論文の内容の要旨

論文題目

Theoretical Study on Universal Few-Body Effects in Ultracold Atomic Gases

(冷却原子気体における普遍的な少数多体効果の理論研究)

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Universality is a crucial concept in modern physics, which allows us to describe phenomena without referring to microscopic details of a system under consideration. Such a universality appears in quantum few-body and many-body systems, where interparticle interactions are near a scattering resonance. The unitary Fermi gas is a prime example, where universal relations and a universal equation of state have been developed and studied in detail. The universal relations follow from the strong short-range correlations due to the two-body collision with the unitarity-limited interaction. They connect short-range correlations to thermodynamic properties. A single quantity, Tan's contact, bridges both sides of the system; this quantity represents the strength of the short-range correlations, and at the same time, it can be written in terms of thermodynamic quantities such as the free energy. They hold true very generally, whether it is a few-body or many-body system, in the ground state or at finite temperatures, in the superfluid phase or the normal phase. Thus, they give strong constraints on the behavior of the unitary Fermi system. The universal equation of state is the most important consequence of the scale invariace of the unitarity-limited interaction. In particular, the ground-state energy is determined by a universal dimensionless number called the Bertsch parameter and the density, as well as the mass of the constituent particle and Planck's constant.

In this thesis, we extend these notions beyond the paradigm of the unitary Fermi gas and investigate universal effects of few-body correlations in resonantly interacting ultracold atomic gases. In particular, we discuss (i) universal relations in a spinless Fermi gas with an anisotropic p-wave resonance and (ii) universality of the few-body spectra and the ground-state properties of an impurity-boson system.

A resonant p-wave interaction has been realized by a magnetic Feshbach resonance in ultracold atomic systems. There, the applied magnetic field explicitly breaks the rotational symmetry into the axisymmetry around the field direction, and lifts the three-fold degeneracy of the p-wave resonance. This can cause, for example, a novel superfluid phase, where the axisymmetry is spontaneously broken. On the other hand, the universal relations in the presence of anisotropic interactions were not fully addressed in the previous studies.

Here, we present the universal relations in an anisotropic p-wave Fermi gas. Starting from the time-independent Schrödinger equation, we find asymptotic power laws in a high-momentum tail of the momentum distribution and a short-range singularity of the density-density correlation function:

$$n_{\boldsymbol{p}} \simeq \sum_{m,m'=-1}^{1} C_{mm'} \frac{Y_1^{m*}(\hat{\boldsymbol{p}}) Y_1^{m'}(\hat{\boldsymbol{p}})}{p^2},$$

$$g(\boldsymbol{r}) \simeq \sum_{m,m'=-1}^{1} C_{mm'} \frac{Y_1^{m*}(\hat{\boldsymbol{r}}) Y_1^{m'}(\hat{\boldsymbol{r}})}{16\pi^2 V r^4},$$

which define the p-wave contact tensor $C_{mm'}$. Here, $Y_1^m(\hat{\boldsymbol{p}})$ is the spherical harmonic function. States at finite temperatures also have this asymptotic form. The above expressions show that the p-wave contact tensor represents the strength of the short-range correlations. On the other hand, we derive the adiabatic sweep theorem, which relates the p-wave contact tensor to the total energy:

$$C_{mm'} = 32\pi^2 M \frac{\partial E}{\partial(-1/v_{mm'})}. \tag{1}$$

Here, $v_{mm'}$ is a generalized p-wave scattering volume, whose diagonal components v_{mm} correspond to the m-dependent p-wave scattering volume and are controlled by a magnetic Feshbach resonance. Its off-diagonal components characterize the scattering amplitude for processes that change the projected relative angular momentum. They cannot be tuned by a magnetic Feshbach resonance. However, we propose an experimental scheme with a two-photon Raman process to control $v_{mm'}$. The derivation of the above general results is based on the axisymmetric interaction, but the expressions for the correlation functions are applicable to states in which the axisymmetry is broken. Of particular interest is the p-wave superfluid phase, where the axisymmetry can be spontaneously broken. We derive the mean-field expression of the p-wave contact tensor, and point out that the presence of the nonzero off-diagonal components of $C_{mm'}$ unambiguously signals the axisymmetry broken superfluid.

The second issue is related to the universal equation of state: do unitary Bose systems have universal equations of state, and if so, what do they look like? To address these questions, we consider an impurity-boson system, where a single impurity interacts via a unitarity-limited interaction with a weakly interacting Bose gas. Such a system has recently been realized by using a mixture of atomic species, and found to be relatively stable and amenable to detailed investigations. This impurity-boson system is known to support an infinite series of Efimov states unless the impurity is infinitely heavy. They are universal three-body bound states, which appear when the impurity-boson interaction is unitarity-limited and asymptotically show discrete scale invariance. To fix the scale of the Efimov states, one needs to introduce a length parameter called the three-body parameter, in addition to the s-wave scattering length. The three-body parameter generally depends on microscopic details of the system. Here, we describe this situation by the r_0-model and the Λ-model. The r_0-model has a finite effective range, while the Λ-model imposes a ultraviolet-cutoff in three-body processes involving the impurity and two bosons. These length parameters serve to determine the three-body parameter. We address the universality in the impurity-boson system by calculating properties of this system using the r_0- and Λ-models and comparing the results. We also compare our results with those of the hard-core model studied by Ardila and Giorgini [Phys. Rev. A **92**, 033612 (2015) and Phys. Rev. A **94**, 063640 (2016)], when the data are available. We consider three setups: the equal-mass few-body system, the few-body system with the infinite-mass impurity, and the impurity immersed in a finite-density Bose-Einstein condensate.

In the first setup, we consider an impurity interacting with one, two, and three non-interacting bosons, where the mass of the impurity is equal to that of bosons. This few-body system is interesting not only in its own right, but also from a point of view of the many-body problem; when we consider an impurity in a many-body Bose gas, the few-body problem describes the limit of the dilute background. By comparing dimensionless ratios constructed from the energies in the ground state and the first-excited state of the three-body and four-body systems, we find that the results of the two models agree within 1%. This goes beyond the original notion of the universality in the Efimov effect, which holds true only for excited states whose radii are much larger than microscopic length scales of interactions. We argue that the equal-mass few-body system has the universal spectrum from the ground state because of a large separation between microscopic scales and the scale of the three-body physics.

The second setup, the limit of a heavy impurity, is special in that Efimov's three-body parameter diverges and that the scale invariance of the unitarity limit is recovered in the three-body sector. In the absence of boson-boson interactions, the problem now looks like free particles interacting with an external central field and is apparently trivial. However, we find that the r_0-model is still nontrivial because of the effective three-body repulsion, which we make explicit by mapping the model to a bosonic extension of Anderson's single impurity model. By analytically solving the three-body problem within the bosonic Anderson model, we find that the limit $r_0/a \to 0$ makes the spectrum trivial, but that the approach of the energy to its limiting value is as slow as $1/\log(r_0/a)$. We numerically solve the Λ-model and find a similar logarithmic correction. We thus conclude that this logarithmic correction is a universal consequence of the effective three-body repulsion.

The last subject of this thesis is the universality of the ground-state properties of a mobile impurity immersed in a Bose gas, which is called a Bose polaron. We concentrate on the case where the masses of the impurity and a boson are equal and the s-wave scattering length associated with the impurity-boson interaction is infinite. We employ a variational wave function, which incorporates three Bogoliubov excitations on top of the background Bose-Einstein condensate. The universal spectrum of the few-body system that we have discussed above ensures the universality of the ground-state polaron energy in the dilute limit. By comparing the ground-state energy calculated within the r_0-model and the Λ-model, we argue that it is a universal function of the three-body parameter even when the density of the Bose gas is high enough to smear out the well-defined Efimov states. The universal function corresponds to the Bertsch parameter in the unitary Fermi gas, although in the Bose polaron, it is no longer a single parameter but a function. We also calculate other observables and discuss their universality. In particular, we find that the quasi-particle residue is strongly suppressed in the limit of $a_{\mathrm{B}} \to 0$, where the background Bose gas is non-interacting. This is consistent with a perturbation theory, in which the infrared divergence leads to the vanishing residue. When we take into account a small repulsive interaction between bosons, we show within the perturbation theory that the divergence is cancelled and that the residue becomes finite. We also confirm from the variational calculation that the residue is finite outside the perturbative regime if a_{B} is nonzero.