論 文 の 内 容 の 要 旨

論文題目　Applications of Accelerated Proximal Gradient Method to Binary Classification and Polynomial Optimization
（加速近接勾配法の2値判別と多項式最適化への応用）

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This thesis concerns with large scale optimization methods for binary classification and polynomial optimization. We develop efficient optimization algorithms based on the accelerated proximal gradient (APG) method and appropriate formulations for the algorithms.

First, we present a unified binary classification method. We develop a unified formulation for various binary classification models (including support vector machines (SVMs), logistic regression, and Fisher's descriminant analisys) and a fast APG (FAPG) algorithm for the formulation. Our FAPG is developed by devising various acceleration techniques such as backtracking line search and adaptive restarting strategy. We also give a theoretical convergence guarantee for the proposed FAPG method. Numerical experiments show that our algorithm is stable and highly competitive to specialized algorithms designed for specific models, e.g., sequential minimal optimization and stochastic coordinate descent for SVM.

Next, we improve the doubly nonnegative (DNN) relaxation method for a class of polynomial optimization problems (POPs) proposed by Kim, Kojima, and Toh. In order to approximate the optimal value of a POP by its lower bound, they proposed a DNN relaxation problem and solved it by the bisection and projection (BP) method. The BP method reduces the problem to a dual optimization problem having a single variable and applies the bisection method to the dual; The feasibility of a given point is determined by the APG method. In this thesis, we propose new DNN relaxation problems which give better lower bounds than theirs and can still be solved by the BP method efficiently. We further improved the BP method by developing an adaptive restarting APG and a new criterion for checking feasibility. Numerical experiments demonstrate the advantage of our BP method over SDPNAL+ which is the state-of-the-art solver for DNN optimization problems, especially for very large and sparse POPs.

Finally, by focusing on a class of combinatorial quadratic optimization problems (QOPs), which are special cases of POPs, we examine how the difference in formulations of QOPs can

affect on the numerical computation of conic relaxation methods. The binary and complementarity conditions of the combinatorial optimization problems can be expressed in several ways, each of which results in different conic relaxations. For the completely positive (CPP), DNN, and semidefinite programming (SDP) relaxations of the combinatorial QOPs, we prove the equivalences and differences among the relaxations by investigating the feasible regions obtained from different representations of the combinatorial condition. We also theoretically study the issue of the primal and dual nondegeneracy, the existence of an interior solution and the size of the relaxations, as a result of different representations of the combinatorial condition. These characteristics of the conic relaxations affect the numerical efficiency and stability of the algorithms.