論 文 の 内 容 の 要 旨

論文題目　Theoretical Analysis on Wave Dynamics in Cellular Chaotic Neural Networks
（セルラーカオスニューラルネットワークの波動ダイナミクスに関する理論的解析）

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Wave dynamics emerge widely in spatially extended dynamical systems, including biological and artificial neural networks. This thesis investigates the wave dynamics in a cellular chaotic neural network (CNN) under simplified settings, dealing with both characterization and control of waves. Specifically, the plane wave dynamics is studied from the point of view of local bifurcation theory with an emphasis on the Bogdanov–Takens (BT) bifurcation; the control of spiral wave dynamics is considered on the basis of a dynamic phase space constraint (DPSC) method.

First, to facilitate the BT bifurcation analysis, computation of its parameter-dependent normal form on the center manifold for n-dimensional, m-parameterized continuous-time systems is studied using the homological method. In the general case, a revision to the existent result on the parameter transformation is obtained, which is necessary for determining the bifurcation diagram to the second order. In the case of enduring equilibria, simple formulas are obtained for the transformation of parameters, enabling the formulation of explicit transversality conditions and bifurcation diagrams to at most the second order. Moreover, in $\mathbf{Z}_2$-symmetric systems, the calculation can be further limited within certain subspaces. These results either revise or simplify existent studies, thereby facilitating quick computation of parametric normal forms for BT bifurcations in application.

Next, the previously derived formulas are used to analyze the plane wave dynamics in the neural field model of the cellular CNN, which is a variant reaction-diffusion system with a singular and nonlinear spatial coupling. The BT bifurcation occurs in the three-dimensional traveling wave

system for all wave speed values, indicating the existence of periodic waves, fold of periodic waves, and solitary waves with relatively high speeds, and periodic waves and single fronts/backs with slow speeds. The occurrence of the Bautin bifurcation reveals the existence of fold of periodic waves with relatively slow speeds as well. Moreover, the approximate dispersion relation for small-amplitude periodic waves is obtained, and the stability of these waves is analyzed. All these results are verified or supplemented by numerical continuation studies. This part of research describes the overall plane wave dynamics in oscillatory media (with a $\mathbf{Z}_2$ symmetry) in the parameter space, thereby serving as a supplement to the existent analysis of plane wave dynamics in typical excitable media.

Finally, the spiral wave dynamics in the simplified cellular CNN is investigated. It is shown by simulation that random initial conditions lead to stably rotating chaotic spiral waves in the network, which demonstrate amplitude reduction near the phase singularity. A DPSC method is proposed for eliminating the spiral waves, where a control signal is constructed to indicate the presence of spiral waves, and a limiting threshold modulated by the control signal is imposed on the refractory internal state of the network. Such a control scheme turns out to be successful in redirecting the network from a spiral wave state into either a plane wave (PW) state or a synchronized oscillation (SO) state. The pre-, intra-, and post-control dynamics exhibit different characteristics in the frequency domain; the PW-inducing and SO-inducing control processes are also distinct. Furthermore, a partial selectivity of the control results between PW and SO by varying the control parameters is discovered. This scheme surpasses existent methods of removing spiral wave in the sense that not only homogeneous states are produced and that the control does not need to be turned off manually.

The results of this thesis provide fundamentals of the traveling wave dynamics in the cellular CNN and may help to facilitate the future application of such networks. These results may also be beneficial to the study of wave dynamics in other spatially extended dynamical systems.