# Abstract

## 論文の内容の要旨

論文題目
Critical Phenomena of the Ising Model with Non-Integer Effective Dimensions
- a Monte Carlo Study
(非整数有効次元をもつイジング模型の臨界現象 - モンテカルロ法による研究)

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Classification of critical phenomena has been one of the most important and central issues in the statistical physics for many-body problems for a long time, and has been studied extensively analytically, experimentally, as well as numerically. Critical phenomena can be classified into "universality classes" by their critical exponents (or critical indices). In general, the critical exponents depends on only a few basic properties of the system, such as the dimensionality, symmetry of the order parameter, and so on. However, the existence of long-range interaction (LRI) and/or randomness sometimes changes the behavior of the system completely. Many materials exhibit nontrivial phenomena in which the long-range nature of the interactions, such as the dipole-dipole interaction and the Ruderman-Kittel-Kasuya-Yoshida (RKKY) interaction, plays an essential role. The randomness also causes various nontrivial behavior such as the spin-glass transition and extremely slow dynamics, and so on. As for the phase transitions, they can alter the critical exponents and thus the universality class. Interestingly, this change of universality can be interpreted as the change of the effective dimension of the system as discussed below.

In the Ising model with long-range interaction, all possible pairs of spins interact with each other.

The simplest and most fundamental long-range interaction is the algebraically decaying interaction characterized by the exponent, $d + \sigma$, where $d$ is the dimension of the lattice and $\sigma$ is called the decay exponent. For sufficiently small $\sigma$, the finite-temperature phase transition is expected to belong to the mean-field universality class. Especially, in the limit of $d + \sigma \to 0$, the system becomes equivalent to the fully connected Ising model. On the other hand, when $\sigma$ is sufficiently large, the nearest-neighbor interaction dominates and the transition belongs to the short-range universality class. In the intermediate regime, between the mean-field and the short-range limits, the critical exponents that characterize the universality class vary continuously as $\sigma$ changes. For the $d$-dimensional system with the long-range interaction, this continuous change of the critical exponents between short-range and mean-field universalities can be interpreted as a continuous change of the effective dimension between $d$ and the upper critical dimension of the corresponding short-range model. Thus, one can say that the long-range interaction effectively increases the dimension of the system.

Similarly, the presence of randomness can also change the effective dimension of the system. The random-field Ising model (RFIM) is one of the representative random systems, which has randomly distributed external field. In contrast to the long-range interaction, however, the random field is considered to decrease the effective dimension. Especially, near the upper critical dimension, it is predicted that the critical behavior of the $d$-dimensional random-field Ising model is the same as the pure system in $d - 2$ dimensions. This phenomena is called "dimensional reduction." Spatial correlation between random fields can also alter the critical behavior. For the spatial correlation that decreases algebraically with exponent $d - \rho$ as the distance between spins increases, the renormalization group study predicts that the upper critical dimension $D_{\mathrm{U}}$ and the lower critical dimension $D_{\mathrm{L}}$ become as $D_{\mathrm{U}} = d_{\mathrm{U}} + \rho$ and $D_{\mathrm{L}} = d_{\mathrm{L}} + \rho$, respectively, where $d_{\mathrm{U}} = 6$ and $d_{\mathrm{L}} = 2$ are the upper and lower critical dimensions of the Ising model with uncorrelated random field. In other words, the correlation in random field makes the effective dimension lower.

An interesting point is that the shift of effective dimension is controlled by the continuous parameter, $\sigma$ or $\rho$. By interpreting these continuous parameters as the shift of dimensionality, we are able to obtain the critical behavior between integral dimensions, i.e., non-integer effective dimension like a fractal. Thus, in spite of the simpleness of the Hamiltonian, both of the long-range interacting Ising model and the random-field Ising model are famous for rich behavior and are also notorious for the difficulty of analysis.

In the present thesis, we investigate the long-range interacting Ising model and the random-field Ising model with algebraically decaying random-field correlation by means of the large-scale Monte Carlo simulations and clarify the universality regime boundaries and and the decay-exponent dependence of the universality class.

In the Ising model with long-range interaction, each spin has $(N-1)$ interactions, where $N$ is a number of spins. Therefore, simulation of Ising model with long-range interaction needs to consider $O(N^2)$ interactions and thus computation cost becomes large quickly as the system size increases. Moreover, the long-range interaction causes extremely large finite-size corrections in critical exponents and critical amplitudes especially near the boundary between the mean-field and the intermediate regimes, and that between the intermediate and the short-range regimes. To overcome the difficulties that make conventional analysis ineffective, we utilized several important techniques: the $O(N)$ Fukui-Todo cluster algorithm, the (self-)combined Binder ratio, the generalized Ewald summation, and a generalization of the improved estimator for higher-order moments of magnetization. We conclude that in two dimension, the lower and upper critical decay exponents are $\sigma = 1$ and $7/4$, respectively.

Systems with randomness, such as the present random-field Ising model, are also known by their difficulties in analytical calculations as well as in numerical simulations. Several previous studies of the Ising model with correlated random field are conducted in the analytical way. The renormalization group analysis and the droplet argument for the random systems are believed to be valid near the upper and lower critical dimensions, respectively, and both are considered to fail to predict correct critical behavior at least quantitatively in the intermediate dimensions. Therefore, the precise numerical simulation is quite important to check the validity of analytic predictions. Main difficulties of the Monte Carlo simulation of random-field Ising model are the requirement of average over large number of random samples and extreme slow convergence of the Markov chain to the equilibrium (c.f. infinite disorder fixed point). To improve relaxation to the thermal equilibrium, we introduce the limited cluster flip method advocated by Newman and Barkema. We generalize the limited cluster flip method so that richer configurations of the limited clusters are realized with simple and less tuning parameters for arbitrary dimensions. Furthermore, our formulation of the limited cluster flip method can be applied straightforwardly to the system with long-range interaction. To conduct Monte Carlo simulation with the correlated random field, another essential technique, generation of correlated random number is needed. We generate random fields with arbitrary correlation matrix and with Gaussian marginal distribution by using diagonalization. Since the computation cost of diagonalization proportional to $O(N^3)$, correlated random-field generation step becomes a bottleneck for sufficient large system. We overcome this difficulty by introducing the parallel eigensolver that works efficiently on the massively parallel supercomputer. We perform Monte Carlo simulations for three and four dimensional random-field Ising models and precisely investigate the change of critical properties and effective dimension, especially the competition between the randomness and correlation of random field, and discuss the validity of the previous analytic predictions.