^4k}{i(2\pi)^4}\frac{1}{k^2+\hat{\alpha}^2}. \qquad \text{(C.31)}
\end{aligned}$$

C.2.6 $\Pi_{\phi A_5 A_5\phi}$

Since

$$\begin{aligned}
\mathcal{L}_{5,\phi} &= -\left(\mathcal{D}^M\phi_1\right)^\dagger\left(\mathcal{D}_M\phi_1\right) - \left(\mathcal{D}^M\phi_2^\dagger\right)^\dagger\left(\mathcal{D}_M\phi_2^\dagger\right) \\
&\supset -\left(\mathcal{D}_5\phi_1\right)^\dagger\left(\mathcal{D}_5\phi_1\right) - \left(\mathcal{D}_5\phi_2^\dagger\right)^\dagger\left(\mathcal{D}_5\phi_2^\dagger\right) \\
&= -\left(\partial_5\phi_1^\dagger + ig\phi_1^\dagger A_5\right)\left(\partial_5\phi_1 - igA_5\phi_1\right) - \left(\partial_5\phi_2 + ig\phi_2 A_5\right)\left(\partial_5\phi_2^\dagger - igA_5\phi_2^\dagger\right) \\
&\supset -g^2\left(\phi_1^\dagger A_5^2\phi_1 + \phi_2 A_5^2\phi_2^\dagger\right) \\
&\supset -\frac{g^2}{2\pi^2R^2}\sum_{n,m=1}^{\infty}\phi_0^\dagger A_{5,n}A_{5,m}\phi_0\sin\hat{n}y\sin\hat{m}y, \qquad \text{(C.32)}
\end{aligned}$$

the relevant terms in 4D Lagrangian are

$$\mathcal{L}_{4,\phi_0 A_5 A_5\phi_0} = -g^2\sum_{n=1}^{\infty}\phi_0^\dagger A_{5,n}^2\phi_0. \qquad \text{(C.33)}$$

Therefore

$$\Pi_{\phi A_5 A_5 \phi} = -g^2 C(N) \sum_{n=1}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{n}^2}. \tag{C.34}$$

C.2.7 $\Pi_{\phi A_\mu A_\mu \phi}$

Since

$$\begin{aligned}
\mathcal{L}_{5,\phi} &= -\left(\mathcal{D}^M \phi_1\right)^\dagger \left(\mathcal{D}_M \phi_1\right) - \left(\mathcal{D}^M \phi_2^\dagger\right)^\dagger \left(\mathcal{D}_M \phi_2^\dagger\right) \\
&\supset -g^2 \left(\phi_1^\dagger A^\mu A_\mu \phi_1 + \phi_2 A^\mu A_\mu \phi_2^\dagger\right) \\
&\supset -\frac{g^2}{2\pi^2 R^2} \sum_{n,m=0}^{\infty} \phi_0^\dagger A_n^\mu A_{\mu,m} \phi_0 \eta_n \eta_m \cos \hat{n} y \cos \hat{m} y,
\end{aligned} \tag{C.35}$$

the relevant terms in 4D Lagrangian are

$$\mathcal{L}_{4,\phi_0 A^\mu A_\mu \phi_0} = -g^2 \sum_{n=0}^{\infty} \phi_0^\dagger A_n^\mu A_{\mu,m} \phi_0. \tag{C.36}$$

Therefore

$$\Pi_{\phi A^\mu A_\mu \phi} = -4g^2 C(N) \sum_{n=0}^{\infty} \int \frac{d^4k}{i(2\pi)^4} \frac{1}{k^2 + \hat{n}^2}. \tag{C.37}$$

D The detailed calculation on the Yukawa interaction on the brane

D.1 The full 4D Yukawa interaction terms

The 4D Yukawa terms are

$$
\begin{aligned}
\mathcal{L}_4^{\text{brane}} &= \int_0^{2\pi R} dy\, \mathcal{L}_5^{\text{brane}} = \int_0^{2\pi R} dy \int d^2\theta\, W^{\text{brane}} \\
&= \int_0^{2\pi R} dy \int d^2\theta \Big[y_t \Phi_{\bar{t}_R} \left(\Phi_{t_L} H_u^0 - \Phi_{b_L} H_u^+ \right) - y_b \Phi_{\bar{b}_R} \left(\Phi_{t_L} H_d^- - \Phi_{b_L} H_d^0 \right) \\
&\qquad\qquad + \mu \left(H_u^+ H_d^- - H_u^0 H_d^0 \right) \Big] \\
&\supset \int_0^{2\pi R} dy\, \delta(y) \Bigg[F_{t_L} \left(y_t \phi_{\bar{t}_R} h_u^0 + y_b \phi_{\bar{b}_R} h_d^- \right) + F_{\bar{t}_R} \left(y_t \phi_{t_L} h_u^0 - y_b \phi_{b_L} h_d^+ \right) \\
&\qquad + F_{b_L} \left(-y_t \phi_{\bar{t}_R} h_u^+ - y_b \phi_{\bar{b}_R} h_d^0 \right) + F_{\bar{b}_R} \left(y_b \phi_{t_L} h_d^- - y_b \phi_{b_L} h_d^0 \right) \\
&\qquad + F_{h_u^0} \left(y_t \phi_{\bar{t}_R} \phi_{t_L} - \mu h_d^0 \right) + F_{h_u^+} \left(-y_t \phi_{\bar{t}_R} \phi_{b_L} + \mu h_d^- \right) \\
&\qquad + F_{h_d^0} \left(-y_b \phi_{\bar{b}_R} \phi_{b_L} - \mu h_u^0 \right) + F_{h_u^+} \left(y_b \phi_{\bar{b}_R} \phi_{t_L} + \mu h_u^+ \right) \Bigg].
\end{aligned}
\tag{D.1}
$$

Here in the last line, we have extracted the components of scalar and auxiliary fields and omitted the fermion interactions, since they are not complicated much when we remove the auxiliary fields and perform the KK-expansion. Next, the relevant bulk term is given by

$$
\begin{aligned}
\mathcal{L}_4^{\text{bulk}} &= \int_0^{2\pi R} dy\, \mathcal{L}_5^{\text{bulk}} \\
&= \int_0^{2\pi R} dy \Bigg\{ \sum_{*=t_L,\bar{t}_R,b_L,\bar{b}_R} \left| F_* - [\partial_5 + ig(A_5 + i\Sigma)] \phi_*^{C\dagger} \right|^2 \\
&\qquad + \delta(y) \sum_{\#=h_u^0,h_u^+,h_d^0,h_d^-} \left| F_\# \right|^2 \Bigg\}.
\end{aligned}
\tag{D.2}
$$

By substituting $F_* = F'_* + [\partial_5 + ig(A_5 + i\Sigma)]\phi_*^{C\dagger}$ and perform the KK expansion,

$$
\begin{pmatrix} \tilde{\phi}_* \\ \tilde{\phi}_*^{C\dagger} \end{pmatrix} = \sum_{n=-\infty}^{\infty} \frac{\phi_{*,n}}{\sqrt{2\pi R}} \begin{pmatrix} \cos \hat{n} y \\ \sin \hat{n} y \end{pmatrix}, \quad \tilde{F}'_* = \sum_{n=-\infty}^{\infty} F'_{*,n} \frac{\cos \hat{n} y}{\sqrt{2\pi R}},
\tag{D.3}
$$

we obtain

$$
\begin{aligned}
\mathcal{L}_4^{\rm total} &= \mathcal{L}_4^{\rm bulk} + \mathcal{L}_4^{\rm brane} \\
&= \sum_{*=t_L,\bar{t}_R,b_L,\bar{b}_R} \sum_{n=-\infty}^{\infty} \left|F'_{*,n}\right|^2 + \sum_{\#=h_u^0,h_u^+,h_d^0,h_d^-} |F_\#|^2 \\
&\quad + \sum_{n,m=-\infty}^{\infty} \Big\{ \left(F_{t_L,n} + (\hat{n}+\hat{\alpha})\,\phi_{t_L,n}\right)\left(y_t\phi_{\bar{t}_R,m}h_u^0 + y_b\phi_{\bar{b}_R,m}h_d^-\right) \\
&\qquad + \left(F_{\bar{t}_R,n} + (\hat{n}+\hat{\alpha})\,\phi_{\bar{t}_R,n}\right)\left(y_t\phi_{t_L,m}h_u^0 - y_t\phi_{b_L,m}h_u^+\right) \\
&\qquad + \left(F_{b_L,n} + (\hat{n}+\hat{\alpha})\,\phi_{b_L,n}\right)\left(-y_t\phi_{\bar{t}_R,m}h_u^+ - y_b\phi_{\bar{b}_R,m}h_d^0\right) \\
&\qquad + \left(F_{\bar{b}_R,n} + (\hat{n}+\hat{\alpha})\,\phi_{\bar{b}_R,n}\right)\left(y_b\phi_{t_L,m}h_d^- - y_b\phi_{b_L,m}h_d^0\right) \Big\} \\
&\quad + F_{h_u^0}\left(y_t \sum_{n,m=-\infty}^{\infty} \phi_{\bar{t}_R,n}\phi_{t_L,m} - \mu h_d^0\right) + F_{h_u^+}\left(-y_t \sum_{n,m=-\infty}^{\infty} \phi_{\bar{t}_R,n}\phi_{b_L,m} + \mu h_d^-\right) \\
&\quad + F_{h_d^0}\left(-y_b \sum_{n,m=-\infty}^{\infty} \phi_{\bar{b}_R,n}\phi_{b_L,m} - \mu h_u^0\right) + F_{h_u^+}\left(y_b \sum_{n,m=-\infty}^{\infty} \phi_{\bar{b}_R,n}\phi_{t_L,m} + \mu h_u^+\right).
\end{aligned}
\tag{D.4}
$$

Here we have replaced $y_t/2\pi R$ and $y_b/2\pi R$ with y_t and y_b respectively (we don't put 5D and 4D labels now for notational simplicity).

By eliminating the auxiliary fields F_*, we 4D interaction terms on the brane are finally given by

$$
\begin{aligned}
\mathcal{L}_4^{\text{total}} = &-\Bigg\{\left|y_t \sum_{n,m=-\infty}^{\infty} \phi_{\bar{t}_R,n}\phi_{t_L,m} - \mu h_d^0\right|^2 + \left|-y_t \sum_{n,m=-\infty}^{\infty} \phi_{\bar{t}_R,n}\phi_{b_L,m} + \mu h_d^-\right|^2 \\
&+ \left|-y_b \sum_{n,m=-\infty}^{\infty} \phi_{\bar{b}_R,n}\phi_{b_L,m} - \mu h_u^0\right|^2 + \left|y_b \sum_{n,m=-\infty}^{\infty} \phi_{\bar{b}_R,n}\phi_{t_L,m} + \mu h_u^+\right|^2 \Bigg\} \\
&- \sum_{n=-\infty}^{\infty} \Bigg\{ \left| \sum_{m=-\infty}^{\infty} y_t \phi_{\bar{t}_R,m} h_u^0 + y_b \phi_{\bar{b}_R,m} h_d^- \right|^2 + \left| \sum_{m=-\infty}^{\infty} y_t \phi_{t_L,m} h_u^0 - y_t \phi_{b_L,m} h_u^+ \right|^2 \\
&\qquad + \left| \sum_{m=-\infty}^{\infty} -y_t \phi_{\bar{t}_R,m} h_u^+ - y_b \phi_{\bar{b}_R,m} h_d^0 \right|^2 + \left| \sum_{m=-\infty}^{\infty} y_b \phi_{t_L,m} h_d^- - y_b \phi_{b_L,m} h_d^0 \right|^2 \Bigg\} \\
&+ \sum_{n,m=-\infty}^{\infty} (\hat{n}+\hat{\alpha}) \Bigg\{ \phi_{t_L,n} \left(y_t \phi_{\bar{t}_R,m} h_u^0 + y_b \phi_{\bar{b}_R,m} h_d^- \right) + \phi_{\bar{t}_R,n} \left(y_t \phi_{t_L,m} h_u^0 - y_t \phi_{b_L,m} h_u^+ \right) \\
&+ \phi_{b_L,n} \left(-y_t \phi_{\bar{t}_R,m} h_u^+ - y_b \phi_{\bar{b}_R,m} h_d^0 \right) + \phi_{\bar{t}_R,n} \left(y_b \phi_{t_L,m} h_d^- - y_b \phi_{b_L,m} h_d^0 \right) + \text{h.c.} \Bigg\}.
\end{aligned}
\tag{D.5}
$$

This is the full scalar interaction terms on the brane. The scalar-fermion-fermion interaction terms we have omitted from the beginning of this subsection are given by

$$
\mathcal{L}_5 \supset \delta(y) y_t \left[-\phi_{t_L} \psi_{\bar{t}_R} \psi_{h_u^0} \right] + \text{h.c.}, \tag{D.6}
$$

and so on. To perform the KK-expansion and the derivation of the 4D Lagrangian are trivial enough.

D.2 The radiative correction to the stop mass from the interactions on the brane

We are now ready to evaluate the radiative corrections from the interactions with Higgs fields on the brane.

Let us first consider the stop mass correction. We now assume that we can neglect the correction from y_b, and seek only for the correction from y_t. We would like to evaluate the mass correction of $\tilde{t}_L$ and $\tilde{t}_R$, and we first focus on $\tilde{t}_L$.

The loop diagrams have ultra-violet divergence but we introduce the cutoff Λ to regulate them, and several formulae in KK-regularization method in the Appendix B are required to perform the momentum integral and the infinite sum from the KK-modes at the same time.

The strategy is pretty much the same as the case of the mass corrections from the gauge interactions, and thus we don't write down the intermediate calculations.

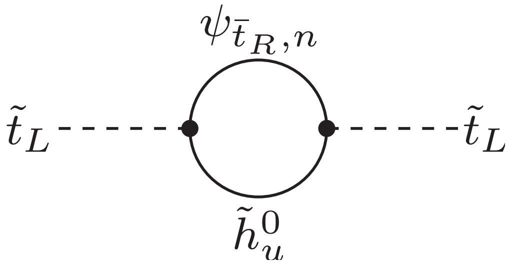
There are roughly two kinds of loops; scalar loop and fermion loop. Therefore what we want to calculate can be written

$$\Pi_{\tilde{t}_L} = \Pi_{\tilde{t}_L}^{\text{scalar}} + \Pi_{\tilde{t}_L}^{\text{fermion}} \tag{D.7}$$

The fermion loop comes from the 3-point interaction terms

$$\mathcal{L}_4 \supset y_t \, \tilde{t}_L \sum_{n=-\infty}^{\infty} \psi_{\bar{t}_R,n} \tilde{h}_u^0 + \text{h.c.} \tag{D.8}$$

and the resulting loop diagram is given in Fig. 11.



(a)

Figure 11: The fermion loop from the top Yukawa coupling.

Taking the external momentum to be p and the loop momentum to be k, it is evaluated as

$$\Pi_{\tilde{t}_L}^{\text{fermion}} = 2\,|y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{(k+p)\cdot k}{[k^2+\mu^2][(k+p)^2+\hat{n}^2]}. \tag{D.9}$$

Let us next consider the scalar loops. There are 4 kinds of scalar loops, two of them are 4-point loops and rest is 3-point, as is shown in Fig. 12.

Taking the external momentum to be p and the loop momentum to be k, it is evaluated

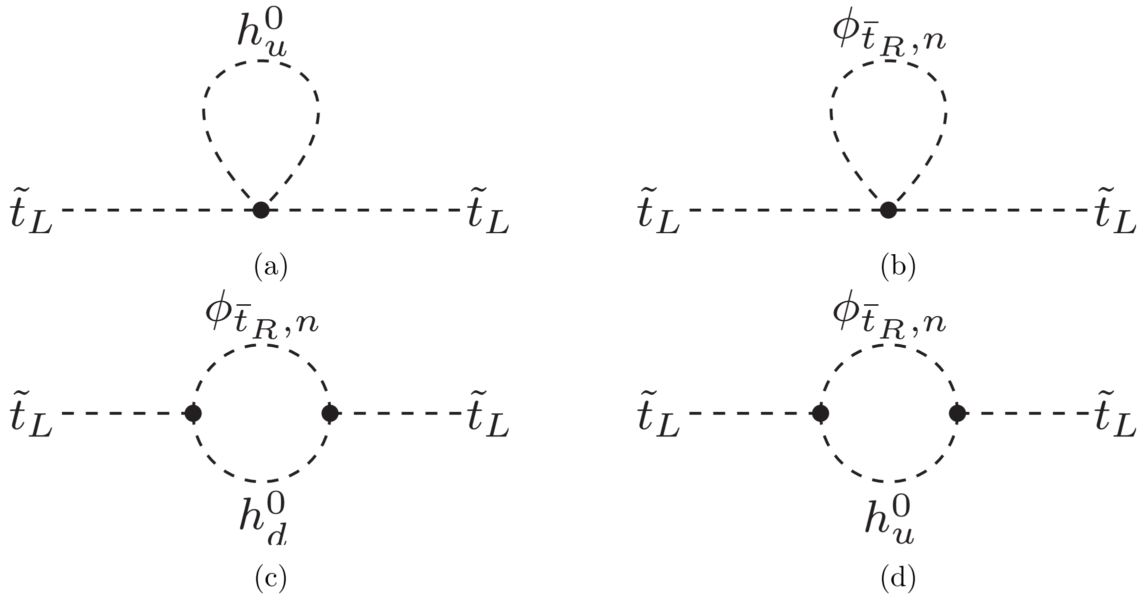


Figure 12: The scalar loops from the top Yukawa couplings.

as

$$\Pi_{\tilde{t}_L}^{\text{scalar}} = -|y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{k^2+\mu^2} \tag{D.10}$$

$$-|y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{k^2+(\hat{n}+\hat{\alpha})^2} \tag{D.11}$$

$$+|y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{k^2+\mu^2} \frac{\mu^2}{(k+p)^2+(\hat{n}+\hat{\alpha})^2} \tag{D.12}$$

$$+|y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{1}{k^2+\mu^2} \frac{(2\hat{\alpha}+\hat{n})^2}{(k+p)^2+(\hat{n}+\hat{\alpha})^2} \tag{D.13}$$

where each line represents the diagrams (a), (b), (c) and (d) in Fig.12 in this order. In total, they can be expressed simply as

$$\Pi_{\tilde{t}_L}^{\text{scalar}} = |y_t|^2 \int \frac{d^4k}{i(2\pi)^4} \sum_{n=-\infty}^{\infty} \frac{2\hat{n}\hat{\alpha}+3\hat{\alpha}^2-k^2-(k+p)^2}{[k^2+\mu^2][(k+p)^2+(\hat{n}+\hat{\alpha})^2]}. \tag{D.14}$$

Both fermion and scalar loops are quadratically divergent (if take the infinite sum with respect to n they have cubic divergence). However both divergence cancels out each other and the resulting mass correction has only logarithmic divergence and infra-red divergence and constant terms. This cancellation of the divergence is thanks to the supersymmetry of the theory, same as in the case of the radiative correction from the gauge interactions.

What we need to calculate is

$$
\begin{aligned}
\Pi_{\tilde{t}_L} &= \Pi_{\tilde{t}_L}^{\text{fermion}} + \Pi_{\tilde{t}_L}^{\text{scalar}} \\
&= |y_t|^2 \int \frac{d^4k}{(2\pi)^2} \frac{1}{k^2+\mu^2} \sum_{n=-\infty}^{\infty} \left\{ \frac{2(k+p)\cdot k}{(k+p)^2+\hat{n}^2} + \frac{2\hat{n}\hat{\alpha}+3\hat{\alpha}^2-k^2-(k+p)^2}{(k+p)^2+(\hat{\alpha}+\hat{n})^2} \right\}, \qquad \text{(D.15)}
\end{aligned}
$$

with the KK-regularization scheme.

E The discussion on $\ln\Lambda$ term in the radiative correction: the comparison with the S^1 case

Looking at the results of the radiative corrections that have $\ln\Lambda$ terms, one may ask the following questions;

- Are $\ln\Lambda$ terms really brane localized?
- Where do $\ln\Lambda$ terms come from?
- Are $\ln\Lambda$ terms gauge independent?
- Why isn't there $\ln\Lambda$ term in Abelian gaugino mass correction?

To answer these, let us consider 5D SUSY gauge theory compactified over S^1 without Z_2 orbifold and compare the result with the orbifold case in section 4.

In the S^1 case, we impose the following twisted boundary conditions to the gaugino and sfermions;

$$\begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix}(y+2\pi R) = \begin{pmatrix} \cos\hat{\alpha}y & \sin\hat{\alpha}y \\ -\sin\hat{\alpha}y & \cos\hat{\alpha}y \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2^\dagger \end{pmatrix}(y), \tag{E.1}$$

$$\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}(y+2\pi R) = \begin{pmatrix} \cos\hat{\alpha}y & \sin\hat{\alpha}y \\ -\sin\hat{\alpha}y & \cos\hat{\alpha}y \end{pmatrix}\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix}(y). \tag{E.2}$$

The 4D spectra in each case are summarized in Fig.13.

Let us first consider the radiative correction to the non-Abelian gaugino mass. There are four types of relevant loop diagrams as in Fig.3. The fermion sfermion loop is

$$\text{Fig.3(a)} = 4g^2 T(F)\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+(\hat{n}+\hat{\alpha})^2}\frac{\not{k}-\not{p}-\hat{n}}{(k-p)^2+\hat{n}^2}. \tag{E.3}$$

This is twice as large as the S^1/Z_2 case since the degree of freedom of fermion and sfermion is twice. The gauge boson loop is

$$\text{Fig.3(b)} = 2g^2 C(A)\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}+2(\hat{n}+\hat{\alpha})}{k^2+(\hat{n}+\hat{\alpha})^2}\frac{1}{(k-p)^2+(\hat{n}+\hat{\alpha})^2}, \tag{E.4}$$

and the rest is given by

$$\text{Fig.3(c)} + \text{Fig.3(d)} = 2g^2 C(A)\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}}{k^2+(\hat{n}+\hat{\alpha})^2}\frac{1}{(k-p)^2+(\hat{n}+\hat{\alpha})^2}. \tag{E.5}$$

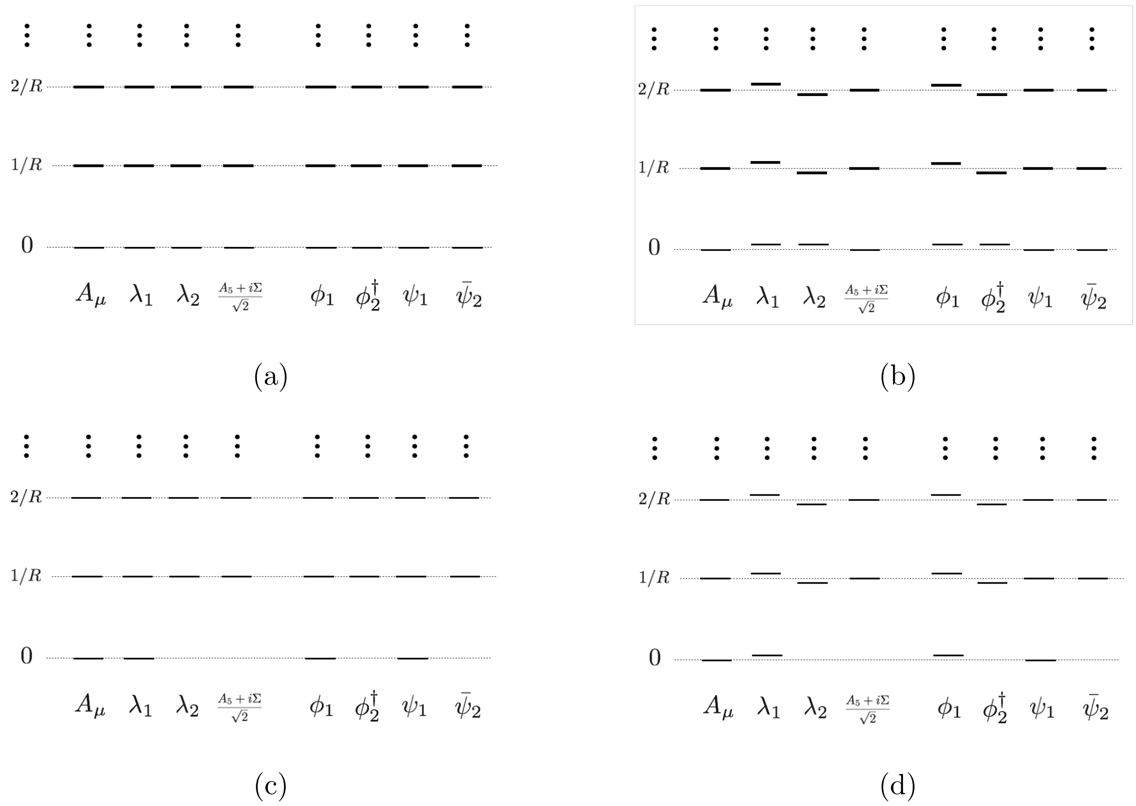


Figure 13: The 4D spectra in various theories. (a) is in S^1 compactification case without any boundary condition. (b) is S^1 with twisted boundary condition. (c) is S^1/Z_2 case and (d) is S^1/Z_2 with twisted boundary condition. Thick lines represent that there are two particles compared with the thin lines.

Therefore the sum of the bosonic loops is

$$\begin{aligned}\Sigma^{\text{boson}} &= \text{Fig.3(b)} + \text{Fig.3(c)} + \text{Fig.3(d)} \\ &= 4g^2 C(A) \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} + \hat{n} + \hat{\alpha}}{k^2 + (\hat{n}+\hat{\alpha})^2} \frac{1}{(k-p)^2 + (\hat{n}+\hat{\alpha})^2}. \end{aligned} \tag{E.6}$$

The both of Eq.(E.3) and Eq.(E.6) can be written in the form of the sum of KK-mode loops. In that case, the divergence vanish in both the KK-regularization scheme and the winding method, which leaves only constant. The resulting mass correction to gaugino is

$$\delta M_\lambda = \frac{g^2}{8\pi^2} \hat{\alpha} \left(2T(F) - 2C(A)\right) \left(\ln[2\pi\alpha] - \frac{3}{2} \right), \tag{E.7}$$

which does not have $\ln \Lambda$ term. This supports the interpretation that $\ln \Lambda$ terms in S^1/Z_2 case are brane localized corrections, since in this S^1 case there is no brane. Note also that the coefficient of the $\ln \alpha$ term is same as the beta function of the gauge coupling in the 4D $\mathcal{N} = 2$ SUSY gauge theory.

Now that we have looked at the S^1 case, we can have better understanding of the S^1/Z_2 case by comparing them. The mass correction in S^1/Z_2 case is basically half of S^1 case since the half of the degree of freedom is dead by Z_2 orbifold. However there is a "mis-match" in zero-mode and thus there is a term that cannot be included in the summation by n as in the last line of Eq.(4.30). That is where the logarithmic divergence comes from and thus its coefficient is scheme independent. Abelian gaugino doesn't have logarithmic divergent mass correction since there is no "mis-match" in the fermion-sfermion loop.

Above discussion has given answers to the questions raised in the beginning of this section, and one can check that the same discussion is valid in the case of sfermion mass correction.

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