TOTAL OPTIMAL SYNTHESIS METHOD FOR FRAME STRUCTURES DEALING WITH SHAPE, SIZING, MATERIAL VARIABLES AND PRESTRESSING

（[illegible]およびプレストレスを設計変数
とした[illegible]に関する研究）

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TOTAL OPTIMAL SYNTHESIS METHOD FOR FRAME STRUCTURES DEALING WITH SHAPE, SIZING, MATERIAL VARIABLES AND PRESTRESSING

(構造形状・断面寸法・部材の材種およびプレストレスを設計変数とした骨組構造物の総合的最適設計法に関する研究)

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CONTENTS

ACKNOWLEDGMENTS i
Chapter 1 INTRODUCTION 1

1-1. STATEMENT OF THE PROBLEM AND OBJECTIVES 1
1-2. REVIEW OF PREVIOUS WORKS 3
1-3. OUTLINE OF THESIS 6
APPENDIX 1-1 General description of dual method using direct and reciprocal variables 11
(1) Optimization algorithm 11
(2) 2-bar truss example 15
REFERENCES 19

Chapter 2 SHAPE AND SIZING OPTIMIZATION OF STEEL CABLE-STAYED BRIDGES 25

2-1. INTRODUCTION 25
2-2. FORMULATION OF OPTIMUM DESIGN PROBLEM 26
(1) Design variables 26
(2) Design constraints 28
(3) Stress analysis and critical stress conditions 30
(4) Formulation of optimum design problem 32
2-3. OPTIMIZATION ALGORITHM 33
(1) Convex and separable approximation 33
(2) Optimization algorithm based on the dual method 34
(3) Calculation of behavior sensitivities 39
2-4. NUMERICAL DESIGN EXAMPLES 39
(1) Design example for steel cable-stayed bridge with 48 cable stays 39
(2) Practical design example of the Swan Bridge at the Tokiwa Park 46
2-5. CONCLUSIONS 53
APPENDIX 2-1 Calculation of sensitivities of stress constraints with respect to design variables 54
REFERENCES 56

Chapter 3 OPTIMUM DESIGN SYSTEM FOR STEEL CABLE-STAYED BRIDGES DEALING WITH SHAPE, SIZING VARIABLES AND CABLE PRESTRESSES ········ 57

3-1. INTRODUCTION ········ 57
3-2. FORMULATION OF OPTIMUM DESIGN PROBLEM ········ 59
(1) Design variables ········ 59
(2) Design constraints ········ 60
(3) Formulation of optimum design problem ········ 62
3-3. OPTIMUM DESIGN METHOD BY TWO-STAGE OPTIMIZATION PROCESS ········ 62
(1) Two-stage optimization process ········ 62
(2) First stage optimization process ········ 66
(3) Second stage optimization process ········ 67
(a) Calculation of sensitivities with respect to pseudo-loads $\mathbf{P}_p$ 67
(b) Formulation of liner programming problem ········ 68
(c) Determination of the best $\Delta\mathbf{P}_p$ by modified LP algorithm ········ 69
3-4. OPTIMUM DESIGN PROCESS OF STEEL CABLE-STAYED BRIDGE ········ 72
3-5. NUMERICAL DESIGN EXAMPLES ········ 73
(1) Design conditions for the elements of main girder, pylon and cables 75
(2) Design example in which $\mathbf{Z}$ is the only design variable ········ 75
(3) Design example with $\mathbf{Z}, \mathbf{X}_C, Y_C$ as design variables ········ 76
(4) Design example with $\mathbf{Z}$ and $\mathbf{P}_p$ as design variables ········ 80
(5) Effect of $\mathbf{P}_p$ on the optimum $\mathbf{X}_C$ and Y_C ········ 80
(6) Effect on $\mathbf{P}_p$, $\mathbf{A}_C$ and TCOST due to changes in $\mathbf{X}_C$ and Y_C ··· 81
(7) Design example in which $\mathbf{X}_C$ and Y_C are modified from aesthetic considerations ········ 82
3-6. CONCLUSIONS ········ 86
REFERENCES ········ 87

Chapter 4 TOTAL OPTIMAL SYNTHESIS METHOD FOR TRUSS STRUCTURES CONSIDERING SHAPE, SIZING AND MATERIAL VARIABLES SUBJECTED TO STATIC LOADS ··· 89

4-1. INTRODUCTION 89
4-2. FORMULATION OF PRIMARY OPTIMUM DESIGN PROBLEM 90
(1) Design variables 90
(2) Stress and displacement constraints 91
(3) Primary optimum design problem 91
4-3. OPTIMAL STRUCTURAL SYNTHESIS METHOD FOR TRUSS STRUCTURES SUBJECTED TO STATIC LOADS 92
(1) Convex and separable approximate subproblem 92
(2) Calculation of behavior sensitivities 93
(3) Improvements of **A**, **Y** and **M** by a two-stage minimization process of the Lagrangian function 95
(a) Two-stage minimization process of the Lagrangian function ... 95
(b) Lagrangian function **L** 96
(c) First stage min.-max. process of **L** with respect to *EA*, **Y** and λ 96
(d) Calculation of search direction **S** by using Cholesky decomposition ... 99
(e) Second stage minimization of **L** with respect to **M** and λ 100
(f) Scaling of initial design variables 103
4-4. NUMERICAL DESIGN EXAMPLES 105
(1) Material sets 105
(2) Effect of the initial material kinds on the optimum solution 107
(3) Optimum solutions for three types of 31-bar simple span trusses 109
(4) Effect of displacement constraints at arbitrary panel points on optimum solution 113
(5) Optimum solutions for 31-bar 2-span continuous trusses 113
4-5. CONCLUSIONS 117
APPENDIX 4-1 Calculation of the *jk*th element of Hessian matrix 118
REFERENCES 119

Chapter 5 TOTAL OPTIMAL SYNTHESIS METHOD FOR TRUSS STRUCTURES SUBJECT TO STATIC AND FREQUENCY CONSTRAINTS 121

5-1. INTRODUCTION 121
5-2. FORMULATION OF PRIMARY OPTIMUM DESIGN PROBLEM ... 122
(1) Design variables 122

(2) Stress, displacement and frequency constraints ········· 123
(3) Primary optimum design problem ········· 123
5-3. OPTIMAL STRUCTURAL SYNTHESIS METHOD FOR TRUSS STRUCTURES SUBJECT TO STATIC AND FREQUENCY CONSTRAINTS ········· 124
(1) Convex and separable approximate subproblem ········· 124
(2) Calculation of sensitivities of frequency constraints with respect to design variables ········· 124
(3) Improvements of **A**, **S** and **M** by a two-stage minimization process of the Lagrangian function ········· 127
(a) Two-stage minimization process of the Lagrangian function ········· 127
(b) Lagrangian function ········· 127
(c) First stage minimization process of the Lagrangian function ········· 128
(d) Second stage minimization process of the Lagrangian function ········· 129
5-4. NUMERICAL DESIGN EXAMPLES ········· 131
(1) Material set ········· 131
(2) 15-bar truss examples ········· 133
5-5. CONCLUSIONS ········· 138
REFERENCES ········· 139

Chapter 6 TOTAL OPTIMAL SYNTHESIS METHOD FOR TRUSS STRUCTURES SUBJECTED TO STATIC AND SEISMIC LOADS ········· 142

6-1. INTRODUCTION ········· 142
6-2. FORMULATION OF PRIMARY OPTIMUM DESIGN PROBLEM ··· 143
(1) Design variables ········· 143
(2) Determinations of **t** and **D** by suboptimization ········· 145
(3) Design constraints ········· 146
(4) Calculation of displacements and axial forces due to seismic loads ········· 148
(5) Formulation of primary optimum design problem ········· 150
6-3. OPTIMAL STRUCTURAL SYNTHESIS METHOD FOR TRUSS STRUCTURES SUBJECTED TO STATIC AND SEISMIC LOADS ········· 151
(1) Convex and separable approximate subproblem ········· 151

(2) Calculation of sensitivities of displacements and axial forces due to seismic loads ··· 152
(3) Improvements of **A**, **S** and **M** by a two-stage minimization process of the Lagrangian function ··· 154
(a) Two-stage minimization process of the Lagrangian function ··· 154
(b) Lagrangian function ··· 154
(c) Improvements of *EA* and **S** by the first stage minimization process ··· 155
(d) Improvements of **A** and **M** by the second stage minimization process 157
6-4. NUMERICAL DESIGN EXAMPLES ··· 161
6-5. CONCLUSIONS ··· 167
REFERENCES ··· 169

Chapter 7 CONCLUSIONS ··· 171

Chapter 1 INTRODUCTION

1-1. STATEMENT OF THE PROBLEM AND OBJECTIVES

In the designs of structures, the most fundamental and significant design variables are the geometry of the structures, mechanical and economical properties of the materials and cross-sectional dimensions that are available for each member element. Furthermore, in the designs of cable-stayed bridges and prestressed concrete bridges the distribution of member forces, such as maximum and minimum bending moments and axial forces in the member elements, can be controlled considerably by introducing prestresses into cables. Therefore, the cable prestresses have also been treated as one of significant design parameters in the practical design. Depending on the characteristics of these design variables, shape and sizing variables (geometry of the structures and cross-sectional dimensions) are dealt with as continuous variables and material variables as discrete variables. The design variables on cable prestresses are dealt with as external loads.

During the past decades, in the field of structural optimization a large number of contributions have been made exclusively to the sizing optimization and, in the last two decades, the shape optimization and topology optimization have been studied considerably. In recent years, some design sensitivity analysis methods which can calculate the sensitivities of structural behaviors with respect to design variables in the optimization process efficiently have also been presented. The evolutional computer technologies such as artificial neural network and genetic algorithm have also been studied and attempted to apply to the field of structural optimization. However, very little attention has been yet paid to total optimization of structures including material variables and cable prestresses in addition to shape and sizing variables for the reason of complexity of design methodology. Therefore, the establishment of total optimal synthesis method for structures, which can optimize the design problem considering above design variables simultaneously, is a significant and challenging task. From these standpoints, this study has two objectives.

The first objective in this study is to propose a total optimal synthesis method for steel cable-stayed bridges which can determine the most economical values of cable anchor positions on the main girder, heights of pylons, cross-sectional dimensions of all member elements and cable prestresses subject to stress constraints specified in the Japanese Specifications for Highway Bridges. The cable-stayed bridge is one of the most attractive types of bridges because of its applicability of a wide range of span lengths from 100m to 800m and its economical and aesthetic excellences. For the reason that the cable-stayed bridge is a highly statically indeterminate structure, its structural behavior and total cost are greatly affected by the cable arrangement and stiffness distribution in the cables, main girder and pylon. Furthermore, the aesthetic view of the bridge is also affected by the cable anchor positions on the main girder and the heights of pylons, therefore, it is very important that the bridge is designed by balancing totally the mechanical, economical and aesthetic characteristics, and the manufacturing and erection conditions. In the practical design of cable-stayed bridges, the design parameters such as cable anchor positions on the main girder, heights of pylons and cross-sectional dimensions of member elements in the main girder, pylons and cables are determined after repeated trial and error by reviewing earlier experiences and bridge design data which have been already constructed. Therefore, the establishment of a rational and efficient optimum design method for cable-stayed bridges, which can determine the optimum values of the design variables mentioned above at various design conditions rigorously and automatically, contributes significantly to the practical design process of the cable-stayed bridge.

From this point of view, a rigorous and efficient optimum design method for steel cable-stayed bridges is developed by utilizing dual method using direct and reciprocal variables, LP algorithm and two-stage optimization technique. The theoretical rigorousness, efficiency and practical usefulness of the proposed optimum design method are demonstrated by giving several numerical design examples of practical-scale steel cable-stayed bridges.

The second objective is to propose a total optimal synthesis method for truss structures which can determine the optimum solutions for the design problems considering the coordinates of the panel points, cross-sectional areas and discrete material kinds of member elements simultaneously as design variables. As mentioned before, very little attention has been paid to total optimization of structures with material variables in addition to shape and sizing variables for the reason of the inherent combinatorial complexity associated with the discrete material-selection problems.

In this study, the total optimal synthesis method is developed by utilizing the two-stage minimization process of the Lagrangian function, dual method using direct and reciprocal variables and discrete sensitivity analysis. The optimal synthesis method is applied to the minimum-cost designs of truss structures subject to stress and displacement constraints due to static loads, and then, the optimal synthesis method is extended to be able to deal with frequency constraints in addition to stress and displacement constraints due to static loads. Finally, the optimal synthesis method is extended to solve the problems of truss structures subjected to static and seismic loads. The structural optimization dealing with shape, sizing and material variables subject to dynamic constraints is the first challenge in the world. The rigorousness, reliability and efficiency of the proposed optimal structural synthesis method are illustrated by applying the method to various minimum-cost design problems of statically indeterminate truss structures. Furthermore, the problem point of optimization algorithm for the cases when the dynamic constrains are taken into account is stated and the solution algorithm for the problem is proposed.

1-2. REVIEW OF PREVIOUS WORKS

In this section, historical background on structural optimization, and previous works related to the topics of this thesis are reviewed.

In 1960, an optimum structural design method coupling a finite element method of structural analysis with mathematical programming techniques was first presented by Schmit[1]. In 1960's, various efficient mathematical programming techniques such as SLP method, gradient projection method, feasible direction method, penalty function methods, etc. were developed and extension of these algorithms to the structural optimization was attempted[2,3]. However, the implementation of structural optimization method incorporating mathematical programming techniques accompanied with some difficulties. The main obstacle was associated with the fact that general structural optimization problems involve large numbers of design variables and design constraints. In early 1970's Venkayya, khot and Berke presented optimality criteria based structural optimization method[4-8]. The design algorithm based on optimality criteria method was very simple and the optimal solutions were obtained quite efficiently for large scale design problems subject to stress and single displacement constraints. However, the essential difficulties in applying the optimality criteria method were the identification of correct critical constraint sets at the optimal solution and the derivation of a set of necessary conditions that must be satisfied at the optimal solution. To overcome these shortcomings the dual method and approximate concepts were presented by Fleury[9-12]. Prasad, Starnes and Haftka[13,14] studied the convex approximation concept and, in 1986, Fleury and Braibant[15] presented a new dual method using direct and reciprocal variables to ensure the convexity of approximate design problem. During the late 1980's and early 1990's the structural optimization algorithm based on the dual method had been updated by Fleury, Svanberg and Zhou[16-21] and the computational efficiency was competed. In the last decade, the sensitivity analysis methods which can calculate the sensitivities of structural behaviors with respect to design variables in the optimization process efficiently have been studied considerably[22-24]. Recently, the evolutional computer technologies such as artificial neural network and genetic algorithm have also been studied and attempted to apply to the field of structural optimization[25-29].

On the other hand, during the 1970's and early 1980's many researchers paid attention to the geometrical optimization of truss structures[30-35]. Most of these works were achieved by using the two separate design spaces for geometrical variables and sizing variables. Ohkubo[36,37] presented an efficient optimum design method using suboptimization concept in which cross-sectional areas of all member elements and height of truss structures were directly optimized by using SLP method. In 1981, Imai and schmit[38] presented a configurational optimization method for truss structures using an augmented Lagrange multiplier method to deal with geometrical variables and sizing variables in same design space. Svanberg[39] proposed a separable convex approximation approach to find the optimum solution of truss structures dealing with geometrical variables and sizing variables by using dual method. Zhou[19] presented an efficient optimization method for geometrical optimization of truss structures by using two-level approximation concept. During the same time a number of contributions to the shape optimization of the boundaries of two- and three-dimensional bodies were also made and those were reviewed in the ref.[40] by Haftka and Grandhi. In the last decade, topology optimizations have been studied considerably[41-44]. However, only a bit of focus has been on optimization with material selection which requires a discrete/continuous formulation of the problem. In earlier studies on this topic, the efforts to solve such a problem were made by applying the discrete or integer programming algorithms[45-48]. Okumura and Ohkubo treated the discrete variables as quasi-continuous variables and solved the problem by using SLP algorithm[49]. In 1980's some extensions of the optimization algorithms were attempted to solve the discrete/continuous formulation problems[50-52]. In 1992 and 1993, Ohkubo et al. presented a hybrid optimal synthesis method in which shape, material and sizing variables of truss structures were optimized simultaneously [53,54].

In the field of the optimum design of cable-stayed bridges, the study was begun in the late 1970's and the research on this topic has been done mainly in Japan. Yamada and Daiguji studied an optimum design method based on the optimality criteria[55]. Kobayashi et al. presented a multilevel optimal design method by using the SLP algorithm and applied it to three types of cable-stayed bridges with different supporting systems[56]. Gimsing[57] investigated the rational cable arrangement of cable-stayed bridges from the structural system analysis viewpoint. All of these works have focused to determine the optimum element sizes in the main girder, pylons and cables.

In the earlier studies of determination method for cable prestresses in steel cable-stayed bridges, Yamada and Daiguji[55] studied a method to determine the optimal cable prestresses on the basis of the element optimization in the main girder. Maeda et al.[58] and Nagai et al.[59] determined the cable prestresses by calculating the support reactions of multispan continuous beam in which the main girder in cable-stayed bridge is considered as the multispan continuous beam with supports at the cable anchor positions. Yamada et al. studied the method for determination of cable prestresses based on the minimum strain energy criterion[60]. Hoshino studied a practical method to determine the cable prestresses on the basis of a structural analysis method using modified cross-sectional properties under the minimum cost criterion[61]. Torii et al.[62] studied a method to determine the cable prestresses without recursive calculation by introducing the relation between the redundant forces in statically indeterminate structures and objective function. Nakamura and Wyatt determined the cable prestresses on the basis of the limit states design code by using a linear programming algorithm[63]. In most of these researches, the cable prestresses are determined so as to reduce the peaks of positive and negative bending moments in the main girder and to average out the bending moment distributions in the main girder.

In the field of optimal structural control and seismic resistant design, optimum design methods with frequency constraints have been studied by many researchers since the earliest study by Turner[64] in 1967. Most of these design methods have been developed on the basis of the optimality criteria methods using cross-sectional areas of member elements as the design variable[65-68]. Felix and Vanderplaats[69] studied the optimum configuration design of truss structures subject to stress, Euler buckling, displacement and natural frequency constraints using a multilevel optimization technique. The methods for computing the derivatives of eigenvalues and eigenvectors have also been studied by many researchers[23,70-75].

With regard to the optimum design of structures subjected to static and seismic loads, a number of contributions have also been made since the earliest study by Pierson[76], but most of the works have focused to determine the optimum member element size distributions in many types of structures [77-86].

1-3. OUTLINE OF THESIS

In this thesis, the structural optimization method based on dual algorithm using direct and reciprocal variables[15] described in Appendix 1-1 is successfully extended to propose total optimal synthesis method for frame structures. The outline of this thesis and summary of each Chapter are follows.

In Chapter 2, a rigorous and efficient optimum design method for steel cable-stayed bridges is presented. In this design method, not only the cross-sectional dimensions of the cables, main girder and pylon elements but also the cable anchor positions on the main girder and the heights of pylons are dealt with as the design variables. The optimization dealing with cable anchor positions on the main girder and the heights of pylons is the first challenge in field of optimization of cable-stayed bridges. The design problem is formulated as a minimum-cost design problem subject to the stress constraints taken from the Japanese Specifications for Highway Bridges[JSHB]. The magnitudes of the dead loads and traffic loads, impact factor, effective widths of the flange plates in the main girder and effective lengths of the pylon elements for bucklings are also taken from the JSHB. The working stress at a structural element is calculated as the sum of the stresses due to dead loads in the cantilever system at the erection closing stage and the stresses due to the traffic loads and a part of the dead loads in the continuous girder system at the service stage. The maximum and minimum values of axial force, shearing force and bending moment at the stress inspection points due to traffic and impact loads are calculated by using the corresponding influence lines.

The cost-minimization problem is approximated by using the first-order partial derivatives of objective function and behavior constraints, and mixed direct/ reciprocal design variables. The approximate subproblem is solved by dual method.

The proposed optimum design method has been applied to the minimum-cost design problems of practical-scale steel cable-stayed bridge with 48 cable stays. The theoretical rigorousness and efficiency of the proposed optimum design method are demonstrated by investigating the optimum solutions at different design conditions. The significance of dealing with the cable anchor positions on the main girder and the heights of pylons is also emphasized for the minimum-cost designs of cable-stayed bridges. Furthermore, the practical usefulness of the proposed optimum design method is also demonstrated by giving the practical design example of the Swan Bridge (Ube city, Yamaguchi) which was designed by using this proposed design method.

In Chapter 3, the optimum design method stated in Chapter 2 is extended to be able to deal with cable prestresses as the design variables, and a general purpose, rigorous and efficient optimum design system for steel cable-stayed bridges is developed. In this design system the pseudo-loads applied to the cables are selected as the design variables with respect to cable prestresses and the optimum cable prestresses are determined from the economical viewpoint. The design problem is formulated as a minimum-cost design problem subject to the stress constraints taken from the JSHB. By investigating a simple design example in which a pseudo-load is dealt with as design variables in addition to cross-sectional dimensions, it is illustrated that the computational effort to obtain the optimum solution of the design problem in which the pseudo-load is dealt with as design variables in addition to cross-sectional dimensions is remarkably increased compared with that of the design problem with only cross-sectional dimensions. This result indicates that the problems of convergency and reliability of the result obtained will arise when the number of pseudo-loads is increased and, furthermore, the cable arrangement is also taken into account as design variables. For this reason, the following powerful two-stage optimum design process is proposed to solve the cost-minimization problem. At the first stage optimization process, the cable arrangement and sizing variables are optimized by the optimization algorithm based on dual method which is developed in Chapter 2. At the second stage optimization process, the optimum values of pseudo-loads, which induce the optimum prestresses into the cables, and the optimum sizing variables are determined so as to minimize the total cost of the bridge further by utilizing the sensitivities of objective function, behavior constraints and cross-sectional dimensions with respect to the pseudo-loads and a modified LP algorithm.

The proposed optimum design method has been applied to the minimum-cost design problems of practical-scale steel cable-stayed bridge with 64 cable stays. The theoretical rigorousness, efficiency and practical usefulness of the proposed optimum design system are demonstrated by giving several numerical design examples and investigations of the optimum solutions at various design conditions. It is also illustrated that 2.6%-4.1% of the total cost of the bridge can be reduced by giving the optimum prestresses in the cables.

In Chapter 4, an optimal structural synthesis method is presented to determine the optimum solutions for the design problems of truss structures considering the coordinates of the panel points, cross-sectional areas and discrete material kinds of member elements simultaneously as design variables. The stress and displacement constraints due to static loads are taken into account in the optimization process.

The primary design problem is transformed into an approximate subproblem of convex and separable form by using mixed direct/reciprocal design variables, shape, material and sizing sensitivities. The approximate subproblem is solved by a dual method, where the separable Lagrangian function for each member element is introduced. In this study, shape and sizing variables are dealt with as continuous variables and material variables as discrete variables. Therefore, the following two-stage minimization process of the Lagrangian function is proposed to solve the design problem including the continuous and discrete variables. At the first stage minimization process, the product of modulus of elasticity E and cross-sectional area A, EA, is treated as one continuous design variable and the optimum values of EA and shape variable are determined by minimizing the Lagrangian function with respect to EA and shape variable. Then, at next stage the shape variable is maintained constant, and the better combination of cross-sectional area and material kind for each member element is searched independently to reduce the Lagrangian function by comparing the values of discretized Lagrangian function while keeping the activeness of the constraints which are determined by the first minimization process. This separable minimization of the element Lagrangian function with respect to material and sizing variables simplifies the inherent combinatorial complexity associated with the discrete material-selection problems.

The generality, rigorousness and reliability of the proposed optimal structural synthesis method are illustrated by applying the method to various minimum-cost design problems of 31-bar trusses subject to stress and displacement constraints and investigations of the optimum solutions at various design conditions. It is also demonstrated that the optimum solutions can be obtained after 15-25 iterations efficiently even when the algorithm is initialized with the worst possible material distribution.

In Chapter 5, the hybrid optimal synthesis method proposed in Chapter 4 is applied to solve the optimum design problem of truss structures subject to both static and dynamic constraints.

In the optimum design method, the primary design problem is transformed into an approximate subproblem of convex and separable form by using mixed direct/reciprocal design variables and the sensitivities of shape, material and sizing variables. The sensitivities of static and frequency constraints with respect to design variables are calculated analytically by using the differentials of the stiffness and mass matrices. The approximate subproblem is solved by utilizing the two-stage minimization process of the Lagrangian function, concepts of convex and linear approximation, dual method and discrete sensitivity analysis proposed in Chapter 4.

The rigorousness, reliability and efficiency of the proposed optimal structural synthesis method are illustrated by applying the method to various minimum-cost design problems of 15-bar truss subject to stress, displacement and natural frequency constraints. It is also emphasized that the vibration mode and frequency of truss structure are very sensitive to the distribution of EA and shape of structure, and the vibration mode might be changed by improvements of cross sections and shape of structure at the first stage of the minimization process. Therefore, it is necessary to calculate the exact vibration mode and frequency and to examine the activeness of frequency constraint at the end of the first stage minimization process to ensure the smooth convergence to the optimum solution.

In Chapter 6, the optimal synthesis method is extended to solve problems of truss structures subjected to static and seismic loads. The structural optimization dealing with shape, sizing and material variables subjected to static and seismic loads is the first challenge in the world. In the optimum design process, all member elements are assumed to be made of circular steel pipes. By applying the concept of suboptimization, the cross-sectional areas of all member elements are dealt with as sizing variables instead of the diameters and plate thicknesses of circular steel pipes. The objective function is the total construction cost of truss structures considering not only the cost of truss structures but also the cost of land of construction site. The stress, displacement and slenderness ratio constraints are considered as behavior and side constraints. From the practical design viewpoint, the allowable stresses of member elements are taken from the JSHB. The stresses of all member elements due to seismic loads are calculated by a response spectrum method using the acceleration response spectrum which is specified in the JSHB.

In the optimum design method, the primary optimum design problem expressed in terms of primary design variables, namely, shape, material and sizing variables, is transformed into an approximate subproblem of convex and separable form by using mixed direct/reciprocal design variables and the sensitivities of behavior constraints with respect to the primary design variables. The sensitivities of stress and displacement constraints due to seismic loads with respect to design variables are calculated analytically by using the sensitivities of eigenvalues, eigenvectors, participation factor and acceleration response spectrum. The separable Lagrangian function is introduced for the approximate subproblem and the Lagrangian function is minimized by the algorithm proposed in Chapters 4 and 5 incorporating suboptimization technique.

In the numerical design examples, the numerical results of minimum-cost design problems of 193-bar transmission tower truss are shown for the three design conditions with different unit costs of land of construction sites. By comparing the optimum solutions, the rigorousness, reliability and efficiency of the optimum design method are demonstrated. It is also emphasized that the optimal configuration, distribution of material kinds and cross-sectional areas of all member elements are significantly influenced by the value of unit cost of land of construction site.

Finally, Chapter 7 is devoted to description of conclusions obtained through Chapters 2-6.

APPENDIX 1-1 General description of dual method using direct and reciprocal variables

In this study, total optimal synthesis method for frame structures is developed on the basis of dual method using direct and reciprocal variables which is originated by Fleury[15]. In this appendix, the structural optimization method based on dual algorithm is described with 2-bar truss example[87].

(1) Optimization algorithm

The primary structural optimization problem can be mathematically formulated as

Find $\mathbf{X}$, which

minimize $\mathrm{W}(\mathbf{X})=\sum_{i=1}^{n}\rho_i l_i(\mathbf{X})A_i(\mathbf{X})$ (A1-1)

subject to $g_j(\mathbf{X})\leq 0 \qquad (j=1,\cdots,m)$ (A1-2)

$X_i^l \leq X_i \leq X_i^u \qquad (i=1,\cdots,n)$ (A1-3)

In the above expressions, $\mathbf{X}$ denote the design variables which correspond to cross-sectional dimensions and geometry of structures. The weight or cost of structure is taken into account as the objective function $\mathrm{W}(\mathbf{X})$ which is expressed as functions of unit cost or weight density ρ_i, member length $l_i(\mathbf{X})$ and cross-sectional area of the ith member element $A_i(\mathbf{X})$. $\mathbf{g}(\mathbf{X})$ are the behavior constraints such as limitations on the stresses and displacements. n and m are, respectively, the numbers of design variables and behavior constraints. Superscripts l and u represent the lower and upper limits of the design variables.

Applying the convex and linear approximation concept, the primary optimum design problem can be approximated by using first-order terms of the Taylor series expansions with respect to the direct variables of $\mathbf{X}$ and their reciprocal variables. In the approximate subproblem, the change of objective function $\Delta\mathbf{W}(\mathbf{X})$ is taken into account instead of $\mathbf{W}(\mathbf{X})$. Then, the following convex and separable approximate subproblem can be derived.

Find $\mathbf{X}$, which

minimize $$\Delta\mathbf{W}(\mathbf{X})=\sum_{i=1}^{n}\left[\omega_{Xi_{(+)}}X_i-\omega_{Xi_{(-)}}\left(X_i^0\right)^2\frac{1}{X_i}\right] \tag{A1-4}$$

subject to $$\bar{g}_j(\mathbf{X})=\sum_{i=1}^{n}\left[c_{ji_{(+)}}X_i-c_{ji_{(-)}}\left(X_i^0\right)^2\frac{1}{X_i}\right]+\bar{U}_j\le 0 \qquad (j=1,\cdots,m) \tag{A1-5}$$

$$X_i^l\le X_i\le X_i^u \qquad (i=1,\cdots,n) \tag{A1-6}$$

where $$\bar{U}_j=g_j\left(\mathbf{X}^0\right)-\sum_{i=1}^{n}X_i^0\left[c_{ji_{(+)}}-c_{ji_{(-)}}\right]$$

$$\omega_{Xi}=\frac{\partial\mathbf{W}(\mathbf{X})}{\partial X_i}, \qquad c_{ji}=\frac{\partial g_j(\mathbf{X})}{\partial X_i}$$

In the above expressions, the symbols (+) and (-) express the signs of the first-order partial derivatives.

A separable Lagrangian function can be introduced for the subproblem as

$$\mathbf{L}(\mathbf{X},\lambda)=\sum_{i=1}^{n}L_i(X_i,\lambda)+\sum_{j=1}^{m}\lambda_j\bar{U}_j \tag{A1-7}$$

where $\lambda_j\ge 0 \qquad (j=1,\cdots,m)$

where L_i is the element Lagrangian function given by (A1-8). λ_j is the Lagrange multiplier(dual variable) for the jth behavior constraint.

$$L_i(X_i,\lambda)=\left[\omega_{Xi_{(+)}}X_i-\omega_{Xi_{(-)}}\left(X_i^0\right)^2\frac{1}{X_i}\right]+\sum_{j=1}^{m}\lambda_j\left[c_{ji_{(+)}}X_i-c_{ji_{(-)}}\left(X_i^0\right)^2\frac{1}{X_i}\right] \tag{A1-8}$$

The solutions of the dual problem $\mathbf{X}^*$ and λ^* can be obtained by maximizing $\mathbf{L}(\mathbf{X},\lambda)$ with respect to λ and minimizing it with respect to $\mathbf{X}$. X_i, which minimizes $L_i(X_i,\lambda)$, is given by the simple expression in eq. (A1-9) which is derived from the necessary condition for the minimum of $L_i(X_i,\lambda)$, namely, $\partial L_i/\partial X_i=0$, and the side constraint.

$$\left.\begin{array}{lll}\text{if} & \left[X_i^l\right]^2<Z_{Xi}<\left[X_i^u\right]^2, & X_i^*=\sqrt{Z_{Xi}}\\ \text{if} & Z_{Xi}\le\left[X_i^l\right]^2, & X_i^*=X_i^l\\ \text{if} & Z_{Xi}\ge\left[X_i^u\right]^2, & X_i^*=X_i^u\end{array}\right\} \tag{A1-9}$$

where

$$\text{if} \quad \omega_{Xi} \geq 0, \qquad Z_{Xi} = \frac{-\sum_{j=1}^{m} \lambda_j c_{ji(-)} \left(X_i^0 \right)^2}{\omega_{Xi_{(+)}} + \sum_{j=1}^{m} \lambda_j c_{ji(+)}},$$

$$\text{if} \quad \omega_{Xi} < 0, \qquad Z_{Xi} = \frac{-\left(\omega_{Xi_{(-)}} + \sum_{j=1}^{m} \lambda_j c_{ji(-)} \right) \left(X_i^0 \right)^2}{\sum_{j=1}^{m} \lambda_j c_{ji(+)}}$$

By introducing the convex and separable approximate subproblem(eqs.(A1-4)-(A1-6)) using the direct and reciprocal design variables, $\mathbf{X}$ are improved analytically in eq.(A1-9) which is expressed explicitly in terms of λ. This is one of advantages of this optimization algorithm.

The minimized Lagrangian function with respect to $\mathbf{X}$ is denoted as $l(\lambda)$:

$$l(\lambda) = \min_{\mathbf{X}} \mathbf{L}(\mathbf{X}, \lambda) \tag{A1-10}$$

Following the minimization process with respect to $\mathbf{X}$, the Lagrangian function $l(\lambda)$ is maximized with respect to the dual variables λ by using a Newton-type algorithm. In the Newton-type algorithm, the dual variables $\bar{\lambda}$ corresponding to the active behavior constraints at the current stage are modified iteratively as

$$\bar{\lambda}^{(t+1)} = \bar{\lambda}^{(t)} + \alpha^{(t)} \cdot \mathbf{S}^{(t)} \tag{A1-11}$$

or in a scalar form

$$\bar{\lambda}_j^{(t+1)} = \bar{\lambda}_j^{(t)} + \alpha^{(t)} \cdot S_j^{(t)} \qquad (j \in \boldsymbol{S}_{AG}) \tag{A1-12}$$

where $\mathbf{S}^{(t)}$ denotes the search direction of λ for active constraints, S_{AG} is the set of active behavior constraints and α represents the step length parameter. The search direction $\mathbf{S}^{(t)}$ is given by

$$\mathbf{S}^{(t)} = -\left[\mathbf{H}\left(\bar{\lambda}^{(t)} \right) \right]^{-1} \cdot \nabla l\left(\bar{\lambda}^{(t)} \right) \tag{A1-13}$$

where $\nabla l(\bar{\lambda})$ is the vector of first derivatives of $l(\lambda)$ with respect to $\bar{\lambda}$ and the components of the vector are simply given by the approximate primary active constrains, namely,

$$\frac{\partial l(\overline{\lambda})}{\partial \overline{\lambda}_j} = \sum_{i=1}^{n}\left[c_{ji(+)}X_i^* - c_{ji(-)}\left(X_i^0\right)\frac{1}{X_i^*}\right] + \overline{U}_j \qquad \text{(A1-14)}$$

H in eq.(A1-13) is the Hessian matrix of $l(\overline{\lambda})$ with respect to $\overline{\lambda}$ and its jkth element is given by

$$H_{jk} = \frac{\partial^2 l(\overline{\lambda})}{\partial \lambda_j \partial \lambda_k} \qquad \left(j, k \in S_{AG}\right) \qquad \text{(A1-15)}$$

The elements of Hessian matrix can be calculated analytically and accurately in eq.(A1-15). It is also one of advantages of this optimization algorithm.

The step length α is first set as 1.0; however, its maximum allowable value is restricted by

$$\alpha_{\max}^{(t)} = \min_{S_j^{(t)}<0} \left|\frac{\overline{\lambda}_j^{(t)}}{S_j^{(t)}}\right| \qquad \left(j \in S_{AG}\right) \qquad \text{(A1-16)}$$

to ensure the non-negativity of λ when $\mathbf{S}^{(t)}$ includes negative components. When α is equal to $\alpha_{\max}^{(t)}$, the value of $\alpha_{\max}^{(t)}$ is investigated by evaluating the directional derivative at $\lambda^{(t+1)}$ expressed in eq.(A1-17).

$$\frac{\partial l(\overline{\lambda}^{(t+1)})}{\partial \alpha} = \frac{\partial l(\overline{\lambda}^{(t+1)})}{\partial \overline{\lambda}_k^{(t+1)}} \cdot \frac{\partial \overline{\lambda}_k^{(t+1)}}{\partial \alpha} = \frac{\partial l(\overline{\lambda}^{(t+1)})}{\partial \overline{\lambda}_k^{(t+1)}} \cdot S_k^{(t)} \qquad \text{(A1-17)}$$

where $\overline{\lambda}_k$ denotes the active constraint used for determination of $\alpha_{\max}^{(t)}$. The value of $\partial l(\overline{\lambda}^{(t+1)})/\partial \overline{\lambda}_k$ is calculated by eq.(A1-14).

The negative value of $\partial l(\overline{\lambda}^{(t+1)})/\partial \alpha$ indicates that $\lambda^{(t)}$ are modified too much along the search direction $\mathbf{S}^{(t)}$. Therefore, if $\alpha = \alpha_{\max}^{(t)}$ and $\partial l(\overline{\lambda}^{(t+1)})/\partial \alpha \leq 0$, then the step size α cut in half, namely, $\alpha = \alpha_{\max}^{(t)}/2$.

Based on the modifications of the dual variables λ through the above search procedure, the primary design variables **X** are improved by eq.(A1-9). The set of active constraints S_{AG} in the currently approximated design space also has to be updated. The min.-mix. process described above is iterated until **X** and λ converge to constant values $\mathbf{X}^*$ and λ^*. Then, $\mathbf{X}^*$ are assumed as new initial values of design variables and a new approximate subproblem is derived. The final optimum solutions can be determined by iterating the above process until $\mathbf{X}^*$ and λ^* converge to constant vales.

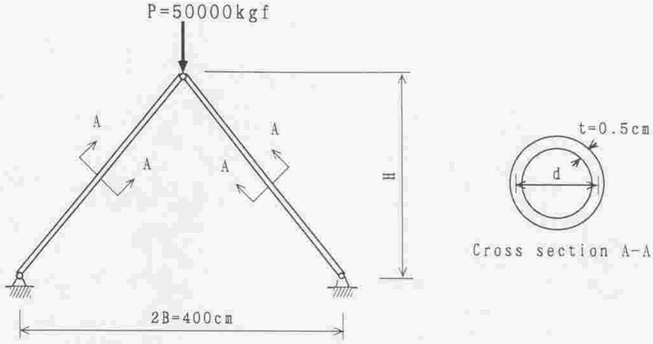

Fig.A1-1 2-bar truss and cross section of member elements

(2) 2-bar truss example[87]

The above structural optimization method is applied to the minimum weight design of 2-bar truss shown in Fig.A1-1. The cross sections of member elements are assumed to be made of circular steel pipes shown in Fig.A1-1. The plate thickness of cross section of a member element, t, and width of truss, 2B, are preassigned as 0.5cm and 400cm. The design variables are the height of truss, H, and averaged diameter of cross section, d. As the behavior constraints, the stress limitations on Euler buckling stress and yield stress, $g_1(\mathrm{H},d)$ and $g_2(\mathrm{H},d)$, are taken into account. The side constraints on H and d are imposed. Considering the above design variables, behavior constraints and side constraints, the minimum weight design of the truss is formulated as

Find H, d, which

minimize
$$\mathbf{W}(\mathrm{H},d)=\rho\cdot 2\pi dt\sqrt{(\mathrm{B}^2+\mathrm{H}^2)} \tag{A1-18}$$

subject to
$$g_1(\mathrm{H},d)=\frac{P}{2\pi td}\frac{\sqrt{(\mathrm{B}^2+\mathrm{H}^2)}}{\mathrm{H}}-\frac{\pi^2\mathrm{E}}{8}\frac{d^2+t^2}{\mathrm{B}^2+\mathrm{H}^2}\le 0 \tag{A1-19}$$

$$g_2(\mathrm{H},d)=\frac{P}{2\pi td}\frac{\sqrt{(\mathrm{B}^2+\mathrm{H}^2)}}{\mathrm{H}}-\sigma_y\le 0 \tag{A1-20}$$

$$\left.\begin{array}{l}\mathrm{H}^l\le\mathrm{H}\le\mathrm{H}^u\\ d^l\le d\le d^u\end{array}\right\} \tag{A1-21}$$

In the numerical example, weight density ρ, modulus of elasticity E, yield stress σ_y, the lower limits H^l and d^l, and the upper limits H^u and d^u are, respectively, assumed as 0.00785 kgf/cm^3, 2100000kgf/cm^2, 3000kgf/cm^2, 30.0cm and 3.0cm, and 600.0cm and 30.0cm. Considering these assumed values and preassigned values the primary optimum design problem can be written as

Find H, d, which

minimize
$$\mathbf{W}(\mathrm{H},d) = 0.024661 \cdot d\sqrt{(40000+\mathrm{H}^2)} \tag{A1-22}$$

subject to
$$g_1(\mathrm{H},d) = 15915.885 \cdot \frac{\sqrt{(40000+\mathrm{H}^2)}}{d \cdot \mathrm{H}} - 2590645 \cdot \frac{d^2+0.25}{40000+\mathrm{H}^2} \leq 0 \tag{A1-23}$$

$$g_2(\mathrm{H},d) = 15915.885 \cdot \frac{\sqrt{(40000+\mathrm{H}^2)}}{d \cdot \mathrm{H}} - 3000.0 \leq 0 \tag{A1-24}$$

$$\left.\begin{array}{l} 30.0 \leq \mathrm{H} \leq 600.0 \\ 3.0 \leq d \leq 30.0 \end{array}\right\} \tag{A1-25}$$

The above optimum design problem is solved by the dual method in which the initial values of H, d, λ_1 and λ_2 are, respectively, assumed as 500.0cm, 15.0cm, 0.01 and 0.01. The design space of the optimum design problem and iteration history to the optimum solution are depicted in Fig.A1-2. At the initial point (H^0, d^0)= (500.0,15.0), sensitivities of $\mathbf{W}(\mathrm{H},d)$, $g_1(\mathrm{H},d)$ and $g_2(\mathrm{H},d)$ with respect to H and d are calculated as

$$\omega_{\mathrm{H}} = \frac{\partial \mathbf{W}(\mathrm{H},d)}{\partial \mathrm{H}} = 0.3435 \tag{A1-26}$$

$$\omega_d = \frac{\partial \mathbf{W}(\mathrm{H},d)}{\partial d} = 13.280 \tag{A1-27}$$

$$c_{1\mathrm{H}} = \frac{\partial g_1(\mathrm{H},d)}{\partial \mathrm{H}} = 6.6154 \tag{A1-28}$$

$$c_{1d} = \frac{\partial g_1(\mathrm{H},d)}{\partial d} = -344.24 \tag{A1-29}$$

$$c_{2\mathrm{H}} = \frac{\partial g_2(\mathrm{H},d)}{\partial \mathrm{H}} = -0.3148 \tag{A1-30}$$

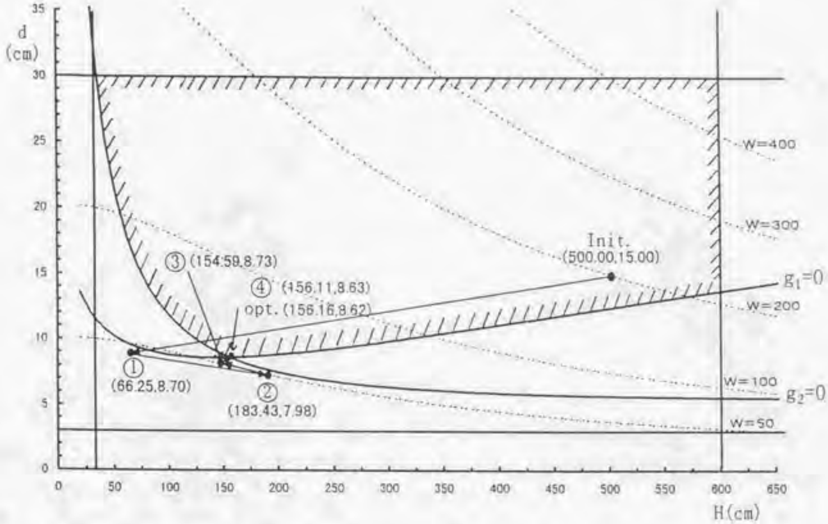

Fig.A1-2 Primary design space and iteration history to the optimum solution

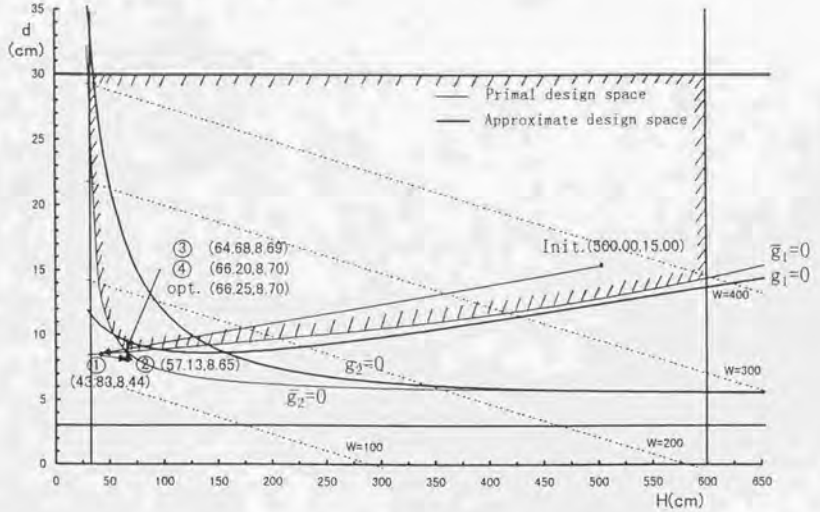

Fig.A1-3 Primary and approximate design spaces at H=500.0, d=15.0 and improvement history in the approximate design space

$$c_{2d} = \frac{\partial g_2(d, \mathrm{H})}{\partial d} = -76.110 \quad \text{(A1-31)}$$

Applying the concept of convex and separable approximation stated in eqs.(A1-4)-(A1-6) and using the sensitivities obtained above, the primary optimum design problem defined in eqs.(A1-22)-(A1-25) can be transformed into the following approximate subproblem at the initial point (H^0, d^0)=(500.0,15.0).

Find H, d, which

minimize $\Delta \mathbf{W}(\mathrm{H}, d) = 0.3435\mathrm{H} + 13.280d$ (A1-32)

subject to $\bar{g}_1(\mathrm{H}, d) = 6.61539\mathrm{H} + 77454.45\frac{1}{d} - 9340.745 \leq 0$ (A1-33)

$$\bar{g}_2(\mathrm{H}, d) = 78700.83\frac{1}{\mathrm{H}} + 17124.81\frac{1}{d} - 3156.26 \leq 0 \quad \text{(A1-34)}$$

$$\left.\begin{array}{l} 30.0 \leq \mathrm{H} \leq 600.0 \\ 3.0 \leq d \leq 30.0 \end{array}\right\} \quad \text{(A1-35)}$$

Solving the above subproblem through the optimization process in eqs.(A1-7)-(A1-17), we can obtain the solutions (H^1, d^1)=(66.25,8.70), $\lambda_1^1 = 0.00808$ and $\lambda_2^1 = 0.022137$. The design space for the currently approximate subproblem and improvement history in the approximate design space are shown in Fig.A1-3. Then, (H^1, d^1) are assumed as new initial values (H^0, d^0) and a new approximate subproblem is derived. This optimization process is iterated until H, d, λ_1 and λ_2 converge to constant values. As shown in Fig.A1-2, we can obtain the optimum solutions $(\mathrm{H}^{\mathrm{opt}}, d^{\mathrm{opt}})$= (156.16,8.62), $\lambda_1^{\mathrm{opt}} = 0.002181$ and $\lambda_2^{\mathrm{opt}} = 0.011463$ after 5 iterations.

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Chapter 2
SHAPE AND SIZING OPTIMIZATION OF STEEL CABLE-STAYED BRIDGES

2-1. INTRODUCTION

The cable-stayed bridge is one of the most attractive types of bridge due to its ability to overcome large spans and its economical and aesthetic excellences. Bridges of this type have been constructed in a wide range of span lengths from 100m to 800m throughout the world. Because the cable-stayed bridge is a highly statically indeterminate structure, its structural behavior and total cost are greatly affected by the cable arrangement and stiffness distribution in the cables, main girder and pylon. Furthermore, the aesthetic view of the bridge is also affected by these design variables, therefore, it is very important that the bridge is designed by balancing totally the mechanical, economical and aesthetic characteristics, and the manufacturing and erection conditions. The establishment of a rational and efficient computer-aided design system for cable-stayed bridges, which can determine the optimum values of the design variables mentioned above at various design conditions rigorously and automatically, contributes significantly to the practical design process of the cable-stayed bridge.

The study of the optimum design of cable-stayed bridges was begun in the late 1970's. Yamada and Daiguji studied an optimum design method based on the optimality criteria[1]. Kobayashi et al. presented a multilevel optimal design method by using the SLP algorithm and applied it to three types of cable-stayed bridges with different supporting systems[2]. Gimsing[3] investigated the rational cable arrangement of cable-stayed bridges from the structural system analysis viewpoint.

In this Chapter, a rigorous and efficient optimum design method for steel cable-stayed bridges is presented. In this design method, not only the cross-sectional dimensions of the cables, main girder and pylon elements but also the cable anchor positions on the main girder and the heights of pylons are dealt with as the design variables. The optimization dealing with cable anchor positions on the main girder and the heights of pylons is the first challenge in field of optimization of cable-stayed bridges. The design problem is formulated as a minimum-cost design problem subject to the stress constraints taken from the Japanese Specifications for Highway Bridges[JSHB][4]. The magnitudes of the dead loads and traffic loads, impact factor, effective widths of the flange plates in the main girder and effective lengths of the pylon elements for bucklings are also taken from the JSHB[4]. The working stress at a structural element is calculated as the sum of the stresses due to dead loads in the cantilever system at the erection closing stage and the stresses due to the traffic loads and a part of the dead loads in the continuous girder system at the service stage. The maximum and minimum values of axial force, shearing force and bending moment at the stress inspection points due to traffic and impact loads are calculated by using the corresponding influence lines.

The cost-minimization problem is approximated by using the first-order partial derivatives of objective function and behavior constraints, and mixed direct/reciprocal design variables. The approximate subproblem is solved by dual method.

The proposed optimum design method has been applied to the minimum-cost design problems of practical-scale steel cable-stayed bridge with 48 cable stays. The theoretical rigorousness and efficiency of the proposed optimum design method are demonstrated by investigating the optimum solutions at different design conditions. The significance of dealing with the cable anchor positions on the main girder and the heights of pylons is also emphasized for the minimum-cost designs of cable-stayed bridges. Furthermore, the practical usefulness of the proposed optimum design method is also demonstrated by giving the practical design example of the Swan Bridge (Ube city, Yamaguchi) which was designed by using this proposed design method.

2-2. FORMULATION OF OPTIMUM DESIGN PROBLEM

(1) Design variables

In this optimum design method, the span lengths, number of cables, height and width of the elements of main girder and pylon, and material types to be used for each structural element are assumed as the preassigned constant design parameters, and the shapes of cross sections of main girder and pylon are assumed as box types depicted in Figs.2-1 (a) and (b), respectively.

The cross sections of main girder can vary in the middle of the cable anchor positions as shown in Fig.2-2. In the pylon, the cross sections can vary at the cable

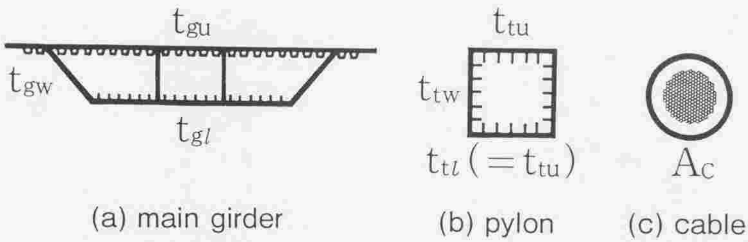

Fig.2-1 Cross sections and sizing variables $t_{gu}, t_{gl}, t_{tu}, t_{tl}$ and A_C in the main girder, pylon and cable elements

anchor positions, and if the lowest cable anchor positions in the pylon, Y_{C1} and Y_{C2} in Fig.2-2, are larger than 20m, the cross sections can vary at the middle of the Y_{C1} and Y_{C2}.

The design variables corresponding to the sizes of cross sections of all member elements are the cross-sectional area of each cable, $\mathbf{A}_C$, and the thicknesses of upper and lower flange plates of each main girder element, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$, and pylon element, $\mathbf{t}_{tu}$ and $\mathbf{t}_{tl}$, as shown in Fig.2-1, where $\mathbf{t}_{tu}$ and $\mathbf{t}_{tl}$ in the pylon are assumed to be the same. The thicknesses of these flange plates are dealt with as the converted thicknesses which include the contributions of the longitudinal stiffeners. These sizing variables are termed by the vector $\mathbf{Z}$:

$$\mathbf{Z} = \left[\boldsymbol{Z}_1^T, \cdots, \boldsymbol{Z}_i^T, \cdots, \boldsymbol{Z}_n^T\right]^T \tag{2-1}$$

where

$$\boldsymbol{Z}_i = \left[Z_{i1}, \cdots, Z_{ir}, \cdots, Z_{iq_i}\right]^T$$

if i denotes the element of main girder: $\boldsymbol{Z}_i = \left[t_{gui}, t_{gli}\right]^T$

if i denotes the element of pylon: $\boldsymbol{Z}_i = \left[t_{tui}, t_{tli}\right]^T$ where $t_{tui} = t_{tli}$

if i denotes the element of cable: $\boldsymbol{Z}_i = A_{Ci}$

$n = ng + nt + nc$,

ng : the total number of elements of main girder,

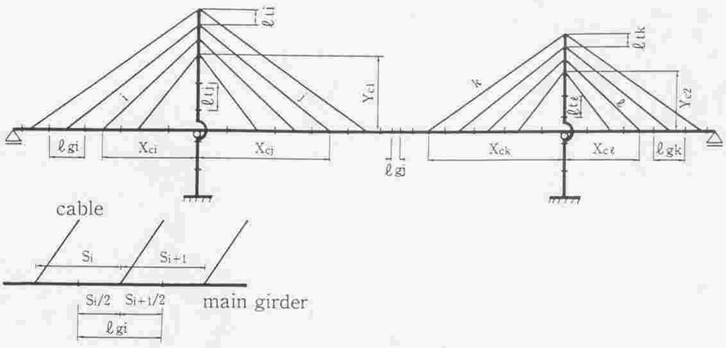

Fig.2-2 Design variables $\mathbf{X}_C, \mathbf{Y}_C$ and element lengths l_{gi} and l_{ti} for the main girder and pylon elements

nt : the total number of elements of pylon,

nc : the total number of cables,

q_i : the total number of design variables in $\boldsymbol{Z}_i$.

The distance from the pylon to each cable anchor position in the main girder, X_{Ck}, and the height of the lowest cable in the pylon from the axis of main girder, Y_C, in Fig.2-2 are dealt with as the design variables corresponding to the cable anchor positions on the main girder and the height of pylon. The distances of each cable in the pylon, l_{ti} and l_{tk}, are assumed as the preassigned constant values. These design variables are termed by the vectors $\mathbf{X}_C$ and $\mathbf{Y}_C$, respectively:

$$\mathbf{X}_C = \left[X_{C1}, X_{C2}, \cdots, X_{CK}\right]^T \tag{2-2}$$

$$\mathbf{Y}_C = \left[Y_{C1}, Y_{C2}, \cdots, Y_{CL}\right]^T \tag{2-3}$$

where K and L are, respectively, the numbers of design variables on $\mathbf{X}_C$ and $\mathbf{Y}_C$.

(2) Design constraints

In this design method, the constraints on stresses are considered as behavioral constraints. The working stresses at cables, elements of the main girder and pylon are summarized as the stresses due to dead loads acting in the cantilever system at the erection closing stage as shown in Fig.2-3 (a), and the stresses due to traffic loads and a part of dead loads acting in the continuous girder system at the service stage as shown in Fig.2-3 (b). The magnitudes of dead loads and traffic loads, impact factor, effective widths of flange plates in the main girder and effective lengths of the pylon elements for bucklings are taken from the JSHB[4].

The following constraints related to the stresses at each cable and elements of main girder and pylon, the slenderness ratios of pylon elements, and upper and lower limits of the design variables are considered in the optimization process.

(a) The stress at the main girder element:

$$g_{\sigma_{gi}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)=\sigma_i(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)-\sigma_{ai}(\mathbf{Z})\le 0 \qquad (i=1,\cdots,\mathrm{m_g}) \tag{2-4}$$

where

$\sigma_i(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the stress due to design loads,

$\sigma_{ai}(\mathbf{Z})$: the allowable compressive stress against local buckling or allowable tensile stress,

$\mathrm{m_g}$: the number of stress constraints at the main girder element.

(b) The stress at the pylon element:

$$g_{\sigma_{t1j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)=\frac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)}{\sigma_{cazj}(\mathbf{Z},\mathbf{Y}_C)}+\frac{\sigma_{bcyj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)}{\sigma_{bagyj}\left(1-\dfrac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)}{\sigma_{eayj}(\mathbf{Z},\mathbf{Y}_C)}\right)}-1\le 0 \quad (j=1,\cdots,\mathrm{m_t}) \tag{2-5}$$

$$g_{\sigma_{t2j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)=\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)+\frac{\sigma_{bcyj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)}{\left(1-\dfrac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)}{\sigma_{eayj}(\mathbf{Z},\mathbf{Y}_C)}\right)}-\sigma_{calj}(\mathbf{Z})\le 0 \quad (j=1,\cdots,\mathrm{m_t}) \tag{2-6}$$

where

$g_{\sigma_{t1j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the design constraints on the working stresses,

$g_{\sigma_{t2j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the constraints on the buckling stability,

$\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the axial compressive stress due to design loads,

$\sigma_{bcyj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the compressive bending stress due to design loads,

$\sigma_{cazj}(\mathbf{Z},\mathbf{Y}_C)$: the allowable axial compressive stress,

σ_{bagyj} : the allowable compressive bending stress not concerning local buckling,

$\sigma_{eayj}(\mathbf{Z},\mathbf{Y}_C)$: the allowable stress for Euler's buckling,

$\sigma_{calj}(\mathbf{Z})$: the allowable compressive bending stress against local buckling in the stiffening plate,

m_t: the number of stress constraints at the pylon element.

(c) The slenderness ratio of the pylon element:

$$g_{lj}(\mathbf{Z},\mathbf{Y}_C) = l_j(\mathbf{Y}_C)/r_j(\mathbf{Z}) - 120 \le 0 \qquad (j=1,\cdots,nt) \qquad (2\text{-}7)$$

where

$l_j(\mathbf{Y}_C)/r_j(\mathbf{Z})$: the slenderness ratio of the jth pylon element.

(d) The stress at the cable element:

$$g_{\sigma_{tk}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) = \sigma_k(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) - \sigma_{ak} \le 0 \qquad (k=1,\cdots,nc) \qquad (2\text{-}8)$$

where

$\sigma_k(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C)$: the tensile stress due to design loads,

σ_{ak} : the allowable tensile stress.

(e) The upper and lower limits of the design variables:

$$\mathbf{Z}^{(l)} \le \mathbf{Z} \le \mathbf{Z}^{(u)},\ \ \mathbf{X}_C^{(l)} \le \mathbf{X}_C \le \mathbf{X}_C^{(u)},\ \ \mathbf{Y}_C^{(l)} \le \mathbf{Y}_C \le \mathbf{Y}_C^{(u)} \qquad (2\text{-}9)$$

The minimum web plate thicknesses of each element of the main girder and pylon are determined so as to satisfy the composite stress criteria on the web plates.

(3) Stress analysis and critical stress conditions

The working stress at a structural element is calculated as the sum of the stresses due to dead loads in the cantilever system at the erection closing stage and the stresses due to traffic loads and a part of dead loads in the continuous girder system at the service stage. The two structure-load systems at erection closing stage and service stage (see Fig.2-3) are analyzed by the finite element method as a 2-dimensional plane frame structure.

In this design method, it is assumed that the cross-sectional dimensions of main girder can vary at the center of the adjoining cable anchor positions. Therefore, the critical stresses are inspected at the six points in the upper and lower flange plates in each element of main girder shown in Fig.2-4 (a). In the pylon element, the critical stresses are inspected at the upper and lower cross sections as shown in Fig.2-4 (b).

Because each elements of the main girder and pylon are subjected to bending

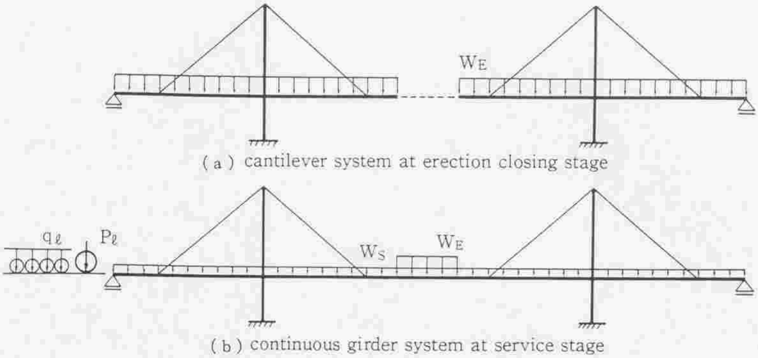

Fig.2-3 Structure-load systems at erection closing and service stages

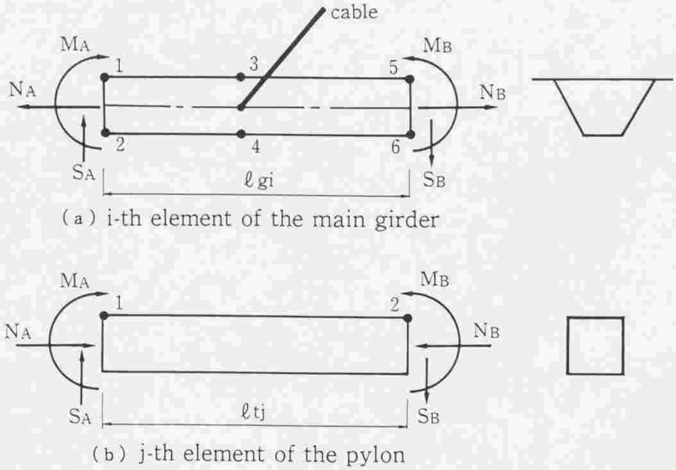

Fig.2-4 Stress inspection points in the elements of main girder and pylon

moment **M**, axial force **N** and shearing force **S**, the most critical composite stress condition for the determination of plate thickness at each stress inspection point must be determined by comparing the resultant stresses at six loading conditions in which **N**, **S**, and **M** are given as maximum and minimum values. The maximum and minimum values of **N**, **S**, and **M** at the stress inspection points due to traffic and impact loads are calculated by using the corresponding influence lines.

(4) Formulation of optimum design problem

By taking into account of the design variables and design constraints, respectively, described in sections 2-2.(1) and (2), the minimum cost design problem of the steel cable-stayed bridge can be formulated as follows:

Find $\quad \mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$, which

$$\text{minimize} \quad \mathrm{TCOST}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) = \sum_{i=1}^{n} \mathrm{W}_i(\mathbf{X}_C,\mathbf{Y}_C)\cdot A_i(\boldsymbol{Z}_i) \tag{2-10}$$

$$\text{subject to} \quad g_{\sigma_{gi}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) \le 0 \qquad (i=1,\cdots,\mathrm{m_g}) \tag{2-11}$$

$$g_{\sigma_{t1j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) \le 0 \qquad (j=1,\cdots,\mathrm{m_t}) \tag{2-12}$$

$$g_{\sigma_{t2j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) \le 0 \qquad (j=1,\cdots,\mathrm{m_t}) \tag{2-13}$$

$$g_{lj}(\mathbf{Z},\mathbf{Y}_C) \le 0 \qquad (j=1,\cdots,nt) \tag{2-14}$$

$$g_{\sigma_{ck}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) \le 0 \qquad (k=1,\cdots,nc) \tag{2-15}$$

$$\mathbf{Z}^{(l)} \le \mathbf{Z} \le \mathbf{Z}^{(u)}, \quad \mathbf{X}_C^{(l)} \le \mathbf{X}_C \le \mathbf{X}_C^{(u)}, \quad \mathbf{Y}_C^{(l)} \le \mathbf{Y}_C \le \mathbf{Y}_C^{(u)} \tag{2-16}$$

where

if $i=1 \sim ng$:

$$\mathrm{W}_i(\mathbf{X}_C,\mathbf{Y}_C) = \rho_{gi}\cdot l_{gi}(\mathbf{X}_C), \qquad A_i(\boldsymbol{Z}_i) = A_{gi}(\boldsymbol{Z}_i);$$

if $i=ng+1 \sim ng+nt$:

$$\mathrm{W}_i(\mathbf{X}_C,\mathbf{Y}_C) = \rho_{t(i-ng)}\cdot l_{t(i-ng)}(\mathbf{Y}_C), \qquad A_i(\boldsymbol{Z}_i) = A_{t(i-ng)}(\boldsymbol{Z}_i);$$

if $i=ng+nt+1 \sim ng+nt+nc$:

$$\mathrm{W}_i(\mathbf{X}_C,\mathbf{Y}_C) = \rho_{c(i-ng-nt)}\cdot l_{c(i-ng-nt)}(\mathbf{X}_C,\mathbf{Y}_C), \qquad A_i(\boldsymbol{Z}_i) = A_{c(i-ng-nt)}(\boldsymbol{Z}_i);$$

$\rho_{gi}, \rho_{ti}, \rho_{ci}$: price per unit volume of the ith member element at main girder, pylon and cable, respectively,

l_{gi}, l_{ti}, l_{ci}: length of the ith member element at main girder, pylon and cable, respectively,

$A_{gi}(\boldsymbol{Z}_i), A_{ti}(\boldsymbol{Z}_i), A_{ci}(\boldsymbol{Z}_i)$: cross-sectional area of the ith member element at main girder, pylon and cable, respectively,

2-3. OPTIMIZATION ALGORITHM[5,6]

(1) Convex and separable approximation

Several types of optimization algorithms can be applied to solve the optimum design problem. In this study, the optimization algorithm developed by Fleury et al.[7-10] is used for the optimization algorithm. Utilizing the convex and linear approximation concepts, the objective function, eq.(2-10), and the behavior constraints, eqs.(2-11)-(2-16), are approximated by using the first-order partial derivatives of behavior constraints, the primary design variables $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ and, or their reciprocal design variables. In the objective function, the constant term can be neglected in the optimization process and only the change in the objective function, $\Delta\mathrm{TCOST}(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C)$, need to be considered. The following approximate subproblem can then be derived:

Find $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$, which

$$\text{minimize} \quad \Delta\mathrm{TCOST}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) = \sum_{i=1}^{n} \mathrm{W}_i \left(\sum_{r=1}^{q_i} \frac{\partial A_{ir}}{\partial Z_{ir}} \cdot Z_{ir} \right) + \sum_{k=1}^{K} \left(\sum_{i=1}^{n} W_{x_{ik}} \cdot A_i\left(\boldsymbol{Z}_i^0\right) \right)_{(+)} \cdot X_{Ck}$$

$$- \sum_{k=1}^{K} \left(\sum_{i=1}^{n} W_{x_{ik}} \cdot A_i\left(\boldsymbol{Z}_i^0\right) \right)_{(-)} \cdot \left(X_{Ck}^0\right)^2 \cdot \frac{1}{X_{Ck}} + \sum_{l=1}^{L} \left(\sum_{i=1}^{n} W_{y_{il}} \cdot A_i\left(\boldsymbol{Z}_i^0\right) \right)_{(+)} \cdot Y_{Cl}$$

$$- \sum_{l=1}^{L} \left(\sum_{i=1}^{n} W_{y_{il}} \cdot A_i\left(\boldsymbol{Z}_i^0\right) \right)_{(-)} \cdot \left(Y_{Cl}^0\right)^2 \cdot \frac{1}{Y_{Cl}} \qquad (2\text{-}17)$$

subject to

$$\bar{g}_j(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C) = \sum_{i=1}^{n} \left\{ \left(\sum_{r=1}^{q_i} a_{(+)j,ir} \cdot Z_{ir} \right) - \left(\sum_{r=1}^{q_i} a_{(-)j,ir} \cdot \left(Z_{ir}^0\right)^2 \cdot \frac{1}{Z_{ir}} \right) \right\}$$

$$+ \sum_{k=1}^{K} \left(b_{(+)jk} \cdot X_{Ck} - b_{(-)jk} \cdot \left(X_{Ck}^0\right)^2 \cdot \frac{1}{X_{Ck}} \right)$$

$$+\sum_{l=1}^{L}\left(c_{(+)jl}\cdot Y_{Cl}-c_{(-)jl}\cdot\left(Y_{Cl}^{0}\right)^{2}\cdot\frac{1}{Y_{Cl}}\right)+\overline{U}_{j}\leq 0 \qquad (j=1,\cdots,\mathrm{m}) \quad (2\text{-}18)$$

$$\mathbf{Z}^{(l)}\leq\mathbf{Z}\leq\mathbf{Z}^{(u)},\ \ \mathbf{X}_{C}{}^{(l)}\leq\mathbf{X}_{C}\leq\mathbf{X}_{C}{}^{(u)},\ \ \mathbf{Y}_{C}{}^{(l)}\leq\mathbf{Y}_{C}\leq\mathbf{Y}_{C}{}^{(u)} \qquad (2\text{-}19)$$

where

$$\overline{U}_{j}=g_{j}(\mathbf{Z}^{0},\mathbf{X}_{C}^{0},\mathbf{Y}_{C}^{0})+\sum_{i=1}^{n}\left\{\left(\sum_{r=1}^{q_i}a_{(-)j,ir}\cdot Z_{ir}^{0}\right)-\left(\sum_{r=1}^{q_i}a_{(+)j,ir}\cdot Z_{ir}^{0}\right)\right\}$$

$$+\sum_{k=1}^{K}\left(b_{(-)jk}\cdot X_{Ck}^{0}-b_{(+)jk}\cdot X_{Ck}^{0}\right)+\sum_{l=1}^{L}\left(c_{(-)jl}\cdot Y_{Cl}^{0}-c_{(+)jl}\cdot Y_{Cl}^{0}\right) \qquad (2\text{-}20)$$

$$\mathrm{W}_{i}=\rho_{i}\cdot l_{i},\ \ W_{Xik}=\frac{\partial\mathrm{W}_{i}}{\partial X_{Ck}},\ \ W_{yil}=\frac{\partial\mathrm{W}_{i}}{\partial Y_{Cl}},\ \ a_{j,ir}=\frac{\partial g_{j}}{\partial Z_{ir}},\ \ b_{jk}=\frac{\partial g_{j}}{\partial X_{Ck}},\ \ c_{jl}=\frac{\partial g_{j}}{\partial Y_{Cl}}$$

$$\overline{\mathbf{g}}=\left[\overline{g}_{1},\cdots,\overline{g}_{m}\right]^{T}$$

$$=\left[\overline{g}_{\sigma_{g(1)}},\cdots,\overline{g}_{\sigma_{g(mg)}},\overline{g}_{\sigma_{t1(1)}},\cdots,\overline{g}_{\sigma_{t1(mt)}},\overline{g}_{\sigma_{t2(1)}},\cdots,\overline{g}_{\sigma_{t2(mt)}},\overline{g}_{l_{(1)}},\cdots,\overline{g}_{l_{(m)}},\overline{g}_{\sigma_{c(1)}},\cdots,\overline{g}_{\sigma_{c(mc)}}\right]^{T}$$

In the above expressions the symbols (+) and (-) express the signs of the first-order partial derivatives. **A** represents the vector of cross section of each member element.

(2) Optimization algorithm based on dual method

The above approximate subproblem is solved by the dual method. The optimization algorithm is as follows :

① Derive a separable Lagrangian function for the approximate subproblem as shown below:

$$\mathbf{L}(\mathbf{Z},\mathbf{X}_{C},\mathbf{Y}_{C},\boldsymbol{\lambda})=\Delta\mathrm{TCOST}(\mathbf{Z},\mathbf{X}_{C},\mathbf{Y}_{C})+\sum_{j=1}^{m}\lambda_{j}\cdot\overline{g}_{j}(\mathbf{Z},\mathbf{X}_{C},\mathbf{Y}_{C})$$

$$=\sum_{i=1}^{n}L_{i}(\boldsymbol{Z}_{i},\boldsymbol{\lambda})+\sum_{k=1}^{K}L_{k}(X_{CK},\boldsymbol{\lambda})+\sum_{l=1}^{L}L_{l}(Y_{Cl},\boldsymbol{\lambda})+\sum_{j=1}^{m}\lambda_{j}\cdot\overline{U}_{j} \qquad (2\text{-}21)$$

$$\text{where} \qquad \lambda_{j}\geq 0 \qquad\qquad (j=1,\cdots,\mathrm{m})$$

λ_j is the Lagrange multiplier for the jth behavior constraint and $\overline{U}_j$ is a constant term. L_i, L_k and L_l are, in turn, given by the following expressions:

$$L_i(\mathbf{Z},\lambda) = \mathrm{W}_i \sum_{r=1}^{q_i} \frac{\partial A_{ir}}{\partial Z_{ir}} \cdot Z_{ir} + \sum_{j=1}^{m} \lambda_j \cdot \left(\sum_{r=1}^{q_i} a_{(+)j,ir} \cdot Z_{ir} - \sum_{r=1}^{q_i} a_{(-)j,ir} \cdot \left(Z_{ir}^0\right)^2 \cdot \frac{1}{Z_{ir}} \right) \tag{2-22}$$

$$\begin{aligned} L_k(X_{CK},\lambda) = & \left(\sum_{i=1}^{n} W_{x_{ik}} \cdot A_i\left(\mathbf{Z}_i^0\right) \right)_{(+)} \cdot X_{Ck} - \left(\sum_{i=1}^{n} W_{x_{ik}} \cdot A_i\left(\mathbf{Z}_i^0\right) \right)_{(-)} \cdot \left(X_{Ck}^0\right)^2 \cdot \frac{1}{X_{Ck}} \\ & + \sum_{j=1}^{m} \lambda_j \cdot \left(b_{(+)jk} \cdot X_{Ck} - b_{(-)jk} \cdot \left(X_{Ck}^0\right)^2 \cdot \frac{1}{X_{Ck}} \right) \end{aligned} \tag{2-23}$$

$$\begin{aligned} L_l(Y_{Cl},\lambda) = & \left(\sum_{i=1}^{n} W_{y_{il}} \cdot A_i\left(\mathbf{Z}_i^0\right) \right)_{(+)} \cdot Y_{Cl} - \left(\sum_{i=1}^{n} W_{y_{il}} \cdot A_i\left(\mathbf{Z}_i^0\right) \right)_{(-)} \cdot \left(Y_{Cl}^0\right)^2 \cdot \frac{1}{Y_{Cl}} \\ & + \sum_{j=1}^{m} \lambda_j \cdot \left(c_{(+)jl} \cdot Y_{Cl} - c_{(-)jl} \cdot \left(Y_{Cl}^0\right)^2 \cdot \frac{1}{Y_{Cl}} \right) \end{aligned} \tag{2-24}$$

② The solutions of dual problem, $\mathbf{Z}^*, \mathbf{X}_C^*, \mathbf{Y}_C^*$ and λ^*, can be obtained by minimizing $\mathbf{L}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\lambda)$ with respect to $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$, and maximizing it with respect to λ.

Eq.(2-21) has a simple form of a summation of the element Lagrangian functions L_i, L_k and L_l, and these functions are formulated in terms of $\mathbf{Z}$ or $\mathbf{X}_C$ or $\mathbf{Y}_C$ and their reciprocals. Therefore, $\mathbf{D}^* = \left[\mathbf{Z}^{*T}, \mathbf{X}_C^{*T}, \mathbf{Y}_C^{*T}\right]^T$ which minimize $\mathbf{L}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\lambda)$ can be obtained analytically from the necessary conditions for the minimums of L_i, L_k and L_l, namely, $\partial L_i / \partial Z_{ir} = 0$, $\partial L_k / \partial X_{Ck} = 0$, $\partial L_l / \partial Y_{Cl} = 0$ and the side constraints:

(a) if $\omega_i = \partial \mathrm{TCOST} / \partial D_i \geq 0$:

$$\left.\begin{aligned} D_i^* &= D_i^{(l)} && \text{if } \; R_i = 0 \;\text{ or }\; D_i \leq D_i^{(l)} \\ D_i^* &= D_i^{(u)} && \text{if } \; \left(\omega_i + V_i\right) = 0 \;\text{ or }\; D_i \geq D_i^{(u)} \\ D_i^* &= D_i && \text{if } \; D_i^{(l)} < D_i < D_i^{(u)} \end{aligned}\right\} \tag{2-25}$$

$$D_i = \sqrt{\frac{R_i}{\omega_i + V_i}}$$

(b) if $\omega_i = \partial \mathrm{TCOST} / \partial D_i < 0$:

$$\left.\begin{aligned} D_i^* &= D_i^{(l)} \quad \text{if} \quad \left(R_i - \omega_i\right) = 0 \quad \text{or} \quad D_i \le D_i^{(l)} \\ D_i^* &= D_i^{(u)} \quad \text{if} \quad V_i = 0 \quad \text{or} \quad D_i \ge D_i^{(u)} \\ D_i^* &= D_i \qquad \text{if} \quad D_i^{(l)} < D_i < D_i^{(u)} \end{aligned}\right\} \tag{2-26}$$

$$D_i = \sqrt{\frac{R_i - \omega_i}{V_i}}$$

where

$$R_i = -\sum_{j=1}^{m} \lambda_j \cdot T_{(-)ji} \cdot \left(D_i^0\right)^2 \tag{2-27}$$

$$V_i = \sum_{j=1}^{m} \lambda_j \cdot T_{(+)ji} \tag{2-28}$$

T_{ji} and ω_i in the above expressions take the following values with respect to $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$:

(i) if $D_i = Z_{ir}$:

$$\left.\begin{aligned} T_{ji} &= a_{jir} \\ \omega_i &= W_i \cdot \partial A_i / \partial Z_{ir} \end{aligned}\right\} \tag{2-29}$$

(ii) if $D_i = X_{Ci}$:

$$\left.\begin{aligned} T_{ji} &= b_{ji} \\ \omega_i &= \left(\sum_{j=1}^{n} W_{x_{ji}} \cdot A_j\left(\boldsymbol{Z}_j^0\right)\right)_{(-)} \cdot \left(X_{Ci}^0\right)^2 \quad \text{if} \quad \omega_i < 0 \\ \omega_i &= \left(\sum_{j=1}^{n} W_{x_{ji}} \cdot A_j\left(\boldsymbol{Z}_j^0\right)\right)_{(+)} \qquad\qquad \text{if} \quad \omega_i \ge 0 \end{aligned}\right\} \tag{2-30}$$

(iii) if $D_i = Y_{Ci}$:

$$\left.\begin{aligned} T_{ji} &= c_{ji} \\ \omega_i &= \left(\sum_{j=1}^{n} W_{y_{ji}} \cdot A_j\left(\boldsymbol{Z}_j^0\right)\right)_{(-)} \cdot \left(Y_{Ci}^0\right)^2 \quad \text{if} \quad \omega_i < 0 \\ \omega_i &= \left(\sum_{j=1}^{n} W_{y_{ji}} \cdot A_j\left(\boldsymbol{Z}_j^0\right)\right)_{(+)} \qquad\qquad \text{if} \quad \omega_i \ge 0 \end{aligned}\right\} \tag{2-31}$$

The minimized Lagrangian function with respect to $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ by the above expressions is denoted as $l(\lambda)$:

$$l(\lambda) = \min_{\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C} \mathbf{L}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\lambda) \tag{2-32}$$

Following the minimization with respect to $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$, the minimized Lagrangian function $l(\lambda)$ is then maximized with respect to λ by using a Newton-type algorithm. In the algorithm, the Lagrange multipliers λ corresponding to the active behavior constraints at the current stage are modified iteratively as:

$$\bar{\lambda}^{(t+1)} = \bar{\lambda}^{(t)} + \alpha_{max}^{(t)} \cdot \mathbf{S}^{(t)} \tag{2-33}$$

or in a scalar form

$$\bar{\lambda}_j^{(t)} = \bar{\lambda}_j^{(t)} + \alpha_{max}^{(t)} \cdot S_j^{(t)} \quad (j \in \boldsymbol{S}_{AG}) \tag{2-34}$$

where $\mathbf{S}^{(t)}$ is the search direction of $\bar{\lambda}^{(t)}$ for active constraints. S_{AG} is the set of active behavior constraints and $\alpha_{max}^{(t)}$ is the step length parameter. $\mathbf{S}^{(t)}$ is given by

$$\mathbf{S}^{(t)} = -\left[\mathbf{H}(\bar{\lambda}^{(t)})\right]^{-1} \cdot \nabla l(\bar{\lambda}^{(t)}) \tag{2-35}$$

where $\nabla l(\bar{\lambda}^{(t)})$ is the vector of first derivatives of $l(\lambda)$ with respect to $\bar{\lambda}$ and the components of the vector are simply given by the values of approximate primary constraints which are active at the current stage.

$\mathbf{H}(\lambda)$ is the Hessian matrix of $l(\lambda)$ with respect to $\lambda_j (j \in S_{AG})$ and the jkth component of the matrix is :

$$H_{jk} = \sum_{i=1}^{n+K+L} B_i \tag{2-36}$$

where

$$B_i = Q_i \cdot (D_i^0)^4 / D_i^3 \quad \text{if} \quad T_{ji} < 0,\ T_{ki} < 0$$

$$B_i = Q_i \cdot (D_i^0)^2 / D_i \quad \text{if} \quad T_{ji} \geq 0,\ T_{ki} < 0$$

$$B_i = Q_i \cdot (D_i^0)^2 / D_i \quad \text{if} \quad T_{ji} < 0,\ T_{ki} \geq 0$$

$$B_i = Q_i \cdot D_i \quad \text{if} \quad T_{ji} \geq 0,\ T_{ki} \geq 0$$

Q_i is given by:

$$
\left.\begin{aligned}
Q_i &= \frac{T_{ji}T_{ki}}{2(\omega_i + V_i)} \qquad \text{if} \qquad \omega_i \geq 0 \\
Q_i &= \frac{T_{ji}T_{ki}}{2V_i} \qquad\qquad \text{if} \qquad \omega_i < 0
\end{aligned}\right\} \tag{2-37}
$$

The step length $\alpha_{max}^{(t)}$ is first set as 1.0, and at later stages its value is taken as

$$
\alpha_{max}^{(t)} = \min_{S_j^{(t)}<0} \left| \frac{\bar{\lambda}_j^{(t)}}{S_j^{(t)}} \right| \qquad (j \in S_{AG}) \tag{2-38}
$$

with the additional restriction that $\alpha_{max}^{(t)} \leq 1.0$ to ensure the non-negativity of $\bar{\lambda}$ when $\mathbf{S}^{(t)}$ include negative components.

When $l(\bar{\lambda}^{(t+1)})$ exceeds the maximum point, $\alpha_{max}^{(t)}$ is reduced and the search is continued until $l(\bar{\lambda}^{(t+1)})$ is maximized. Based on the modification of λ, the primary variables $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ are improved by eqs.(2-25)-(2-31) and the set of active constraints S_{AG} is also updated. The min.-max. process described above is iterated until $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ and λ converge to constant values $\mathbf{Z}^*, \mathbf{X}_C^*, \mathbf{Y}_C^*$ and λ^*.

③ By using $\mathbf{Z}^*, \mathbf{X}_C^*, \mathbf{Y}_C^*$, the minimum web thicknesses of main girder and pylon elements, $\mathbf{t}_W$, are improved so as to satisfy the corresponding stress constraints on $\mathbf{t}_W$.

④ $\mathbf{Z}^*, \mathbf{X}_C^*, \mathbf{Y}_C^*$ are assumed as new initial values of the design variables and a new approximate subproblem is derived. The final optimum solutions can be determined by iterating steps ①-③ until $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ and λ converge to constant values.

In the above optimization algorithm, it should be noted that if the changing rates in $\mathbf{X}_C$ and $\mathbf{Y}_C$ calculated as per eqs.(2-25)-(2-31) are too large in any one iteration of the improvement process, the successive solutions may oscillate and in some cases smooth convergence may not be obtained. For this reason, the adaptive move limit constraints are restricted such that the maximum rates of change in $\mathbf{X}_C$ and $\mathbf{Y}_C$ are limited to 10 %. It is, also, important to note that if two or more stress constraints given by eq.(2-18) become active with respect to one flange plate in a main girder element and $a_{j,ir}$, b_{jk} and c_{jl} in the constraints have almost the same values, the constraints become linearly dependent on each other and consequently the Hessian matrix $\mathbf{H}(\bar{\lambda})$ becomes singular. If this is the case, λ can be successfully improved by considering only the most critical stress constraint as active and deleting the other stress constraints on the same flange plate from the set of active constraints S_{AG}.

(3) Calculation of behavior sensitivities

a_{jir}, b_{jk} and c_{jl} in eq.(2-18) are calculated by using the first-order partial derivatives of $\mathbf{N}_E$, $\mathbf{M}_E$, $\mathbf{N}_S$ and $\mathbf{M}_S$ with respect to Z_{ir}, X_{Ck} and Y_{Cl}, respectively, where $\mathbf{N}_E$ and $\mathbf{M}_E$ denote the axial forces and bending moments in the cantilever system at the erection closing stage and $\mathbf{N}_S$ and $\mathbf{M}_S$ denote those in the continuous girder system at the service stage. In this study, the partial derivatives are calculated by a finite difference method. In the calculation, the changes in the dead loads due to the changes in $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ and the changes in the loading positions of traffic loads due to the changes in $\mathbf{X}_C$ are evaluated. However, the changes in the loading positions of traffic loads due to the changes in $\mathbf{Z}$ and $\mathbf{Y}_C$ are neglected. The detailed calculations of sensitivities of stress constraints with respect to design variables are explained in Appendix 2-1.

2-4. NUMERICAL DESIGN EXAMPLES

(1) Design example for steel cable-stayed bridge with 48 cable stays

Various minimum-cost design problems of practical-scale steel cable-stayed bridges have been solved by the proposed design method. In this section, the numerical results for a three-span steel cable-stayed bridge with 48 cable stays shown in Fig.2-5 under various design conditions are presented to demonstrate the general purpose, rigorousness, reliability and efficiency of the proposed optimum design method. The significances of dealing with $\mathbf{X}_C$, $\mathbf{Y}_C$ as the design variables are, also, clarified.

The design constants used in the numerical examples, such as the moduli of elasticity of the steel plate and cable, E, the unit prices of the materials, ρ, the allowable tensile and shearing stresses, σ_{ta} and τ_a, the effective widths of upper and lower flange plates in the main girder, B_{eu} and B_{el}, the effective lengths of the pylon elements for buckling in longitudinal and transverse directions, l_1 and l_2, the minimum plate thicknesses, t_u^l, t_l^l, t_w^l, are tabulated in Table 2-1. The dead loads at the cantilever erection closing stage and the traffic loads at service stage on one half

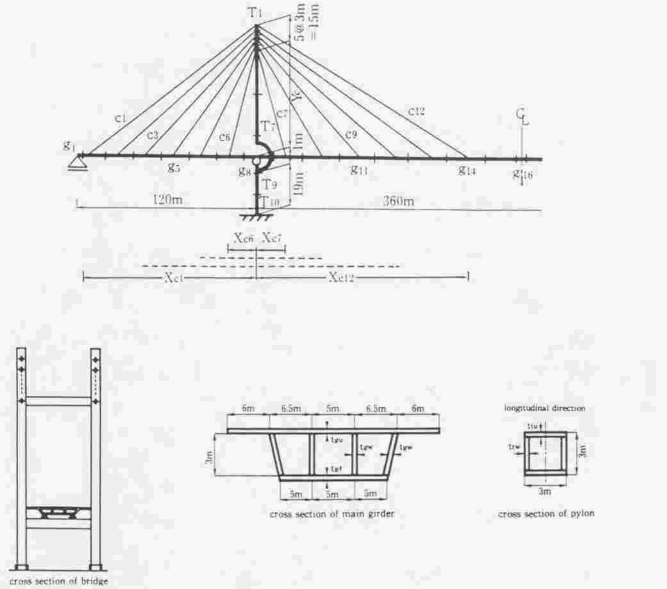

Fig.2-5 Three span steel cable-stayed bridge with 48 cable stays

Table 2-1 Material properties and minimum plate thicknesses used for the elements of main girder, pylon and cables

No. of member element	1) $E(kg/cm^2)$	2) $\rho(YEN/m^3)$	3) σ_{ta}	4) τ_a	5) B_{eu}, 6) $\ell_1(m)$	7) $B_{e\ell}$, 8) $\ell_2(m)$	9) $t_u^\ell(mm)$	10) $t_\ell^\ell(mm)$	11) $t_w^\ell(mm)$
g_1-g_6					29.70 5)	15.00 7)			
g_7					29.25	14.94			
g_8	2.1×10^6	500×10^4	1400	800	28.38	14.81	18.5	15.3	12.0
g_9					29.47	14.94			
g_{10}-g_{16}					30.00	15.00			
T_1-T_5			1900	1100	3.0 6)	3.0 8)	26.0	26.0	28.0
T_6-T_8	2.1×10^6	700×10^4	2600	1500	(Y_C+1)	(Y_C+1)	26.0	26.0	28.0
T_9-T_{10}			2600	1500	38.0	15.2	26.0	26.0	28.0
C_1-C_{12}	2.0×10^6	900×10^4	5100	—	—	—	$A_C^\ell=0.00001\ (m^2)$		

1) Modulus of elasticity 2) Price per unit volume 3) Allowable tensile stress (kg/cm^2)
4) Allowable shearing stress (kg/cm^2) 5) Effective width of the upper flange plates
6) Effective length for buckling in longitudinal direction 7) Effective width of the lower flange plates
8) Effective length for buckling in transverse direction
9) Converted minimum upper flange plate thickness including longitudinal stiffeners
10) Converted minimum lower flange plate thickness including longitudinal stiffeners
11) Converted minimum web plate thickness including longitudinal stiffeners

Table 2-2 Dead loads and traffic loads at erection closing and service stages

Erection closing stage	D. load [1]	main girder [3]	4.0 tf/m
		pylon [3]	2.0 tf/m
		steel weight	7.85 tf/m^3
Service stage	D. load [1]	main girder [4]	3.4 tf/m
	T. load [2]	uniform	2.25 tf/m
		line	54.1 tf
		impact	0.10

1) Dead load 2) Traffic load
3) Dead load due to cable anchors etc.
4) Dead load due to asphalt pavement etc.

of the cross section of bridge are given in Table 2-2.

Since the structure is symmetric, the numbers of independent design variables $\mathbf{Z}$, $\mathbf{X}_C$, $\mathbf{Y}_C$ in the optimization process are 52,14,1, respectively, and the number of constraints is 158. In the design problem, the lower limit on the cross-sectional areas of cables is set at 0.1cm^2 and the objective is to determine the optimum cable arrangement.

To investigate the significance of dealing with $\mathbf{X}_C$ and Y_C as the design variables, the optimum solutions for the cases in which $\mathbf{Z}$ only and $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C are dealt with as the design variables are compared. The global optimality of the optimum solutions obtained by the proposed design method is confirmed by comparing the optimum solutions which are obtained by assuming the initial values of Y_C^0 as 75m and 45m, respectively.

Table 2-3 summarizes the optimum solutions for the above four cases, namely two for $Y_C^0 = 75m$ and another two for $Y_C^0 = 45m$. The cable arrangement, maximum and minimum bending moments and axial forces distributions, M_{max}, M_{min}, N_{max} and N_{min}, and upper and lower flange plate thicknesses distributions, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$, at the optimum solutions in which Y_C^0 is assumed to be 75m are shown in Fig.2-6. The distributions of M_{max}, M_{min}, N_{max}, N_{min} and the cross-sectional areas in the pylon at the above optimum solutions are depicted in Fig.2-7. The values of bending moments and axial forces shown in Fig.2-6 express the magnitudes of these acting on the half of the cross-sectional area of main girder depicted in Fig.2-5, while the total cost expresses the cost in the whole system.

Table 2-3 Comparison of the optimum solutions for the cases in which **Z** only and $\mathbf{Z}, \mathbf{X}_C, Y_C$ are dealt with as design variables

No. of Cables C_i, Y_C, H_t	$Y_C^0 = 75m$				$Y_C^0 = 45m$			
	Z only 1)		$\mathbf{Z}, \mathbf{X}_C, Y_C$ 1)		**Z** only 1)		$\mathbf{Z}, \mathbf{X}_C, Y_C$ 1)	
	$X_C, Y_C(m)$	$A_C(cm^2)$	$X_C, Y_C(m)$	$A_C(cm^2)$	$X_C, Y_C(m)$	$A_C(cm^2)$	$X_C, Y_C(m)$	$A_C(cm^2)$
C_1	119.00	422	119.00	391	119.00	553	119.00	379
C_2	99.00	94	114.07	157	99.00	141	114.06	186
C_3	79.00	77	93.28	102	79.00	19	85.70	107
C_4	59.00	122	58.22	116	59.00	178	56.35	100
C_5	39.00	64	38.87	133	39.00	162	38.23	133
C_6	19.00	0	15.57	0	19.00	0	16.02	0
C_7	20.00	0	16.15	0	20.00	0	16.46	0
C_8	45.00	66	36.56	89	45.00	154	36.10	92
C_9	70.00	128	54.10	127	70.00	143	53.56	119
C_{10}	95.00	95	78.83	121	95.00	115	77.19	122
C_{11}	120.00	77	109.33	214	120.00	122	107.06	215
C_{12}	145.00	324	152.40	326	145.00	430	152.38	334
Y_C	75.00	—	57.52	—	45.00	—	57.45	—
H_t 2)	90.00	—	72.52	—	60.00	—	72.45	—
ITE 3)	9		15		9		13	
Cost of girder	306313.1		300548.9		312711.4		300026.1	
Cost of pylons	150653.4		104558.9		99628.5		104063.0	
Cost of cables	72166.3		78783.0		84420.3		79223.8	
TCOST	529132.8		483890.7		496760.3		483312.9	
R.TCOST 4)	1.000		0.914		1.000		0.973	

1) Design variables 2)Total pylon height: total length from main girder to the top of pylon 3)Number of iterations

4) Ratio between TCOST (**Z** only) and TCOST (**Z**,Xc,Yc)

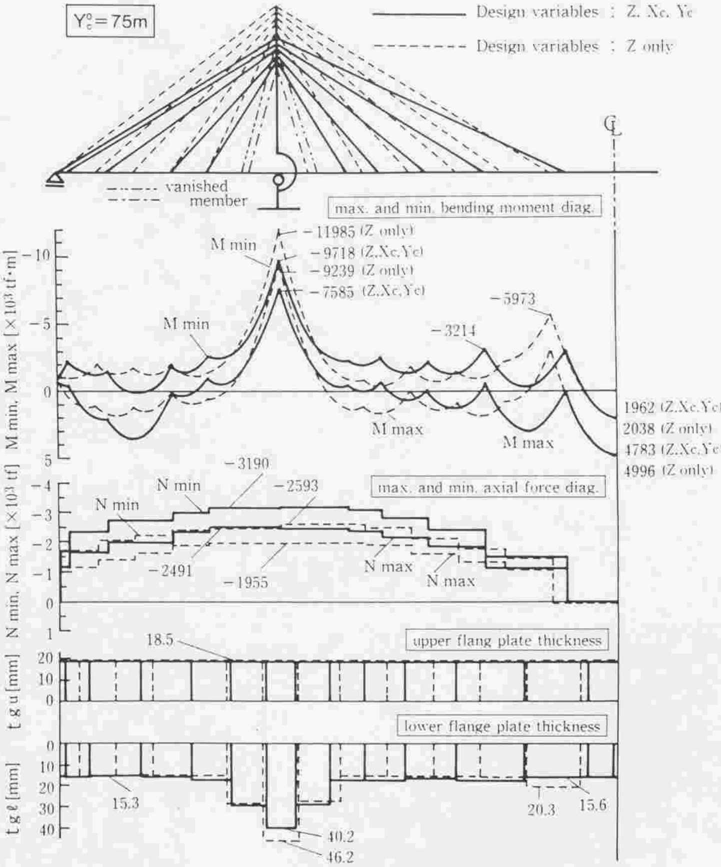

Fig.2-6 Comparisons of optimum cable arrangement, M_{max}, M_{min}, N_{max}, N_{min}, t_{gu} and t_{gl} in the main girder for the cases in which **Z** only and **Z**, $\mathbf{X}_C$ and Y_C are dealt with as design variables ($Y_C^0 = 75m$)

As clearly seen from Table 2-3, the optimum solutions can be obtained efficiently after 15 and 13 iterations, respectively, for the cases of $Y_C^0 = 75m$ and 45m even if the adaptive move limit constraints, maximum 10%, are imposed on $\mathbf{X}_C$ and Y_C. The optimum heights of the lowest cable, Y_{Copt}, are determined as 57.52m and 57.45m, respectively, for $Y_C^0 = 75m$ and 45m. The optimum cable anchor positions on the main girder, $\mathbf{X}_{Copt}$, are also almost same for the two cases, although a slight difference of 7.6m is observed in X_{C3}. The difference of the minimum total costs for both cases is 0.12%. From theses results, it can be said that quite similar optimum solutions, the global optimum solutions, can be obtained by the proposed design method even if the optimization process is started from extremely different initial values of $\mathbf{Z}$ and Y_C.

At the optimum cable arrangement, the top two cables in the side span are parallel and are anchored at the end support. Their cross-sectional areas are 3.6-1.4 times larger than those of the middle cables. In the center span, all cables are distributed as the geometric series and the cross-sectional areas of top two cables are also 2.6-1.7 times larger than those of the middle cables. The cross-sectional areas of lowest cables in the side and center spans are determined as $0.1cm^2$ by the lower limit constraints, which indicates that the lowest cables in the side and center spans are unnecessary from the static optimization view point.

In the optimum solutions in which $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C are dealt with as the design variables, the cable arrangements are determined so as to decrease the critical local peaks of the maximum and minimum bending moments in the main girder and pylon. As clearly seen from Fig.2-6, the local peaks of the minimum bending moments at the middle support and near the center point in the main girder are reduced to 81% and 54%, respectively, compared with the ones for the case in which $Y_C^0 = 75m$ and $\mathbf{Z}$ only is dealt with as the design variable. At the center point of the main girder, it seems that a large maximum bending moment still acts on the cross section, although the cable arrangement is optimized, however, the upper and lower flange plate thicknesses at this point are determined to be the same as the lower limit plate thicknesses. This means that it is not necessary to decrease the maximum bending moment at this point. This result also shows the reliability and rigorousness of the proposed design method. In the optimum solutions in which $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C are dealt with as the design variables, the horizontal components of the tensions in the left and right cables at each set in the pylon due to dead load are well balanced, and the magnitudes of the maximum and minimum bending moments in the pylon are reduced drastically as shown in Fig.2-7, with 28% reduction in the maximum bending moment at the main girder position, and are well averaged throughout the pylon compared with the optimum bending moment distributions in which **Z** only is dealt with as the design variable.

As the consequence of the improvements mentioned above due to the changes in $\mathbf{X}_C$ and Y_C, the total cost of the bridge decreases by 8.6% compared with that for the case in which **Z** only is dealt with as the design variable.

Similar comparisons can be made for the two optimum solutions obtained from $Y_C^0 = 45m$ which are given in Table 2-3. In this case, 2.7% reduction in the total cost of the bridge is observed when $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C are dealt with as the design variables.

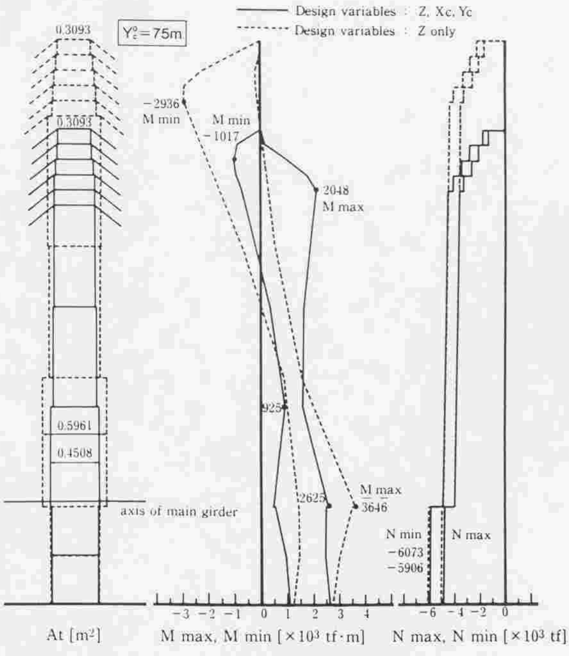

Fig.2-7 Comparisons of $\mathbf{A}_t$, M_{max}, M_{min}, N_{max}, N_{min} in the pylon for the cases in which **Z** only and **Z**, $\mathbf{X}_C$ and Y_C are dealt with as design variables ($Y_C^0 = 75m$)

From the investigations of the optimum solutions for various design conditions, it is clear that the proposed optimum design method can determine the optimum cable anchor positions on the main girder, the height of pylon and the optimum cross-sectional dimensions of all member elements of a steel cable-stayed bridge superstructure efficiently and rigorously. Furthermore, the significance of dealing with cable anchor positions on the main girder and the height of pylon as the design variables is clarified for the minimum cost design of steel cable-stayed bridges.

(2) Practical design example of the Swan Bridge at the Tokiwa Park[11,12]

The Swan Bridge, crossing the lake Tokiwa in Ube city, was constructed in June, 1992. The bridge was designed as a 3-span steel cable-stayed pedestrian bridge with

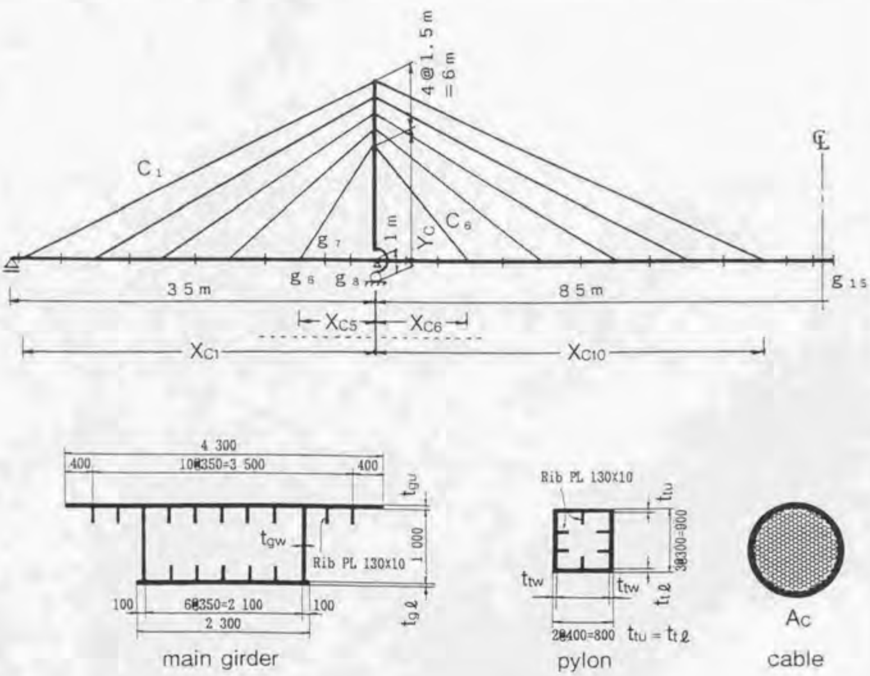

Fig.2-8 Design variables $\mathbf{Z} = [\mathbf{t}_{gu}^T, \mathbf{t}_{gl}^T, \mathbf{t}_{tu}^T, \mathbf{A}_C^T]^T$, $\mathbf{X}_C$ and Y_C for the Swan Bridge

Table 2-4 Material kind, modulus of elasticity and minimum plate thickness (cross-sectional area) for the elements of main girder, pylon and cables

Member element	Material Kind	E(Kgf/cm^2)	Minimum plate thickness, Minimum cross-sectional area
Main girder	SS400	2.1×10^6	12.7, 12.8, 9.0$^{1)}$ (mm)
Pylon	SS400	2.1×10^6	10.0, 9.0$^{2)}$ (mm)
Cable	SWPR7A	1.95×10^6	970.1$^{3)}$ (mm^2)

1) Minimum plate thicknesses of t_{gu}, t_{gl}, and t_{gw} in the main girder.
2) Minimum plate thicknesses of t_{tu} and t_{tw} in the pylon.
3) Minimum cross-sectional area in the cable

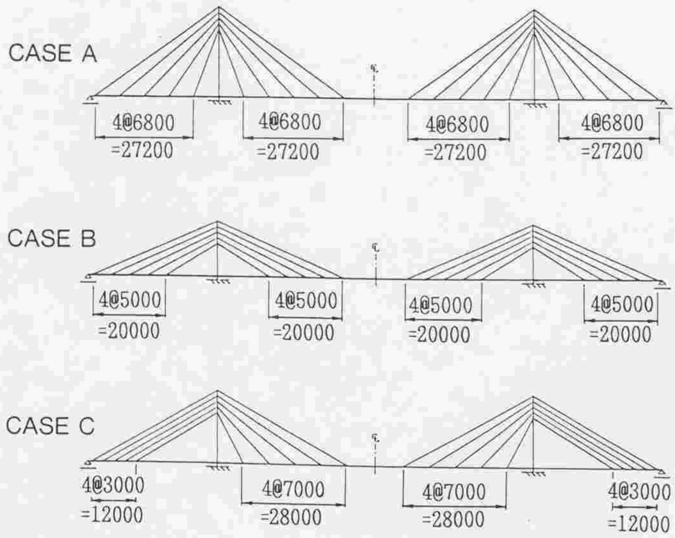

Fig.2-9 Initial cable arrangements and pylon heights for cases A, B and C

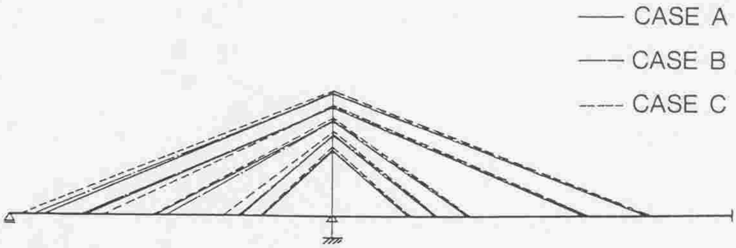

Fig.2-10 Comparisons of the optimum cable arrangements and pylon heights for cases A, B and C

Table 2-5 Comparisons of the optimum solutions for cases A, B and C

Design variables	CASE A		CASE B		CASE C	
	init[1]	opt.[2]	init.[1]	opt.[2]	init.[1]	opt.[2]
X_{C1}	34.00m	31.24m	34.00m	32.30m	34.00m	34.00m
X_{C2}	27.20	26.60	29.00	26.94	31.00	24.30
X_{C3}	20.40	19.11	24.00	17.72	28.00	19.40
X_{C4}	13.60	10.00	19.00	10.00	25.00	11.90
X_{C5}	6.80	8.00	14.00	8.00	22.00	8.00
X_{C6}	6.80	8.00	14.00	8.00	7.00	8.00
X_{C7}	13.60	10.89	19.00	11.04	14.00	10.92
X_{C8}	20.40	14.02	24.00	14.64	21.00	14.39
X_{C9}	27.20	26.71	29.00	27.22	28.00	27.23
X_{C10}	34.00	33.00	34.00	33.59	35.00	33.33
Y_C	19.00	6.95	10.00	6.85	15.00	7.25
TCOST	$180{,}758\times 10^3$(YEN)		$180{,}755\times 10^3$(YEN)		$180{,}967\times 10^3$(YEN)	

1) Initial values of cable arrangement and pylon height.
2) Optimum solutions of cable arrangement and pylon height.

span lengths 35m+85m+35m symbolizing the swans in the lake Tokiwa. In the design process, the proposed design method was applied as the main design tool for the first time. Comparisons of the optimized designs for several design conditions were made and final decision-making could be accomplished quite easily and efficiently by utilizing the proposed design method.

The design variables, $\mathbf{Z}=[\mathbf{t}_{gu}^T, \mathbf{t}_{gl}^T, \mathbf{t}_{w}^T, \mathbf{A}_C^T]^T$, $\mathbf{X}_C$ and Y_C, considered in the design process are shown in Fig.2-8. The price per unit volume of the ith member element at main girder, pylon and cable are, respectively, assumed as 5.9x10^5 yen/tf, 6.1x10^5 yen/tf and 31.0x10^5 yen/tf. Table 2-4 summarizes the material kind, modulus of elasticity and minimum plate thickness or cross-sectional area used for the elements of main girder, pylon and cables. The design variables $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C are optimized by the proposed design method in which three initial values of $\mathbf{X}_C$ and Y_C, cases A, B and C shown in Fig.2-9, are assumed to confirm the global optimality of optimal solutions obtained.

The comparisons of the optimum solutions for cases A, B and C are summarized in Table 2-5. Fig.2-10 shows the comparisons of the optimum cable arrangements and pylon heights for cases A, B and C. As clearly seen from Table 2-5 and Fig.2-10, the optimum $\mathbf{X}_C, Y_C$ and total cost obtained are quite similar for cases A, B and C. Therefore, we can say that the global optimum solution is obtained by the proposed optimum design method. Based on this result, the process of decision for the final optimum design has been made. The optimum design process is depicted in Fig.2-11.

Step 1: Even when the initial values of $\mathbf{X}_C$ are assumed as the unsymmetrical values to the pylon shown in Fig.2-9(case C), the optimum cable arrangement obtained is almost symmetrical to the pylon as shown in Fig. 2-11(Step1) and Table 2-6(Step1). This result shows that the symmetrical cable arrangement to the pylon is most economical for this cable-stayed bridge. Furthermore, the optimal configuration leads that a set of three cable stays replacing the lower three cable stays into one cable stay is advantageous from the economical viewpoint.

Step 2: Considering the optimum solution in Step1 and aesthetic view as a landmark in the Tokiwa Park, the set of cable stays is selected as original five and the cables are arranged symmetrically to the pylon as shown in Fig.2-11(Step2).

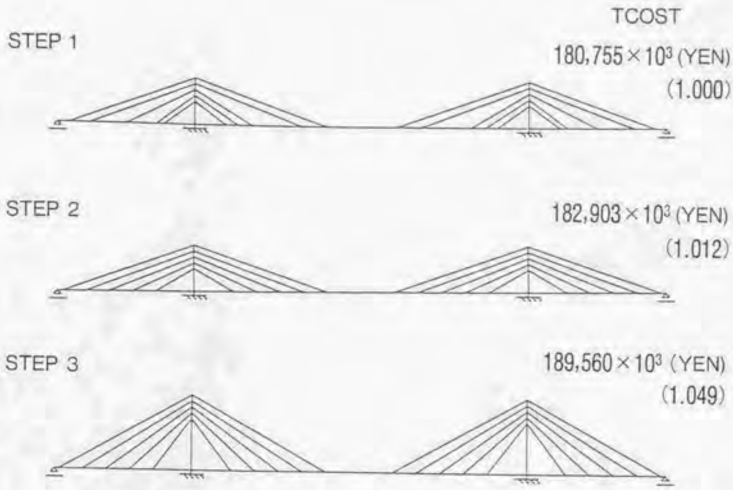

Fig.2-11 Optimum design process

Table 2-6 Comparisons of the optimum solutions for STEPS 1, 2 and 3

Design Variables	STEP 1	STEP 2	STEP 3
X_{C1}	32.30m	34.00m	34.00m
X_{C2}	26.94	28.00	28.00
X_{C3}	17.72	22.00	22.00
X_{C4}	10.00	16.00	16.00
X_{C5}	8.00	10.00	10.00
X_{C6}	8.00	10.00	10.00
X_{C7}	11.04	16.00	16.00
X_{C8}	14.64	22.00	22.00
X_{C9}	27.22	28.00	28.00
X_{C10}	33.59	34.00	34.00
Y_C	6.85	6.89	14.00
TCOST(YEN)	$180,755 \times 10^3$	$182,903 \times 10^3$	$189,560 \times 10^3$
RCOST[1)]	1.000	1.012	1.049

1) Ratio of TCOST for each STEP to TCOST for STEP 1

The distance between adjacent cable anchor positions is set at 6m by taking into account the restriction related to the design of diaphragm specified in JSHB[4]. Then, the bridge is re-optimized in which the design variables are **Z** and Y_C. The optimum solution is summarized in Table 2-6 (Step2). The optimum Y_C for Step1 and Step2 are, respectively, 6.85m and 6.89m. The total cost for Step2 increases 1.2% larger than that for Step1.

Step3 : From the optimum solution in Step2 it is clear that the most economical height of pylon(Y_C+9m) is around 16m. However, in Step3 the height of pylon is modified as 23m to emphasize the aesthetic feeling for symbolization in the Tokiwa Park and then, the bridge is optimized again in which the design variables are **Z** only. The optimum solution is summarized in Table 2-6(Step3). At the optimum solution, all cross-sectional dimensions, $\mathbf{t}_{gu}$, $\mathbf{t}_{gl}$, $\mathbf{t}_{tu}$, $\mathbf{A}_C$, are determined by the minimum plate thicknesses and minimum cross-sectional area. The total cost for Step3 increases 4.9% and 3.6% larger than those for Steps1 and 2, respectively.

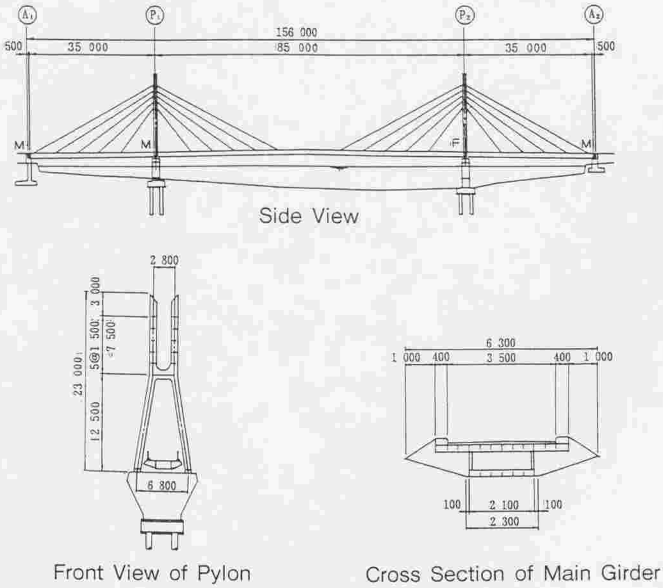

Fig.2-12 General view of the Swan Bridge

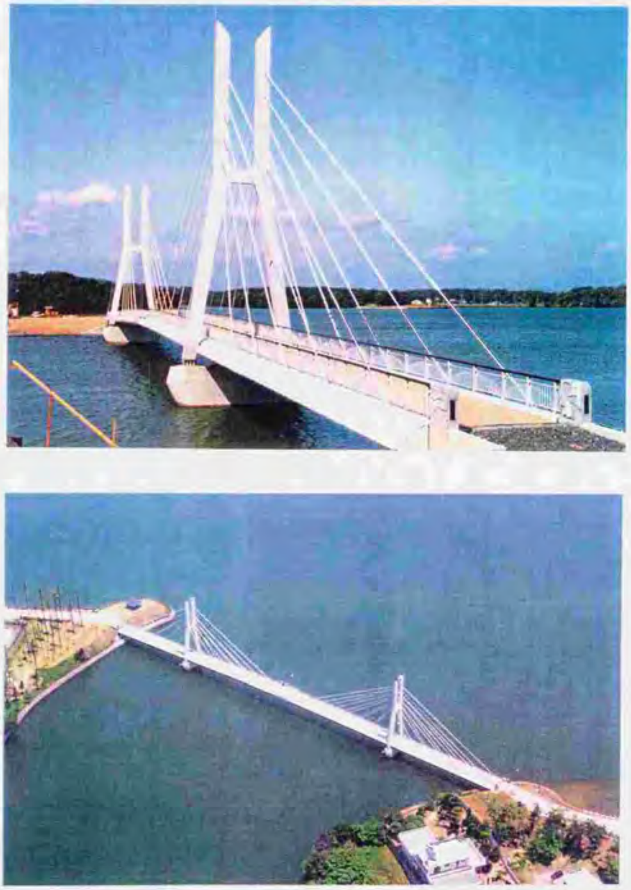

Fig.2-13 Viewing of the Swan Bridge in the lake Tokiwa
(by courtesy of Dr. Yasuo Watada of Ube Industries Ltd.)

By mutually comparing the optimum solutions for Steps1, 2 and 3, we concluded that the configuration for Step3 is most preferable as the landmark at the Tokiwa Park from the standpoint of total optimization considering not only cost minimization but also aesthetic feeling. The general view of the Swan Bridge including side view, front view of pylon and cross section of main girder is depicted in Fig.2-12. The photos of viewing of the Swan Bridge are shown in Figs.2-13.

2-5. CONCLUSIONS

In this Chapter, a rigorous and efficient optimum design method for steel cable-stayed bridges is presented. In this design method, not only the cross-sectional dimensions of cables, main girder and pylon elements but also the cable anchor positions on the main girder and the heights of pylons are dealt with as the design variables. The proposed optimum design method has been applied to the minimum-cost design problem of steel cable-stayed bridge with 48 cable stays and the practical design problem of the Swan Bridge at the Tokiwa Park. The theoretical rigorousness, efficiency and practical usefulness of the proposed optimum design method are demonstrated by investigating the optimum solutions at various design conditions.

The following conclusions can be drawn from this study:

(1) The global optimum solutions can be determined in 9-15 iterations quite efficiently by the proposed optimum design method.
(2) The optimum values of pylon height, cable anchor positions on the main girder, steel plate thicknesses of each main girder and pylon elements, and cross-sectional area of each cable appear to be reasonable and well balanced.
(3) In the numerical examples of steel cable-stayed bridge with 48 cable stays, the reduction of total cost from 2.7%-8.6% can be observed by dealing with the cable anchor positions on the main girder and the height of pylon as the design variables. Therefore, the treatment of cable arrangement as the design variables is extremely significant in the optimum design problem of steel cable-stayed bridges.
(4) From structural mechanics consideration, with regard to the optimum cable arrangement in the numerical example of steel cable-stayed bridge with 48 cable stays, the top two cables are parallel and are anchored at the end support in the side span, on the other hand, the cables are distributed as the geometric series in the center span. The cross-sectional areas of top two cables in the side and center spans are determined to be 3.6-1.4 times larger than those of the middle cables. The cross-sectional areas of unnecessary cables at the optimum solutions are found to be the imposed lower limit automatically by the proposed optimum design method.

(5) By applying the proposed method to the practical design of the Swan Bridge at the Tokiwa Park, the final decision-making could be accomplished quite easily and efficiently from the standpoint of total optimization considering not only the cost minimization but also the aesthetic feeling. Therefore, we can conclude that the proposed design method is quite useful in the practical design of steel cable-stayed bridges.

APPENDIX 2-1 Calculation of sensitivities of stress constraints with respect to design variables

The sensitivities of stress constraints for main girder, pylon and cable with respect to cross-sectional dimension can be calculated by eqs.(A2-1)-(A2-5).

(1) Sensitivities of g_{σ_g} in the main girder with respect to Z_{ir}

(a) For stress constraint at the upper flange plate;

$$a_{jir}=\left(\pm\frac{\partial \mathrm{N}_j}{\partial Z_{ir}}\right)\cdot\frac{1}{A_{gi}}-\left(\pm\frac{\mathrm{N}_j}{A_{gi}^2}\right)\cdot\frac{\partial A_{gi}}{\partial Z_{ir}}-\left(\pm\frac{\partial \mathrm{M}_j}{\partial Z_{ir}}\right)\cdot\frac{\mathrm{y}_{gi}}{\mathrm{I}_{gi}}$$

$$+\left(\pm\frac{\mathrm{M}_j}{\mathrm{I}_{gi}^2}\right)\cdot\mathrm{y}_{gi}\cdot\frac{\partial \mathrm{I}_{gi}}{\partial Z_{ir}}-\left(\pm\frac{\mathrm{M}_j}{\mathrm{I}_{gi}^2}\right)\cdot\frac{\partial \mathrm{y}_{gi}}{\partial Z_{ir}}-\frac{\partial \sigma_{ai}}{\partial Z_{ir}} \tag{A2-1}$$

(b) For stress constraint at the lower flange plate;

$$a_{jir}=\left(\pm\frac{\partial \mathrm{N}_j}{\partial Z_{ir}}\right)\cdot\frac{1}{A_{gi}}-\left(\pm\frac{\mathrm{N}_j}{A_{gi}^2}\right)\cdot\frac{\partial A_{gi}}{\partial Z_{ir}}+\left(\pm\frac{\partial \mathrm{M}_j}{\partial Z_{ir}}\right)\cdot\frac{\mathrm{y}_{gi}}{\mathrm{I}_{gi}}$$

$$-\left(\pm\frac{\mathrm{M}_j}{\mathrm{I}_{gi}^2}\right)\cdot\mathrm{y}_{gi}\cdot\frac{\partial \mathrm{I}_{gi}}{\partial Z_{ir}}+\left(\pm\frac{\mathrm{M}_j}{\mathrm{I}_{gi}^2}\right)\cdot\frac{\partial \mathrm{y}_{gi}}{\partial Z_{ir}}-\frac{\partial \sigma_{ai}}{\partial Z_{ir}} \tag{A2-2}$$

where I_{gi}, y_{gi} are, respectively, the moment of inertia of the *i*th member element and the distance from the neutral axis to top or bottom fiber of cross section in the main girder. The sign $\pm$ in ( ) indicates that the identical sign (+ or -) to stress due to N_j and M_j should be chosen.

(2) Sensitivities of $g_{\sigma_{t1j}}$ and $g_{\sigma_{t2j}}$ in the pylon with respect to Z_{ir}

(a) For constraint $g_{\sigma_{t1j}}$;

$$a_{j,ir} = \left\{\left(\pm\frac{\partial \mathrm{N}_j}{\partial Z_{ir}}\right).\frac{1}{A_{tj}} - \frac{|\mathrm{N}_j|}{A_{tj}^2}.\frac{\partial A_{tj}}{\partial Z_{ir}}\right\}.\frac{1}{\sigma_{cazj}} - \frac{\sigma_{cj}}{\sigma_{cazj}^2}.\frac{\partial \sigma_{cazj}}{\partial Z_{ir}} + \frac{1}{\sigma_{bagyj}.\mathrm{P}_j}.\left\{\left(\pm\frac{\partial \mathrm{M}_j}{\partial Z_{ir}}\right).\frac{\mathrm{y}_{tj}}{\mathrm{I}_{tj}}\right.$$

$$\left. - \frac{|\mathrm{M}_j|}{\mathrm{I}_{tj}^2}.\mathrm{y}_{tj}.\frac{\partial \mathrm{I}_{tj}}{\partial Z_{ir}} + \frac{|\mathrm{M}_j|}{\mathrm{I}_{tj}^2}.\frac{\partial \mathrm{y}_{tj}}{\partial Z_{ir}}\right\} - \frac{\sigma_{bcyj}}{\sigma_{bagyj}.\mathrm{P}_j^2.\sigma_{eayj}}.\left\{\frac{1}{A_{tj}}.\left(\pm\frac{\partial \mathrm{N}_i}{\partial Z_{ir}}\right)\right.$$

$$\left. + \frac{|\mathrm{N}_j|}{A_{tj}^2}.\frac{\partial A_{tj}}{\partial Z_{ir}} + \frac{\sigma_{cj}}{\sigma_{eayj}}.\frac{\partial \sigma_{eayj}}{\partial Z_{ir}}\right\} \qquad \text{(A2-3)}$$

(b) For constraint $g_{\sigma_{t2j}}$;

$$a_{j,ir} = \left\{\left(\pm\frac{\partial \mathrm{N}_j}{\partial Z_{ir}}\right).\frac{1}{A_{tj}} - \frac{|\mathrm{N}_j|}{A_{tj}^2}.\frac{\partial A_{tj}}{\partial Z_{ir}}\right\} + \frac{1}{\mathrm{P}_j}.\left\{\left(\pm\frac{\partial \mathrm{M}_j}{\partial Z_{ir}}\right).\frac{\mathrm{y}_{tj}}{\mathrm{I}_{tj}}\right.$$

$$\left. - \frac{|\mathrm{M}_j|}{\mathrm{I}_{tj}^2}.\mathrm{y}_{tj}.\frac{\partial \mathrm{I}_{tj}}{\partial Z_{ir}} + \frac{|\mathrm{M}_j|}{\mathrm{I}_{tj}^2}.\frac{\partial \mathrm{y}_{tj}}{\partial Z_{ir}}\right\} - \frac{\sigma_{bcyj}}{\mathrm{P}_j^2.\sigma_{eayj}}.\left\{\frac{1}{A_{tj}}.\left(\pm\frac{\partial \mathrm{N}_i}{\partial Z_{ir}}\right)\right.$$

$$\left. + \frac{|\mathrm{N}_j|}{A_{tj}^2}.\frac{\partial A_{tj}}{\partial Z_{ir}} + \frac{\sigma_{cj}}{\sigma_{eayj}}.\frac{\partial \sigma_{eayj}}{\partial Z_{ir}}\right\} - \frac{\partial \sigma_{ealj}}{\partial Z_{ir}} \qquad \text{(A2-4)}$$

where $\mathrm{P}_j = \{1 - (\sigma_{cj}/\sigma_{eayj})\}$, $\sigma_{cj} = \frac{|\mathrm{N}_j|}{A_{tj}}$, $\sigma_{bcyj} = \frac{|\mathrm{M}_j|}{\mathrm{I}_{tj}}.\mathrm{y}_{tj}$

$\mathrm{I}_{ti}, \mathrm{y}_{ti}$ are, respectively, the moment of inertia of the ith member element and the distance from the neutral axis to top fiber of cross section in the pylon. The sign $\pm$ in ( ) indicates that the identical sign (+ or -) to N_j or M_j should be chosen.

(3) Sensitivities of $g_{\sigma_{cj}}$ in the cable with respect to Z_{ir}

$$a_{j,ir} = \frac{\partial \mathrm{N}_j}{\partial Z_{ir}}.\frac{1}{A_{cj}} - \frac{\mathrm{N}_j}{A_{cj}^2}.\frac{\partial A_{cj}}{\partial Z_{ir}} \qquad \text{(A2-5)}$$

In the above expressions, the positive directions of member forces are defined in Fig. 2-4. The calculations of sensitivities of stress constraints with respect to X_{Ck} and Y_{Cl} can be carried out in the same manner by replacing Z_{ir} into X_{Ck} and Y_{Cl} in the above expressions.

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Chapter 3

OPTIMUM DESIGN SYSTEM FOR STEEL CABLE-STAYED BRIDGES DEALING WITH SHAPE, SIZING VARIABLES AND CABLE PRESTRESSES

3-1. INTRODUCTION

In Chapter 2, it is illustrated that the total cost of steel cable-stayed bridge is greatly affected by the cable anchor positions on the main girder and the heights of pylons, therefore, the treatment of the cable anchor positions on the main girder and the heights of pylons as the design variables is extremely significant in the optimization of steel cable-stayed bridges.

By the way, the distribution of member forces, such as maximum and minimum bending moments and axial forces in the main girder and pylon, and cable tensions, can be controlled considerably by giving prestresses into cables. Therefore, the cable prestresses have also been treated as one of significant design parameters in the practical design of cable-stayed bridges.

In the earlier studies of determination method for cable prestresses in steel cable-stayed bridges, Yamada and Daiguji[1] studied a method to determine the optimal cable prestresses on the basis of the element optimization in the main girder. Maeda et al.[2] and Nagai et al.[3] determined the cable prestresses by calculating the support reactions of multispan continuous beam in which the main girder in cable-stayed bridge is considered as the multispan continuous beam with supports at the cable anchor positions. Yamada et al. studied the method for determination of cable prestresses on the basis of the minimum strain energy criterion[4]. Hoshino studied a practical method to determine the cable prestresses based on a structural analysis method using modified cross-sectional properties under the minimum cost criterion[5]. Torii et al.[6] studied a method to determine the cable prestresses without recursive calculation by introducing the relation between the redundant forces in statically indeterminate structures and objective function. Nakamura and Wyatt determined the cable prestresses on the basis of the limit states design code by using a linear programming algorithm[7]. In most of these researches, the cable prestresses are determined so as to reduce the peaks of positive and negative bending moments in the main girder and to average out the bending moment distributions in the main girder.

In this Chapter the optimum design method stated in Chapter 2 is extended to be able to deal with cable prestresses as the design variables, and a general purpose, rigorous and efficient optimum design system for steel cable-stayed bridges is developed. In this design system the pseudo-loads applied to the cables are selected as the design variables with respect to cable prestresses and the optimum cable prestresses are determined from the economical viewpoint. The design problem is formulated as a minimum-cost design problem subject to the stress constraints taken from the Japanese Specifications for Highway Bridges[JSHB][8]. By investigating a simple design example in which a pseudo-load is dealt with as design variables in addition to cross-sectional dimensions, it is illustrated that the computational effort to obtain the optimum solution of the design problem in which the pseudo-load is dealt with as design variables in addition to cross-sectional dimensions is remarkably increased compared with that of the design problem with only cross-sectional dimensions. This result indicates that the problems of convergency and reliability of the result obtained will arise when the number of pseudo-loads is increased and, furthermore, the cable arrangement is also taken into account as design variables. For this reason, the following powerful two-stage optimum design process is proposed to solve the cost-minimization problem. At the first stage optimization process, the cable arrangement and sizing variables are optimized by the optimization algorithm based on dual method which is developed in Chapter 2. At the second stage optimization process, the optimum values of pseudo-loads, which induce the optimum prestresses into the cables, and the optimum sizing variables are determined so as to minimize the total cost of the bridge further by utilizing the sensitivities of objective function, behavior constraints and cross-sectional dimensions with respect to the pseudo-loads and a modified LP algorithm.

The proposed optimum design method has been applied to the minimum-cost design problems of practical-scale steel cable-stayed bridge with 64 cable stays. The theoretical rigorousness, efficiency and practical usefulness of the proposed optimum design system are demonstrated by giving several numerical design examples and investigations of the optimum solutions at various design conditions. It is also illustrated that 2.6%-4.1% of the total cost of the bridge can be reduced by giving the optimum prestresses in the cables.

3-2. FORMULATION OF OPTIMUM DESIGN PROBLEM

(1) Design variables

In this optimum design system, the pseudo-loads $\mathbf{P}_P$ applied to each cable as seen in Fig.3-1 are taken into account as the design variables with respect to the cable prestresses in addition to the design variables described in Chapter 2. Namely, the shapes of cross sections of main girder and pylon are assumed as the box types depicted in Figs.2-1 (a) and (b), respectively. The span lengths, number of cables, height and width of cross sections of main girder and pylon, and material types to be used for each structural element are assumed as the preassigned constant design parameters. The cross sections in the main girder and pylon can be varied at the same positions described in section 2-2.(1). The design variables related to the cross-sectional dimensions of all member elements are the cross-sectional area of each cable, $\mathbf{A}_C$, and the thicknesses of upper and lower flange plates of each main girder element, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$, and pylon element, $\mathbf{t}_{tu}$ and $\mathbf{t}_{tl}$, as shown in Fig 2-1, where $\mathbf{t}_{tu}$ and $\mathbf{t}_{tl}$ in the pylon are assumed to be the same. The thicknesses of these flange plates are dealt with as the converted thicknesses which include the contributions of the longitudinal stiffeners. These sizing variables are denoted as $\mathbf{Z}$, hereafter.

$$\mathbf{Z} = \left[\boldsymbol{Z}_1^T, \cdots, \boldsymbol{Z}_i^T, \cdots, \boldsymbol{Z}_n^T\right]^T \tag{3-1}$$

The distance from the pylon to each cable anchor position on the main girder, X_{Ck}, and the height of the lowest cable in the pylon from the axis of main girder, Y_C, in Fig.2-2 are dealt with as the design variables with respect to the cable anchor

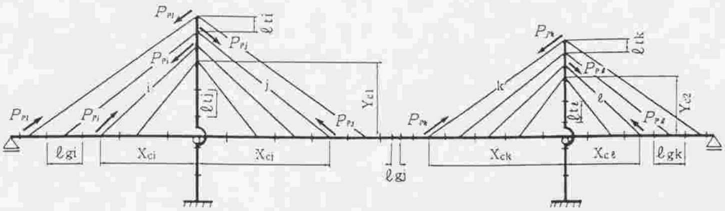

Fig.3-1 Design variables $\mathbf{X}_C$, $\mathbf{Y}_C$ and $\mathbf{P}_P$

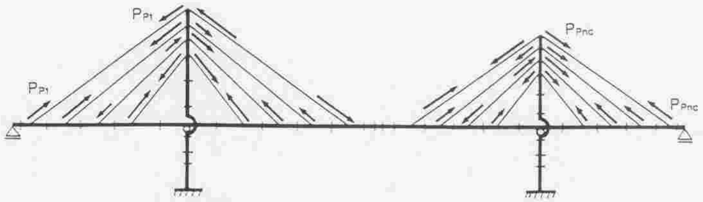

Fig.3-2 Determination of cable prestresses

positions on the main girder and the height of pylon, and these design variables are termed by the vectors $\mathbf{X}_C$ and $\mathbf{Y}_C$, respectively.

$$\mathbf{X}_C = \left[X_{C1}, X_{C2}, \cdots, X_{CK}\right]^T \tag{3-2}$$

$$\mathbf{Y}_C = \left[Y_{C1}, Y_{C2}, \cdots, Y_{CL}\right]^T \tag{3-3}$$

where K and L are, respectively, the numbers of design variables with respect to $\mathbf{X}_C$ and $\mathbf{Y}_C$.

As the design variables with respect to the cable prestresses, we select the pseudo-loads applied to each cable, $\mathbf{P}_P$, as seen in Fig.3-1.

$$\mathbf{P}_P = \left[P_{P1}, P_{P2}, \cdots, P_{Pnc}\right]^T \tag{3-4}$$

where nc denotes the total number of cables.

The optimum prestresses to be given to the cables, $\mathbf{P}_S$, can be determined as the resultant forces of $\mathbf{P}_P^*$ and the axial forces in the cables, $\mathbf{N}_C$, which are obtained by analyzing the bridge subjected to $\mathbf{P}_P^*$ only as shown in Fig.3-2 .

$$\mathbf{P}_S = \mathbf{P}_P^* + \mathbf{N}_C \tag{3-5}$$

(2) Design constraints

In this design system, the following constraints related to the stresses at each cable and elements of main girder and pylon, slenderness ratios of the pylon elements, and upper and lower limits of the design variables, which are specified in the JSHB[8], are considered in the optimization process.

(a) The stress at the main girder element:

$$g_{\sigma_{gi}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)=\sigma_i(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)-\sigma_{ai}(\mathbf{Z})\le 0 \qquad (i=1,\cdots,m_g) \qquad (3\text{-}6)$$

(b) The stress at the pylon element:

$$g_{\sigma_{t1j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)=\frac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)}{\sigma_{cazj}(\mathbf{Z},\mathbf{Y}_C)}-\frac{\sigma_{bcyj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)}{\sigma_{bagyj}\left(1-\dfrac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)}{\sigma_{eayj}(\mathbf{Z},\mathbf{Y}_C)}\right)}-1\le 0$$

$$(j=1,\cdots,m_t) \qquad (3\text{-}7)$$

$$g_{\sigma_{t2j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)=\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)+\frac{\sigma_{bcyj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)}{\left(1-\dfrac{\sigma_{cj}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)}{\sigma_{eayj}(\mathbf{Z},\mathbf{Y}_C)}\right)}-\sigma_{calj}(\mathbf{Z})\le 0$$

$$(j=1,\cdots,m_t) \qquad (3\text{-}8)$$

(c) The slenderness ratio of the pylon element:

$$g_{lj}(\mathbf{Z},\mathbf{Y}_C)=l_j(\mathbf{Y}_C)/r_j(\mathbf{Z})-120\le 0 \qquad (j=1,\cdots,nt) \qquad (3\text{-}9)$$

(d) The stress at the cable element:

$$g_{\sigma_{ck}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)=\sigma_k(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_P)-\sigma_{ak}\le 0 \qquad (k=1,\cdots,nc) \qquad (3\text{-}10)$$

(e) The upper and lower limits of the design variables:

$$\mathbf{Z}^{(l)}\le\mathbf{Z}\le\mathbf{Z}^{(u)},\ \ \mathbf{X}_C{}^{(l)}\le\mathbf{X}_C\le\mathbf{X}_C{}^{(u)},\ \ \mathbf{Y}_C{}^{(l)}\le\mathbf{Y}_C\le\mathbf{Y}_C{}^{(u)},\ \ \mathbf{P}_P{}^{(l)}\le\mathbf{P}_P\le\mathbf{P}_P{}^{(u)} \qquad (3\text{-}11)$$

The indexes in the above expressions are explained in section 2-2.(2).

The minimum web plate thicknesses of each elements of main girder and pylon are determined so as to satisfy the composite stress criteria on the web plates.

The working stress at a structural element is calculated as the sum of the stresses due to dead loads in the cantilever system at the erection closing stage and the stresses due to traffic loads and a part of dead loads in the continuous girder system at the service stage. The two structure-load systems at erection closing stage and service stage (see Figs.2-3 (a) and (b)) are analyzed by the finite element method as a 2-dimensional plane frame structure.

The maximum and minimum values of **N**, **S**, and **M** at the stress inspection points shown in Figs. 2-4 (a) and (b) due to traffic and impact loads are calculated by using the corresponding influence lines.

(3) Formulation of optimum design problem

By taking into account of the design variables and design constraints described in sections 3-2.(1) and 3-2.(2), the minimum cost design problem of steel cable-stayed bridge can be formulated as follows:

Find $\quad \mathbf{Z},\ \mathbf{X}_C, \mathbf{Y}_C, \mathbf{P}_\mathrm{P},\quad$ which

minimize
$$\mathrm{TCOST}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_\mathrm{P}) = \sum_{i=1}^{n} \mathrm{W}_i(\mathbf{X}_C,\mathbf{Y}_C)\cdot A_i(\mathbf{Z}_i) + \sum_{k=1}^{nc} T_{Pk}\cdot P_{Pk} \qquad (3\text{-}12)$$

subject to
$$g_{\sigma_{gi}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_\mathrm{P}) \le 0 \qquad (i=1,\cdots,\mathrm{m_g}) \qquad (3\text{-}13)$$
$$g_{\sigma_{t1j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_\mathrm{P}) \le 0 \qquad (j=1,\cdots,\mathrm{m_t}) \qquad (3\text{-}14)$$
$$g_{\sigma_{t2j}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_\mathrm{P}) \le 0 \qquad (j=1,\cdots,\mathrm{m_t}) \qquad (3\text{-}15)$$
$$g_{lj}(\mathbf{Z},\mathbf{Y}_C) \le 0 \qquad (j=1,\cdots,nt) \qquad (3\text{-}16)$$
$$g_{\sigma_{ck}}(\mathbf{Z},\mathbf{X}_C,\mathbf{Y}_C,\mathbf{P}_\mathrm{P}) \le 0 \qquad (k=1,\cdots,nc) \qquad (3\text{-}17)$$
$$\mathbf{Z}^{(l)} \le \mathbf{Z} \le \mathbf{Z}^{(u)},\ \ \mathbf{X}_C^{\ (l)} \le \mathbf{X}_C \le \mathbf{X}_C^{\ (u)},$$
$$\mathbf{Y}_C^{\ (l)} \le \mathbf{Y}_C \le \mathbf{Y}_C^{\ (u)},\ \ \mathbf{P}_\mathrm{P}^{\ (l)} \le \mathbf{P}_\mathrm{P} \le \mathbf{P}_\mathrm{P}^{\ (u)} \qquad (3\text{-}18)$$

where T_{Pk} is the cost for unit loading of P_{Pk} and as it is reasonable to assume $T_{Pk} \approx 0.0$. In practice its value is taken as 0.0 in our design system.

3-3. OPTIMUM DESIGN METHOD BY TWO-STAGE OPTIMIZATION PROCESS

(1) Two-stage optimization process[9-11]

When we attempt to solve the optimum design problem defined in eqs.(3-12)-(3-18), it will be found that the computational effort to obtain the optimum solution is remarkably increased compared with the design problem with only cross-sectional dimensions. This matter is illustrated with a simple design example of cable-stayed system shown in Fig.3-3.

In this problem, the width of cross section in the beam, B, a cross section of

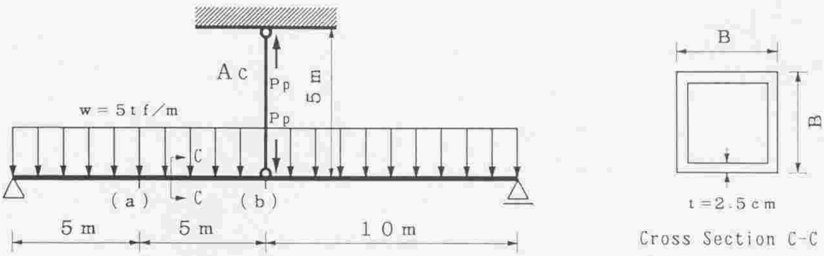

Fig.3-3 A cable-stayed system

cable, A_C, and a pseudo-load applied to the cable, P_P, are dealt with as the design variables. The stress due to positive bending moment at point (a), stresses due to positive and negative bending moments at point (b) in the beam, and stress in the cable are taken into account as the behavior constraints, $g_a(B,P_P)$, $g_b^{(+)}(B,P_P)$, $g_b^{(-)}(B,P_P)$ and $g_C(A_C,P_P)$. The objective is to minimize the volume of the structure. Then, the optimum design problem for the structure shown in Fig.3-3 can be formulated as:

Find $\quad B$, A_C, P_P, which

minimize $\quad \mathbf{V}(B,A_C)=\rho_g(20000B-50000)+\rho_c\cdot 500\cdot A_C \qquad (3\text{-}19)$

subject to

$$g_a(B,P_P)=\frac{600000\cdot M_a(P_P)}{B^4-(B-5)^4}B-\sigma_{ab}\le 0 \qquad (3\text{-}20)$$

$$g_b^{(+)}(B,P_P)=\frac{600000\cdot M_b(P_P)}{B^4-(B-5)^4}B-\sigma_{ab}\le 0 \qquad (3\text{-}21)$$

$$g_b^{(-)}(B,P_P)=-\frac{600000\cdot M_b(P_P)}{B^4-(B-5)^4}B-\sigma_{ab}\le 0 \qquad (3\text{-}22)$$

$$g_C(A_C,P_P)=\frac{T_C(P_P)}{A_C}-\sigma_{ac}\le 0 \qquad (3\text{-}23)$$

where the bending moments at points (a) and (b), $M_a(P_P)$, $M_b(P_P)$, and cable tension $T_C(P_P)$ are, respectively, given by

$$M_a(P_P)=187.5-2.5\cdot T_C(P_P) \qquad (3\text{-}24)$$

$$\mathrm{M}_b(\mathrm{P_P}) = 250.0 - 5 \cdot \mathrm{T}_C(\mathrm{P_P}) \tag{3-25}$$

$$\mathrm{T}_C(\mathrm{P_P}) = \frac{\dfrac{5wl^4}{384\mathrm{EI}} + \dfrac{\mathrm{P_P}l^3}{48\mathrm{EI}}}{\dfrac{l_C}{\mathrm{E}_C\mathrm{A}_C} + \dfrac{l^3}{48\mathrm{EI}}} - \mathrm{P_P} \tag{3-26}$$

In the problem, the following values are assumed, namely, E=2100000kgf/cm², E_c=2000000kgf/cm², l=20m, l_c=5m, σ_{ab}=1900kgf/cm², σ_{ac}=5000kgf/cm², ρ_g=1.0, ρ_c=2.0. The sensitivity of objective function with respect to P_P is zero, namely, the contribution of P_P to the objective function is not considered. By solving this

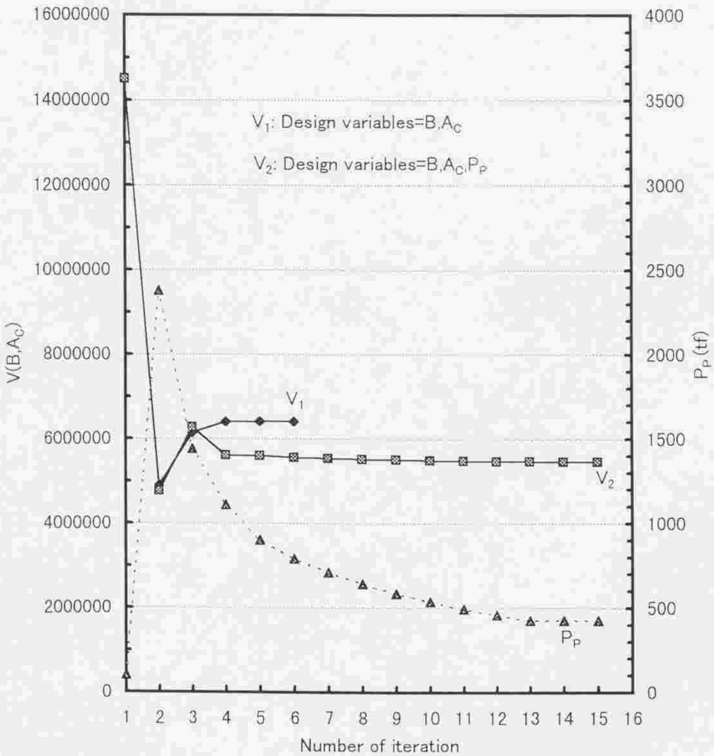

Fig.3-4 Iteration histories for the problems in which design variables are B and A_C, and B, A_C and P_P

problem with the aid of dual method, we can obtain the optimum solutions as B=29.2cm, A_C=11.67cm², P_P=423.55tf and $T_C(P_P)$=58.33tf. To investigate the effect of treatment of design variable P_P on the computational effort to obtain the optimum solution the design problem with only cross-sectional dimensions, B and A_C, are also solved. For this problem the optimum solutions obtained are B=33.95cm, A_C= 12.34cm², $T_C(P_P)$=61.68tf. The iteration histories for the problems in which design variables are B and A_C, and B, A_C and P_P are compared in Fig.3-4. As clearly seen from Fig.3-4, the optimum solution for the problem in which design variables are B and A_C can be obtained after 5 iterations quite efficiently, while, for the problem with B, A_C and P_P, 15 iterations are required until P_P converges to the constant value. This result indicates that the problems of convergency and reliability of the result obtained will arise when the number of pseudo-loads is increased and, furthermore, the cable arrangement is also dealt with as design variables. For this reason, the following two-stage optimization process is proposed in this study to solve the optimum design problem in eqs.(3-12)-(3-18).

At the first stage optimization process, $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ or the selected design variables among $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ by the designer are dealt with as the design variables and the optimum solutions of those design variables are determined by the optimization algorithm developed in Chapter 2. Namely, applying the convex and linear approximation concept the objective function and the behavior constraints are approximated by using the first-order partial derivatives and the primary design variables, $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$, and their reciprocal design variables. The approximate subproblem is solved by dual method. The optimized TCOST, $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ under the design loads $\mathbf{P}_R$ at this stage are denoted as $\mathrm{TCOST}^*(\mathbf{P}_R)$, $\mathbf{Z}^*(\mathbf{P}_R)$, $\mathbf{X}_C^*(\mathbf{P}_R)$ and $\mathbf{Y}_C^*(\mathbf{P}_R)$.

As it will be described in the design examples, the effects of the optimum pseudo-loads on the optimum cable anchor positions $\mathbf{X}_C^*$ and $\mathbf{Y}_C^*$ are negligibly small. Therefore, after the first stage optimization process, $\mathbf{X}_C^*(\mathbf{P}_R)$ and $\mathbf{Y}_C^*(\mathbf{P}_R)$ are fixed, and a finite value of pseudo-load $\overline{\Delta P_{Pi}}$ is applied to the ith cable in addition to the design loads $\mathbf{P}_R$ and the pseudo-loads $\mathbf{P}_P$ at the current stage. The cable-stayed bridge is optimized again dealing with $\mathbf{Z}$ only by utilizing the first stage optimization algorithm described in Chapter 2. The optimized TCOST, $\mathbf{g}$ and $\mathbf{Z}$ by this process are denoted as $\mathrm{TCOST}^0(\mathbf{P}_R+\mathbf{P}_P+\overline{\Delta P_{Pi}})$, $\mathbf{g}^0(\mathbf{P}_R+\mathbf{P}_P+\overline{\Delta P_{Pi}})$ and $\mathbf{Z}^0(\mathbf{P}_R+\mathbf{P}_P+\overline{\Delta P_{Pi}})$. The approximate sensitivities of TCOST, $\mathbf{g}$ and $\mathbf{Z}$ with respect to P_{Pi} are, then, calculated by a finite difference formula using the two optimum solutions obtained under the loads $\mathbf{P}_{\mathrm{R}}+\mathbf{P}_{\mathrm{P}}$ and $\mathbf{P}_{\mathrm{R}}+\mathbf{P}_{\mathrm{P}}+\overline{\Delta P}_{Pi}$. Then, a linear programming problem in terms of the finite increments or decrements of pseudo-loads, $+\Delta\mathbf{P}_{\mathrm{P}}^{K}$ or $-\Delta\mathbf{P}_{\mathrm{P}}^{K}$ (where K denotes the iteration number) is formulated utilizing the sensitivities obtained above and the move limit constraints on $+\Delta\mathbf{P}_{\mathrm{P}}^{K}$ and $-\Delta\mathbf{P}_{\mathrm{P}}^{K}$. The increments or decrements of pseudo-loads, $+\Delta\mathbf{P}_{\mathrm{P}}^{K}$ or $-\Delta\mathbf{P}_{\mathrm{P}}^{K}$, to be applied to the cables for minimizing the objective function are determined with the aid of a modified LP algorithm, and the changes in $\mathbf{Z}$ due to $+\Delta\mathbf{P}_{\mathrm{P}}^{K}$ or $-\Delta\mathbf{P}_{\mathrm{P}}^{K}$, $\Delta\mathbf{Z}^{K}$, are calculated from the sensitivities of $\mathbf{Z}$ with respect to P_{Pi}. The cable-stayed bridge with $\mathbf{Z}^{K}\left(=\mathbf{Z}^{K-1}+\Delta\mathbf{Z}^{K}\right)$ is re-optimized for the combined effect of the design loads $\mathbf{P}_{\mathrm{R}}$ and the improved pseudo-loads $\mathbf{P}_{\mathrm{P}}^{K}\left(=\mathbf{P}_{\mathrm{P}}^{K-1}+\Delta\mathbf{P}_{\mathrm{P}}^{K}\right)$. The improvements of $\mathbf{P}_{\mathrm{P}}$ are iterated until TCOST converges to the minimum value.

(2) First stage optimization process[12,13]

At the first stage optimization process, $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$, or the selected design variables among $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ by the designer are dealt with as the design variables. Utilizing the convex and linear approximation concept, the primary design problem (eqs.(2-10)-(2-16)) is transformed into the approximate subproblem with the objective function and behavior constraints, the primary design variables $\mathbf{X}_C$, $\mathbf{Y}_C$ and their reciprocal design variables. In the objective function, the constant term can be neglected in the optimization process and only the change in the objective function, $\Delta\mathrm{TCOST}\left(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C\right)$, need to be considered. The approximate subproblem is solved by dual method. In the optimization algorithm based on dual method, a separable Lagrangian function $\mathbf{L}(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C, \lambda)$(eq.2-21) is introduced and the optimum solutions of $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ and λ can be obtained by minimizing $\mathbf{L}\left(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C, \lambda\right)$ with respect to $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ and maximizing it with respect to λ. The values of $\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C$ which minimize the $\mathbf{L}\left(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C, \lambda\right)$ are given by the simple expressions (eqs.(2-25)-(2-31)) analytically. The maximization of $\mathbf{L}\left(\mathbf{Z}, \mathbf{X}_C, \mathbf{Y}_C, \lambda\right)$ is carried out by the Newton-type algorithm (eqs.(2-33)-(2-38)). In this optimization process, the adaptive move limit constraints, maximum 10%, are imposed on the changing rates of $\mathbf{X}_C$ and $\mathbf{Y}_C$ to ensure smooth convergence to the optimum solutions. The detailed algorithm of first stage optimization process is explained in Chapter 2.

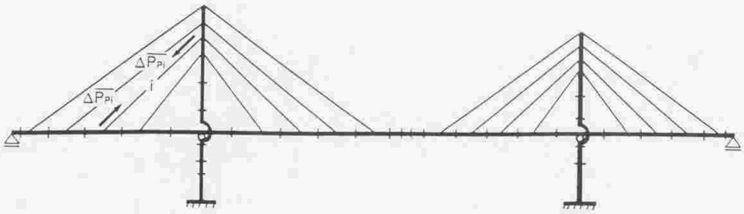

Fig.3-5 A finite pseudo-load $\overline{\Delta P}_{Pi}$

(3) Second stage optimization process[9-11]

(a) Calculation of sensitivities with respect to pseudo-loads $\mathbf{P}_\mathrm{P}$

As described in section 3-3.(1), the optimum pseudo-loads $\mathbf{P}_\mathrm{P}$ are determined by using a modified LP algorithm. For the formulation of linear programming problem, the sensitivities of total cost, stress constraints and cross-sectional dimensions with respect to the pseudo-loads $\mathbf{P}_\mathrm{P}$ need to be calculated.

The optimum cable anchor positions $\mathbf{X}_C^*(\mathbf{P}_\mathrm{R})$ and $\mathbf{Y}_C^*(\mathbf{P}_\mathrm{R})$ determined during the first stage optimization process by considering only the design loads $\mathbf{P}_\mathrm{R}$ are found to be scarcely affected by the optimum pseudo-loads as seen in the design examples. Therefore, $\mathbf{X}_C^*(\mathbf{P}_\mathrm{R})$ and $\mathbf{Y}_C^*(\mathbf{P}_\mathrm{R})$ are fixed in the second stage optimization process. In the calculation of the sensitivities with respect to the *i*th pseudo-load P_{Pi}, a finite value of pseudo-load $\overline{\Delta P}_{Pi}$ is applied to the *i*th cable, as shown in Fig.3-5, in addition to the design loads $\mathbf{P}_\mathrm{R}$ and the current pseudo-loads $\mathbf{P}_\mathrm{P}$. The cost minimization problem of the cable-stayed bridge subjected to $\mathbf{P}_\mathrm{R}$, $\mathbf{P}_\mathrm{P}$ and $\overline{\Delta P}_{Pi}$ can then be formulated as:

Find $\quad \mathbf{Z}(\mathbf{P}_\mathrm{R}+\mathbf{P}_\mathrm{P}+\overline{\Delta P}_{Pi})$, which

minimize

$$\mathrm{TCOST}(\mathbf{P}_\mathrm{R}+\mathbf{P}_\mathrm{P}+\overline{\Delta P}_{Pi}) = \sum_{j=1}^{n} \mathrm{W}_j \cdot A_j(Z_j(\mathbf{P}_\mathrm{R}+\mathbf{P}_\mathrm{P}+\overline{\Delta P}_{Pi})) + T_{Pi}(\mathbf{P}_\mathrm{P}+\overline{\Delta P}_{Pi}) \qquad (3\text{-}27)$$

subject to

$$g_j(\mathbf{P}_\mathrm{R}+\mathbf{P}_\mathrm{P}+\overline{\Delta P}_{Pi}) \le 0 \qquad (j=1,\cdots,m) \qquad (3\text{-}28)$$

$$\mathbf{Z}^{(l)} \le \mathbf{Z}(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi}) \le \mathbf{Z}^{(u)} \tag{3-29}$$

In the above formulation, $\mathbf{Z}$ is the only design variable. g_j is the jth constraint in the set of design constraints in eqs.(3-13)-(3-17). As described in section 3-2.(3), the value of T_{P_i} is taken as 0.0 in our design system.

The optimum design problem formulated in eqs.(3-12)-(3-18) can be solved quite readily by using the first stage optimization algorithm. The cost of computing the final optimum pseudo-load P_{P_i} is affected considerably by the magnitude of $\overline{\Delta P}_{Pi}$. After an initial investigation of the convergence to the optimum $\mathbf{P}_P$, we determined the upper limit of the magnitude of $\overline{\Delta P}_{Pi}$ as 5% of the maximum cable tension produced by the design loads in the ith cable. The optimum values of TCOST, g and $\mathbf{Z}$ obtained by solving the optimum design problem in eqs.(3-27)-(3-29) are denoted, respectively, as $\mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi})$, $\mathbf{g}^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi})$ and $\mathbf{Z}^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi})$.

The approximate sensitivities of TCOST, g_j and Z_{kr} with respect to P_{Pi}, denoted as T_i, d_{ji} and e_{kri}, are calculated using the following finite difference formula :

$$T_i = \frac{\partial \mathrm{TCOST}(\overline{\Delta P}_{Pi})}{\partial P_{Pi}}$$

$$\approx \frac{\mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi}) - \mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P)}{\overline{\Delta P}_{Pi}} \tag{3-30}$$

$$d_{ji} = \frac{\partial g_j(\overline{\Delta P}_{Pi})}{\partial P_{Pi}} \approx \frac{g_j^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi}) - g_j^0(\mathbf{P}_R + \mathbf{P}_P)}{\overline{\Delta P}_{Pi}} \tag{3-31}$$

$$e_{kri} = \frac{\partial Z_{kr}(\overline{\Delta P}_{Pi})}{\partial P_{Pi}} \approx \frac{Z_{kr}^0(\mathbf{P}_R + \mathbf{P}_P + \overline{\Delta P}_{Pi}) - Z_{kr}^0(\mathbf{P}_R + \mathbf{P}_P)}{\overline{\Delta P}_{Pi}} \tag{3-32}$$

where $\mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P)$, $g_j^0(\mathbf{P}_R + \mathbf{P}_P)$ and $Z_{kr}^0(\mathbf{P}_R + \mathbf{P}_P)$ are, respectively, the optimum values of TCOST, g_j and Z_{kr} of the bridge subjected to the design loads $\mathbf{P}_R$ and pseudo-loads $\mathbf{P}_P$.

(b) Formulation of linear programming problem

Utilizing the above sensitivities, the objective function and the design constraints, TCOST and $\mathbf{g}$, are approximated to the linear expressions on $\Delta\mathbf{P}_P$.

Consequently, a linear programming problem is derived for the determination of the improvements of $\mathbf{P}_{\mathrm{P}}$, $\Delta\mathbf{P}_{\mathrm{P}}$, so as to reduce the total cost of the bridge. This can be stated as:

Find $\quad \Delta\mathbf{P}_{\mathrm{P}}$, which

minimize $$\Delta\mathrm{TCOST}(\Delta\mathbf{P}_{\mathrm{P}}) = \sum_{i=1}^{nc} T_i \cdot \Delta P_{Pi} \tag{3-33}$$

subject to $$\overline{g}_j(\Delta\mathbf{P}_{\mathrm{P}}) = \sum_{i=1}^{nc} d_{ji} \cdot \Delta P_{Pi} + g_j(\mathbf{P}_{\mathrm{R}} + \mathbf{P}_{\mathrm{P}}) \le 0 \qquad (j=1,\cdots,\mathrm{m}) \tag{3-34}$$

$$|\ \Delta P_{Pi}\ | < \xi \cdot \overline{\Delta P}_{Pi} \qquad (i=1,\cdots,nc) \tag{3-35}$$

where ξ is the adaptive move limit parameter on $\Delta\mathbf{P}_{\mathrm{P}}$. Based on our investigations of the convergence of $\mathbf{P}_{\mathrm{P}}$ to the final optimum solutions, its value is assumed as 4.0 in our design system.

(c) Determination of the best $\Delta\mathbf{P}_{\mathrm{P}}$ by modified LP algorithm

In the design of cable-stayed bridge, the cables are prestressed by not only the tensile force but also the force to reduce the tensile stress if it is found to be effective in lowering the total cost of the bridge.

Consequently, in the linear programming problem, eqs.(3-33)-(3-35), we introduce new variables $\Delta P'_{Pi}$ and $\Delta P''_{Pi}$ in place of ΔP_{Pi}. ΔP_{Pi} is defined as

$$\Delta P_{Pi} = \Delta P'_{Pi} - \Delta P''_{Pi} \qquad (i=1,\cdots,nc) \tag{3-36}$$

where $\quad \Delta P'_{Pi} \ge 0, \quad \Delta P''_{Pi} \ge 0$

and the linear programming problem is re-formulated as follows:

Find $\quad \Delta\mathbf{P}'_{\mathrm{P}},\ \Delta\mathbf{P}''_{\mathrm{P}}$, which

minimize $$\Delta\mathrm{TCOST}(\Delta\mathbf{P}'_{\mathrm{P}}, \Delta\mathbf{P}''_{\mathrm{P}}) = \sum_{i=1}^{nc} T_i \cdot \Delta P'_{Pi} - \sum_{i=1}^{nc} T_i \cdot \Delta P''_{Pi} \tag{3-37}$$

subject to $$\overline{g}_j(\Delta\mathbf{P}'_{\mathrm{P}}, \Delta\mathbf{P}''_{\mathrm{P}}) = \sum_{i=1}^{nc} d_{ji} \cdot \Delta P'_{Pi} - \sum_{i=1}^{nc} d_{ji} \cdot \Delta P''_{Pi} + g_j(\mathbf{P}_{\mathrm{R}} + \mathbf{P}_{\mathrm{P}}) \le 0 \qquad (j=1,\cdots,mp) \tag{3-38}$$

	ΔP_{P1}			ΔP_{Pi}			ΔP_{Pnc}			
	$\Delta P'_{P1}$	$\Delta P''_{P1}$	$\cdots$	$\Delta P'_{Pi}$	$\Delta P''_{Pi}$	$\cdots$	$\Delta P'_{Pnc}$	$\Delta P''_{Pnc}$		
	T_1	$-T_1$		T_i	$-T_i$		T_{nc}	$-T_{nc}$	$0\cdots0$	$0\cdots0$
$-\overline{g}_1$	d_{11}	$-d_{11}$	$\cdots$	d_{1i}	$-d_{1i}$	$\cdots$	d_{1nc}	$-d_{1nc}$	1	
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	
$-\overline{g}_j$	d_{j1}	$-d_{j1}$	$\cdots$	d_{ji}	$-d_{ji}$	$\cdots$	d_{jnc}	$-d_{jnc}$	1	
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	
$-\overline{g}_{mp}$	d_{mp1}	$-d_{mp1}$	$\cdots$	d_{mpi}	$-d_{mpi}$	$\cdots$	$d_{mp\,nc}$	$d_{mp\,nc}$	1	
$\xi\Delta P^0_{P1}$	1	-1								1
$\vdots$										$\ddots$
$\xi\Delta P^0_{Pi}$				1	-1					1
$\vdots$										$\ddots$
$\xi\Delta P^0_{Pnc}$							1	-1		1

Fig.3-6 Simplex tableau for determination of $\Delta\mathbf{P}_{\mathrm{P}}$

$$\left|\Delta P'_{Pi} - \Delta P''_{Pi}\right| < \xi \cdot \overline{\Delta P}_{Pi} \qquad (i = 1, \cdots, nc) \qquad (3\text{-}39)$$

where *mp* is the total number of active constraints during the first stage optimization process and the calculation of the sensitivities with respect to $\mathbf{P}_{\mathrm{P}}$ in section 3-3.(3).(a).

The best improvements of $\mathbf{P}_{\mathrm{P}}$, $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$, which result in the highest reduction of the total cost of the bridge can be obtained by continuously pivoting the rows and columns of the pivot elements in the simplex tableau depicted in Fig.3-6. These elements are selected based on the following decision criteria:
Determine column r, such that

$$E_r = \min_i \{E_i \mid \ E_i < 0, \quad i = 1, \cdots, nc + nc\} \qquad (3\text{-}40)$$

where
$$E_i = \begin{Bmatrix} T_{(i+1)/2} & \text{if } i \text{ is odd} \\ -T_{i/2} & \text{if } i \text{ is even} \end{Bmatrix} \qquad (3\text{-}41)$$

Determine row s, such as

$$\frac{-\bar{g}_s}{C_{sr}} = \min_j \left\{ \frac{-\bar{g}_j}{C_{jr}} \middle| \ -\bar{g}_j > 0,\ C_{jr} > 0, \quad j = 1, \cdots, mp \right\} \tag{3-42}$$

where
$$C_{ir} = \begin{Bmatrix} d_{j(r+1)/2} & \text{if } r \text{ is odd} \\ -d_{jr/2} & \text{if } r \text{ is even} \end{Bmatrix} \tag{3-43}$$

We can determine $\Delta\mathbf{P}'_{\mathrm{P}}$ and $\Delta\mathbf{P}''_{\mathrm{P}}$ as the best improvements of $\mathbf{P}_{\mathrm{P}}$ when the total cost does not reduce any more from further pivoting. The best $\Delta\mathbf{P}'_{\mathrm{P}}$ and $\Delta\mathbf{P}''_{\mathrm{P}}$ obtained in this manner are denoted as $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$, respectively.

Once $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$ are known, the feasibility with $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$ for the remaining approximate constraints in eq.(3-34) which were not considered in the linear programming problem in eqs.(3-37)-(3-39) needs to be investigated. If some of these constraints are not satisfied, they will have to be added to the linear programming problem in eqs.(3-37)-(3-39), and $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$ need modification so as to satisfy all approximate constraints $\bar{\mathbf{g}}$. In the modified linear programming problem the decision criteria used in the selection is:

Determine row s, such that

$$-\bar{g}_s = \min_j \left\{ -\bar{g}_j \middle| \ -\bar{g}_j < 0, \quad j \in \mathbf{S}_{VG} \right\} \tag{3-44}$$

Determine column r, such as

$$\frac{E_r}{C_{sr}} = \max_i \left\{ \frac{E_i}{C_{si}} \middle| \ C_{si} < 0, \quad i = 1, \cdots, nc + nc \right\} \tag{3-45}$$

where $\mathbf{S}_{VG}$ is the set of the constraints which are not satisfied with $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$.

$$C_{si} = \begin{Bmatrix} d_{s(i+1)/2} & \text{if } i \text{ is odd} \\ -d_{si/2} & \text{if } i \text{ is even} \end{Bmatrix} \tag{3-46}$$

E_i is given by eq.(3-41).

By using $\Delta\mathbf{P}'^{0}_{\mathrm{P}}$ and $\Delta\mathbf{P}''^{0}_{\mathrm{P}}$, $\Delta\mathbf{P}^{0}_{\mathrm{P}}$ is calculated from eq.(3-36) and the pseudo-loads $\mathbf{P}^{0}_{\mathrm{P}}$ are improved by

$$\mathbf{P}^{0}_{\mathrm{P}} = \mathbf{P}^{0}_{\mathrm{P}} + \Delta\mathbf{P}^{0}_{\mathrm{P}} \tag{3-47}$$

The approximate improvements $\mathbf{Z}$ for $\Delta\mathbf{P}^{0}_{\mathrm{P}}$ can be carried out using the sensitivities of $\mathbf{Z}$ with respect to $\mathbf{P}_{\mathrm{P}}$, e_{kri}, as shown below:

$$Z_{kr}^{0} = Z_{kr}^{0} + \sum_{i=1}^{nc} e_{kr\,i} \cdot \Delta P_i^0 \quad (k = 1,\cdots,n;\ r = 1,\cdots,q_k) \tag{3-48}$$

Once the improvements of $\mathbf{P}_P^0$ and $\mathbf{Z}^0$ are accomplished, we formulate the following optimum design problem:

$$\text{Find} \qquad \mathbf{Z}(\mathbf{P}_R + \mathbf{P}_P^0), \qquad \text{which}$$

$$\text{minimize} \qquad \mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P^0) \tag{3-49}$$

$$\text{subject to} \qquad g_j(\mathbf{P}_R + \mathbf{P}_P^0) \le 0 \qquad (j = 1,\cdots,\mathrm{m}) \tag{3-50}$$

$$\mathbf{Z}^{(l)} \le \mathbf{Z}(\mathbf{P}_R + \mathbf{P}_P^0) \le \mathbf{Z}^{(u)} \tag{3-51}$$

The above optimization problem is solved by using the first stage optimization algorithm. The optimum TCOST, $\mathbf{g}$ and $\mathbf{Z}$ under design loads $\mathbf{P}_R$ and pseudo-loads $\mathbf{P}_P^0$ are denoted as $\mathrm{TCOST}^0(\mathbf{P}_R + \mathbf{P}_P^0)$, $\mathbf{g}^0(\mathbf{P}_R + \mathbf{P}_P^0)$ and $\mathbf{Z}^0(\mathbf{P}_R + \mathbf{P}_P^0)$, respectively.

The final optimum TCOST and $\mathbf{Z}$ of the cable-stayed bridge subjected to the design loads $\mathbf{P}_R$ and the optimum pseudo-loads $\mathbf{P}_P^*$, denoted as $\mathrm{TCOST}^*(\mathbf{P}_R + \mathbf{P}_P^*)$ and $\mathbf{Z}^*(\mathbf{P}_R + \mathbf{P}_P^*)$, respectively, can be determined by iterating the processes described in 3-3.(3).(a)-3-3.(3).(c) until TCOST converges to its minimum value.

3-4. OPTIMUM DESIGN PROCESS OF STEEL CABLE-STAYED BRIDGE

The algorithm for the optimum design of steel cable-stayed bridges subjected to static design loads and pseudo-loads is summarized below.

① Set the design conditions and assume initial values of $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$. The cable-stayed bridge subjected to the design loads $\mathbf{P}_R$ is analyzed and the sensitivities of TCOST and $\mathbf{g}$ with respect to $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ are calculated.

② Formulate the first stage optimum design problem, eqs.(2-10)-(2-16), and solve the design problem by using the optimization routine described in Chapter 2. The optimum values at this step are $\mathrm{TCOST}^*(\mathbf{P}_R)$, $\mathbf{Z}^*(\mathbf{P}_R)$, $\mathbf{X}_C^*(\mathbf{P}_R)$, $\mathbf{Y}_C^*(\mathbf{P}_R)$ and $\mathbf{g}^*(\mathbf{P}_R)$.

③ If any modifications of $\mathbf{X}_C^*(\mathbf{P}_R)$ and $\mathbf{Y}_C^*(\mathbf{P}_R)$ are necessary from an aesthetic, fabrication, erection or other considerations, do so and then fix $\mathbf{X}_C^*(\mathbf{P}_R)$ and $\mathbf{Y}_C^*(\mathbf{P}_R)$ as constant design parameters. Then, the bridge is optimized again by using the optimization procedure described in Chapter 2. The design variables at this step is $\mathbf{Z}$ only.

④ The cable-stayed bridge subjected to the design loads $\mathbf{P}_R$ and pseudo-loads $\mathbf{P}_P$ (at the first iteration $\mathbf{P}_P=0$) is optimized by applying the optimization routine described in Chapter 2. During this stage, $\mathbf{X}_C$ and $\mathbf{Y}_C$ are fixed and the design variable is $\mathbf{Z}$ only. The optimized values at this step are denoted as $\mathrm{TCOST}^0(\mathbf{P}_R+\mathbf{P}_P^0)$, $\mathbf{g}^0(\mathbf{P}_R+\mathbf{P}_P^0)$ and $\mathbf{Z}^0(\mathbf{P}_R+\mathbf{P}_P^0)$.

⑤ Apply a finite pseudo-load $\overline{\Delta P}_{Pi}$ to the ith cable on top of $\mathbf{P}_R$ and $\mathbf{P}_P$, and carry out an optimization with respect to $\mathbf{Z}$ only. The optimum values at this step are $\mathrm{TCOST}^0\left(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi}\right)$, $\mathbf{g}^0\left(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi}\right)$ and $\mathbf{Z}^0\left(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi}\right)$.

⑥ Calculate the sensitivities of TCOST, $\mathbf{g}$ and $\mathbf{Z}$ with respect to P_{Pi} using eqs.(3-30)-(3-32) and the known values of $\mathrm{TCOST}^0(\mathbf{P}_R+\mathbf{P}_P^0)$, $\mathbf{g}^0(\mathbf{P}_R+\mathbf{P}_P^0)$, $\mathbf{Z}^0(\mathbf{P}_R+\mathbf{P}_P^0)$ and $\mathrm{TCOST}^0(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi})$, $\mathbf{g}^0\left(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi}\right)$, $\mathbf{Z}^0\left(\mathbf{P}_R+\mathbf{P}_P^0+\overline{\Delta P}_{Pi}\right)$.

⑦ Utilizing the sensitivities $\mathbf{T}$, $\mathbf{d}$, $\mathbf{e}$ calculated in step ⑥, formulate the linear programming problem as per eqs.(3-33)-(3-35) and solve it for the determining the best improvements $\Delta\mathbf{P}_P^0$ by using the procedure described in 3-3.(3).(c). $\mathbf{P}_P^0$ and $\mathbf{Z}^0$ are improved by eqs.(3-47) and (3-48), respectively.

⑧ If TCOST^0 does not converge to its minimum value, repeat steps ④－⑦ to minimize TCOST^0. The final optimum values of TCOST, $\mathbf{P}_P$ and $\mathbf{Z}$ under design loads and optimum pseudo-loads are denoted as $\mathrm{TCOST}^*(\mathbf{P}_R+\mathbf{P}_P^*)$, $\mathbf{P}_P^*$, and $\mathbf{Z}^*(\mathbf{P}_R+\mathbf{P}_P^*)$.

⑨ The optimum cable prestress for the ith cable, P_{si}, can be determined as the resultant force of P_{Pi}^* and the axial force N_{Ci}, which is obtained by analyzing the bridge subjected to $\mathbf{P}_P^*$ only.

3-5. NUMERICAL DESIGN EXAMPLES

Various minimum-cost design problems of practical-scale steel cable-stayed bridges have been solved by the proposed design system. In this section, the numerical results for a three-span steel cable-stayed bridge with 64 cable stays shown in Fig.3-7 under various design conditions are presented to demonstrate the general purpose, rigorousness, reliability and efficiency of the proposed optimum design system. The significance of dealing with $\mathbf{X}_C, \mathbf{Y}_C$ and $\mathbf{P}_P$ as the design variables is, also, clarified.

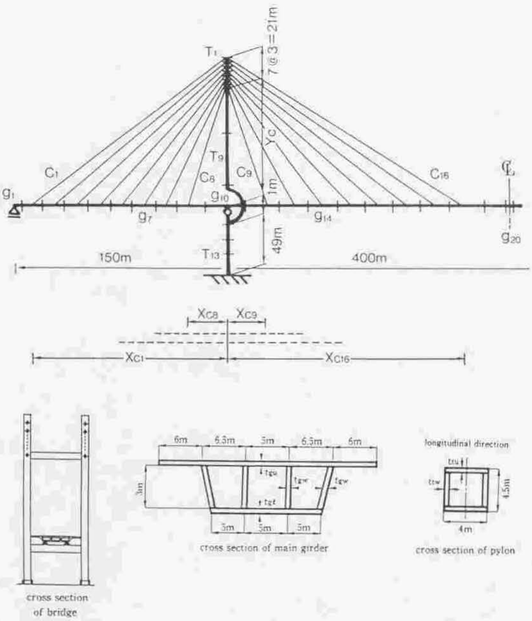

Fig.3-7 Three span cable-stayed bridge with 64 cable stays

Table 3-1 Material properties and minimum plate thicknesses used for the elements of main girder, pylon and cables

No. of member element	[1] $E(kg/cm^2)$	[2] $\rho(YEN/m^3)$	[3] σ_{ta}	[4] τ_a	[5] B_{eu}, [6] $\ell_1(m)$	[7] $B_{e\ell}$, [8] $\ell_2(m)$	[9] t_u^ℓ (mm)	[10] t_ℓ^ℓ (mm)	[11] t_w^ℓ (mm)
g_1~g_8					30.00 [5]	15.00 [7]			
g_9					29.60	15.00			
g_{10}	2.1×10^6	500×10^4	1400	800	28.80	14.90	18.5	15.5	12.0
g_{11}					29.60	15.00			
g_{12}~g_{20}					30.00	15.00			
T_1~T_7			1900	1100	3.0 [6]	3.8 [8]	26.0	26.0	26.0
T_8~T_{10}	2.1×10^6	700×10^4	2600	1500	(Y_C+1)	(Y_C+1)	28.0	28.0	32.0
T_{11}~T_{13}			2600	1500	98.0	39.2	28.0	28.0	32.0
C_1~C_{16}	2.0×10^6	900×10^4	5100	—	—	—	$A_C^\ell=0.00001$ (m^2)		

1) Modulus of elasticity 2) Price per unit volume 3) Allowable tensile stress (kg/cm^2)
4) Allowable shearing stress (kg/cm^2) 5) Effective width of the upper flange plates
6) Effective length for buckling in longitudinal direction 7) Effective width of the lower flange plates
8) Effective length for buckling in transverse direction
9) Converted minimum upper flange plate thickness including longitudinal stiffeners
10) Converted minimum lower flange plate thickness including longitudinal stiffeners
11) Converted minimum web plate thickness including longitudinal stiffeners

Table 3-2 Dead load and traffic load at erection closing and service stages

Erection closing stage	D. load [1]	main girder [3]	4.0 tf/m
		pylon [3]	2.0 tf/m
		steel weight	7.85 tf/m^3
Service stage	D. load [1]	main girder [4]	3.4 tf/m
	T. load [2]	uniform	2.25 tf/m
		line	54.1 tf
		impact	0.10

1) Dead load 2) Traffic load
3) Dead load due to cable anchors etc.
4) Dead load due to asphalt pavement etc.

(1) Design conditions for the elements of main girder, pylon and cables

The design constants used in the numerical examples, such as the moduli of elasticity of steel plate and cable, E, the unit prices of materials, ρ, the allowable tensile and shearing stresses, σ_{ta} and τ_a, the effective widths of upper and lower flange plates in the main girder, B_{eu} and B_{el}, the effective lengths of the pylon elements for buckling in longitudinal and transverse directions, l_1 and l_2, the minimum plate thicknesses, t_u^l, t_l^l, t_w^l, are tabulated in Table 3-1. The dead loads at the cantilever erection closing stage and traffic loads at service stage on one half of the cross section of the bridge are given in Table 3-2.

Since the structure is symmetrical about the center line, the numbers of independent design variables, $\mathbf{Z}$, $\mathbf{X}_C$, $\mathbf{Y}_C$ and $\mathbf{P}_P$ in the optimization process are 69,16,1 and 16, respectively, and the number of constraints is 201. In the design problem, the lower limit on the cross-section areas of cables is set at 0.1cm^2 and the objective is to determine the optimum cable arrangement.

(2) Design example in which $\mathbf{Z}$ is the only design variable

The significance of design variables $\mathbf{X}_C$ and Y_C is investigated by comparing the optimum solutions for the cases in which the design variables are $\mathbf{Z}$, $\mathbf{X}_C$, Y_C and $\mathbf{Z}$ only.

The optimum solution for the case in which $\mathbf{Z}$ is the only design variable is summarized as case A in Table 3-3, in this design example $\mathbf{X}_C$ and Y_C are fixed as shown in case A. At the optimum solution, the maximum and minimum bending moments and axial force distributions, M_{max}, M_{min}, N_{max}, N_{min}, upper and lower flange plate thicknesses distributions in the main girder, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$, the distributions of M_{max}, M_{min}, N_{max}, N_{min}, and the cross-sectional areas in the pylon, $\mathbf{A}_t$, are depicted in Figs.3-8 and 3-9 by dotted lines. The bending moments and axial forces shown in Fig.3-8 are those acting on one half of the cross-sectional area of the main girder depicted in Fig.3-7, while the total cost expresses the cost for the whole system.

As seen from Table 3-3, the optimum solution is reached after 6 iterations and the total cost of the bridge converges to 707234.3x10^4 yen. However, the critical local peaks -9955 tf·m near the end support and -11652 tf·m at the middle support are observed in the max. or min. bending moment distributions in the main girder. In the pylon the critical local peaks, -6308 tf·m and 6937 tf·m occur near the top and at the girder position, respectively.

(3) Design example with $\mathbf{Z}$, $\mathbf{X}_C$, Y_C as design variables

The optimum solution with $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C as design variables is summarized as case B in Table 3-3. The distributions of M_{max}, M_{min}, N_{max}, N_{min}, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$ in the main girder and the distributions of M_{max}, M_{min}, N_{max}, N_{min}, and $\mathbf{A}_t$ in the pylon at the optimum solution are depicted in Figs.3-8 and 3-9 by the chained lines. For this case, the initial values of $\mathbf{X}_C$ and Y_C, $\mathbf{X}_C^0$ and Y_C^0, are assumed to be the same as those in case A.

The optimum solution is reached after 19 iterations. Y_C^0 was assumed as 85m, however, its optimum value, Y_C^*, is determined as 56.90m which is 33% lower than Y_C^0. With regard to the optimum $\mathbf{X}_C$, the top two cables in the side span are seen to be parallel and are both anchored at the end support. Almost all the cables, except X_{C3} in the side span, are distributed in a geometric ratio. In the center span all the cables are distributed in a geometric ratio of 1:1.3. The cross-sectional areas of the top cables in the center and side spans are much larger than those of the middle cables. The cross-sectional areas of the lowest cables in the side and center spans are found to be 0.1cm^2 which is equal to the lower limit constraint and this indicates that these cables are unnecessary from a structural optimization viewpoint. It is, therefore, clear that the optimum topological cable arrangement can, also, be determined by our proposed optimum design method.

Figs.3-8 and 3-9 show that the cables are arranged so as to reduce the critical

Table 3-3 Comparison of optimum solutions for cases A, B, C, D and E

Case	A			B			C			D			E			
Design variables	Z			Z, X_C, Y_C			Z, P_P			Z, X_C, Y_C			Z, P_P			
Loads	Design loads			Design loads			Design loads + Pseudo loads			Design loads + Pseudo loads			Design loads + Pseudo loads			
Optimization process	First stage			First stage			Second stage			First stage			Second stage			
No. of cable, Y_C	X_C^0, Y_C^0(m)	A_c(cm²)	σ/σ_a [2]	X_C, Y_C(m)	A_c(cm²)	σ/σ_a [2]	A_c(cm²)	P_f(tf)	σ/σ_a [2]	X_C, Y_C(m)	A_c(cm²)	σ/σ_a [2]	A_c(cm²)	P_f(tf)	P_s(tf) [1]	σ/σ_a [2]
C_1	140.00	483	1.00	149.00	432	1.00	508	982.8	1.00	148.69	521	1.00	515	1161.4	60.9	1.00
C_2	124.00	50	0.92	143.27	111	0.97	40	79.3	1.00	143.27	41	1.00	44	101.9	10.4	1.00
C_3	108.00	16	0.96	115.48	47	1.00	23	38.5	1.00	113.78	25	1.00	58	110.7	-11.8	1.00
C_4	92.00	39	0.99	103.66	141	1.00	163	273.1	1.00	98.24	173	1.00	144	260.3	-44.3	1.00
C_5	76.00	49	0.95	73.64	96	0.94	89	170.1	1.00	70.61	89	1.00	91	180.7	-26.6	1.00
C_6	60.00	110	0.80	53.18	126	0.74	96	259.4	1.00	48.36	91	1.00	86	247.4	48.8	1.00
C_7	44.00	86	0.58	36.16	116	0.47	78	285.2	1.00	29.58	70	1.00	76	317.9	182.7	1.00
C_8	28.00	9	0.30	16.45	0 [3]	0.08	0 [3]	0.0	0.03	13.67	0 [3]	0.03	0 [3]	0.0	-0.1	0.04
C_9	28.00	8	0.35	13.30	0 [3]	0.06	0 [3]	0.0	0.00	9.60	0 [3]	0.05	0 [3]	0.0	0.0	0.04
C_{10}	48.00	93	0.68	28.01	58	0.37	20	80.5	1.00	20.70	18	1.00	9	38.2	31.3	1.00
C_{11}	68.00	102	0.86	43.63	109	0.64	100	300.4	1.00	41.08	99	1.00	92	289.8	139.7	1.00
C_{12}	88.00	66	0.97	59.43	84	0.83	70	143.1	1.00	60.73	78	1.00	89	189.6	45.4	1.00
C_{13}	108.00	101	1.00	80.76	82	0.96	78	80.3	1.00	81.52	82	1.00	117	135.7	-1.8	1.00
C_{14}	128.00	75	1.00	103.45	140	1.00	170	55.2	1.00	103.63	168	1.00	117	43.7	-35.8	1.00
C_{15}	148.53	70	0.99	133.66	227	1.00	175	-46.0	1.00	134.37	189	1.00	197	-5.7	-49.9	1.00
C_{16}	168.00	289	1.00	172.67	328	1.00	350	-123.8	1.00	173.59	341	1.00	352	2.5	40.0	1.00
Y_C	85.00	—	—	56.90	—	—	—	—	—	56.03	—	—	—	—	—	—
ITE [4]	6			19			10			7			13			
TCOST (YEN)	707234.3x10⁴ (1.085) [5]			651918.9 x10⁴ (1.000) [5]			635109.9x10⁴ (0.974) [5]			633750.1x10⁴ (0.972) [5]			633218.2x10⁴ (0.971) [5]			

1) Cable prestress 2) Ratio of actual tensile stress and allowable tensile stress 3) Minimum cross-sectional area
4) Number of iterations at the first stage and second stage optimization process 5) Ratio of the TCOST to the TCOST of case B

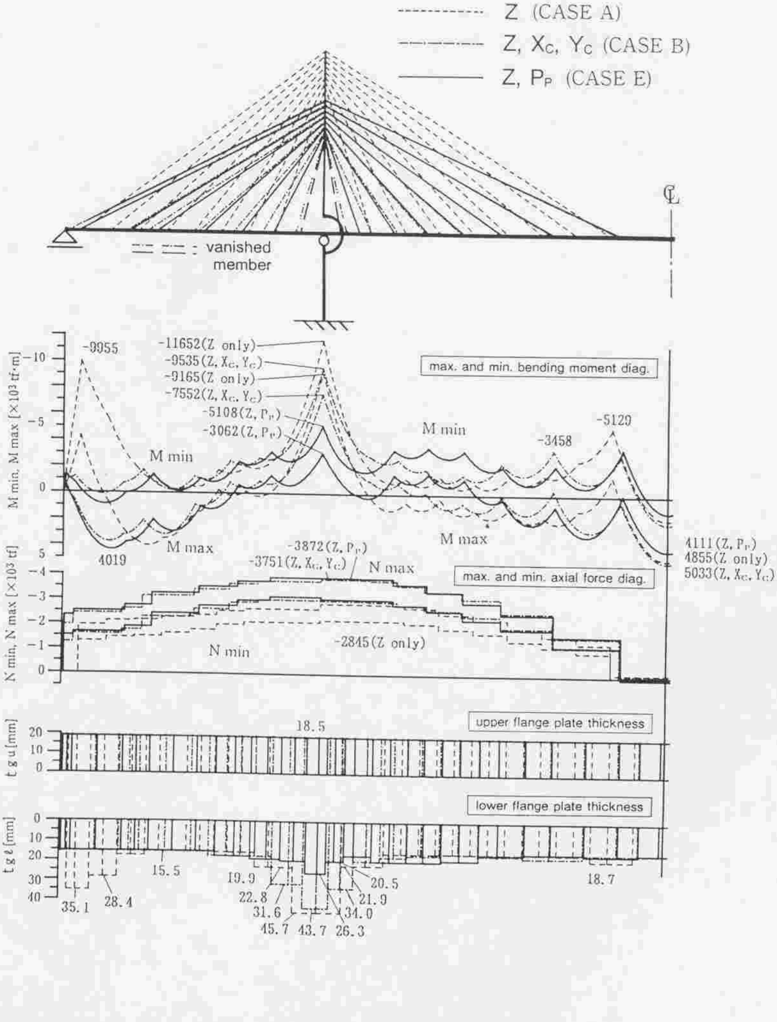

Fig.3-8 Comparisons of optimum cable arrangement, M_{max}, M_{min}, N_{max}, N_{min}, t_{gu} and t_{gl} in the main girder for cases A, B and E

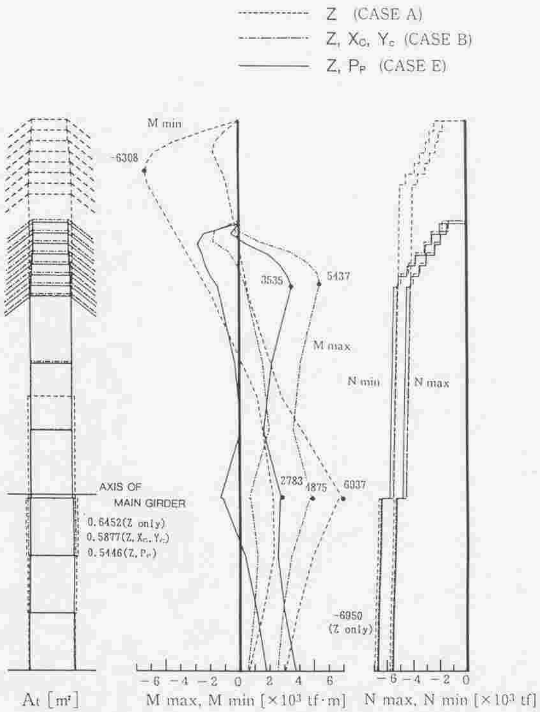

Fig.3-9 Comparisons of $\mathbf{A}_t$, M_{max}, M_{min}, N_{max} and N_{min} in the pylon for cases A, B and E

local peaks of the max. and min. bending moments in the main girder and pylon. The local peak of the min. bending moment near the end support in the main girder for case A is totally absent and the local peaks of the min. bending moment at the middle support and near the center point are reduced to 82% and 68%, respectively, compared with their corresponding values for case A. At the center of the main girder, a large max. bending moment acts on the cross section, however, the upper and lower flange plate thicknesses are determined as the same as the lower limit plate thicknesses. This means that it is not necessary to reduce the max. bending moment at this point and this result emphasizes the reliability and rigorousness of the proposed design method.

In the pylon, the max. and min. bending moments are reduced to 86% and 70%, respectively, compared with the ones for case A.

The cross-sectional areas of cables are generally higher than those for case A. In one special case, the cross-sectional area of C_4 is 3.6 times larger than its corresponding value in case A.

As a consequence of the changes in $\mathbf{X}_C$ and Y_C, the total cost of the bridge is 7.8% less than that for case A.

This investigation of the optimum solutions establishes the significance of the cable anchor positions on the main girder and the height of pylon in the minimum cost design of steel cable-stayed bridges.

(4) Design example with $\mathbf{Z}$ and $\mathbf{P}_P$ as design variables

In this design example $\mathbf{X}_C$ and Y_C are fixed at the optimum values obtained for case B, and $\mathbf{P}_P$ and $\mathbf{Z}$ are dealt with as design variables for investigating the significance of prestresses in cables. After 10 iterations of the sensitivity calculations and improvements of $\mathbf{P}_P$ by the second stage optimization process described in 3-3.(2).(c), the optimum values $\mathbf{P}_P$ and $\mathbf{Z}$ are obtained. These are summarized as case C in Table 3-3. The total number of the active constraints, *mp*, in the linear programming problem in eqs.(3-37)-(3-39) is 45. All the cables are fully stressed by providing the optimum pseudo-loads. The total cost of the bridge decreases by 2.6% compared with case B. This means that 16800×10^4 yen can be saved by giving the optimum prestresses in the cables.

(5) Effect of $\mathbf{P}_P$ on the optimum $\mathbf{X}_C$ and Y_C

In this design example, the effect of $\mathbf{P}_P$ on the optimum values of $\mathbf{X}_C$ and Y_C are investigated. The bridge is optimized treating $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C as design variables under the design loads $\mathbf{P}_R$ and optimum pseudo-loads $\mathbf{P}_P$ which are obtained for case C. The optimum $\mathbf{Z}$, $\mathbf{X}_C$ and Y_C tabulated as case D in Table 3-3 are obtained after 7 iterations by using the first stage optimization process.

By comparing the optimum $\mathbf{X}_C$ for cases B and D, the relative differences of 7.31m and 6.58m are observed at the lower cables C_{10} and C_7, respectively. However, $\mathbf{X}_C$ for the upper and middle cables in the both cases are almost similar to differences limited to 0.00m-5.42m. Furthermore, the difference in Y_C is only

0.87m. These results lead to the conclusion that for this specific design problem the optimum values of $\mathbf{X}_C$ and Y_C are negligibly affected by the optimum pseudo-loads $\mathbf{P}_P$. The minimum total cost is only 0.2% less than that for case C in which the optimum $\mathbf{P}_P$ are determined with fixed $\mathbf{X}_C$ and Y_C obtained from case B.

(6) Effects on $\mathbf{P}_P$, $\mathbf{A}_C$ and TCOST due to changes in $\mathbf{X}_C$ and Y_C

In this design example, the effects on $\mathbf{P}_P$, $\mathbf{A}_C$ and TCOST due to small changes in $\mathbf{X}_C$ and Y_C are investigated. $\mathbf{X}_C$ and Y_C are fixed at the optimum $\mathbf{X}_C$ and Y_C obtained from case D (which are slightly different from the optimum $\mathbf{X}_C$ and Y_C for case B as noted in section 3-5.(5)).

The design of the bridge is optimized by using the second stage optimization process with $\mathbf{P}_P$ and $\mathbf{Z}$ as design variables. The optimum values of $\mathbf{P}_P$, $\mathbf{A}_C$, TCOST and the prestresses of the cables corresponding to the optimum $\mathbf{P}_P$, $\mathbf{P}_S$, are tabulated as case E in Table 3-3.

The optimum $\mathbf{P}_P$ and $\mathbf{Z}$ are determined after 13 iterations of sensitivity calculations and improvements of $\mathbf{P}_P$ by the modified LP algorithm. Comparing the cross-sectional areas of cables $\mathbf{A}_C$ for cases E and C, relatively large changes ,from +42.7% to -84.5%, are observed. A similar behavior is seen in the distribution of $\mathbf{P}_P$ at the cables. However, in spite of the large changes in the distributions of $\mathbf{A}_C$ and $\mathbf{P}_P$, TCOST is only 0.3% and 0.08% less than that for cases C and D, respectively.

Therefore, it is clear that the effect of including $\mathbf{P}_P$ and $\mathbf{A}_C$ does not result in a unique set of values for the pair and these may be combined in many ways. Also, the effect on TCOST as a result of changes in $\mathbf{X}_C$ and Y_C is seen to be small. By inspecting TCOST for cases C, D and E, it can be concluded that TCOST for case E is almost equal to the exact minimum cost of the bridge. We can, therefore, save 2.9% of the total cost of the bridge by providing the optimum pseudo-loads, namely by giving the optimum prestresses in the cables.

By the way, the optimum TCOST for case C which is obtained in only one iteration of the first and second stage optimization routines is only 0.3% larger than that for case E. For this reason, for practical design problems we can adopt the optimum values of the design variables $\mathbf{Z}$, $\mathbf{X}_C$, Y_C and $\mathbf{P}_P$ for case C as the theoretically almost final optimum solutions.

The optimum bending moment distributions in the main girder for case E are depicted by the solid line in Fig.3-8. The local peaks of M_{min} and M_{max} at the middle support are 53.6% and 40.5% of their corresponding values in case B. The smaller bending moments lead to lower flange plate thicknesses with reductions of 17.4-11.7mm. The bending moment distributions and flange plate thicknesses distributions also appear to be averaged out and as a result the magnitudes are minimized throughout the bridge. The bending moment distributions in the pylon are depicted by the solid line in Fig.3-9. Near the top of pylon and at the main girder position, M_{max} are reduced to 65.0% and 57.1%, respectively, of their corresponding values in case B. A similar reduction is seen in the cross-sectional areas of pylon.

In the cables, the prestresses of -49.9 to +182.7tf are given and the cross-sectional areas of cables change to the range from +42.7% to -84.5% of the values for case B. Furthermore, all the cables are fully stressed. On the other hand, stress margins of more than 50% exist in some cables for case B.

(7) Design example in which $\mathbf{X}_C$ and Y_C are modified from aesthetic considerations
In the design examples C, D and E, the design variables are $\mathbf{Z}$, $\mathbf{X}_C$, Y_C and $\mathbf{P}_P$, and theoretically exact optimum values of the design variables are determined totally from structural mechanics considerations.

In this design example, the number of cables, $\mathbf{X}_C$ and Y_C, of the bridge are slightly modified, as shown in terms of $C_1 - C_{12}$, $\mathbf{X}_C^0$ and Y_C^0 in Table 3-4 and Fig.3-10, from aesthetic considerations.

The number of cables is reduced from 64 to 48 and the cable anchor positions $\mathbf{X}_C$ are fixed as $\mathbf{X}_C^0$ shown in Table 3-4. The anchor positions expand in a geometric ratio of 1:1.14 in the side span and 1:1.18 in the center span. Y_C is assumed as 62.00m. The height of top cables at the pylon is the same as for cases D and E. The design of the bridge is optimized for two cases, one in which only $\mathbf{Z}$ and the other in which $\mathbf{Z}$ and $\mathbf{P}_P$ are treated as design variables. The optimized values of the design variables for both cases are tabulated as cases F and G in Table 3-4. The optimum distributions of M_{max}, M_{min}, N_{max}, N_{min}, $\mathbf{t}_{gu}$ and $\mathbf{t}_{gl}$ in the main girder for cases F and G are depicted in Fig.3-10 by dotted lines and solid lines, respectively.

The optimized values of $\mathbf{P}_P$ and $\mathbf{Z}$ for case G are obtained after 13 iterations of the sensitivity calculations and improvements of $\mathbf{P}_P$ by LP algorithm. Similar to the comparisons of the optimum $\mathbf{A}_C$ for cases B and C, the optimum $\mathbf{A}_C$ for cases F and G changed considerably by prestressing the cables. All cables are fully stressed with the exception of the C_6 cable whose cross-sectional area A_{C6} is smallest as $9cm^2$

Table 3-4 Comparison of optimum solutions for cases F and G

Case	F			G			
Design variables	Z			Z, P_P			
Loads	Design loads			Design loads + Pseudo loads			
Optimization process	First stage			Second stage			
No. of cable, (X_C), (Y_C)	X_C^*, (Y_C^*)(m)	A_C(cm²)	σ_t/σ_a [2]	A_C(cm²)	P_P(tf)	P_S(tf) [1]	σ_t/σ_a [2]
C_1 (X_{C1})	149.00	496	1.00	527	743.8	46.34	1.00
C_2 (X_{C2})	118.59	142	1.00	79	98.8	-28.27	0.96
C_3 (X_{C3})	91.92	90	1.00	165	302.6	-23.13	1.00
C_4 (X_{C4})	68.52	126	0.84	66	166.9	15.44	1.00
C_5 (X_{C5})	48.00	157	0.62	128	425.7	140.37	1.00
C_6 (X_{C6})	30.00	51	0.34	9	2.9	-10.70	0.27
C_7 (X_{C7})	30.00	63	0.38	34	165.9	84.79	1.00
C_8 (X_{C8})	50.00	154	0.88	120	203.1	101.85	1.00
C_9 (X_{C9})	73.60	107	0.90	92	39.5	-18.40	1.00
C_{10} (X_{C10})	101.45	144	1.00	148	-13.7	-76.32	0.99
C_{11} (X_{C11})	134.31	241	1.00	190	42.5	-45.22	1.00
C_{12} (X_{C12})	173.11	323	1.00	369	263.4	90.77	1.00
(Y_C)	(Y_C^*) 62.00	—	—	—	—	—	—
ITE [3]	6			13			
TCOST (YEN)	651026.5×10⁴ (0.999) [4]			634301.2×10⁴ (0.973) [4]			

1) Cable prestress 2) Ratio of actual tensile stress and allowable tensile stress
3) Number of iterations at the first stage and second stage optimization process
4) Ratio of the *TCOST* to the *TCOST* of case B

and stress ratio is 0.27. Looking at the optimum bending moment distributions in the main girder for case G shown in Fig.3-10, the local peak of M_{min} at the middle support is 39% larger than that for case E, while the local peaks of M_{max} and M_{min} and in the center span are 13-18% smaller than those for case E. In the bending moment distributions in the pylon depicted in Fig.3-11, the bending moment at the main girder position for case G is 1.6 times as large as that for case E. Also, the distribution of the pseudo-loads $\mathbf{P}_P$ underwent a change, especially compared with case E, but at the same time the minimum total cost for case G is only 0.17% larger than that for case E.

This design example shows that even if the optimized design variables $\mathbf{X}_C$ and Y_C are modified slightly from considerations such as aesthetics, fabrication, erection, etc., we can still design the bridge with almost the same minimum cost by including $\mathbf{P}_P$ and $\mathbf{Z}$ as design variables.

In this section seven design examples with various design conditions were

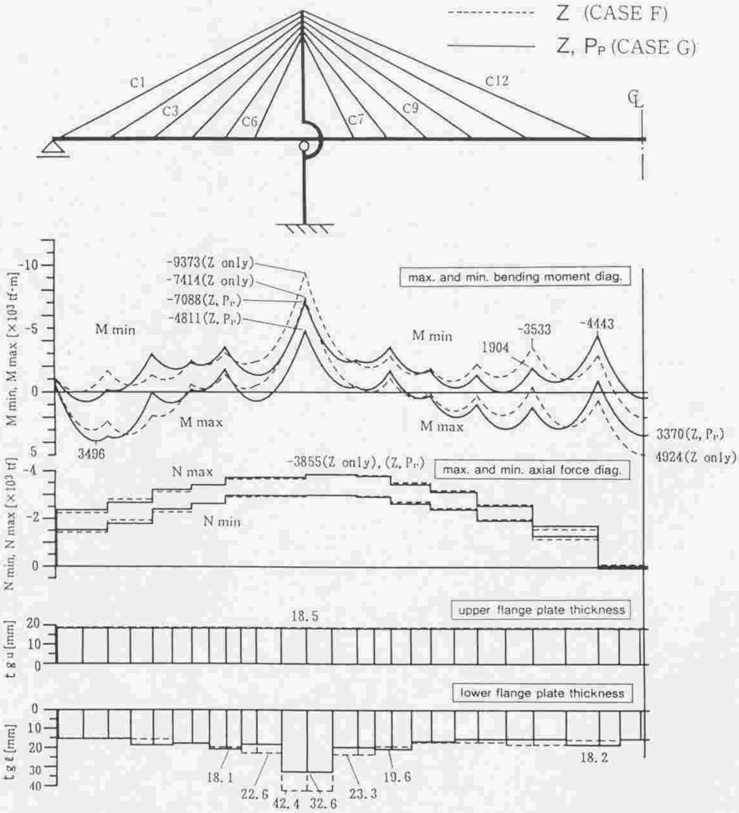

Fig.3-10 Comparisons of M_{max}, M_{min}, N_{max}, N_{min}, t_{gu} and t_{gl} in the main girder for cases F and G

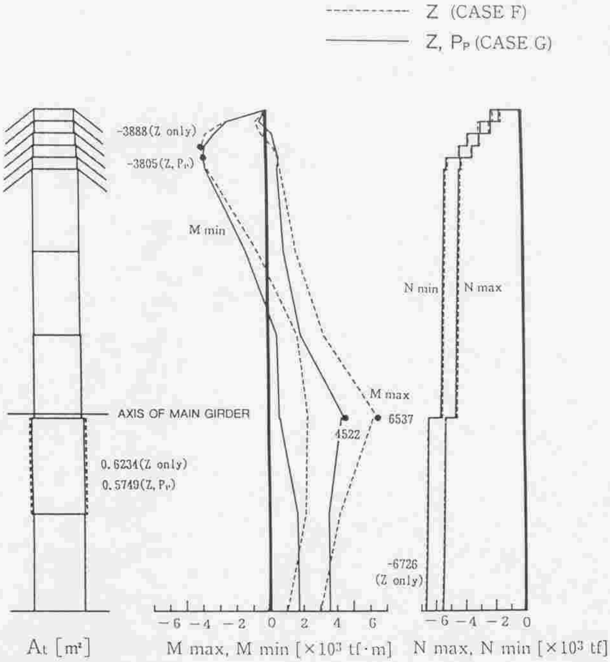

Fig.3-11 Comparisons of $\mathbf{A}_t$, M_{max}, M_{min}, N_{max}, N_{min} in the pylon for cases F and G

described. From the investigations of their optimum solutions, it is clear that the proposed optimum design algorithm can determine the cable prestresses besides the cross-sectional dimensions, such as upper and lower flange plate thicknesses of each member element in the main girder and pylon, cross-sectional areas of cables, and cable arrangement etc., quite rigorously and efficiently. We can, therefore, conclude that the proposed optimum design system is quite useful for practical design of the steel cable-stayed bridge at all design stages, from the planning stage to the detailed design stage.

3-6. CONCLUSIONS

In this Chapter, a general purpose, rigorous and efficient optimum design system for steel cable-stayed bridges is developed. In this design system, not only can the cable anchor positions on the main girder and the height of pylon, and the cross-sectional dimensions of cables, main girder and pylon elements be dealt with as design variables, but also the pseudo-loads applied to the cables which induce the prestresses into the cables. The cost-minimization problem is solved by a powerful two-stage optimum design process. The proposed optimum design method has been applied to the minimum-cost design problems of practical-scale steel cable-stayed bridge with 64 cable stays. The theoretical rigorousness, efficiency and practical usefulness of the proposed optimum design system are demonstrated by giving several numerical design examples and investigating the optimum solutions at various design conditions.

The conclusions that can be drawn from this study are:

(1) The global optimum solutions of steel cable-stayed bridges for various design conditions and combinations of the design variables $\mathbf{Z}$, $\mathbf{X}_C$, $\mathbf{Y}_C$ and $\mathbf{P}_P$ can be determined quite rigorously and efficiently by the proposed two-stage optimum design method.

(2) The significance of dealing with cable anchor positions on the main girder $\mathbf{X}_C$ and the height of pylon $\mathbf{Y}_C$ as the design variables in the optimum design of steel cable-stayed bridges is also confirmed from the design examples in this Chapter.

(3) The optimum solutions of $\mathbf{Z}$ only or $\mathbf{Z}$, $\mathbf{X}_C$ and $\mathbf{Y}_C$ can be obtained in 6-19 iterations of the first stage optimization process theoretically and efficiently. Following the optimum solutions in the first stage optimization process, after 10-14 iterations of the second stage optimization process the theoretical optimum solutions of $\mathbf{Z}$ and $\mathbf{P}_P$ can be obtained quite efficiently.

(4) The optimum cable anchor positions on the main girder $\mathbf{X}_C$ and the height of pylon $\mathbf{Y}_C$ determined during the first stage optimization process by considering only the design loads are found to be scarcely affected by the optimum pseudo-loads from the design example. Therefore, we can obtain the final optimum solutions of $\mathbf{Z}$, $\mathbf{X}_C$, $\mathbf{Y}_C$ and $\mathbf{P}_P$ after only one repetition of two-stage

optimization process, namely determination of the optimum solutions of Z, X_c, Y_c subjected to only the design loads by the first stage optimization process and determination of the optimum solutions of Z and P_P subjected to design loads and pseudo-loads by the second stage optimization process.

(5) By giving the optimum prestresses to the cables, the local peaks of min. and max. bending moments at the middle support in the main girder are reduced to 53.6 − 40.5% and the cross-sectional areas of cables change to the range from +42.7% to -84.5%, and all nontrivial cables are fully stressed. As a result, 2.9% reduction in the total cost of the bridge is observed by giving the optimum cable prestresses in the design examples. From various design examples, it can be said that we can save 2.6%-4.1% of the total cost of the bridge by giving the optimum prestress in the cables.

(6) The proposed optimum design system is quite useful for practical design of the steel cable-stayed bridge at all design stages, from the planning stage to the detailed design stage.

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