Three Tales of Subgame-Perfection
（部分ゲーム完全性に関する3つの話）

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Contents

Chapter 1

Overview

The subgame-perfect equilibrium, originated by Selten (1975), is one of the most popular equilibrium concepts in industrial organization. Its idea is very simple. Consider the simple entry game where an incumbent firm is confronted by a potential entrant. First, the potential entrant decides whether or not to enter the market. It must sink the fixed cost of $30 to enter the market. If entry does not occur, the incumbent earns $100 and the game ends. If entry occurs, the incumbent has two options: to compete fiercely with the entrant, or to accommodate with it. The former option brings each firm the gross profit of $10. The latter option brings each the gross profit of $40.

This game has two Nash equilibria. One is that the incumbent competes fiercely and entry does not occur (because the entrant loses $20 net of the fixed cost if it enters the market). However, this equilibrium is fragile to the entrant's 'mistake'. Suppose the entrant happens to enter the market. Then the incumbent has an incentive to accommodate with the entrant since it can earn not $10 but $40. Namely the incumbent's threat to compete intensely is not credible. To eliminate this unrealistic equilibrium, we have only to consider the strategy profile that is a Nash equilibrium in all subgames. This is the main concept of the subgame perfection. The second and more intuitive equilibrium is that entry occurs and the incumbent accommodates with the entrant. This is the unique subgame-perfect equilibrium of the above entry game.

Next let us consider a similar entry game in a horizontally differentiated market. Schmalensee (1978) argues that the incumbent can commit itself to tough competition

5

by choosing many varieties of products before entry occurs. However, Judd (1985) shows in the two-goods model that this strategy is not credible when the exit cost is small. The reason is that the incumbent's profit increases by withdrawal of competing product with the entrant. Chapter 2 of this dissertation extends the Judd's argument to the infinite-goods case using Hotelling's (1929) model with zero exit cost. It shows that the incumbent cannot deter entry by choosing any variety of products on condition that it cannot deter entry by one product.

Is there any way for the incumbent to deter entry, then? Chapter 2 demonstrates a new commitment device for the incumbent: it can commit to intense competition by letting other firm in the market intentionally. Suppose an incumbent firm is confronted by two potential entrants, a weak firm and a strong firm. The incumbent can deter entry of the weak firm, but cannot deter entry of the strong firm by itself. When the weak firm moves before the strong firm, the incumbent may be able to block entry of the strong firm by allowing the entry of the weak firm and filling up the market.

Chapter 3 analyzes the vertically differentiated market, and proved the robustness of the results in Chapter 2. Moreover it shows that a policy that enhances competition in a vertically differentiated market may be harmful to the society. The intuition is that, as the entrant becomes more dangerous, the incumbent cuts down the quality of its product in order to reduce the niche in the low-quality market.

The simplest way to find the subgame-perfect equilibrium is to solve the game from the terminal branches, i.e. to use the backward induction. Chapter 4 extends this method to the signaling game described below (More precisely, it analyzes the sequential equilibrium of this game). Consider the economists who predict the trend of the economy in turn. Competent economists obtain common information on business

forecasts, and incompetent economists obtain independent information. No one knows who is able. Then young economists mimic others because a forecast different from others indicates inability when it proves wrong. An older economist, however, can infer his ability from past information. Therefore those who got useful information in the past stop herding to signal their ability when economists are heterogeneous. On the other hand, all economists herd together when economists are homogeneous, because the merit from signaling is small. The latter half of Chapter 4 investigates the empirical data, and shows that Japanese economists are more homogeneous than American.

Chapter 2

Weak Entrants Are Welcome*

Abstract

This paper investigates the decision problem of an incumbent firm confronted by both a weak and strong entrant in a differentiated market. Suppose that the incumbent can deter entry of the weak firm, but cannot deter entry of the strong firm by itself. Then the incumbent may allow entry of the weak firm and use it to alter the strong firm's entry decision. The present paper formalizes this idea, and it sheds new light on the fact that domestic firms are sometimes able to block strong foreign firms after trade liberalization. The idea also explains why a dominant firm lets fringe firms be in the market.

1. Introduction

This paper demonstrates that an incumbent firm may intentionally allow entry of a weak firm to stop entry of a strong firm. In other words, an incumbent firm that is confronted by a strong entrant may *welcome* a weak entrant. Consider the incumbent firm confronted by both a weak and strong entrant in a differentiated market. Suppose the incumbent can deter entry of the weak firm, but cannot commit to deter entry of the strong firm by itself. Then, the incumbent has two options if the weak firm moves before the strong firm: to prevent entry of the weak firm, or to allow it. The former option leads to entry of the strong firm. On the other hand, the latter option may make it possible to block entry of the strong firm, since the market is now crowded. This article clarifies the conditions in which the incumbent will choose the latter option.

This argument can explain many interesting phenomena. Let us imagine a developing country. The incumbent domestic firms are too weak to stop entry of strong foreign firms by themselves. What if the government temporarily restricts trade? Then domestic entrants can move before foreign entrants, and the incumbents will allow entry of weak domestic firms to fill up the market. This will block the entry of foreign firms after trade liberalization since the entrants cannot earn enough profit. [1] This argument fits well the Japanese car industry around 1960 and the Japanese bearing industry.

In 1960, the Japanese government announced that it would gradually remove import quotas and reduce tariff rates of cars and trucks after the mid-1960's. Mitsubishi, Mazda, and Honda entered the car industry successfully in the early 1960's. In contrast, the strong foreign firms such as the Big 3 failed to enter the Japanese market after the reduction of tariff rates and the dollar depreciation in 1971. Taking account of the large technological gap between the Big 3 and the Japanese firms, it is clear that Toyota and Nissan, the incumbents, could not prevent entry of the Big 3 by themselves. Thus it is reasonable to argue that Toyota and Nissan used the domestic entrants to deter the entry of the Big 3.

Next let us consider the Japanese bearing industry. The Japanese domestic firms, NSK and NTN, were so weak before World War 2 that Japan depended heavily on foreign firms such as Timken and AB SKF. After World War 2, Japan restricted imports

and that caused the growth of the new domestic firms, Koyo Seiko and Nachi-Fujikoshi. When Japan removed import quotas and reduced tariff rates in 1970's, Timken and AB SKF were still strong and they occupied 35% of the world market. Nonetheless they could not reenter the Japanese market since Koyo Seiko and Nachi-Fujikoshi filled it up.

These arguments give rise to one question: why does the incumbent need a weak entrant to deter the entry of a strong firm? Why can it not block the entry of a strong firm by making additional plants itself instead of depending on the weak firm? Richard Schmalensee (1978) suggests that an incumbent firm can prevent entry by filling up the product spectrum. Giacomo Bonanno (1987) formally calculates the incumbent's optimal choice of product variety. Kenneth L. Judd (1985), however, shows this strategy is not credible if an exit cost is small. The reason is the incumbent has an ex post incentive to withdraw some products in response to entry by another firm. To illustrate this point, Judd considers a simple example with two close substitutes, say tea and coffee. All firms can produce them at the same constant marginal cost after they bear fixed costs, and they compete on price. Suppose that the incumbent produces both goods, and that entry occurs in coffee. If the incumbent continues to produce coffee, it will earn zero gross profit from coffee, and the price war in coffee reduces demand for tea. In contrast, if an exit cost is not high then the incumbent can do better by stopping production of coffee, since this raises the price of coffee and profit from tea. Thus it will leave the coffee market and consequently entry by a new firm will occur.

The above argument simplifies the real world on two important points; the incumbent may not be the same type of firm as the entrant, and there are many substitutes such as juice and cocoa in reality. An incumbent is usually mature and has a wide variety of products, while a newcomer has a limited variety of products. [2] Therefore the incumbent may be able to deter entry by clustering its products. Suppose the incumbent can choose any variety of products and the entrant can choose one kind only. At first, the incumbent may choose many products all of which are close substitutes. If entry occurs in one of them, the incumbent withdraws the directly competing product as Judd suggests but may keep the other products. Then, surrounded by many substitutes, the entrant may not be able to earn enough profit. Consequently it may not enter the market.

We shall examine this possibility in the standard Hotelling model with quadratic

transportation costs X. Martinez-Giralt and Neven (1987) shows that quadratic transportation costs make competition between similar products so severe that a firm prefers to have one product rather than two. The same logic shows that, even if an incumbent firm can fill up the product spectrum, it has an ex post incentive to withdraw not only the directly competing product but also other products near to the strong firm. Therefore an incumbent firm cannot deter entry of a strong firm without help of a weak entrant if there is no exit cost. [3]

If the exit cost is positive, on the other hand, the incumbent firm that clusters its products may keep them because withdrawal of one of them may not mitigate price competition enough to recover the exit cost. Yet, this strategy leaves broad room for an entrant in the market ends because the products must be sufficiently close together in order for the incumbent to keep them. Accordingly the incumbent needs a weak entrant to fill the market and deter entry of a strong firm.

A brief review of the previous literature is in order now. A considerable number of studies have been made of entry deterrence over the past few decades, and some articles consider the idea of sequential entry. Among them, Edward C. Prescott and Michael Visscher (1977), James A. Brander and Jonathan Eaton (1984), and Damien J. Neven (1987) investigate how the possibility of further entry affects each firm's optimal product choice in a differentiated market. Gillian K. Hadfield (1991) considers spatial preemption and argues that an incumbent manufacturer can escape the commitment problem by delegating pricing authority to independent franchisees. [4] Richard Gilbert and Xavier Vives (1986) show that oligopoly firms facing a potential entrant never under-invest in entry deterrence if they can commit to the quantity they produce. Vives (1988) shows that the incumbent(s) will either allow all the potential entrants in or keep them out when there is a large pool of potential entrants. Other articles take account of the entrant's strength explicitly. Judith R. Gelman and Steven C. Salop (1983) argue that an entrant can elicit a less aggressive reaction from an incumbent by limiting its own capacity. Nancy T. Gallini (1984), and Claude Crampes and Abraham Hollander (1993) consider R&D competition. Gallini (1984) shows that an incumbent may license its technology to reduce an entrant's incentive to develop better technology. Crampes and Hollander (1993) suggest that an incumbent may raise the profits of an entrant by selling at a high price

instead of licensing.

Nevertheless, all of these studies fail to grasp the essence of the actual market that it consists of a variety of entrants. In contrast to their models, which contain only one type of entrant(s), in reality different types of entrants coexist; some are strong, and others are weak. Therefore, for a more realistic analysis it is necessary to integrate the heterogeneity of entrants into the model. The present paper addresses this issue.

Furthermore, we can analyze the incumbent's response to changes in the relative strength of the entrants. We shall obtain the following results by gradually increasing the marginal cost of the weak firm. At first, the incumbent and the weak entrant locate at opposite ends of the market to avoid competition. Then, as the strong entrant becomes dangerous, they commit themselves to compete intensely: they choose closer products to deter entry between them. More precisely, the incumbent moves inward to secure its market and the weak firm stays at the market end. Finally, the weak firm becomes too weak for entry deterrence and the incumbent stops using the weak firm.

The paper is organized as follows. Section 2 describes the model. Section 3 explains why an incumbent does not proliferate its brands. Section 4 identifies the conditions for the incumbent to permit entry of the weak firm. Section 5 and Section 6 discuss some further issues, and Section 7 concludes this paper. The Appendix A contains the formal proofs of lemmata.

2. The model

We use a variant of Harold Hotelling's (1929) model with three players: A denotes an incumbent, S denotes a strong entrant, and W denotes a weak entrant.[5] The potential product range is represented by the unit-segment $[0,1]$. A location on this segment corresponds to the attribute of the product.

At first each firm chooses a finite set of products $X_i \subset [0,1]$ ($i = A, S, W$) with a fixed cost F per product. If firm i does not enter the market, $X_i = \phi$. Next each decides the product(s) it reserves. The timing of each firm's action, which is common knowledge, is as follows.

At date 1, the incumbent A chooses a set X_A and sinks a fixed cost. A is assumed to

be mature and can produce a number of products: $X_A = \{x_A^1, \cdots, x_A^n\}$. We assume $x_A^1 < \cdots < x_A^n \leq 1 - x_A^1$ (and consequently $0 \leq x_A^1 \leq 0.5$) without loss of generality (See Figure 2-1).

At date 2, W observes X_A and chooses X_W. It sinks a fixed cost if it enters the market. At date 3, S observes X_A and X_W, and chooses X_S. It sinks a fixed cost if it enters. We suppose that W and S are new, inexperienced firms which can choose at most one product, so that $X_i = \{x_i\}$ or ϕ for $i = W, S$ [6] If $X_i = \{x_i\}$ for $i = A, W, S$, we slightly abuse the notation and write $X_i = x_i$.

At date 4, each firm observes the product choice (X_A, X_S, X_W), and simultaneously selects a set of products to withdraw.[7] Each firm can withdraw its product(s) with no additional cost, but cannot recover the fixed cost. This assumption, which will play a central role in the analysis, is natural and essential since a firm can get out of the market at will in reality.[8] Let $\hat{X}_i$ be the set of products firm i does not withdraw ($\hat{X}_i \subseteq X_i$). If $\hat{X}_i = \{x_i\}$, we write $\hat{X}_i = x_i$.

At date 5, each firm observes $(\hat{X}_A, \hat{X}_S, \hat{X}_W)$, and simultaneously selects prices of its products.[9] Each pays variable costs and earns sales revenue. Each firm makes goods at constant marginal cost C_i. Let us assume $C_W > C_S = C_A$.[10]

Consumers are uniformly distributed with density 1 on the segment $[0,1]$, and their locations correspond to their favorite products. Each of them always purchases one unit of the good for which her indirect utility is maximized;[11] the utility of a consumer in $y \in [0,1]$ who bought x_i^j at price P_i^j is

$$U(x_i^j, y) = a - t(x_i^j - y)^2 - P_i^j \quad (t > 0)$$

where $t(x_i^j - y)^2$ is the disutility of distance ("transportation costs") in the attribute space. It means that the marginal disutility of the consumer increases as the product she bought locates farther from her favorite one.

The equilibrium concept we adopt is a weak refinement of subgame-perfect Nash equilibrium that assumes no weakly dominated strategy is played in equilibrium.[12] Let

$\Pi_i(\hat{X}_A, \hat{X}_S, \hat{X}_W)$ be the equilibrium profit of firm i (gross of the fixed cost) given $(\tilde{X}_A, \hat{X}_S, \hat{X}_W)$ in the subgame after date 5. Entry occurs when Π_i is equal to or larger than the fixed cost. This paper does not take into account mergers or the formation of cartels.

3. Why does the incumbent need the weak firm?

Suppose an incumbent cannot deter entry of the strong firm by choosing one product. Then, this section will show that no matter how many products it chooses, the incumbent cannot deter entry of the strong firm without the weak firm. If an incumbent chooses more than one product (instead of letting the weak firm in), it faces the commitment problem: it cannot commit to keep its products after entry occurs since withdrawal of competing products relaxes price competition and raises profits from the remaining products. Hence choosing two or more products is useless for entry deterrence.

Bonanno (1987) calculates the equilibrium prices of two firms when one firm produces two goods and another firm produces one good. Martinez-Giralt and Neven (1988) uses them and shows that both firm choose one product only and locate at opposite ends if two firms simultaneously choose their product varieties. We shall extend their results to the n-goods case.

Lemma 1.

Suppose A chooses n products, W does not enter, S chooses $x_S = 1$, and they withdraw no products. Then in the unique equilibrium of this subgame
(a) all products have positive sales, and
(b) each firm chooses

$$P_S = C_S + \frac{t}{3}(1 - x_A^n)(3 - x_A^n),$$

$$P_A^n = C_A + \frac{t}{3}(1 - x_A^n)(3 + x_A^n), \text{ and}$$

$$P_A^i = P_A^n + \frac{t}{2}(x_A^n + x_A^i)(x_A^n - x_A^i) \text{ for } i = 1, \cdots, n-1.$$

The proofs of all lemmata can be found in the Appendix A. Though profit from each product generally depends on all other products' locations and prices, Lemma 1 shows that each product has positive sales and competes only with its two neighbors in equilibrium. The reason is firm A can raise its profits by cutting P_A^j when nobody buys x_A^j. Lemma 2 uses this result and shows that A reserves only one product if W does not enter and S chooses the end of the market. Note that "keeping all products and charging very high prices near S" is not equal to "withdrawing products nearer to S"; A cannot commit itself to the former strategy but can commit to the latter.

Lemma 2.

Suppose A chooses n (≥ 2) products, W does not enter the market, and S chooses $x_S = 1$. Then the unique equilibrium of the subgame after this history is

(a) S does not withdraw its product, and

(b) A withdraws $n-1$ products and keeps x_A^1, namely the farthest product from x_S.

The outline of the proof is as follows. We shall calculate equilibrium payoffs from Lemma 1, and show that A's profit increases when it withdraws x_A^n. On the other hand, S keeps its product since it can earn zero or positive gross profit. By induction, A never reserves two or more products in the equilibrium of this subgame. A keeps x_A^1 since $\Pi_A(x_A,1,\varphi)$ is decreasing in x_A.

Keeping x_A^n, the nearest product to the other firm, has two opposing effects on A's profit; it enlarges its market share, but it intensifies competition. The assumption of "quadratic transportation costs" implies that the marginal disutility of "moving" along the product line increases (decreases) as a consumer buys the farther (closer) product to her favorite one. Hence this assumption makes the latter negative effect so strong that the incumbent withdraws products nearer to the strong firm.

This is also true of each entrant: choosing the product nearer to the incumbent reduces its profits. Lemma 3 shows that each entrant chooses the farthest product from

the incumbent if the incumbent chooses one product and the other entrant does not enter. The combination of Lemma 2 and Lemma 3 yields Proposition 1.

Lemma 3.

(a) $\forall x_A \leq 0.5$ $\arg\max_{\{x_S\}} \Pi_S(x_A, x_S, \phi) = \arg\max_{\{x_W\}} \Pi_W(x_A, \phi, x_W) = 1$.

(b) $\Pi_S(x_A, 1, \phi) > \Pi_S(x'_A, 1, \phi)$ for $x_A < x'_A$.

Proposition 1.

Suppose W does not enter the market. If A cannot deter entry of S by choosing one product, then A cannot deter entry of S by choosing any number of products.

Proof.

If A cannot deter entry of S by choosing one product, for any x_A there exists some x_S such that $\Pi_S(x_A, x_S, \phi) \geq F$. We show A cannot deter entry of S by itself in this case. Suppose A chooses n products at date 1. If S chooses $x_S = 1$, Lemma 2 shows that A keeps x_A^1 and withdraws the rest at date 4. Lemma 3 (a) shows that $\forall x_S$ $\Pi_S(x_A^1, 1, \phi) \geq \Pi_S(x_A^1, x_S, \phi)$. Consequently S can enter the market by choosing $x_S = 1$ and keeping it at date 4 regardless of the products A chooses. Q.E.D.

4. When to allow entry of the weak firm?

We shall confine our attention to the case in which the incumbent can stop entry of the weak firm but cannot stop entry of the strong firm by itself (Other cases are trivial). The last section showed that the incumbent cannot deter entry of the strong firm by itself when it cannot deter entry by choosing one product. From Lemma 3 (b), A cannot prevent entry of S by itself if and only if $\Pi_S(0.5, 1, \phi) \geq F$. Note that S chooses the farthest product from x_A to avoid competition and that choosing $x_A = 0.5$ minimizes the remaining space. By Lemma 1, $\Pi_S(x_A, x_S, \phi) = \frac{t}{18}(x_S - x_A)(4 - x_A - x_S)^2$ for $x_S \geq x_A$, and $\Pi_S(0.5, 1, \phi) \geq F$ is equivalent to

$$f \leq \frac{25}{144} \quad \text{where} \quad f \equiv \frac{F}{t} \tag{1}$$

(F is the fixed cost per product and t is the coefficient of the disutility of distance). Similarly, A can block entry of W by itself if and only if $\Pi_w(0.5, \phi, 1) < F$. That is,

$$f > \frac{(5 - 4\Delta)^2}{144} \quad \text{or} \quad \Delta \geq 1.25, \quad \text{where} \quad \Delta \equiv \frac{C_w - C_S}{t}. \tag{2}$$

Now we are ready to determine the conditions for the incumbent to permit entry of the weak firm. This section considers the sufficient conditions for simplicity (Section 5 shows the necessary and sufficient conditions). Let us examine the decision of each firm in reverse order, starting with the decision problem of the strong firm.

Define $x_S^* \equiv \arg\max \Pi_S(x_A, x_S, x_W)$. Suppose A chooses $x_A = 0$ and W chooses $x_W = 1$. Then S does not enter the market if

$$\Pi_S(0, x_S^*, 1) < F, \tag{3}$$

where

$$\Pi_S(0, x_S, 1) = \begin{cases} \dfrac{tx_S[\Delta + 3(1 - x_S)]^2}{18(1 - x_S)} & \text{if} \quad x_S \leq \dfrac{3(1 - \Delta)}{3 - \Delta}, \\[3ex] \dfrac{t}{4x_S}(1 + 2x_S - \Delta)\left[\Delta - (1 - x_S)^2\right] & \\[1ex] \qquad \text{if} \quad x_S > \dfrac{3(1 - \Delta)}{3 - \Delta} \quad \text{and} \quad 3(1 - \Delta) - 2x_S(1 - x_S) > 0, \\[3ex] \dfrac{tx_S(4 - x_S)^2}{18} & \text{otherwise.} \end{cases}$$

Next suppose $\Pi_w(0, \phi, 1) \geq F$, namely

$$f \leq \frac{(3 - \Delta)^2}{18} \tag{4}$$

Then Proposition 2 below shows that in the unique equilibrium A allows entry of W and deters entry of S by use of W if $\Delta \leq 1$ and the (Δ, f) pair satisfies all of (1), (2), (3) and (4). The proof will make use of the following preliminaries.

Lemma 4.

Suppose that A keeps n (≥ 2) products and that $x_W \leq x_A^n$ or $x_A^n \geq 0.5$. Then

$$\Pi_W\left(\left\{x_A^1, \cdots, x_A^n\right\}, \phi, x_W\right) \leq \Pi_W(0.5, \phi, 1).$$

Lemma 5.

$$\Pi_i\left(\hat{X}_A, x_S, x_W\right) \leq \Pi_i\left(\hat{X}_A, \phi, x_W\right) \text{ for any } i \in \{A, W\}, \ \hat{X}_A, \ x_W, \text{ and } x_S.$$

Lemma 6.

S chooses $x_S = 1$ if (1) holds and W does not enter the market.

Lemma 7.

Suppose A chooses n (≥ 2) products and $x_A^n < 0.5$. Then

(a) $\Pi_A\left(\left\{x_A^1, \cdots, x_A^n\right\}, \phi, 1\right) > \Pi_A\left(\left\{x_A^1, \cdots, x_A^n\right\}, \phi, x_W\right)$ for any $x_W \in \left(x_A^n, 1\right)$,

(b) $\Pi_A\left(\left\{x_A^1, \cdots, x_A^{n-1}\right\}, \phi, 1\right) > \Pi_A\left(\left\{x_A^1, \cdots, x_A^{n-1}, x_A^n\right\}, \phi, 1\right)$ for $\Delta \leq 1$, and

(c) $\Pi_A\left(x_A, \phi, 1\right)$ is decreasing in x_A.

Proposition 2.

Suppose all of (1), (2), (3), (4), and $\Delta \leq 1$ are satisfied. Then the unique equilibrium outcome is that A chooses $x_A = 0$, W chooses $x_W = 1$, and S does not enter the market.

Proof.

First we show that A earns $\Pi_A(0, \phi, 1) - F$ if it chooses $x_A = 0$ at date 1. Note that S does not enter the market if A chooses $x_A = 0$, W chooses $x_W = 1$, and (3) holds. W chooses $x_W = 1$ if A chooses $x_A = 0$ and both of (3) and (4) hold. The reason is

$$\Pi_W\left(0, x_S, x_W\right) \leq \Pi_W\left(0, \phi, x_W\right) \text{ (from Lemma 5)}$$

$$\leq \Pi_W\left(0, \phi, 1\right) \text{ (from lemma 3 (a)).}$$

Thus A earns $\Pi_A(0, \phi, 1) - F$ if it chooses $x_A = 0$ at date 1 and keeps it at date 4. We show Π_A decreases if A deviates from this.

Consider the subgame where A chooses $x_A > 0$ at date 1 and W enters the market. We will determine an upper bound of A's payoff. Lemma 5 shows

$\Pi_A(x_A, x_S, x_W) \leq \Pi_A(x_A, \phi, x_W)$. Calculation shows that Π_A decreases as A and W choose products nearer to each other, namely $\Pi_A(x_A, \phi, x_W) < \Pi_A(0, \phi, 1)$. Thus A's payoff must be smaller than $\Pi_A(0, \phi, 1) - F$ in this subgame.

Next consider the subgame where A chooses n (≥ 2) products at date 1 and W enters. Lemma 5 shows $\Pi_W(\{x_A^1, \cdots, x_A^n\} x_S, x_W) \leq \Pi_W(\{x_A^1, \cdots, x_A^n\} \phi, x_W)$. Lemma 4 shows $\Pi_W(\{x_A^1, \cdots, x_A^n\} \phi, x_W) \leq \Pi_W(0.5, \phi, 1)$ if $x_A^n \geq 0.5$ or $x_W \leq x_A^n$. Therefore both of $x_A^n < 0.5$ and $x_W > x_A^n$ must hold in order for W to enter the market. Then

$$\Pi_A(\{x_A^1, \cdots, x_A^n\} x_S, x_W) \leq \Pi_A(\{x_A^1, \cdots, x_A^n\} \phi, x_W) \text{ (from Lemma 5)}$$
$$\leq \Pi_A(\{x_A^1, \cdots, x_A^n\} \phi, 1) \text{ (from Lemma 7(a))}$$
$$< \Pi_A(x_A^1, \phi, 1) \text{ for } \Delta \leq 1 \text{ (from Lemma 7(b))}$$
$$\leq \Pi_A(0, \phi, 1) \text{ (from Lemma 7(c))},$$

and A's payoff must be smaller than $\Pi_A(0, \phi, 1) - nF$ ($< \Pi_A(0, \phi, 1) - F$) in this subgame regardless of the product(s) it keeps at date 4.

Lastly, we consider the remaining subgame where A chooses $n(\geq 1)$ product(s) at date 1 and W does not enter the market. Then Lemma 2 and Lemma 6 show that S chooses $x_S = 1$ and keeps it at date 4, and that A keeps x_A^1 only at date 4. Since $\Pi_A(x_A, 1, \phi)$ is decreasing in x_A, we conclude that A can earn $\Pi_A(0, 1, \phi) - nF$ ($< \Pi_A(0, \phi, 1) - F$) or less in this subgame. Q.E.D.

Proposition 2 shows that A chooses $x_A = 0$ and lets the weak firm enter if all of (1), (2), (3), (4), and $\Delta \leq 1$ are satisfied. The intuition is as follows. Now that A cannot stop entry of S without W from (1), it is better for A to allow entry of W and use it for entry deterrence. Then choosing $x_A = 0$ maximizes Π_A from Lemma 1, and W chooses $x_W = 1$ to avoid competition. This blocks entry of S because of (3).

The question is whether there exists any set of parameters that meets all conditions of Proposition 2. Theorem 1 shows the existence of such sets.

Theorem 1.

Define $\Delta \equiv \dfrac{C_w - C_s}{t}$ and $f \equiv \dfrac{F}{t}$. Then

(a) there exists a (Δ, f) pair that meets all of the conditions in Proposition 2, and

(b) the following statements are true under such a (Δ, f) pair;

   (b1) A lets W enter to deter entry of S in the unique equilibrium.

   (b2) A could not deter entry of S if S moved before W.

   (b3) A would deter entry of W if S did not exist.

Proof.

     Proposition 2 shows that (b1) is true if (1), (2), (3), and (4) are satisfied and $\Delta \le 1$.
(b2) and (b3) are true if (1) and (2) hold. Hence we have only to prove the existence of a
(Δ, f) pair that satisfies all of (1), (2), (3), (4), and $\Delta \le 1$. See Figure 2-2.

     The boundaries of (3) and (4),

$$tf = \Pi_s\left(0, x_s^*, 1\right) \tag{3'}$$

$$\text{and } f = \frac{(3-\Delta)^2}{18}, \tag{4'}$$

are continuous curves in the (Δ, f) plane and they are not vertical lines. If $\Delta = 0$,
$\Pi_s(0, x_s, 1) = 0.5 t x_s(1 - x_s)$. Thus (3') crosses the vertical axis at 0.125. (4') crosses it

at 0.5, and the boundaries of (1) and (2) cross it at $\dfrac{25}{144}$. Consequently, for sufficiently

small Δ there must be a point on the boundary of (1) that is above (3') and below (4').
All of (1), (2), (3), and (4) hold under such a (Δ, f) pair. Q.E.D.

5. Discussion

Theorem 1 shows that the incumbent blocks entry of the strong firm by use of the weak
firm if (Δ, f) lies in the shaded area in Figure 2-2. The incumbent benefits from the
existence of the weak firm because it could not deter entry of the strong firm without the
weak firm. The weak firm benefits from the existence of the strong firm because the
incumbent would not permit its entry without the strong firm.

     Let us discuss Figure 2-2 in detail. Remember that t is the coefficient of the disutility

of distance in the attribute space, Δ is the difference of marginal cost between W and S normalized by t, and that f is the fixed cost normalized by t.

A cannot deter entry of S by itself if (Δ, f) lies below (1). (1) is horizontal because the size of C_W is irrelevant. A can stop entry of W if (Δ, f) lies above (2). The reason why the slope of (2) is negative is obvious. To block entry of W, f must be large enough. The stronger W is (i.e. the smaller Δ is), the larger f must be. (1) and (2) cut the vertical axis at the same point because $\Delta = 0$ implies $C_W = C_S$.

A and W can prevent entry of S if (Δ, f) lies above the graph of (3'). (3') is lower than (1) at $\Delta = 0$ because the presence of W reduces the niche for S. (3') slopes upward because S can earn more gross profit as W becomes weaker.

W can enter the market if A chooses $x_A = 0$ and (Δ, f) lies below the graph of (4'). (4') is higher than (2) at $\Delta = 0$ since the remaining space for W decreases in x_A. (4') slopes downward because f must become smaller as W becomes weaker in order to keep W's profit non-negative.

Since Theorem 1 gives sufficient conditions, actual area where A can deter entry of S is larger than the shaded area in Figure 2-2. If (Δ, f) lies just below (3'), A and W can prevent entry of S by narrowing their distance. Either way below will do for entry deterrence; A moves inward enough and W stays at the end, or A forces W to move in. Proposition 3 shows that A chooses the former in the unique equilibrium. The intuition is, provided that A and W must choose closer products, it is better for A to move inward and secure customers than to stay and lose the market (The Appendix A contains the proof).

Lemma 8.

W does not enter the market if $\Pi_S\left(x_A, x_S^*(x_A, x_W), x_W\right) \geq F$ for any x_W.

Proposition 3.

Suppose some x_A satisfies all of the following inequalities;

$$\Pi_S\left(x_A, x_S^*(x_A), 1\right) \leq F,$$

$$\Pi_A\left(x_A, \phi, 1\right) \geq \Pi_A\left(0, 1, \phi\right),$$

and $\Pi_W(x_A, \phi, 1) \geq F$.

Also suppose that (Δ, f) pair satisfies (1) and (2). Define

$$x_A^* \equiv \min\left\{x_A \in [0, 0.5] \mid \Pi_S(x_A, x_S^*(x_A), 1) \leq F\right\}.$$

Then A chooses x_A^*, W chooses $x_W = 1$, and S does not enter the market in the unique equilibrium.

Proposition 3 shows that the incumbent and the weak firm choose similar products and commit themselves to tough competition when entry threat is severe. Theorem 2 derives the necessary and sufficient conditions under which A uses W for entry deterrence. Figure 2-3 shows the conditions graphically.

Theorem 2.

(a) The following inequalities are the necessary and sufficient conditions for A to let W enter intentionally in the unique equilibrium:

$$\Pi_S(0.5, 1, \phi) \geq F, \tag{1}$$

$$\Pi_W(0.5, \phi, 1) < F, \tag{2}$$

$$\Pi_S(0.5, x_S^*, 1) < F, \tag{5}$$

$$\Pi_W(x_A^*, \phi, 1) \geq F, \tag{6}$$

and $\quad \Pi_A(x_A^*, \phi, 1) \geq \Pi_A(\bar{x}_A, 1, \phi) \tag{7}$

where $\bar{x}_A \equiv \min x_A$ such that $\Pi_S(x_A, x_S^*(x_A, x_W), x_W) \geq F$

for any x_W that satisfies $\Pi_W(x_A, \phi, x_W) \geq F$.

(b) If A uses W for entry deterrence, it chooses $x_A = x_A^*$ and W chooses $x_W = 1$.

(c) $\dfrac{\partial x_A^*}{\partial \Delta} \geq 0$ and $\dfrac{\partial x_A^*}{\partial f} \leq 0$. The equalities hold if $\Pi_S(0, x_S^*, 1) < F$.

Proof.

If (1) is satisfied, A cannot deter entry of S by itself. If (2) is satisfied, A can stop entry of W. There exists x_A^* if (5) is satisfied. Then A can use W to prevent entry of S by

22

choosing $x_A = x_A^*$ provided that (6) is satisfied and W can enter the market. $\Pi_A(\bar{x}_A, 1, \phi)$ is the maximum profit A can earn when it chooses x_A such that W cannot deter entry of S. Using W for entry deterrence is profitable if (7) is satisfied. Therefore (1), (2), (5), (6) and (7) are the sufficient conditions.

If (1) is not satisfied, A can deter entry of S without W. If (2) is not satisfied, A cannot stop entry of W. If (5) is not satisfied, A and W cannot prevent entry of S. If (6) is not satisfied, A cannot use W for entry deterrence. If A intends not to deter entry of S, it can earn $\Pi_A(\bar{x}_A, 1, \phi)$ because Lemma 8 shows that W does not enter when $x_A = \bar{x}_A$. Accordingly A does not have an incentive to prevent entry of S if (7) is not satisfied. Therefore (1), (2), (5), (6) and (7) are the necessary conditions.

(b) and (c) are directly derived from Proposition 3. Q.E.D.

Figure 2-3 reveals three important points. The first point is that the incumbent does not allow entry of W if it is too weak (i.e., Δ is too large). A uses W and deters entry of S if (Δ, f) lies in the shaded area in Figure 2-3. A horizontal move from the left to the right in Figure 2-3 shows an increase in C_W given $C_S (= C_A)$ and f. The intuition is that an entrant which is too weak is useless in deterring the entry of a strong firm.

The second point is that all lines in Figure 2-3 shift upward in direct proportion to the density of consumers. This implies that the shaded area exists regardless of the density of consumers, i.e. the market size.

The third point is that t is irrelevant to the shape of the shaded area in Figure 2-3. This is due to the assumption that a consumer always buys one product, because it makes each firm's optimal product homogeneous of degree zero. If a (reservation price) is sufficiently large compared to C_W and t, this assumption is satisfied and we can conclude that the incumbent may use the weak firm for entry deterrence regardless of the degree of substitutability between products.

Figure 2-3 has some implications for a developing country where imports are temporarily forbidden and an incumbent firm is confronted by a strong foreign firm and a weak domestic firm. Suppose the domestic entrant is so weak that the incumbent cannot deter entry of the strong foreign firm even if it allows entry of the weak domestic firm.

Then the incumbent has an incentive to provide technical assistance for the domestic entrant before trade liberalization in order to lower C_{ir} and use it for entry deterrence. They can block entry of the strong firm if the incumbent succeeds in moving the (Δ, f) pair into the shaded area in Figure 2-3. Since the incumbent gains from this provision of technical assistance, the incumbent may even *pay* for it!

6. Extensions

6-1. Positive exit costs

Our argument until now has assumed zero exit cost. What will happen if the exit cost, E, is positive? Suppose the incumbent clusters its products. Then withdrawing one of them may not mitigate price competition enough to recover exit cost. Hence the incumbent may keep all products after entry occurs, and it may be able to deter entry by itself. Proposition 4 shows that, for any positive exit cost, there exists a set of parameters where the incumbent can deter entry of the strong firm by itself only if it chooses two products.

Proposition 4.

    Suppose $E > 0$ and (1) is satisfied. Then there exists a set of parameters under which A deters entry of S by choosing two products in the unique equilibrium.

Proof.

    Suppose A chooses two products. Then $x_A^2 = 1 - x_A^1 = x > 0.5$ minimize the market niche. Calculation in the proof of Lemma 2 shows that

$$\Pi_A\left(\{x_A^1,\cdots,x_A^{n-1}\}x_S,\phi\right) - \Pi_A\left(\{x_A^1,\cdots,x_A^{n-1},x_A^n\}x_S,\phi\right)$$

$$= \frac{t\left(x_A^n - x_A^{n-1}\right)}{72}\left[16 - 4(x_S)^2 + (16 + 4x_S)\left(x_A^{n-i} + x_A^n\right) - 5\left(x_A^{n-i}\right)^2 - 14x_A^{n-i}x_A^n - 5\left(x_A^n\right)^2\right]$$

and it is minimum at $x_S = 1$. Thus A can commit itself to keep both products by choosing x that satisfies

$$\frac{t}{72}(2x - 1)\left(27 - 4x + 4x^2\right) < E. \tag{8}$$

Note that (8) is satisfied for any positive E if x is sufficiently close to 0.5. Define

$$\bar{x} \equiv \left\{ x \in (0.5,1) \left| \frac{t}{72}(2x-1)(27-4x+4x^2) = E \right. \right\}.$$

Also suppose a (the reservation price of consumers) is so large that

$$\Pi_A((1-x,x),\phi,\phi) - F > \Pi_A(0,\phi,1) \text{ for some } x \in (0.5,\bar{x}).$$

Define $x' \equiv \arg\max \Pi_A((1-x,x),\phi,\phi)$ subject to $x \in (0.5,\bar{x})$. Then, if (F,t) satisfies

$$\frac{25t}{144} > F > \frac{t}{18}(1-x')(3-x')^2 = \Pi_S((1-x',x'),1,\phi),$$

A can deter entry of S by choosing $x_A^2 = 1 - x_A^1 = x'$ but cannot deter entry with one product. It is clear that such (F,t) exists. Q.E.D.

Proposition 4 shows that choosing two or more brands may be useful for entry deterrence if they are thickly clustered. Yet this strategy leaves broad room for an entrant in the market ends. Therefore the incumbent may need a weak entrant to fill it up. Theorem 3 shows that, for moderate exit costs, there exists a set of parameters under which A chooses only one product and uses W to deter entry of S in the unique equilibrium.

Theorem 3.

There exists a set of parameters under which

(a) A chooses one product and lets W enter to deter entry of S in the unique equilibrium;

(b) A would deter entry of W if S did not exist;

(c) A could not deter entry of S if S moved before W; and

(d) $E > 0.25F$.

Proof.

Suppose $C_A = C_S = 0$ to simplify the argument. First we derive the conditions where A never chooses more than three goods in equilibrium. Define

$$x_W^* \equiv \arg\max \Pi_W(x_A, \phi, x_W)$$

and $x_A^* \equiv \arg\max \Pi_A(x_A, \phi, x_W^*(x_A))$.

Suppose that A can choose one product only, and that

$$\Pi_W\left(x_A^*, \phi, x_W^*\left(x_A^*\right)\right) \geq F \tag{9}$$

$$\text{and } \Pi_S\left(x_A^*, x_S^*\left(x_A^*, x_W^*\right), x_W^*\left(x_A^*\right)\right) < F \tag{10}$$

Then, since $\Pi_A\left(x_A^*, \phi, x_W^*\left(x_A^*\right)\right) > \max \Pi_A\left(x_A, x_S^*(x_A), \phi\right)$, the same argument as Proposition 2 shows that A chooses x_A^*, W chooses $x_W^*\left(x_A^*\right)$, and S does not enter the market in the unique equilibrium. If A chooses four goods,

$$\max \Pi_A\left(\hat{X}_A, \hat{X}_S, \hat{X}_W\right) = \Pi_A\left(\left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}, \phi, \phi\right) = a - \frac{t}{64}.$$

If A chooses five goods,

$$\max \Pi_A\left(\hat{X}_A, \hat{X}_S, \hat{X}_W\right) = \Pi_A\left(\{0.1, 0.3, 0.5, 0.7, 0.9\}, \phi, \phi\right) = a - \frac{t}{100}.$$

Therefore A never chooses more than four goods in equilibrium if (9), (10),

$$F > \frac{9t}{1600}, \text{ and } \Pi_A\left(x_A^*, \phi, x_W^*\left(x_A^*\right)\right) > a - \frac{t}{64} - 3F \tag{11}$$

are satisfied.

Next we consider the case where (9), (10), and (11) are satisfied. Then the best A can do for entry deterrence is to choose $X_A = \{1 - \hat{x}, 0.5, \hat{x}\}$, where

$$\hat{x} \equiv \max\left\{x \mid E \geq \frac{(2x-1)}{576}\left(83 + 52x - 20x^2\right)\right\}.$$

Accordingly, A cannot stop entry of S by itself if and only if

$$\Pi_S\left(\{1 - \hat{x}, 0.5, \hat{x}\}, 1, \phi\right) = \frac{t}{18}(1 - \hat{x})(3 - \hat{x})^2 \geq F. \tag{12}$$

A can prevent entry of W if and only if

$$\Pi_W\left(\{1 - \hat{x}, 0.5, \hat{x}\}, \phi, 1\right) = \frac{t}{18(1 - \hat{x})}\left[(1 - \hat{x})(3 - \hat{x}) - \Delta\right]^2 < F. \tag{13}$$

A has no incentive to choose more than one product if

$$\Pi_A\left(x_A^*, \phi, x_W^*\left(x_A^*\right)\right) > \max \Pi_A\left(\{x_A^1, x_A^2\}, \phi, 1\right) - F. \tag{14}$$

Finally, we show the set of parameters that satisfies all of (9), (10), (11), (12), (13), and (14). Suppose $a = 0.8t$, $E \leq 0.034098t$, $F = 0.131444t$, and Δ is sufficiently small. Then all of the above conditions are satisfied, and A chooses $x_A^* = 0.2695$, W chooses $x_W^*\left(x_A^*\right) = 0.94697$, and S does not enter the market in the unique equilibrium.

Q.E.D.

If the exit cost is moderate, A cannot cover the whole market with its products because of the commitment problem. Thus it needs W to deter entry of S. The fact that A uses W even if the exit cost is as much as a quarter of the fixed cost demonstrates generality of our argument.

6-2. Cost differences

Next consider the case that $C_A \neq C_S$. See Figure 2-2. (1) and (2) always cut the vertical axis at the same point because $C_W = C_S$ at $\Delta = 0$. Given C_S, all lines in Figure 2-2 move upward as C_A increases. The reason is S and W can earn more as A becomes weaker. Similarly, (1), (2), and (3') move downward as C_S increases given C_A. Therefore, on condition that the difference between C_A and C_S is sufficiently small, the shaded area in Figure 2-2 exists when $C_A > C_W > C_S$ and $C_W > C_S > C_A$.

6-3. Several incumbents

Finally, we shall consider the case where two or more incumbents are confronted by a weak and a strong entrant. Suppose two incumbents, A and B, locate on a circle and they cannot deter entry of a strong firm by themselves. Namely suppose they cannot deter entry when A chooses three o'clock and B chooses nine o'clock (this minimizes the market niche). Then they may choose closer products and allow entry of a weak firm on the other side of the market. For example, suppose A chooses two o'clock and B chooses ten o'clock. Then a weak entrant fills in six o'clock, and there remains no room for the strong firm. This is what has happened to the Japanese car market around 1960. The upper half of the circle represents the market for medium-class cars. The lower half represents the market for mini cars. Since the Big 3 intended to enter twelve o'clock (the medium and large car market) after trade liberalization, Toyota and Nissan chose two o'clock and ten o'clock respectively (produced medium cars) and intentionally allowed entry of Honda in the micro-mini car market.

7. Conclusions

We have considered a differentiated market where an incumbent firm is confronted by a weak entrant and a strong entrant. If the weak entrant moves before the strong entrant, the incumbent concedes the product space intentionally and permits the weak firm in the market. Then the market is filled up and entry of the strong firm is prevented. The first-mover's advantage is so strong in this model that the fittest, i.e. the strong entrant, may not survive in the market. From the weak firm's point of view, its successful entry is due to the strong firm because the incumbent would prevent the weak firm's entry without the threat of the strong firm. Past literature has not focused on this idea.

We show moreover that the incumbent firm and the weak firm choose closer products and commit themselves to compete fiercely as the threat of entry becomes severe. This explains very well one of the notable features of the Japanese market, "tough competition between domestic firms with similar products". When Japan gradually removed import quotas and reduced tariff rates in the 1960's and 1970's, Japan was still a developing country and its domestic firms were weak. Confronted by strong foreign firms, the Japanese domestic firms chose similar products and committed themselves to intense competition. As a result, the Japanese market became unattractive and foreign firms did not enter.

Our results can be applied to a homogeneous market if each firm commits itself to a variable like investment level (in capacity or in R&D) before production. By committing itself to a low investment level, the incumbent can persuade the weak firm to enter the market. Thus it can use the weak firm to prevent entry of the strong firm.

There are many arguments about why a dominant firm does not drive fringe firms out of the market. [13] The result here offers a new explanation of this puzzle: the dominant firm uses weak fringe firms to deter entry of strong potential competitors.

28

Notes

1. The government has an incentive to put this policy into action because it raises the social welfare of the domestic country by the net profit of the weak firm.

2. Mitsubishi, Mazda, and Honda all specialized in "micro-mini" cars in the early stage of entry.

3. Ashiya (1999) obtains the same result in a vertically differentiated market.

4. This assumes homogeneous retailers at regular intervals who are eager to join in the franchise. The incumbent deters entry by combining these retailers.

5. We can make similar arguments with two or more incumbent firms. See Section 6.

6. To let each entrant choose two or more products does not alter the conclusion; the entrant chooses one product in the unique equilibrium.

7. Judd (1985 p.156 fn. 3) argues that "this is the correct static approximation of a truly dynamic analysis of entry into a growing market. Intuitively, in a continuous-time analysis, no one firm can commit itself to staying since tomorrow will give another chance to exit."

8. An exit cost is the cost arising only because of the act of exit. One example is a printing cost of a new catalogue (from which withdrawn products are deleted). Note that irreversible investment in product-specific capital is a sunk cost and is not an exit cost. We shall consider the case of positive exit costs in Section 6.

9. A unique pure strategy equilibrium always exists in this subgame because of the quadratic "transportation costs". See Lemma 1 in Section 3.

10. We shall extend our argument to other cases (e.g. $C_A > C_B > C_S$) in Section 6.

11. A very similar analysis can be done with elastic demand.

12. Consider the Judd's example in the introduction. Suppose the incumbent chooses tea and coffee, and entry occurs in coffee. Then it is also a subgame-perfect Nash equilibrium after this history that the incumbent keeps both products and the entrant exits from coffee. We need further refinement to exclude this unrealistic equilibrium.

13. For example, myopic behavior of fringe firms (Peter Berck and Jeffrey M. Perloff (1988)), demand uncertainty (E. Appelbaum and C. Lim (1985)), and the possibility of antitrust action.

Figure 2-1: The product line of A.
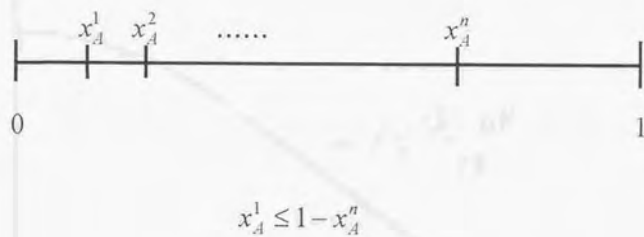
Figure 2-2: The sufficient conditions in which A allows entry of W.

$f \equiv \dfrac{F}{t}$

(4) $f \leq \dfrac{(3-\Delta)^2}{18}$

(3) $\Pi_S\left(0, x_S^*, 1\right) < F$

(1) $f \leq \dfrac{25}{144}$

(2) $f > \dfrac{(5-4\Delta)^2}{144}$

0

$\Delta \equiv \dfrac{C_W - C_S}{t}$
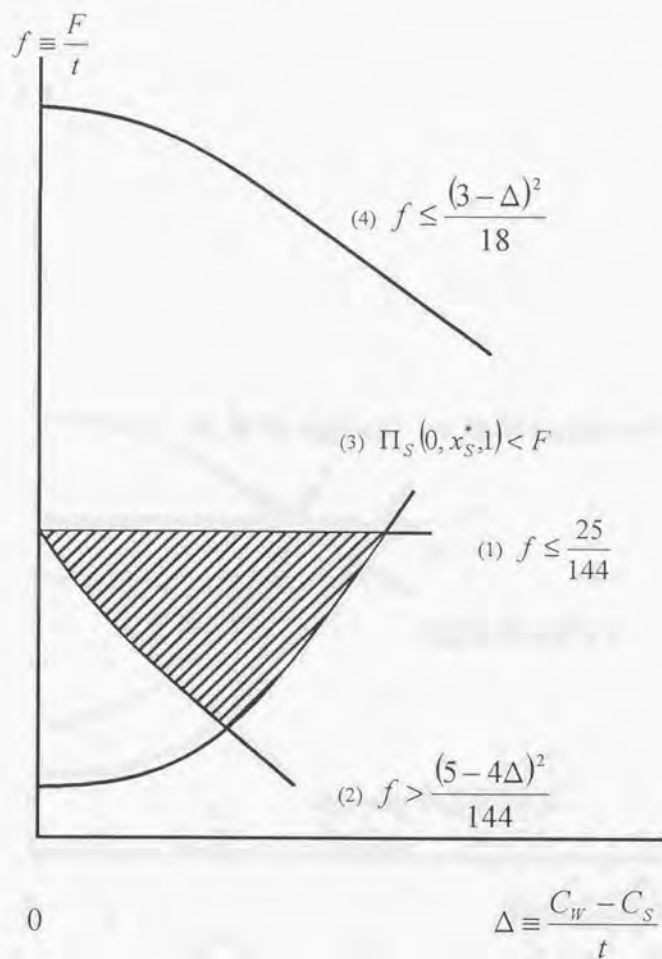
Figure 2-3: The necessary and sufficient conditions in which A allows entry of W

$$f \equiv \frac{F}{t}$$

$\Pi_S\left(0.34, x_S^*, 1\right) < F$ (5) $\Pi_S\left(0.5, x_S^*, 1\right) < F$

(1)

(7)

$\Pi_W\left(0.34, \phi, 1\right) \geq F$

(2) $\Pi_W\left(0.5, \phi, 1\right) \geq F$

0

$$\Delta \equiv \frac{C_W - C_S}{t}$$
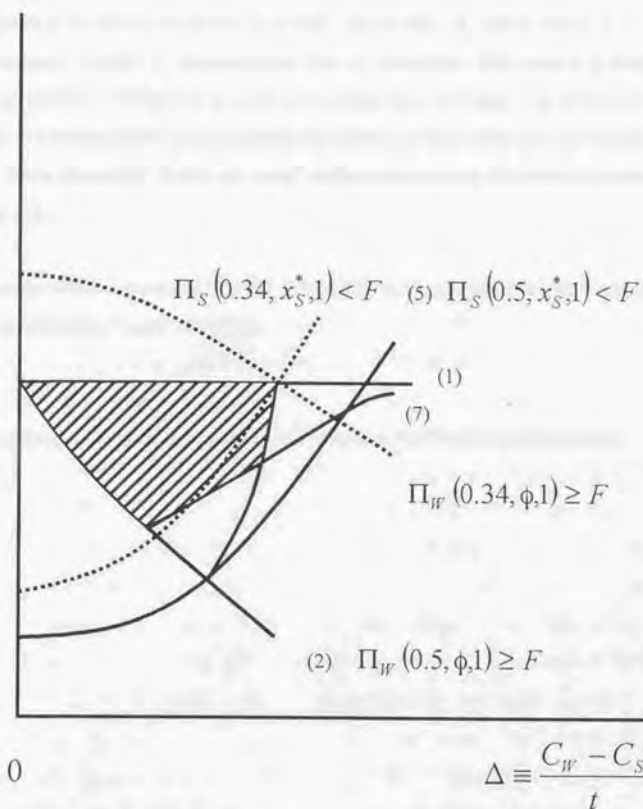
Chapter 3

Brand Proliferation Is Useless to Deter Entry*

Abstract

This paper considers an incumbent firm that is faced with a potential entrant in a vertically differentiated market. It demonstrates that an incumbent firm cannot prevent entry through product proliferation because of a commitment problem. The incumbent always makes one product only, and it degrades the quality to deter entry of a low-quality firm if entry is not blockaded. Hence the social welfare decreases as the entrant becomes more dangerous.

Keywords: Entry deterrence; Vertical differentiation; Brand proliferation; Commitment. JEL Classification Codes: D43, L13.

1. Introduction

This paper considers an incumbent firm that is faced with a potential entrant in a vertically differentiated market. It shows that to make two or more kinds of products and fill up the market is *not* an effective measure of entry deterrence. Suppose an incumbent firm makes some kinds of products and entry occurs near to one of them. Keeping all products enables the incumbent to discriminate among its customers, but it induces tough competition with the entrant. Withdrawal of the products near to the entrant, on the other hand, relaxes competition and increases the incumbent's profit from the remaining products. Thus the incumbent withdraws competing products and consequently entry occurs in equilibrium.

Ironically, choosing two or more kinds of products makes entry *easier*: it warrants an entrant a large market because the incumbent is forced to withdraw all products near to the entrant. Therefore, faced with an entrant, the incumbent always chooses one kind of product in equilibrium. [1] It selects the good of the highest quality when the fixed cost is large and entry is blockaded. It chooses a product of middle quality to prevent entry of a low-quality firm when the fixed cost decreases. Finally, when the fixed cost is too small to deter entry by one product, it produces the highest quality to secure its profit.

The above result indicates that the social welfare *increases* in the fixed cost when the incumbent deters entry. As the fixed cost decreases, the incumbent cuts down the quality of the product and each customer obtains less utility from consuming it. Therefore both of the incumbent's profit and consumer's surplus decreases. This argument demonstrates that a policy intervention that enhances competition is harmful to the society when no entry occurs afterwards.

There are a considerable number of studies about product-line selection in a differentiated market. We can classify them into three types. The studies of the first type assume that two firms choose their product varieties simultaneously. Gal-Or (1983) and Wernerfelt (1986) investigate the optimal product line when firms compete in quantity. Brander and Eaton (1984) analyze the optimal product choice when each firm chooses two varieties. Martinez-Giralt and Neven (1988) consider the horizontal differentiation model of d'Aspremont, Gabszewicz, and Thisse (1979) (Hotelling (1929)'s model with

quadratic transportation costs: we call it 'Horizontal model' hereafter) and show that firms choose only one product each even if they can select any number. Martinez-Giralt (1989) obtains the same result in the vertically differentiated Hotelling model with quadratic transportation costs. [2] Champsaur and Rochet (1989) and Cremer and Thisse (1991) prove that the vertical differentiation model of Mussa and Rosen (1978) is mathematically equivalent to Horizontal model if marginal cost is a quadratic function of the quality. Therefore both firms choose one product each in the model of Mussa and Rosen (1978). However, the studies of this type leave out of account that an incumbent firm can choose its product line in advance of an entrant.

The papers of the second type assume that firms move sequentially. Schmalensee (1978) argues that an incumbent firm can successfully prevent entry by producing enough varieties. Bonanno (1987) uses Horizontal model and shows that an incumbent firm can stop entry by changing location of its products. Constantatos and Perrakis (1997) obtains a similar result in the vertical differentiation model of Gabszewicz and Thisse (1979). Nevertheless, these papers have two shortcomings: they cannot explain why monopoly is rare in reality, and they do not check whether or not the incumbent's strategy is credible.

The papers of third type explicitly deal with the commitment problem of an incumbent firm after entry occurs. Judd (1985) shows with a two-goods model that, if an exit cost is small, the incumbent cannot stop entry by choosing both goods since it has an ex post incentive to withdraw the product entry occurs. Ashiya (forthcoming) proves that choosing two or more products does not help the incumbent deter entry in Horizontal model. This paper demonstrates robustness of their results: when the exit cost is small, brand proliferation is useless for an incumbent to prevent entry in a vertically differentiated market.

The paper is organized as follows. Section 2 describes the model. Section 3 shows that the incumbent withdraws products near to the entrant. Section 4 investigates the optimal product of the incumbent given the fixed cost. Section 5 analyzes the social welfare, and Section 6 considers the extended model where three firms move sequentially. Section 7 concludes this paper. The Appendix B contains the formal proofs.

2. The model

We use the vertical differentiation model of Gabszewicz and Thisse (1979): A denotes an incumbent and B denotes an entrant. [3] The timing of each firm's action, which is common knowledge, is as follows. At date 1, A chooses a set of products $Q_A = \{q_1, \cdots, q_n\}$ ($q_1 < \cdots < q_n$) from a technologically feasible range of qualities $(1, Q]$. We assume $Q = 6$ for tractability. [4] At date 2, B observes Q_A and chooses $Q_B = \{q_{B1}, q_{B2}, \cdots q_{Bm}\}$ ($q_{B1} < \cdots < q_{Bm}$). Each firm sinks a fixed cost F per product if it enters the market. If firm i does not enter the market, $Q_i = \phi$.

At date 3, each firm observes (Q_A, Q_B) and simultaneously selects a set of products to withdraw. Since a firm can get out of the market at will in reality, each firm can withdraw its product(s) with no additional cost (It cannot recover the fixed cost). [5] Let $\hat{Q}_i$ be the set of products firm i does not withdraw ($\hat{Q}_i \subseteq Q_i$). If $\hat{Q}_i = \{q_k\}$, we write $\hat{Q}_i = q_k$.

At date 4, each firm observes $(\hat{Q}_A, \hat{Q}_B)$, and simultaneously selects prices of its products. Each pays variable costs and earns sales revenue. Each firm makes goods at constant marginal cost, which is assumed to be zero regardless of the quality.

Consumers are identical in tastes but differing in income. Their incomes are uniformly distributed on the segment $[1, h]$. Shaked and Sutton (1982) show that only two firms of the highest and the second highest quality can earn positive gross profit if $2 < h < 4$. Only one firm of the highest quality can earn positive profit if $h < 2$. Thus we assume $h = 3$ (Consequently, consumers are distributed with density 0.5). Each consumer purchases one unit of the good for which her indirect utility is maximized, or buys nothing if it is better. The utility of a consumer of income $y \in [1, 3]$ who bought q_i at price P_i is

$$U(q_i; y) = q_i(y - P_i).$$

If she bought nothing, her utility is

$$U(0; y) = y. \quad [6]$$

The equilibrium concept we adopt is a weak refinement of subgame-perfect Nash

equilibrium that assumes no weakly dominated strategy is played in equilibrium [7] Let $\Pi_i(\hat{Q}_A, \hat{Q}_B)$ be the equilibrium profit of firm i gross of the fixed cost. Entry occurs when Π_i is larger than the fixed cost.

3. The equilibrium profits

Gabszewicz and Thisse (1979) calculate the equilibrium prices when A and B choose one product each. We derive B's optimal product using it. Lemma 3-1 shows that B will locate its product far apart from A to mitigate competition. (The proofs of all Lemmata and propositions can be found in the Appendix B).

Lemma 3-1.

(a) Suppose $q_1 < 6$. Then $\Pi_B(q_1, 6) > \Pi_B(q_1, q_B)$ for any $q_B \in [q_1, 6)$.

(b) Suppose $q_1 \leq 3$. Then $\Pi_B(q_1, 0.25(q_1 + 3)) \geq \Pi_B(q_1, q_B)$ for any $q_B \leq q_1$.

(c) Suppose $q_1 > 3$. Define

$$q_B^*(q_1) \equiv \frac{-1 - q_1 + (q_1 - 1)\sqrt{1 + q_1}}{q_1 - 3}.$$

Then $\Pi_B(q_1, q_B^*(q_1)) \geq \Pi_B(q_1, q_B)$ for any $q_B \leq q_1$

Lemma 3-2 considers the case that A chooses n (≥ 2) goods and B chooses higher quality than A. Since we assume consumers' incomes (i.e. willingness to pay for quality) are similar, everyone prefers 'an expensive but high-quality good' to 'a cheap but low-quality good'. Accordingly A's products have no sales in equilibrium except the highest quality. Then it is better for A to withdraw products near to B and relax competition.

Lemma 3-2.

Suppose A chooses n (≥ 2) goods ($q_1 < \cdots < q_n$) and B chooses $q_B > q_n$. Define

$$q_M \equiv \max\{q_i | \Pi_A(q_i, q_B) \geq \Pi_A(q_j, q_B) \quad \forall j\}.$$

Then A withdraws at least all products larger than q_M, and B earns $\Pi_B(q_M, q_B)$ or more in this subgame.

Lemma 3-3 considers the case that B chooses a low quality. If A keeps its all products, it can separate the market and operate discrimination. The gain from it is small, however, because consumers' tastes are similar in our model. Hence A withdraws the products near to B in order to avoid competition. Lemma 3-2 and Lemma 3-3 show that choosing two or more products is useless for entry deterrence.

Lemma 3-3.

(a1) Suppose $q_{n-1} \leq q_B < q_n$ and $q_B \leq 0.25(3q_{n-1} + q_n)$. Then A keeps only q_n and B earns $\Pi_B(q_n, q_B)$ in the unique equilibrium of this subgame.

(a2) Suppose $q_{n-1} \leq q_B < q_n$ and $q_B > 0.25(3q_{n-1} + q_n)$. Then B earns $\Pi_B(q_n, q_B)$ in any equilibrium of this subgame.

(b) Suppose $q_{k-1} \leq q_B < q_k < \cdots < q_n$ and $2 \leq q_B \leq 0.25(3 + q_n)$. Then A keeps only q_n and B earns $\Pi_B(q_n, q_B)$ in the unique equilibrium of this subgame.

4. The optimal product of the incumbent

The last section proved that an incumbent faces the commitment problem if it has two or more products. It cannot commit to keep them after entry occurs since withdrawal of competing products relaxes competition and raises profits from the remaining products.

This section shows that the incumbent never chooses more than one product in equilibrium. Figure 3-1 indicates the optimal product of the incumbent as a function of the fixed cost. When the fixed cost is large enough to blockade entry, A chooses the product of the highest quality (The proofs can be found in the Appendix B).

Proposition 3-1 (Blockaded entry).

Suppose $F \geq \Pi_B(6, q_B^*(6))$. Then A chooses $Q_A = 6$ and B does not enter the market in the unique equilibrium.

Proposition 3-2 investigates the case that A cannot deter entry by choosing the

highest quality. It shows that, in order to prevent entry of a low-quality firm, A degrades the quality of its product as the fixed cost decreases.

Lemma 3-4.

Define q_1^* such that

$$\Pi_B(q_1^*, 6) = \Pi_B(q_1^*, q_B^*(q_1^*)).$$

Then A can deter entry by choosing one product if and only if $F \geq \Pi_B(q_1^*, 6)$.

Proposition 3-2 (Deterred entry).

Suppose $\Pi_B(q_1^*, 6) \leq F < \Pi_B(6, q_B^*(6))$. Define $\bar{q}_1$ such that

$$\Pi_B(\bar{q}_1, q_B^*(\bar{q}_1)) = F.$$

Then A chooses $Q_A = \bar{q}_1$ and B does not enter the market in the unique equilibrium.

When the fixed cost is too small to deter entry, A chooses the highest quality to secure its profit.

Proposition 3-3 (Allowed entry).

Suppose $F < \Pi_B(q_1^*, 6)$. Then A chooses $Q_A = 6$ and B chooses $Q_B = q_B^*(6)$ in the unique equilibrium.

The combination of Proposition 3-1, 3-2, and 3-3 yields Theorem 3-1: the incumbent always chooses one product because it has an ex post incentive to withdraw all but one product after entry occurs. Calculation shows that A would choose two products if there were no entrant and $F < \Pi_A(\{\sqrt{6}, 6\}, \phi) - \Pi_A(6, \phi)$. Therefore, contrary to the argument of Schmalensee (1978), the incumbent *stops* proliferating its brand when there is an entrant.

Theorem 3-1.

A chooses one product in equilibrium regardless of F.

Corollary of Theorem 3-1.

The number of products A chooses when faced with an entrant is equal to or smaller than that in the absence of the entrant.

5. Welfare analysis

Let us define the social welfare, W, as the sum of consumer's surplus and each firm's net profit. This section investigates how the social welfare changes as the fixed cost decreases. See Figure 3-2.

When the fixed cost is so large that entry is blockaded, the incumbent firm always chooses the same quality and price. Thus the social welfare increases by the same amount as the decrease of the fixed cost.

When the fixed cost becomes small and entry is not blockaded, the incumbent degrades the quality as the fixed cost decreases. Consumer's surplus decreases since each buyer obtains lower utility and calculation shows that the market served by the incumbent does not change. Therefore, if the incumbent deters entry, the social welfare *decreases* as the fixed cost decreases (i.e. as the entry threat is strengthened).

Finally, when the fixed cost is so small that the incumbent cannot deter entry, each firm chooses the fixed product ($Q_A = 6$ and $Q_B = q_B^*(6)$). Consequently the social welfare increases by the same amount as the decrease of the fixed cost. The social welfare under duopoly is larger than that under monopoly because competition drives the prices down and more people buy the top quality good.

Proposition 3-4.

Define W as the sum of consumer's surplus and each firm's net profit. Then

(a) $\dfrac{dW}{dF} = -1$ if $F \geq \Pi_B\left(6, q_B^*(6)\right)$,

(b) $\dfrac{dW}{dF} > 0$ if $\Pi_B\left(q_1^*, 6\right) \leq F < \Pi_B\left(6, q_B^*(6)\right)$,

(c) $\dfrac{dW}{dF} = -2$ if $F < \Pi_B\left(q_1^*, 6\right)$,

(d) W under duopoly is always larger than that under monopoly.

Proposition 3-4 warns that a policy that lowers the entry barrier reduces the social welfare if it fails to establish the second firm in this market. Suppose the government deregulates the monopolized market and tries to enhance competition. If the new entrant is not strong enough, the incumbent deters it by cutting down the quality of its product. This causes a negative effect to the social welfare. Although the government can help the entrant, a small amount of subsidy makes matters worse. The subsidy (or deregulation) must be large enough so that the newcomer can enter the market successfully.

6. Extensions

This section extends the model and assumes that the third firm, C, moves after firm B. Then A and B change the qualities of their products as the fixed cost decreases. When the fixed cost is large, entry of C is blockaded, and A and B choose $Q_A = 6$ and $Q_B = q_B^*(6)$. Otherwise A continues to choose the top quality in order to charge a high price, and B is forced to upgrade its product in order to deter entry of C between them. Since we assume income dispersion is small, firm C cannot enter the market for any positive fixed cost [8]

Proposition 3-5.

Suppose the third firm, C, moves after firm B. Let $\Pi_i(q_A, q_B, q_C)$ be the profit function of firm i. Then

(a) C never enters the market in equilibrium.

(b) $(q_A, q_B) = (6, q_B^*(6))$ in the unique equilibrium if

$$\Pi_C(6, q_B^*(6), 0.6(4 + q_B^*(6))) \leq F < \Pi_B(q_1^*, 6, \phi).$$

(c) $(q_A, q_B) = (6, \overline{q}_B)$ where $\Pi_C(6, \overline{q}_B, 0.6(4 + \overline{q}_B)) = F$ in the unique equilibrium if

$$\Pi_C(6, 2.25, 3.75) \leq F < \Pi_C(6, q_B^*(6), 0.6(4 + q_B^*(6))).$$

(d) $(q_A, q_B) = (6, \overline{q}_B)$ is an equilibrium if $F < \Pi_C(6, 2.25, 3.75)$.

7. Conclusions

After entry occurs, it may be profitable for an incumbent firm to withdraw products near to the entrant and relax competition. We have explicitly dealt with this commitment problem, and have proved that choosing two or more kinds of products cannot deter entry in a vertically differentiated market. This result is quite robust since Ashiya (forthcoming) obtains the same result in the Hotelling's model with quadratic transportation costs.

We have also shown that the entry threat causes the incumbent firm to *avoid* brand proliferation: the incumbent always chooses one good in equilibrium. When the incumbent deters entry, it degrades the quality as the entrant becomes dangerous. Hence a competition-enhancing policy such as deregulation or subsidy to the entrant must be comprehensive enough for the newcomer to enter the market successfully.

Notes

1. If there were no entrant and the fixed cost were small, the incumbent would choose two or more products to screen its customers.

2. It assumes that consumers are located on $[0,1]$ and products are located on $[1,\infty)$.

3. Section 6 considers the model with three firms.

4. Other studies assume narrower ranges than ours. For example, Constantatos and Perrakis (1997) consider the cases of $Q \in [1,3,5]$. Our argument can be easily extended when Q takes other values.

5. An exit cost is the cost arising only because of the act of exit. One example is a printing cost of a new catalogue (from which withdrawn products are deleted). Note that irreversible investment in product-specific capital is a sunk cost and is not an exit cost.

6. This model differs from Horizontal model in that all consumers choose the product of the highest quality when prices of all products are equal to their marginal costs.

7. Consider the subgame where both of the incumbent and the entrant enter the market. If an exit cost is zero, it is also a subgame-perfect Nash equilibrium after this history that the entrant exits and the incumbent keeps all products at date 3. We need further refinement to exclude this rather unrealistic equilibrium.

8. If consumers' incomes are uniformly distributed on $[1,h]$ and $h > 4$, firm C is viable and $q_C < q_B < q_A = 6$ in equilibrium for a sufficiently small fixed cost.

Figure 3-1: The optimal product of the incumbent
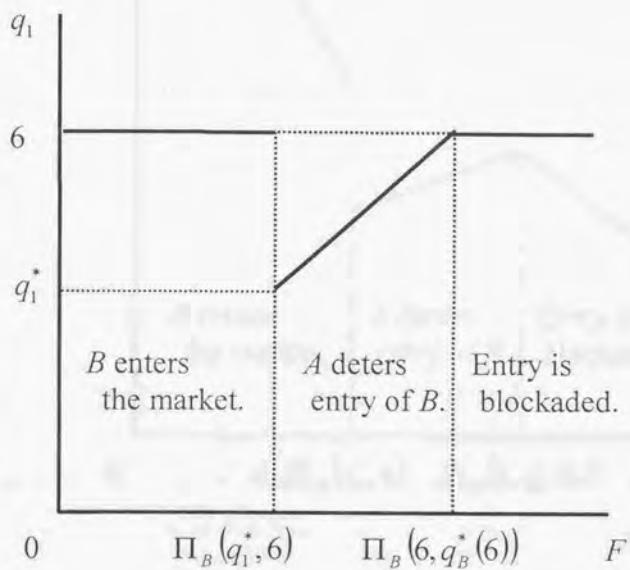
Figure 3-2: The social welfare

B enters
the market.

A deters
entry of B.

Entry is
blockaded.

$$0 \qquad \Pi_B\left(q_1^*,6\right) \quad \Pi_B\left(6,q_B^*(6)\right) \qquad F$$
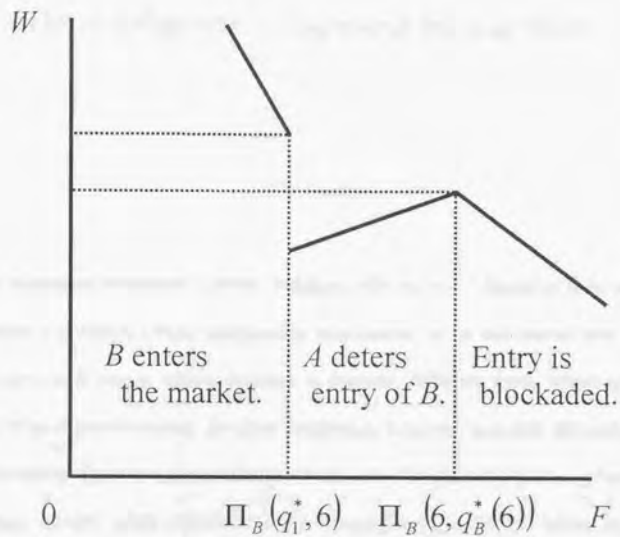
Chapter4

Herd Behavior of Japanese Economists

Abstract

Suppose competent economists obtain common information on business forecasts, and incompetent economists obtain independent information. If no one knows who is able, young economists mimic others because a forecast different from others indicates inability when it proves wrong. An older economist, however, can infer his ability from past information. Therefore those who got useful information in the past stop herding to signal their ability when economists are heterogeneous. On the other hand, all economists herd together when economists are homogeneous, because the merit from signaling is small. The empirical result suggests that Japanese economists are more homogeneous than American.

1. Introduction

People point out 'herd behavior' as a notable feature of Japanese workers. They say that Japanese workers lack originality and always mimic colleagues or workers of other firms. This paper analyzes whether Japanese workers really herd together using macroeconomic forecast data of Japanese economists. It is the first empirical study on herd behavior in Japan.

Scharfstein and Stein (1990) suggest that an economist may herd to keep his reputation.[1] Suppose economists with high ability obtain common information on business forecasts, and economists with low ability obtain independent information.[2] If no one knows who is able, economists are evaluated by their forecasts. Then they have an incentive to imitate each other since a forecast different from others indicates inability when it proves wrong.

This argument fits well with a young economist since he has no way to prove his ability. An old economist, however, may stop herding when forecasts are made repeatedly. Since an economist gradually understands the value of his information, he has an incentive to make forecasts based on his information only if it was valuable in the past. Therefore an older economist can signal his ability by following his own information.

When economists are heterogeneous, the benefit from signaling is large. Accordingly, there exists a separating equilibrium in which only those who got useful information in the past stop herding. Then forecasts tend to be more dispersed as economists grow older.[3] When economists are homogeneous, on the other hand, old economists continue herding since the merit from signaling is small.

The latter half of this paper investigates the relation between economists' age and

degree of herding. It uses the forecast data of Japanese economists on Japanese real growth rate in "Monthly Statistics (Tokei Geppo)". The dependent variable is the degree of herding, which is defined as the distance between a forecast of an economist and the forecast average excluding him. The independent variable is 'age' of the economist (the years while he starts forecasting). Then the coefficient of 'age' is found to be small and statistically insignificant. It suggests that Japanese economists continue herding as they grow older.

Lamont (1995) makes the same regression using American data, and obtains the result that older economists stop herding.[4] Compared with the implication of our model, these results suggest that Japanese economists are more homogeneous than American.

This paper is organized as follows. Section 2 provides the model in which two economists forecast business fluctuation across two periods. Section 3 analyzes the equilibrium. Section 4 explains data and reports the estimated result. Section 5 concludes this paper. All proofs of propositions are summarized in the Appendix C.

2. The Model

We modify the model of Scharfstein and Stein (1990). The structure of this model is common knowledge to all players. There are two risk-neutral economists, A and B, who predict the trend of the economy through two periods. In each period, they collect information independently and report whether the economy will have an upward tendency (u) or it will take a downward turn (d). The prior probability of boom is assumed to be 0.5 in each period, and there is no correlation between business conditions of period 1 and 2. Each of A and B may or may not be a good forecaster, but no one (including himself) knows who is a good forecaster. The market revises the

evaluation of each economist based on fitness of his forecast, and each economist tries to obtain a higher evaluation.

At the beginning of period 1, the ability of A and B are independently determined. The prior probability that economist i is able is θ (the probability that he is not able is $1 - \theta$). No one knows who is able.

At date 1 in period 1, A and B receives a signal $s_1^i \in S_1 \equiv \{u_1, d_1\}$ $(i = A, B)$. No one except i can observe s_1^i. If economist i is able and receives u_1 (d_1), a boom occurs with the probability p $(1-p)$ and a recession occurs with the probability $1-p$ (p). If economist i is not able, a boom occurs with probability 0.5 regardless of his signal. If both are able, they always receive the same signal. Otherwise s_1^A and s_1^B are independent. The conclusion does not change if we suppose competent economists receive imperfectly correlated signals. [5]

At date 2 in period 1, A reports his forecast $R_1^A = r_1^A(s_1^A) \in S_1$. At date 3, B observes R_1^A and reports $R_1^B = r_1^B(R_1^A, s_1^B) \in S_1$. [6] Let $r_1 \equiv (r_1^A, r_1^B)$. Define I_t^i as

$$I_t^i(s_t^i) = s_t^i \text{ for any } s_t^i$$

and let $I_t \equiv (I_t^A, I_t^B)$. I_t^i is the strategy to follow its own information. Also define m_t^B, mimicking strategy, as

$$m_t^B(R_t^A) = R_t^A \text{ for any } R_t^A$$

At the last day of period 1, the true state of the economy, $o_1 \in S_1$, is revealed. Define $h_1 \equiv (R_1^A, R_1^B, o_1)$. Let $\theta_1^i = \theta_1^i(s_1^A, s_1^B, o_1)$ be the *objective* probability at the last day of period 1 that economist i is able. Let $\hat{\theta}_1^i$ be the *subjective* evaluation of the market at the last day of period 1 that economist i is able. Then, since the forecast of

49

other economist is useful in evaluation,

$$\hat{\theta}_1^i = \hat{\theta}_1^i(h_1, r_1) \ \text{if} \ \Pr(h_1|r_1) > 0,$$

$$= \hat{\theta}_1^i(h_1, I_1) \ \text{if} \ \Pr(R_1^A|r_1^A) = 0, \text{and}$$

$$= \hat{\theta}_1^i(h_1, (r_1^A, I_1^B)) \ \text{if} \ \Pr(R_1^A|r_1^A) > 0 \ \text{and} \ \Pr(h_1|r_1) = 0.$$

Each economist obtains utility from higher evaluation. For simplicity, the utility of economist i in period 1, V_1^i, is assumed to be

$$V_1^i = \hat{\theta}_1^i. \ ^7$$

At date 1 in period 2, each economist receives a signal $s_2^i \in S_2 = \{u_2, d_2\}$. At date 2, A reports $R_2^A = r_2^A(h_1, s_1^A, s_2^A) \in S_2$. At date 3, B observes R_2^A and reports $R_2^B = r_2^B(h_1, R_2^A, s_1^B, s_2^B) \in S_2$. We make the same assumptions about the signals as that in period 1. Define $r_2 \equiv (r_2^A, r_2^B)$. Then the updated evaluation of the market at date 3, $\hat{\theta}_{12}^i$, is

$$\hat{\theta}_{12}^A = \hat{\theta}_{12}^A(h_1, R_2^A, r_1, r_2^A) \ \text{if} \ \Pr(h_1|r_1)\Pr(R_2^A|r_2^A) > 0,$$

$$= \hat{\theta}_{12}^A(h_1, R_2^A, r_1^A, I_1^B, r_2^A) \ \text{if} \ \Pr(R_1^A|r_1^A)\Pr(R_2^A|r_2^A) > 0 \ \text{and} \ \Pr(h_1|r_1) = 0,$$

$$= \hat{\theta}_1^A \ \text{otherwise};$$

$$\hat{\theta}_{12}^B = \hat{\theta}_{12}^B(h_1, R_2^A, R_2^B, r_1, r_2) \ \text{if} \ \Pr(h_1|r_1)\Pr(R_2^A, R_2^B|r_2) > 0,$$

$$= \hat{\theta}_1^B \ \text{otherwise}.$$

At the last day of period 2, the true state of the economy, $o_2 \in S_2$, is revealed. Define $h_2 \equiv (h_1, R_2^A, R_2^B, o_2)$. Then the market evaluation of i becomes

$$\hat{\theta}_2^i = \hat{\theta}_2^i(h_2, r_1, r_2) \ \text{if} \ \Pr(h_1|r_1)\Pr(h_2|r_2) > 0,$$

$$= \hat{\theta}_2^i(h_2, I_1, I_2) \text{ if } \Pr\left(R_1^A \middle| r_1^A\right) = 0,$$

$$= \hat{\theta}_2^i(h_2, r_1, I_2) \text{ if } \Pr(h_1 | r_1) > 0 \text{ and } \Pr\left(R_2^A \middle| r_2^A\right) = 0,$$

$$= \hat{\theta}_2^i\left(h_2, \left(r_1^A, I_1^B\right), \left(r_2^A, m_2^B\right)\right) \text{ if } \Pr\left(R_1^A \middle| r_1^A\right)\Pr\left(R_2^A \middle| r_2^A\right) > 0, \; \Pr(h_1 | r_1) = 0, \text{ and } R_2^A = R_2^B,$$

$$= \hat{\theta}_2^i\left(h_2, \left(r_1^A, I_1^B\right), \left(r_2^A, I_2^B\right)\right) \text{ if } \Pr\left(R_1^A \middle| r_1^A\right)\Pr\left(R_2^A \middle| r_2^A\right) > 0, \; \Pr(h_1 | r_1) = 0, \text{ and } R_2^A \neq R_2^B,$$

$$= \hat{\theta}_2^i\left(h_2, \left(r_1^A, I_1^B\right), I_2\right) \text{ if } \Pr\left(R_1^A \middle| r_1^A\right) > 0, \; \Pr(h_1 | r_1) = 0, \text{ and } \Pr\left(R_2^A \middle| r_2^A\right) = 0,$$

$$= \hat{\theta}_2^i\left(h_2, r_1, \left(r_2^A, m_2^B\right)\right) \text{ if } \Pr(h_1 | r_1)\Pr\left(R_2^A \middle| r_2^A\right) > 0, \; \Pr(h_2 | r_2) = 0, \text{ and } R_2^A = R_2^B,$$

$$= \hat{\theta}_2^i\left(h_2, r_1, \left(r_2^A, I_2^B\right)\right) \text{ if } \Pr(h_1 | r_1)\Pr\left(R_2^A \middle| r_2^A\right) > 0, \; \Pr(h_2 | r_2) = 0, \text{ and } R_2^A \neq R_2^B$$

from sequential rationality. The utility of economist i in period 2 is $V_2^i = \hat{\theta}_2^i$. Let

$$\theta_2^i = \theta_2^i\left(s_1^A, s_1^B, o_1, s_2^A, s_2^B, o_2\right)$$

$$= \theta_2^i\left(s_2^A, s_2^B, o_2 \middle| \theta_1^A, \theta_1^B\right)$$

be the objective probability at the last day of period 2 that economist i is able.

The discount rate is supposed to be unity for simplicity. Since each economist is risk-neutral, he maximizes the expectation of $\Pi^i \equiv \hat{\theta}_1^i + \hat{\theta}_2^i$. He maximizes the probability to be correct in case of a tie. [8] We adopt a pure strategy sequential equilibrium as the equilibrium concept.

3. The equilibrium

There are two equilibria, a pooling equilibrium and a separating one, in this game. In either equilibrium, economist A reports what his information suggests in both periods, and economist B mimics A in period 1. The only difference between the two equilibria is B's strategy in period 2; B always mimics A in the pooling equilibrium; B relies on his

information if and only if it was correct in period 1 in the separating equilibrium. There exists a pooling equilibrium when economists are homogeneous (i.e. p is small or θ is large). There exists a separating equilibrium when p and θ take middle values.

3-1. The objective probability to be able

The following argument extends that of Scharfstein and Stein (1990). Let us define $\Pr\left(s_t^A, s_t^B \mid o_t\right)$ as the conditional probability that the economists obtain $\left(s_t^A, s_t^B\right)$ when o_t is the true state. Suppose the objective probability that economist i is able before he collects information is θ_i. Then

$$\Pr\left(u_t, u_t \mid u_t\right) = 0.25\left(1-\theta_A\right)\left(1-\theta_B\right) + 0.5p\theta_A\left(1-\theta_B\right) + 0.5p\theta_B\left(1-\theta_A\right) + p\theta_A\theta_B$$

$$= 0.25 + 0.5\left(\theta_A + \theta_B\right)\left(p - 0.5\right) + 0.25\theta_A\theta_B = \Pr\left(d_t, d_t \mid d_t\right),$$

$$\Pr\left(d_t, d_t \mid u_t\right) = \Pr\left(u_t, u_t \mid d_t\right) = 0.25 - 0.5\left(\theta_A + \theta_B\right)\left(p - 0.5\right) + 0.25\theta_A\theta_B,$$

$$\Pr\left(u_t, d_t \mid u_t\right) = \Pr\left(d_t, u_t \mid d_t\right)$$

$$= 0.25\left(1-\theta_A\right)\left(1-\theta_B\right) + 0.5p\theta_A\left(1-\theta_B\right) + 0.5\theta_B\left(1-p\right)\left(1-\theta_A\right)$$

$$= 0.25 + 0.5\left(\theta_A - \theta_B\right)\left(p - 0.5\right) - 0.25\theta_A\theta_B, \text{ and}$$

$$\Pr\left(d_t, u_t \mid u_t\right) = \Pr\left(u_t, d_t \mid d_t\right) = 0.25 - 0.5\left(\theta_A - \theta_B\right)\left(p - 0.5\right) - 0.25\theta_A\theta_B$$

(Note that $s_t^A = s_t^B$ if both are able). The conditional probability that o_t is realized, $\Pr\left(o_t \mid s_t^A, s_t^B\right)$, is

$$\Pr\left(u_t \mid u_t, u_t\right) = \frac{0.5 + \left(\theta_A + \theta_B\right)\left(p - 0.5\right) + 0.5\theta_A\theta_B}{1 + \theta_A\theta_B} = \Pr\left(d_t \mid d_t, d_t\right),$$

$$\Pr\left(d_t \mid u_t, u_t\right) = \Pr\left(u_t \mid d_t, d_t\right) = \frac{0.5 - \left(\theta_A + \theta_B\right)\left(p - 0.5\right) + 0.5\theta_A\theta_B}{1 + \theta_A\theta_B},$$

$$\Pr(u_t|u_t,d_t) = \Pr(d_i|d_i,u_t) = \frac{0.5 + (\theta_A - \theta_B)(p - 0.5) - 0.5\theta_A\theta_B}{1 - \theta_A\theta_B}, \text{ and}$$

$$\Pr(u_t|d_t,u_t) = \Pr(d_i|u_t,d_t) = \frac{0.5 - (\theta_A - \theta_B)(p - 0.5) - 0.5\theta_A\theta_B}{1 - \theta_A\theta_B}.$$

Therefore the objective probability in the end of period t that economist i is able is

$$\theta_t^i(s_t^A, s_t^B, o_t) = \frac{2p\theta_i(1 + \theta_j)}{1 + (\theta_i + \theta_j)(2p - 1) + \theta_i\theta_j} \quad \text{if } s_t^i = s_t^j = o_t,$$

$$= \frac{2\theta_i(1 + \theta_j)(1 - p)}{1 - (\theta_i + \theta_j)(2p - 1) + \theta_i\theta_j} \quad \text{if } s_t^i = s_t^j \neq o_t,$$

$$= \frac{2p\theta_i(1 - \theta_j)}{1 + (\theta_i - \theta_j)(2p - 1) - \theta_i\theta_j} \quad \text{if } s_t^i = o_t \neq s_t^j, \text{ and}$$

$$= \frac{2\theta_i(1 - \theta_j)(1 - p)}{1 - (\theta_i - \theta_j)(2p - 1) - \theta_i\theta_j} \quad \text{if } s_t^i \neq s_t^j = o_t.$$

θ_1^i (θ_2^i) is obtained by substituting θ (θ_1^i and θ_1^j) for θ_i and θ_j. Two remarks are in order. First, at least one of them is incompetent when $s_t^A \neq s_t^B$. Hence

$$\theta_t^i(s_t^i = s_t^j = o_t) - \theta_t^i(s_t^i = o_t \neq s_t^j)$$

$$= \frac{8\theta_i\theta_jp(1 - p)(1 - \theta_i)}{[1 + (\theta_i + \theta_j)(2p - 1) + \theta_i\theta_j][1 + (\theta_i - \theta_j)(2p - 1) - \theta_i\theta_j]} > 0$$

and similarly $\theta_t^i(s_t^i = s_t^j \neq o_t) > \theta_t^i(s_t^i \neq o_t = s_t^j)$.

Secondly, an incorrect forecast implies incompetency. Hence

$$\theta_t^i(s_t^i = s_t^j = o_t) > \theta_t^i(s_t^i = s_t^j \neq o_t) \text{ and}$$

$$\theta_t^i(s_t^i = o_t \neq s_t^j) > \theta_t^i(s_t^i \neq s_t^j = o_t).$$

3-2. B's equilibrium strategy

This subsection assumes that A always reports what he obtains, namely $r_t^A = I_t^A$ for any $t = 1, 2$. The last subsection shows that, given accuracy of the forecast, an economist is favorably evaluated when the other economist makes the same forecast as he. Since accuracy of s_1^A and s_1^B are the same for B in period 1, he does not have an incentive to make a different forecast from A when $s_1^B \neq R_1^A$. Lemma 1 shows that B always mimics A in period 1 in equilibrium (See the Appendix C for the proof).

Lemma 4-1.

Suppose $r_t^A = I_t^A$ for any $t = 1, 2$. Then B chooses $r_1^B = m_1^B$ in equilibrium.

Next we consider the subgame after $R_1^B = R_1^A$. Proposition 4-1 shows that, if p is small or θ is large, there exists a pooling equilibrium in which B always mimics A in period 2.

Proposition 4-1

Suppose $r_t^A = I_t^A$ for $t = 1, 2$ and $R_1^B = R_1^A$. Suppose also that both of

$$\theta \geq \frac{1}{3} \text{ or } p \leq \frac{(1-\theta)^2}{2(1 - 2\theta - \theta^2)} \qquad (C1)$$

and $p \leq \dfrac{-1 + \theta - 2\theta^2 + \sqrt{-8\theta^3 + 9\theta^2 + 2\theta + 1}}{4\theta(1 - \theta)} \qquad (C2)$

are satisfied. Then $r_2^B = m_2^B$ is an equilibrium of this subgame.

Figure 4-1 shows these conditions graphically. When θ is large, the probability that R_2^A proves right is high. When p is small, s_2^B is not accurate enough even if $s_1^B = o_1$. Therefore the merit to follow s_2^B is small when θ is large or p is small.

Next we consider whether there is an equilibrium in which B makes his forecast based on his own information. Lemma 4-2 shows that there is no equilibrium in which B always reports what his information suggests in period 2. The reason is B has an incentive to mimic A in period 2 when his information proved wrong in period 1.

Lemma 4-2.

Suppose $r_t^A = I_t^A$ for any t. Then $r_2^B = I_2^B$ cannot be part of an equilibrium.

Because of the same reason as Lemma 4-1, $r_2^B = u_2$ and the reverse of $r_2^B = I_2^B$ cannot be part of an equilibrium. Hence only two behavioral strategies of B in period 2 are candidates for an equilibrium strategy. One strategy corresponds to Proposition 4-1, in which B continues mimicking A in period 2. In the other strategy, B follows his information in period 2 if and only if it was correct in period 1. Proposition 4-2 shows that this strategy is an equilibrium of the subgame after $R_1^B = R_1^A$ for some (p, θ).

Proposition 4-2.

Suppose $r_t^A = I_t^A$ for any t and $R_1^B = R_1^A$ in period 1. Then there exists a (p, θ) pair under which $r_2^B = \begin{cases} I_2^B & \text{if } s_1^B = o_1 \\ m_2^B & \text{otherwise} \end{cases}$ is an equilibrium of this subgame.

Figure 4-2 shows the set of (p, θ) under which the above equilibrium exists. Since B has only two options (reporting the same forecast as A or reporting the opposite of A), this separating equilibrium does not exist if his incentive to follow s_2^B is too small or too large. This is the reason why the set of (p, θ) under which the separating equilibrium exists is very small. [9] Let us investigate how (p, θ) affects B's incentive.

Suppose B mimics A in both periods. Then the market cannot update B's evaluation from θ (the ex ante probability that B is able). If θ is large, B does not dare to stop herding in period 2 because he can earn enough utility by mimicking A and the probability that R_2^A proves right is high. If θ is small, B's utility is low as long as he mimics A. Hence he makes a forecast different from R_2^A in period 2 even if $s_1^B \neq o_1$. Therefore θ must take middle values for the separating equilibrium to exist.

If p is small, s_2^B is not so accurate even if $s_1^B = o_1$. Thus B does not have an incentive to follow it. If p is large, on the other hand, B follows s_2^B even if $s_1^B \neq o_1$ because the merit of being regarded as able is large. Consequently p must also take middle values.

3-3. Two types of equilibria

The argument in the last subsection assumes that A always follows his information. This section considers A's incentive and shows that he always follows his own information in equilibrium. Hence there are two equilibria in this game. Proposition 4-3 shows the pooling equilibrium in which B mimics A in both periods. Proposition 4-4 shows the separating equilibrium in which B follows his information if and only if it proved right in period 1 (Lemma 4-1 and Lemma 4-2 shows that there is no other equilibrium). We use Lemma 4-3 to prove them.

Lemma 4-3.

Suppose $r_t^A = I_t^A$ for any t and $R_1^B \neq R_1^A$. Then $r_2^B = m_2^B$ is the unique equilibrium of this subgame.

Proposition 4-3.

Suppose (p, θ) satisfies (1) and (2). Then the following strategy is an equilibrium;

A: $r_t^A = I_t^A$ for any $t = 1, 2$;

B: $r_t^B = m_t^B$ for any $t = 1, 2$.

Proposition 4-4.

There exists a (p, θ) pair under which the following is an equilibrium;

A: $r_t^A = I_t^A$ for any $t = 1, 2$;

B: $r_1^B = m_1^A$ and $r_2^B = \begin{cases} I_2^B & \text{if } s_1^B = o_1 \\ m_2^B & \text{otherwise} \end{cases}$

There exists the separating equilibrium identified in Proposition 4-4 if (p, θ) lies in the shaded area of Figure 4-2. These propositions demonstrate that there are two equilibria in this game depending on the value of p (the probability that an able economist obtains correct information) and θ (ex ante probability that an economist is able). B makes the same forecast as A in period 1 in either equilibrium. The reason is that the accuracy of R_1^A and s_1^B is the same for B in period 1 when $s_1^B \neq R_1^A$ and that he obtains higher utility when both economists make the same forecast given the fitness

57

of his forecast. Proposition 4-3 shows that B always makes the same forecast as A in period 2 when economists are homogeneous, i.e. when p is small or θ is large. Proposition 4-4 shows that B follows his information in period 2 if and only if $s_1^B = o_1$ when p and θ take moderate values. Theorem 4-1 summarizes these results.

Theorem 4-1.

B continues herding in both periods if the economists are homogeneous. B stops herding in period 2 if $s_1^B = o_1$ and the economists are heterogeneous.

4. Data and results

Toyo Keizai Inc. publishes forecasts of about 70 Japanese economists in the January or February issue of "Monthly Statistics (Tokei Geppo)" every year from 1987. We use the forecasts of the Japanese real GDP growth rate for the next fiscal year from 1987 to 1998. [10] Since we exclude all economists who participate in less than five surveys, the sample contains 69 economists. Total number of forecasts is 623, and the average number of observations per economist is 9.03 (Table 4-1 shows summary statistics).

Let f_i^t be the forecast of individual i in year t, and $\bar{f}_{-i}^t$ be the forecast average excluding individual i (Figure 4-3 shows the distribution of f_i^t). Then $y_i^t \equiv \left| f_i^t - \bar{f}_{-i}^t \right|$, the forecast deviation, indicates the degree of i's herding in year t. [11] For instance, smaller y_i^t implies that individual i makes a forecast similar to others in year t. We use y_i^t as the dependent variable of our regression. Its average is 0.4601% points, and its standard deviation is 0.4235% points (Figure 4-4 shows its distribution).

Let us define age_i^t as the years at t while individual t participates in the survey.

We use age_i^t as an independent variable to investigate the effect of aging on herd behavior. We add $\bar{y}_{-i}^t$, the average of y_i^t excluding economist i, as an independent variable to eliminate specific factors in each year. Since larger $\bar{y}_{-i}^t$ means that forecasts of other economists are more dispersed in year t, we expect that the coefficient of $\bar{y}_{-i}^t$ is positive. [12] We also add individual dummies d_i to eliminate individual factors.

The regression is as follows; [13]

$$y_i^t = \beta_a age_i^t + \beta_y \bar{y}_{-i}^t + \sum_j \beta_j d_j ,$$ (E1)

The positive coefficient β_a indicates that economists stop herding as they grow older. Insignificant β_a, on the other hand, indicates that old economists do not change their behavior. According to the analysis of Section 3, β_a is positive if the probability that an able economist obtains accurate information is high or the share of able economists is small. β_a is insignificant if economists are homogeneous.

The result of the regression (E1) is summarized in Table 4-2 (the coefficients of individual dummies are not reported). The parentheses indicate the t-values using the consistent covariance of White (1980). The left and the middle columns in Table 4-2 show our estimates of the fixed effect model (E1) and the random effect model. Since we obtain almost the same estimates in both models, we concentrate on the fixed effect model. The coefficient of age_i^t, β_a, is 0.00658. It suggests that aging ten years widens the distance between his forecast and the market's average by 0.0658%pts. The coefficient is much smaller than the average of y_i^t (0.46%pts), and its t-value is not significant. This demonstrates that Japanese economists continue herding as they become older.

The right column in Table 4-2 shows the estimation of (E1) in Lamont (1995). It uses the data of American economists in "Business Week" from 1971 to 1992. The coefficient of age_i^t (0.018) is about three times that of Japanese, and t-value is significant. Namely, an old economist in America makes a more independent forecast than that of a young economist. Table 4-2 indicates that Japanese economists are more homogeneous than American.

Next we change the independent variable from age_i^t to $time_i^t$, which is the cumulative number of forecasts economist i reports up to year t. Table 4-3 shows the result, which is almost the same as Table 4-2. We obtain the same result when we use $y_i^t - \bar{y}_{-i}^t$ as the dependent variable and when we use logarithm of y_i^t and $\bar{y}_{-i}^t$. We also run the regression using the forecast data on the ongoing fiscal years, but the result does not change (Table 4-4).

5. Concluding remarks

Suppose economists collect information and report forecasts independently. No one knows the ability of each economist, but competent economists obtain common information. Then an economist has an incentive to mimic others because competent economists report the same forecast based on common information. This incentive leads a young economist to herd since he knows nothing about his ability.

An old economist, however, obtains private information about his ability by making forecasts repeatedly. This causes him to signal his ability by following his own information when the merit from it is large, namely when there are few competent economists or when the difference of forecasting ability between the competent

economist and the incompetent one is large. On the other hand, an old economist continues herding when the merit from signaling is small, namely when economists are homogeneous.

We analyze Japanese data based on this argument, and obtain the result that Japanese economists continue mimicking others as they grow older. This result presents a striking contrasts to the American result, in which an old economist stops herding. Our argument suggests that Japanese economists are more homogeneous than American economists.

Notes

1. Trueman (1994), Zwiebel (1995), and Prendergast and Stole (1996) also point out this possibility. Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) argue that herding occurs as a result when people use the information obtained from the decisions of others.

2. This assumption implies that competent economists pay attention to similar data such as unemployment rate, but that incompetent economists uses uninformative data such as N. Y. Yankees' percentage of victories.

3. No pure strategy equilibrium exists if the degree of heterogeneity of economists is too large. The reason is those who got uninformative data in the past have an incentive to conduct themselves as if they got informative data.

4. Laster, Bennett, and Geoum (1999) analyze the relation between forecasts and the market evaluation. They obtain the result that the economists whose wages are closely tied to publicity produce independent forecasts. Kraus and Stoll (1972), Lakonishok, Schleifer, and Vishny (1992), and Ehrbeck and Waldman (1996) empirically study herd behavior in financial markets. Zarnowitz and Braun (1992) examine accuracy of forecasts.

5. According to Scharfstein and Stein (1990), this amounts to assume that "there are systematically unpredictable factors affecting the future state that nobody can know anything about" (p.468). See also note 2.

6. Chamley and Gale (1994), Gul and Lundholm (1995), and Zhang (1997) endogenize the order of players' action.

7. Suppose that an employer gains $F + B$ if the forecast of his employee is correct and that he gains F if it is incorrect. Then an economist whose evaluation is $\hat{\theta}$

earns $W(\hat{\theta}) = F + B(0.5 + \hat{\theta}(p - 0.5))$ on condition that the labor market is competitive. Since an economist is risk neutral, his utility is a linear function of wage. Namely, the utility is a linear function of $\hat{\theta}$.

8. Without this assumption, there are many perverse equilibria. For example, the reverse of an equilibrium strategy is also an equilibrium strategy.

9. If B's strategy space is richer than that in our model, he can make a sufficiently different forecast from A's one when $s_1^B = o_1$. Thus there exists a separating equilibrium in a broader range of parameters.

10. "Monthly Statistics" also contains forecasts for the ongoing fiscal year from 1987 to 1998. Using this data does not change the result (See Table 4-4).

11. We regard the mean forecast as R_t^A since an economist refers to the market consensus for making his forecast.

12. We cannot use year dummies because age_t^i increases every year by one.

13. This is a fixed effect model. We obtain almost the same estimates in a random effect model (See Table 4-2 and Table 4-3).

Figure 4-1: The conditions in which B mimics A in period 2.
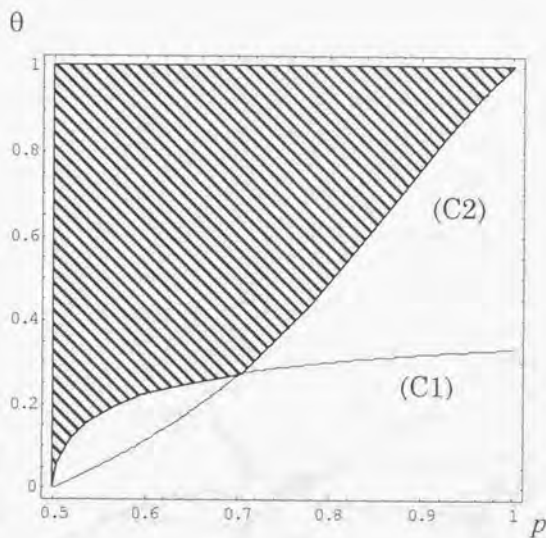
Figure 4-2: The conditions that B follows his own information in period 2 if and only if it was correct in period 1.
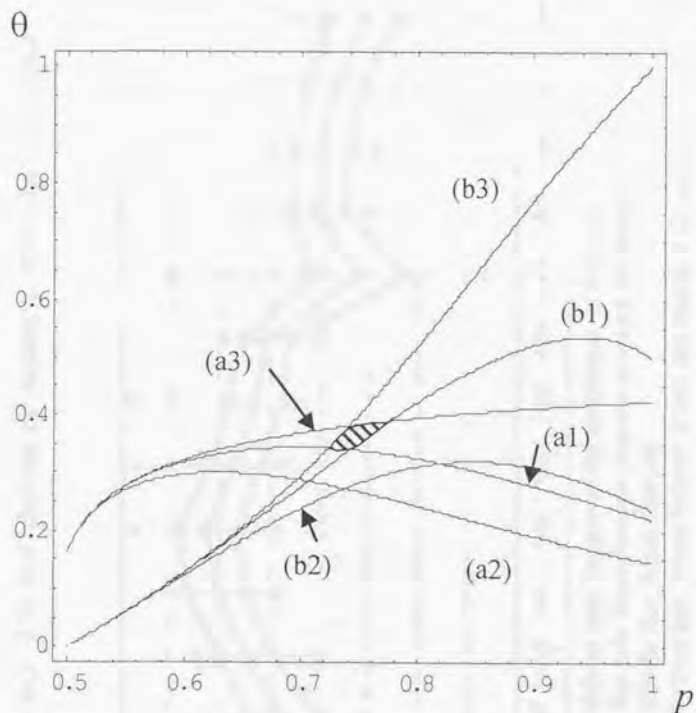
Figure 4-3: The distribution of Japanese real GDP forecasts.
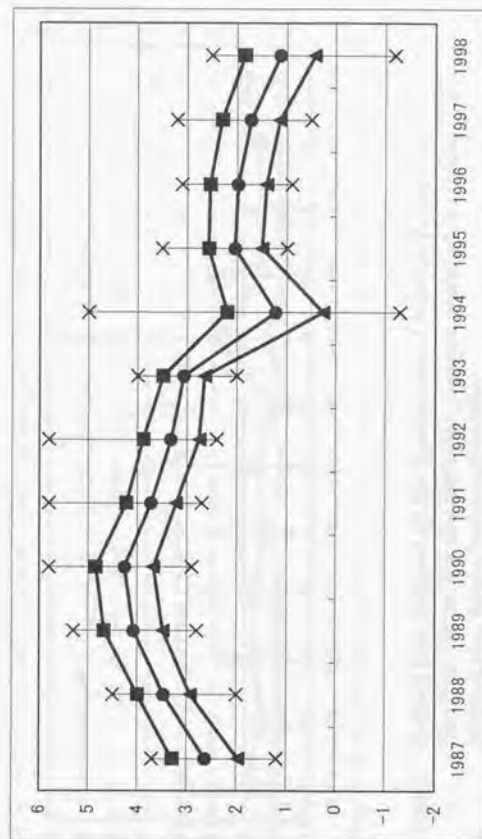
[Note] Vertical line: Support of the distribution of forecasts
Top line: Mean forecast plus one standard deviation
Middle line: Mean forecast
Bottom line: Mean forecast minus one standard deviation

Figure 4-4: The distribution of forecast deviations.
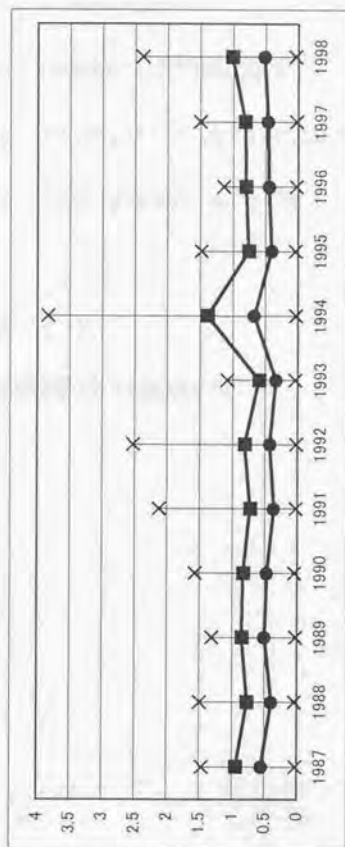
[Note] Vertical line: Support of the distribution of forecast deviations
Top line: Mean forecast deviation plus one standard deviation
Bottom line: Mean forecast deviation

Table 4-1: Summary statistics

No. of forecasters	69
Total number of forecasts	623
Avg. observations per forecaster	9.03
Avg. observations per year	51.9
Avg. of y_i^t	0.4601
Standard deviation of y_i^t	0.4235

Table 4-2: The effect of aging

Dependent variable: y_i' (in %)

	fixed effect model	random effect model	Lamont
age	0.00658	0.00679	0.0180
(t-value)	(1.36)	(1.42)	(2.44)
$\bar{y}_{-i}'$	0.688	0.659	0.77
(t-value)	(4.21)	(4.04)	(7.54)
No. of samples	623	623	728
$\bar{R}^2$	0.256	0.016	0.43
Avg. of y_i'	0.4601	0.4601	0.7381

Hausman test

H_0: random effect vs. H_1: fixed effect

$\chi^2(2)$	5.5674
P-value	0.062

Table 4-3: The effect of experience

Dependent variable: y_i^t (in %)

	fixed effect model	random effect model
time	0.00774	0.00796
(t-value)	(1.48)	(1.54)
$\bar{y}_{-i}^t$	0.686	0.657
(t-value)	(4.20)	(4.04)
No. of samples	623	623
$\bar{R}^2$	0.256	0.016

Hausman test

H_0: random effect vs. H_1: fixed effect

$\chi^2(2)$	5.5695
P-value	0.062

Table 4-4: The estimation using the data for the ongoing year

Dependent variable: y_i^t

	fixed effect model	fixed effect model
age	0.00437	
(t-value)	(0.90)	
time		0.00457
(t-value)		(0.91)
$\bar{y}_{-i}^t$	0.759	0.758
(t-value)	(4.63)	(4.65)
No. of samples	538	538
$\bar{R}^2$	0.214	0.214

Appendix A: Formal proofs of lemmata in Chapter2.

Proof of Lemma 1.

The outline.

We show first that both firms must have positive sales in equilibrium. Next define $P_A \equiv \left(P_A^1, \cdots, P_A^n\right)$, Φ as the set of price vector $\left(P_A, P_S\right)$ on which both firms make positive sales, and Φ_M as the set of $\left(P_A, P_S\right)$ on which products x_S and x_A^i have positive sales for any $i \in M \subseteq \{1, \cdots, n\}$ but no consumer buys x_A^j for any $j \notin M$. Then $\Phi_M \cap \Phi_{M'} = \phi$ for $M \neq M'$ and $\bigcup_{M \subseteq \{1, \cdots, n\}} \Phi_M = \Phi$. Π_A is a concave quadratic function in P_A^i $(i \in M)$ on Φ_M, for any given M.

A's best response correspondence on Φ_M, BR_A^M, is

$$BR_A^M\left(P_S\right) \equiv \arg\max_{\{P_A\}} \Pi_A\left(P_A, P_S\right) \quad \text{s.t.} \quad P_A \in \Phi_M \text{ for given } P_S.$$

S's best response function on Φ_M, BR_S^M, is

$$BR_S^M\left(P_A\right) \equiv \arg\max_{\{P_S\}} \Pi_S\left(P_A, P_S\right) \quad \text{s.t.} \quad P_S \in \Phi_M \text{ for given } P_A.$$

We show that there is an essentially unique equilibrium when the strategy space is limited to one of Φ_M, and calculate the equilibrium strategy $P_i(M) \in BR_i^M\left(P_j(M)\right)$ $(i, j = A, S$ $i \neq j)$. Define $P_M \equiv \left(P_A(M), P_S(M)\right)$.

If it is an equilibrium that some customers buy x_S and x_A^i for any $i \in M$ but no consumer buys x_A^j for any $j \notin M$, it must be an equilibrium when each firm's strategy space is restricted to Φ_M. Therefore P_M $(M \subseteq \{1, \cdots, n\})$ only are candidates for equilibria. However, we will show that for any $M \neq \{1, \cdots, n\}$ $P_A(M) \notin \arg\max_{\{P_A\}} \Pi_A\left(P_A, P_S(M)\right)$ and P_M is not an equilibrium. If $M = \{1, \cdots, n\}$, in contrast,

$$P_A(M) \in \arg\max_{\{P_A\}} \Pi_A\left(P_A, P_S(M)\right)$$

and $P_S(M) \in \arg\max_{\{P_S\}} \Pi_S\left(P_A(M), P_S\right)$.

Consequently $P_{\{1, \cdots, n\}}$ is the unique equilibrium when each firm's strategy is unlimited.

Formal proof.

72

In equilibrium both firms must have positive sales. If x_S only has positive sales, A can increase its profits by choosing P'_A such that $C_A < P'_A \le P_S + t(1 - x'_A)^2$ because P_S must be larger than $C_S (= C_A)$. If x_S has no sales and people at the right end of the market buy x'_A, S can increase its profit by choosing P_S such that $C_S < P_S \le P'_A + t(1 - x'_A)^2$ because P'_A must be larger than C_A. We shall focus on the case that $\forall i \ P^i_A \ge C_A$, $P_S \ge C_S$, and both firms sell something.

Next divide each firm's strategy space in the following manner. Define $P_A \equiv (P^1_A, \cdots, P^a_A)$, Φ as the set of price vector (P_A, P_S) on which both firms make positive sales, and Φ_M as the set of (P_A, P_S) on which x_S and x'_A have positive sales for any $i \in M \subseteq \{1, \cdots, n\}$ but no consumer buys x'_A for any $j \notin M$. Then $\Phi_M \cap \Phi_{M'} = \phi$ for $M \ne M'$ and $\bigcup_{M \subseteq \{1, \cdots, n\}} \Phi_M = \Phi$.

Suppose there are m of A's products that have positive sales, i.e. $\#M = m$. Let us rename them $x_1, \cdots, x_m$ ($x_1 < \cdots < x_m$). Define P_i as the price of x_i,

$$\alpha(i, j) \equiv \begin{cases} 0 & \text{if } P_j \le P_i - t(x_j + x_i)(x_j - x_i) \\ 1 & \text{if } P_j \ge P_i + t(x_j - x_i)(2 - x_i - x_j) \\ \dfrac{P_j - P_i + t(x_j + x_i)(x_j - x_i)}{2t(x_j - x_i)} & \text{otherwise} \end{cases}$$

for $i, j \in \{1, \cdots, m, S\}$ and $i \ne j$,

$\alpha(0, 1) \equiv 0$,

$\alpha(m, m+1) \equiv \alpha(S-1, S) \equiv \alpha(m, S)$, and

$\alpha(S, S+1) \equiv 1$.

Then the argument below shows that all consumers in $[\alpha(i-1, i), \alpha(i, i+1)]$ for any $i \in \{1, \cdots, m, S\}$ buy x_i when each firm's strategy is restricted to Φ_M. x_{i+1} cannot have positive sales in $[0, \alpha(i, i+1)]$ by definition. If x_j ($j > i+1$) has positive sales in $[\alpha(i-1, i), \alpha(i, i+1)]$,

$$P_{i+1} + t(x_{i+1} - \alpha(i, i+1))^2 > P_i + t(x_j - \alpha(i, i+1))^2.$$

This contradicts our assumption that some consumers buy x_{i+1} because it means that

$$\forall y \in [\alpha(i, i+1), 1] \quad P_{i+1} + t(x_{i+1} - y)^2 > P_j + t(x_j - y)^2.$$

The same argument shows x_k ($k < i$) cannot have positive sales in $[\alpha(i-1, i), \alpha(i, i+1)]$.

Φ_M is the set of (P_A, P_S) that satisfy

$$0 < \alpha(1, 2) < \cdots < \alpha(m-1, m) < \alpha(m, S) < 1 \text{ and}$$

$$\forall i \in \{1, \cdots, m, S\}, \quad \forall j \notin M \ x_A^j < x_i, \quad P_A^j > P_i + t(x_A^j - x_i)(2\alpha(i-1, i) - x_i - x_A^j)$$

A's gross profit function on Φ_M, Π_A^M, is

$$\Pi_A^M = \sum_{i=1}^m (\alpha(i, i+1) - \alpha(i-1, i)) P_i - \alpha(m, S) C_A.$$

The Hessian matrix of Π_A^M with respect to P_i ($i = 1, \cdots, m$) is

$$\begin{bmatrix}
\frac{-1}{t(x_2 - x_1)} & \frac{1}{t(x_2 - x_1)} & 0 & \cdots & 0 \\
\frac{1}{t(x_2 - x_1)} & \frac{-(x_3 - x_1)}{t(x_2 - x_1)(x_3 - x_2)} & \frac{1}{t(x_3 - x_2)} & 0 & \vdots \\
0 & \frac{1}{t(x_3 - x_2)} & \ddots & & 0 \\
\vdots & 0 & & \ddots & \frac{-(x_m - x_{m-2})}{t(x_{m-1} - x_{m-2})(x_m - x_{m-1})} & \frac{1}{t(x_m - x_{m-1})} \\
0 & \cdots & 0 & \frac{1}{t(x_m - x_{m-1})} & \frac{-(1 - x_{m-1})}{t(x_m - x_{m-1})(1 - x_m)}
\end{bmatrix}$$

To calculate the determinant, repeat the following operations. Add the first row to the second row. Secondly add the first column to the second column. Thirdly add the second row to the third row... Then we obtain

$$\begin{bmatrix}
\frac{-1}{t(x_2 - x_1)} & 0 & \cdots & 0 \\
0 & \ddots & & \vdots \\
\vdots & & \frac{-1}{t(x_m - x_{m-1})} & 0 \\
0 & \cdots & 0 & \frac{-1}{t(1 - x_m)}
\end{bmatrix}$$

All diagonal terms are negative, and all non-diagonal terms are zero. Hence Π_A^M is strictly concave with respect to P_i ($i = 1, \cdots, m$) on Φ_M.

Differentiating Π_A^M with P_i ($i = 1, \cdots, m$) yields the FOCs,

$$2P_2 - 2P_1 + t(x_2 + x_1)(x_2 - x_1) = 0,$$

$$2(x_{i+1} - x_i)P_{i-1} - 2(x_{i+1} - x_{i-1})P_i + 2(x_i - x_{i-1})P_{i+1} + t(x_i - x_{i-1})(x_{i+1} - x_i)(x_{i+1} - x_{i-1}) = 0$$

$$\text{for } i = 2, \cdots, m-1, \text{ and}$$

$$(x_m - x_{m-1})(P_S + C_A) + 2(1 - x_m)P_{m-1} - 2(1 - x_{m-1})P_m + t(x_m - x_{m-1})(1 - x_m)(1 - x_{m-1}) = 0.$$

Since Φ_M is a convex set, A's best response is

$$P_m = \frac{P_S + C_A + t(1 + x_m)(1 - x_m)}{2} \quad \text{and}$$

$$P_i = P_m + \frac{t}{2}(x_m + x_i)(x_m - x_i) \quad \text{for } i = 1, \cdots, m-1.$$

As for Π_S, $\Pi_S = (P_S - C_S)\dfrac{\left[P_m - P_S + t(1 - x_m)^2\right]}{2t(1 - x_m)}$ on Φ_M and the FOC is

$$P_m - 2P_S + C_S + t(1 - x_m)^2 = 0.$$

The unique Nash equilibrium on Φ_M, which we call P_M, is

$$P_S = C_S + \frac{t}{3}(1 - x_m)(3 - x_m),$$

$$P_m = C_A + \frac{t}{3}(1 - x_m)(3 + x_m),$$

$$P_i = C_A + \frac{t}{6}\left[6 - 4x_m + (x_m)^2 - 3(x_i)^2\right] \quad \text{for } i = 1, \cdots, m-1$$

and $\forall i \in \{1, \cdots, m, S\}$, $\forall j \in M$ $x_A^j < x_i$, $P_A^j > P_i + t(x_A^j - x_i)(2\alpha(i-1,i) - x_i - x_A^j)$

(They satisfy the conditions of Φ_M since $\alpha(m, S) = \dfrac{3 + x_m}{6}$ and $\alpha(i, i+1) = \dfrac{x_i + x_{i+1}}{4}$).

P_M ($M \subseteq \{1, \cdots, n\}$) are the only candidates for equilibria when both firm's strategy space is unlimited.

If A can choose any price, however, for any $M \neq \{1, \cdots, n\}$ P_M is not an equilibrium since A can increase its profit by decreasing P_A^j ($j \notin M$) and making x_A^j have positive sales. If $x_A^j < x_1$, choosing $P_A^j = C_A + \dfrac{t}{6}\left[6 - 4x_m + (x_m)^2 - 3(x_A^j)^2\right] \equiv P^j$ raises Π_A by

$\dfrac{t}{8}(x_1 + x_A^j)^2(x_1 - x_A^j)$. If $x_{i-1} < x_A^j < x_i$ ($i = 2, \cdots, m$), choosing $P_A^j = P^j$ raises Π_A by

$\frac{1}{8}\left(x_A^j - x_{i-1}\right)\left(x_i - x_{i-1}\right)\left(x_i - x_A^j\right)$. If $x_A^j > x_m$ and $\Omega \equiv 6 + 4x_m - (x_m)^2 - 12x_A^j + 3\left(x_A^j\right)^2 \geq 0$,

choosing $P_A^j = P'$ raises Π_A by $\dfrac{t(1 - x_m)\left(x_A^j - x_m\right)\left(6 - x_m - 3x_A^j\right)^2}{72\left(1 - x_A^j\right)}$. If $x_A^j > x_m$ and

$\Omega < 0$, choosing $P_A^j = C_A + \dfrac{t}{3}\left[6x_A^j - 4x_m + (x_m)^2 - 3\left(x_A^j\right)^2\right]$ raises Π_A by

$\dfrac{t(1 - x_m)(3 - x_m)\left[\left(1 - x_A^j\right)(3 - x_m) - \Omega\right]}{18\left(x_A^j - x_m\right)}$.

Finally we show that the remaining candidate, $P_{(1,\cdots,n)}$, is the unique Nash equilibrium. Namely x_S and all of A's products have positive sales in the unique equilibrium. Suppose A deviates from $P_{(1,\cdots,n)}$. Π_A decreases as long as some products are sold from all of A's product lines because Π_A is strictly concave on $\Phi_{(1,\cdots,n)}$. The following argument shows that Π_A decreases if some of A's products have no sales. Suppose x_j sells nothing for any $j \in L \subseteq \{1,\cdots,n\}$ and x_k is the nearest product to x_S that has positive sales. Then A's best response for x_i $(i \notin L)$ is

$$P_k = \frac{1}{2}\left[P_S + C_A + t(1 + x_k)(1 - x_k)\right]$$

$$= C_A + \frac{t}{6}\left[6 - 4x_n + (x_n)^2 - 3(x_k)^2\right] \text{ and}$$

$$P_i = P_k + \frac{t}{2}(x_k + x_i)(x_k - x_i)$$

$$= C_A + \frac{t}{6}\left[6 - 4x_n + (x_n)^2 - 3(x_i)^2\right] \text{ for each } i \notin L.$$

Π_A decreases at least

by $\dfrac{t}{8}(x_2 + x_1)^2(x_2 - x_1)$ if $1 \in L$;

by $\dfrac{t}{8}(x_j - x_{j-1})(x_{j+1} - x_{j-1})(x_{j+1} - x_j)$ if $j \in L$ $(1 < j < n)$;

by $\dfrac{t(1 - x_n)(x_n - x_{n-1})(6 - 3x_{n-1} - x_n)^2}{72(1 - x_{n-1})}$ if $n \in L$.

If S chooses P_S such that $P_S - C_S \geq \frac{t}{6}\left[(x_n)^2 - 4x_n + 3(x_1)^2\right]$, some of A's product(s) have positive sales. Then, since $\forall i \neq n$ and $\forall \delta < \frac{x_{n-1} + x_n}{4}$

$$P_i + t(x_i - \delta)^2 < P_n + t(x_n - \delta)^2,$$

$$\Pi_S \leq (P_S - C_S)\frac{\left[P_n - P_S + t(1 - x_n)^2\right]}{2t(1 - x_n)}$$

$$= \frac{(P_S - C_S)}{6t(1 - x_n)}\left[3(C_S - P_S) + 2t(1 - x_n)(3 - x_n)\right]$$

(the equality holds for $P_S \geq C_S + \frac{t}{6}(1 - x_n)(3x_{n-1} - x_n)$). If P_S satisfies

$P_S - C_S < \frac{t}{6}\left[(x_n)^2 - 4x_n + 3(x_1)^2\right](< 0)$, $\Pi_S < 0$. Therefore each firm cannot profit by deviation.

Let us rewrite $P_{(1,\cdots,n)}$ precisely. The unique Nash equilibrium is

$$P_S = C_S + \frac{t}{3}(1 - x_A^n)(3 - x_A^n),$$

$$P_A^n = C_A + \frac{t}{3}(1 - x_A^n)(3 + x_A^n), \text{ and}$$

$$P_A^i = C_A + \frac{t}{6}\left[6 - 4x_A^n + (x_A^n)^2 - 3(x_A^i)^2\right] \text{ for } i = 1, \cdots, n-1. \quad \text{Q.E.D.}$$

Proof of Lemma 2.

The strategy that S keeps its product and chooses P_S such that $P_S > C_S$ weakly dominates the strategy that S withdraws its product. Therefore S does not withdraw its product in equilibrium.

Next we show that A does not keep more than one product in equilibrium. If A does not withdraw its products, Lemma 1 shows that the equilibrium prices are

$$P_S = C_S + \frac{t}{3}(1 - x_A^n)(3 - x_A^n) \text{ and}$$

$$P_A^i = C_A + \frac{t}{6}\left[6 - 4x_A^n + (x_A^n)^2 - 3(x_A^i)^2\right] \text{ for } i = 1, 2, \cdots, n.$$

The market boundary between x_A^n and x_S, denoted by $\alpha(n, S)$, is $\dfrac{3 + x_A^n}{6}$. The boundary between x_A^{i-1} and x_A^i ($i = 2, 3, \cdots, n$), denoted by $\alpha(i-1, i)$, is $\dfrac{x_A^{i-1} + x_A^i}{4}$.

If A withdraws x_A^n, the prices become

$$P_S = C_S + \frac{t}{3}\left(1 - x_A^{n-1}\right)\left(3 - x_A^{n-1}\right) > C_S \text{ and}$$

$$P_A^i = C_A + \frac{t}{6}\left[6 - 4x_A^{n-1} + \left(x_A^{n-1}\right)^2 - 3\left(x_A^i\right)^2\right] \quad \text{for } i = 1, 2, \cdots, n-1.$$

The market boundary between x_A^{n-1} and x_S, $\alpha(n-1, S)$, is $\dfrac{3 + x_A^{n-1}}{6}$. The boundary between x_A^{i-1} and x_A^i ($i = 2, 3, \cdots, n-1$) is the same as $\alpha(i-1, i)$.

$$\Pi_A\left(\{x_A^1, \cdots, x_A^{n-1}\}, 1, \phi\right) - \Pi_A\left(\{x_A^1, \cdots, x_A^{n-1}, x_A^n\}, 1, \phi\right)$$

$$= \Delta\Pi_A \text{ in } [0, \alpha(n-1, n)] + [\alpha(n-1, n), \alpha(n-1, S)] + [\alpha(n-1, S), \alpha(n, S)]$$

$$= \frac{\left(x_A^{n-1} + x_A^n\right) t\left(x_A^n - x_A^{n-1}\right)\left(4 - x_A^{n-1} - x_A^n\right)}{6} + \frac{\left(6 - x_A^{n-1} - 3x_A^n\right) t\left(x_A^n - x_A^{n-1}\right)\left(2 + x_A^{n-1} + x_A^n\right)}{12} \cdot \frac{1}{3}$$

$$- \frac{\left(x_A^n - x_A^{n-1}\right) t\left(1 - x_A^n\right)\left(3 + x_A^n\right)}{6} \cdot \frac{1}{3}$$

$$= \frac{t\left(x_A^n - x_A^{n-1}\right)}{72}\left[12 + 20x_A^{n-1} - 5\left(x_A^{n-1}\right)^2 + 20x_A^n - 14x_A^{n-1}x_A^n - 5\left(x_A^n\right)^2\right] > 0$$

and A has an ex post incentive to withdraw x_A^n.

Now it is clear that A and S keep one product each in equilibrium. If A keeps x_A^1,

$$P_S = C_S + \frac{t}{3}\left(1 - x_A^1\right)\left(3 - x_A^1\right),$$

$$P_A^1 = C_A + \frac{t}{3}\left(1 - x_A^1\right)\left(3 + x_A^1\right), \text{ and}$$

$$\Pi_A = \frac{t}{18}\left(1 - x_A^1\right)\left(3 + x_A^1\right)^2.$$

Since $\dfrac{d\Pi_A}{dx_A^1} = -\dfrac{t}{18}\left(3 + x_A^1\right)\left(1 + 3x_A^1\right) < 0$, reserving x_A^1 maximizes Π_A. Q.E.D.

Proof of Lemma 3

From Lemma 1,

$$\Pi_S(x_A, x_S, \phi) = \begin{cases} \dfrac{t}{18}(x_S - x_A)(4 - x_S - x_A)^2 & \text{for} \quad x_S \geq x_A \\[2mm] \dfrac{t}{18}(x_A - x_S)(2 + x_A + x_S)^2 & \text{otherwise} \end{cases}$$

Since $\dfrac{\partial \Pi_S}{\partial x_S} > 0$ and $\dfrac{\partial \Pi_S}{\partial x_A} < 0$ for $x_S \geq x_A$, $\Pi_S(x_A, 1, \phi) \geq \Pi_S(x'_A, 1, \phi)$ for $x_A < x'_A$

and $\arg\max_{\{x_S\}} \Pi_S(x_A, x_S, \phi) = 1$ for $x_A \leq 0.5$.

Similarly, $\Pi_W(x_A, \phi, x_W) = \dfrac{t[-\Delta + (x_W - x_A)(4 - x_W - x_A)]^2}{18(x_W - x_A)}$ for $x_W \geq x_A$, where

$\Delta \equiv \dfrac{C_W - C_S}{t}$ $\quad (\Pi_W = 0 \quad$ if $\quad \Delta > (x_W - x_A)(4 - x_W - x_A))$. Therefore

$\arg\max_{\{x_W\}} \Pi_W(x_A, \phi, x_W) = 1$ for $x_A \leq 0.5$. Q.E.D.

Proof of Lemma 4.

If $x_W > x_A^n$, $P_W = C_W - \dfrac{t\Delta}{3} + \dfrac{t}{3}(x_W - x_A^n)(4 - x_W - x_A^n)$,

$P_A^n = C_A + \dfrac{t\Delta}{3} + \dfrac{t}{3}(x_W - x_A^n)(2 + x_W + x_A^n)$, and

$P_A^i = P_A^n + \dfrac{t}{2}(x_A^n + x_A^i)(x_A^n - x_A^i)$ for $i = 1, 2, \cdots, n-1$.

Since $\forall i \neq n$ x_A^i is irrelevant for Π_W, $\Pi_W(\{x_A^1, \cdots, x_A^n\}, \phi, x_W) = \Pi_W(x_A^n, \phi, x_W)$. Then

$\Pi_W(x_A^n, \phi, x_W) \leq \Pi_W(0.5, \phi, 1)$ for any $x_A^n \geq 0.5$ from Lemma 3.

If $x_W \in [x_A^i, x_A^{i+1}]$ $(i = 1, \cdots, n-1)$,

$\Pi_W(\{x_A^1, \cdots, x_A^n\}, \phi, x_W) = \Pi_W(\{x_A^i, x_A^{i+1}\}, \phi, x_W)$

$\leq \Pi_W(\{0, 1\}, \phi, 0.5) = \begin{cases} \dfrac{t}{72}(3 - 4\Delta)^2 & \text{for} \quad \Delta < 0.75 \\[2mm] 0 & \text{otherwise} \end{cases}$

$< \Pi_W(0.5, \phi, 1)$.

If $x_W < x_A^1 (\leq 0.5)$, $\Pi_W(\{x_A^1, \cdots, x_A^n\}, \phi, x_W) = \Pi_W(x_A^1, \phi, x_W) \leq \Pi_W(0.5, \phi, 1)$. Q.E.D.

79

Proof of Lemma 5.

Define Case N as $x_S = \phi$ and Case E as $x_S \neq \phi$. Raising P_A' (P_W) has two effects for A (W): the price margin becomes larger but some customers may go to other firm(s). Though the former positive effect is the same in Case E as in Case N, the latter negative effect in Case E is equal to or stronger than that in Case N since S may deprive A (W) of customers. Thus the equilibrium prices of A and W in Case E are equal to or lower than those in Case N, and the equilibrium profit of A and W in Case E are equal to or smaller than those in Case N. Q.E.D.

Corollary of Lemma 5.

Suppose $\hat{X}_A \neq \phi$. Let us exclude x_A' from $\hat{X}_A$ and define the remainder as $\hat{X}_A'$. Then $\Pi_S(\hat{X}_A, x_S, \phi) \leq \Pi_S(\hat{X}_A', x_S, \phi)$ for any x_S and x_A'.

Proof.

Π_S weakly increases if the competing product is withdrawn. Q.E.D.

Proof of Lemma 6

Lemma 2 shows that S earns $\Pi_S(x_A', 1, \phi)(\geq F)$ if it chooses $x_S = 1$. We show that Π_S decreases if S deviates from this strategy. Suppose A chooses n product(s) and S chooses $x_S \neq 1$. The corollary of Lemma 5 shows that Π_S is largest for given x_S when A reserves one product. From Lemma 3, $\dfrac{\partial}{\partial x_A}\Pi_S(x_A, x_S, \phi)$ is negative if $x_A < x_S$ and positive otherwise. Thus S can earn at most $\max\{\Pi_S(x_A', x_S, \phi), \Pi_S(x_A^n, x_S, \phi)\}$, which is smaller than $\Pi_S(x_A', 1, \phi)$. The reason is that $x_A^n \leq 1 - x_A'$ and that $\dfrac{\partial}{\partial x_S}\Pi_S(x_A, x_S, \phi)$ is positive if $x_A \leq x_S$ and negative otherwise. Q.E.D.

Proof of Lemma 7.

(a) If $x_W > x_A^n$,

$$P_A^n = C_A + \frac{t\Delta}{3} + \frac{t}{3}\left(x_W - x_A^n\right)\left(2 + x_W + x_A^n\right),$$

$$P_A^i = P_A^n + \frac{t}{2}\left(x_A^n + x_A^i\right)\left(x_A^n - x_A^i\right) \quad \text{for} \quad i = 1,2,\cdots,n-1,$$

and the boundary of the market between x_W and x_A^n is $\dfrac{\Delta + \left(x_W - x_A^n\right)\left(2 + x_W + x_A^n\right)}{6\left(x_W - x_A^n\right)}$.

Thus Π_A increases as x_W increases.

(b) $\Pi_A\left(\left\{x_A^1,\cdots,x_A^{n-1}\right\},\phi,1\right) - \Pi_A\left(\left\{x_A^1,\cdots,x_A^{n-1},x_A^n\right\},\phi,1\right)$

$$= \frac{t\left(x_A^n - x_A^{n-1}\right)Y}{72\left(1 - x_A^n\right)\left(1 - x_A^{n-1}\right)},$$

where $Y \equiv \left(1 - x_A^n\right)\left(1 - x_A^{n-1}\right)\left[12 + 20x_A^{n-1} - 5\left(x_A^{n-1}\right)^2 + 20x_A^n - 14x_A^{n-1}x_A^n - 5\left(x_A^n\right)^2\right]$

$$- 4\Delta\left[\Delta + 2\left(1 - x_A^n\right)\left(1 - x_A^{n-1}\right)\right],$$

and it is positive for $\Delta \leq 1$. The reason is that $\dfrac{\partial^2 Y}{\partial K^2} < 0$ for $K \in \left\{\Delta, x_A^n, x_A^{n-1}\right\}$, that

$Y > 0$ for $\Delta = 0$, and that $Y > 0$ for $\Delta = 1$ and $\left(x_A^{n-1}, x_A^n\right) \in \left\{(0,\varepsilon), (0,0.5), (0.5,0.5)\right\}$.

(c) $\Pi_A\left(x_A,\phi,1\right) = \dfrac{t\left[\Delta + \left(1 - x_A\right)\left(3 + x_A\right)\right]^2}{18\left(1 - x_A\right)}$ and it is decreasing in x_A. Q.E.D.

Proof of Lemma 8.

S always enters when $\Pi_S\left(x_A, x_S^*\left(x_A, x_W\right), x_W\right) \geq F$. If $x_A \leq x_W \leq x_S$,

$$\max \Pi_W\left(x_A, x_S, x_W\right) = \Pi_W\left(0, 1, 0.5\right) = \frac{t}{72}\left(3 - 4\Delta\right)^2$$

$$\leq \Pi_W\left(0.5, \phi, 1\right) < F.$$

If $x_A \leq x_S < x_W$, $\Pi_W\left(0, x_S, 1\right) = \dfrac{t}{72\left(1 - x_A\right)}\left[3\left(1 - x_A\right) - \Delta\left(3 - x_A\right)\right]^2$ and

$$\max \Pi_W\left(x_A, x_S, x_W\right) = \Pi_W\left(0, 0, 1\right) = \frac{t}{8}\left(1 - \Delta\right)^2$$

$$\leq \Pi_W\left(0.5, \phi, 1\right) < F.$$

If $x_A > x_S$, $\max \Pi_W\left(x_A, x_S, x_W\right) = \Pi_W\left(0, 0, 1\right)$. Thus W cannot earn positive profit

if S enters after W. Q.E.D.

Proof of Proposition 3.

If some x_A satisfies all of $\Pi_S\left(x_A, x_S^*(x_A), 1\right) \leq F$, $\Pi_A(x_A, \phi, 1) \geq \Pi_A(0, 1, \phi)$, and $\Pi_W(x_A, \phi, 1) \geq F$, then $\Pi_W\left(x_A^*, \phi, 1\right) \geq F$ and $\Pi_A\left(x_A^*, \phi, 1\right) \geq \Pi_A(0, 1, \phi)$ because $\Pi_W(x_A, \phi, 1)$ and $\Pi_A(x_A, \phi, 1)$ are decreasing in x_A.

If (3) is satisfied, $x_A^* = 0$ and A chooses $x_A = 0$ from Lemma 2. We shall assume below that (3) is not satisfied.

Case 1: $x_S > x_A$ and $\Delta \leq \dfrac{(x_W - x_S)(x_A(x_A + 2x_W - 8) + 2x_S(1 + x_A - x_W) + 3x_W(2 - x_W))}{3x_W - 2x_A - x_S} \equiv \Delta_1$.

$$\Pi_S(x_A, x_S, x_W) = \frac{t(x_S - x_A)[\Delta + (2 - x_A + x_W)(x_W - x_S)]^2}{18(x_W - x_S)(x_W - x_A)} \equiv \Pi_S^1, \text{ and}$$

$$x_S^* = \frac{x_A(2 - x_A - 2x_W) + 3x_W(2 + x_W) - \sqrt{(1 + x_W - x_A)^2 - 1}\sqrt{(1 + x_W - x_A)^2 - 1 - 8\Delta}}{4(2 - x_A + x_W)}$$

We shall analyze this case after Case 4.

Case 2: $x_S > x_A$ and $\Delta \geq \dfrac{1}{3}\left[3x_W(2 - x_W) - 2x_S(1 - x_S) - x_A(4 - x_A)\right] \equiv \Delta_2$.

$$\Pi_S(x_A, x_S, x_W) = \frac{t(x_S - x_A)(4 - x_S - x_A)^2}{18}, \text{ and } A \text{ must move in to deter entry of } S.$$

Since $\Pi_A(x_A, \phi, x_W)$ is decreasing in x_A, A chooses x_A^* in this case.

Case 3: $x_S > x_A$ and $\Delta_1 < \Delta < \Delta_2$.

$$\Pi_S(x_A, x_S, x_W) = \frac{t[\Delta - (x_W - x_S)(2 - x_W - x_S)][x_A(x_A - 4) + 2x_S + x_W(2 - x_W) - \Delta]}{4(x_S - x_A)} \equiv \Pi_S^3,$$

and A must move inward to deter entry of S because

$$\frac{\partial}{\partial x_W}\Pi_S = -\frac{(1 - x_W)\left(-2\Delta - 4x_A + (x_A)^2 + (x_S)^2 + 2x_W(2 - x_W)\right)}{2(x_S - x_A)} \leq 0.$$

Since $\Pi_A(x_A, \phi, x_W)$ is decreasing in x_A, A chooses x_A^* in this case.

Case 4: $x_S \leq x_A$.

Π_S is maximized when $(x_S, x_W) = (0,1)$. Since $\Pi_S(x_A, 0, 1) = \dfrac{x_A}{72}(\Delta + 3)^2$,

$$\forall x_S \quad \Pi_S(0.3, x_S, 1) < \Pi_W(0.5, \phi, 1) \, (< F) \text{ for } \Delta \leq 0.2$$

and $\Pi_S(0.5, 0, 1) < \Pi_S(0.5, 1, 1)$ for $\Delta \geq 0.2$.

Therefore S does not choose $x_S \leq x_A$ in equilibrium.

Next we shall return to the Case 1. Define

$$K \equiv \frac{\partial}{\partial x_W}\Pi_A(x_A, \phi, 1) \Big/ \frac{\partial}{\partial x_W}\Pi_S^1(x_A, x_S^*, 1) - \frac{\partial}{\partial x_A}\Pi_A(x_A, \phi, 1) \Big/ \frac{\partial}{\partial x_A}\Pi_S^1(x_A, x_S^*, 1)$$

$$= \frac{(3 - x_A)[3 + \Delta - x_A(2 + x_A)]H}{9(1 - x_A)\sqrt{rs}(r + \sqrt{rs})^2}$$

where $r \equiv 3 - x_A(4 - x_A) > 0$, $s \equiv r - 8\Delta$, and

$$H \equiv (3 - x_A)^2(1 - x_A)^3(5 - 3x_A)(r + \sqrt{rs}) + 16\Delta^3[2(1 - x_A)(1 + x_A) - \sqrt{rs}]$$

$$+ 4\Delta^2(1 - x_A)[-27 + 57x_A - 37(x_A)^2 + 7(x_A)^3 - (15 - 9x_A)\sqrt{rs}]$$

$$+ 4\Delta(1 - x_A)^2(3 - x_A)[-27 + 51x_A - 29(x_A)^2 + 5(x_A)^3 - (4 - 2x_A)\sqrt{rs}].$$

If $K > 0$, A moves inward until $\Pi_S^1(x_A, x_S^*, 1) \leq F$ is satisfied. Calculation shows that $K > 0$ for any $\Delta \leq 0.257$ and $x_A \leq 0.25$, and that $\Pi_S^1 \leq \Pi_S^3$ for $\Delta \geq 0.257$. Thus A moves inward at least until $x_A = 0.25$. If $\Delta \leq 0.18$, $\Pi_S^1(0.25, x_S^*, 1) < \Pi_W(0.5, \phi, 1) < F$ and choosing $x_A = 0.25$ is enough for entry deterrence. If $\Delta \geq 0.18$, $\Pi_S(x_A, x_S^*, 1) \neq \Pi_S^1$ for any $x_A > 0.25$. Consequently A chooses x_A^* if it intends to deter entry of S.

The above argument shows that A chooses x_A^* if it wants to deter entry of S. Since $\Pi_A(x_A^*, \phi, 1) \geq \Pi_A(0, 1, \phi)$ from our assumption, to deter entry of S is more profitable for A than not to deter. Therefore A chooses x_A^*, W chooses $x_W = 1$ (since this maximizes $\Pi_W(x_A^*, \phi, x_W)$) and S does not enter the market in the unique equilibrium. Q.E.D.

Appendix B: Formal proofs of lemmata in Chapter 3.

Proof of Lemma 3-1.

From Gabszewicz and Thisse (1979),

$$\Pi_B(q_1, q_B) = \frac{(3q_B - 2q_1 - 1)^2}{8q_B(q_B - q_1)} \text{ if } q_B \geq 4q_1 - 3,$$

$$= \frac{25(q_B - q_1)}{18q_B} \text{ if } q_1 \leq q_B \leq 4q_1 - 3;$$

$$= \frac{(q_1 - q_B)}{18q_B} \text{ if } 0.25(q_1 + 3) \leq q_B \leq q_1, \text{ and}$$

$$= \frac{(q_B - 1)(q_1 - 2q_B + 1)}{4q_B(q_1 - q_B)} \equiv \pi_1 \text{ if } q_B \leq 0.25(q_1 + 3)$$

(Note that the density of consumers is 0.5 in our model). Then

$$\frac{\partial \pi_1}{\partial q_B} = \frac{q_1^2 + q_1 - 2q_1 q_B - q_1 q_B^2 + 3q_B^2 - 2q_B}{4q_B^2(q_1 - q_B)^2}$$

and it is positive at $q_B = 0.25(q_1 + 3)$ if and only if $q_1 < 3$. Thus it is straightforward to show (a), (b), and (c). Q.E.D.

Proof of Lemma 3-2.

B always keeps q_B (and charges $P_B > 0$) because it is the weakly dominant strategy. Define $q_Z \equiv \max\{q_i | q_i \in \hat{Q}_A\}$. Then $P_i = 0$ for any $i \neq \{Z, B\}$ and

$$\Pi_A(\hat{Q}_A, q_B) = \Pi_A(q_Z, q_B) = \begin{cases} \dfrac{(q_B - q_Z)}{18q_Z} & \text{if } q_Z \geq 0.25(q_B + 3) \\ \dfrac{(q_Z - 1)(1 + q_B - 2q_Z)}{4q_Z(q_B - q_Z)} & \text{otherwise} \end{cases}$$

in equilibrium. Thus A withdraws any q_i that satisfies $\Pi_A(q_i, q_B) < \Pi_A(q_j, q_B)$ for some $j < i$. Consequently $q_Z \leq q_M$ must hold in equilibrium. Since $\Pi_B(\hat{Q}_A, q_B)$ $(= \Pi_B(q_Z, q_B))$ is decreasing in q_Z, B earns $\Pi_B(q_M, q_B)$ or more. Q.E.D.

Proof of Lemma 3-3 (a1).

Suppose A keeps some products smaller than q_B. Define

$$q_X \equiv \max\left\{q_i \,\middle|\, q_i \in \hat{Q}_A \text{ and } q_i \le q_B\right\}$$

Then $P_B = \dfrac{q_B - q_X}{q_B}$, $P_n = \dfrac{3q_n - 2q_B - q_X}{2q_n}$, and $P_i = 0$ for any $i \ne \{n, B\}$ in

equilibrium. Hence $\Pi_A\left(\hat{Q}_A, q_B\right) = \dfrac{(3q_n - 2q_B - q_X)^2}{8q_n(q_n - q_B)}$. Since Lemma 3-1 shows that

$$\Pi_A(q_n, q_B) = \begin{cases} \dfrac{25(q_n - q_B)}{18q_n} & \text{if } q_B \ge 0.25(q_n + 3) \\[3mm] \dfrac{(3q_n - 2q_B - 1)^2}{8q_n(q_n - q_B)} & \text{otherwise} \end{cases}$$

A has an incentive to withdraw all products except q_n. Q.E.D

Proof of Lemma 3-3 (a2)

When $q_B > 0.25(3q_{n-1} + q_n)$ and A keeps q_n, B chooses $P_B = \dfrac{q_n - q_B}{3q_B}$ and earns

$\Pi_B(q_n, q_B)$ even if A keeps some products smaller than q_B. Q.E.D

Proof of Lemma 3-3 (b)

If A keeps some $q_i \le q_B$, $P_i = 0$ in equilibrium and it causes negative effect on P_B. Therefore A withdraws any $q_i \le q_B$ in equilibrium.

Suppose A keeps q_k and q_{k+1}. Then in equilibrium

$$P_B = 0.5q_B^{-1}Z^{-1}\left(2q_k^3 - q_k q_B(q_k + q_{k+1}) + q_B^2(q_{k+1} - q_k)\right),$$

$$P_k = Z^{-1}(3q_k + 2q_{k+1})(q_k - q_B),$$

$$P_{k+1} = 0.5Z^{-1}\left(2q_k(q_k + 4q_{k+1}) + q_B(q_{k+1} - 11q_k)\right),$$

and $\Pi_A\left(\{q_k, q_{k+1}\}, q_B\right) = 0.125\,Z^{-2}V$

where $Z \equiv q_k(q_k + 2q_{k+1}) + q_B(q_{k+1} - q_k)$

and $V \equiv 12q_k^4 + q_k^3(40q_{k+1} - 42q_B) + 2q_k^2\left(24q_{k+1}^2 - 43q_{k+1}q_B + 15q_B^2\right)$

$$+ 7q_k q_{k+1} q_B(4q_{k+1} - 5q_B) + 5q_{k+1}^2 q_B^2.$$

Define $\Delta\Pi_{21} \equiv \Pi_A\left(\{q_k, q_{k+1}\}, q_B\right) - \Pi_A(q_{k+1}, q_B)$. Then $\Delta\Pi_{21}$ is maximized at $q_B = 2$

because $\dfrac{\partial^2}{\partial q_B^2}\Delta\Pi_{21} > 0$ and $\dfrac{\partial}{\partial q_B}\Delta\Pi_{21} < 0$ at $q_B = 2.25$. Given $q_B = 2$, $\Delta\Pi_{21} < 0$ at

$q_k = q_B$, $\Delta\Pi_{21} = 0$ at $q_k = q_{k+1}$, and $\dfrac{\partial^2}{\partial q_k^2}\Delta\Pi_{21} > 0$. Consequently $\Delta\Pi_{21} < 0$ is

always satisfied and A withdraws q_k in equilibrium.

Next suppose A keeps n products. Then $\Pi_A(\{q_k,\cdots,q_n\},q_B) - \Pi_A(q_n,q_B)$ is decreasing in q_n for given q_B. The reason is that a change in q_n has a direct effect on P_B in the latter case, while it has an indirect effect on P_B in the former case. Hence

$$\Pi_A(\{q_k,\cdots,q_{n-1},q_n\},q_B) - \Pi_A(q_n,q_B)$$
$$< \Pi_A(\{q_k,\cdots,q_{n-1},q_{n-1}\},q_B) - \Pi_A(q_{n-1},q_B)$$
$$= \Pi_A(\{q_k,\cdots,q_{n-1}\},q_B) - \Pi_A(q_{n-1},q_B)$$
$$< \Pi_A(\{q_k,\cdots,q_{n-2}\},q_B) - \Pi_A(q_{n-2},q_B)$$
$$< \cdots < \Pi_A(\{q_k,q_{k+1}\},q_B) - \Pi_A(q_{k+1},q_B) = \Delta\Pi_{21} < 0$$

for given q_B. Namely A withdraws $\{q_k,\cdots,q_{n-1}\}$ in equilibrium. Q.E.D.

Proof of Lemma 3-4.

Since $\Pi_B(3,6) > \Pi_B(3,1.5)$, $q_1^* > 3$ must hold. Then Π_B is maximized at either $q_B = 6$ or $q_B = q_B^*(q_1)$ from Lemma 3-1. Hence choosing q_1^* deters entry if $F \geq \Pi_B(q_1^*,6)$. If $F < \Pi_B(q_1^*,6)$, B can enter either $q_B = 6$ or $q_B = q_B^*(q_1)$. Q.E.D.

Proof of Proposition 3-1.

If $Q_A = 6$, B does not enter the market because Lemma 3-1 and Lemma 3-2 show that it can earn $\Pi_B(6,q_B^*(6)) - F$ at most. Then A earns $\Pi_A(6,\phi) - F$. If $Q_A = q_1 < 6$, Π_A decreases because

$$\Pi_A(q_1,\phi) = 1.125(1 - q_1^{-1})$$

If A produces the second product,

$$\Pi_A(\{q_1,q_2\},\phi) = \frac{4.5q_1(q_2-1)}{q_2 + 3q_1q_2 - q_1 + q_1^2}$$

86

and calculation shows that it can earn at most

$$\max_{Q_A} \Pi_A(\{q_1, q_2\}, \phi) - 2F = \Pi_A(\{\sqrt{6}, 6\}, \phi) - 2F$$

$$(\leq \Pi_A(\{\sqrt{6}, 6\}, \phi) - \Pi_B(6, q_B^*(6)) - F < \Pi_A(6, \phi) - F).$$

Therefore A never chooses two or more products. Q.E.D.

Proof of Proposition 3-2.

If $Q_A = \overline{q}_1$, B does not enter the market and A earns $\Pi_A(\overline{q}_1, \phi) - F$. We shall prove Π_A decreases when A deviates from this. First suppose that A chooses one product other than $\overline{q}_1$. If $Q_A = q_1 < \overline{q}_1$, Π_A decreases because $\Pi_A(q_1, \phi)$ is increasing in q_1. If $Q_A = q_1 > \overline{q}_1$, B enters $q_B^*(q_1)$ because $\Pi_B(q_1, q_B^*(q_1)) > F$ and Lemma 3-2 shows that choosing other product(s) decreases Π_B. Then A earns $\Pi_A(q_1, q_B^*(q_1)) - F$ $(\leq \Pi_A(6, q_B^*(6)) - F)$, and numerical calculation shows that $\Pi_A(6, q_B^*(6)) < \Pi_A(\overline{q}_1, \phi)$.

Next suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$, and that B does not enter the market. Lemma 3-2 shows that, in order to deter entry of B, q_k and q_{k-1} must satisfy

$$\Pi_B(q_k, 6) = \frac{25(6 - q_k)}{108} \leq F(\leq \Pi_B(6, q_B^*(6)))$$

and $\Pi_A(q_{k-1}, 6) < \Pi_A(q_k, 6)$.

Define $\overline{q}_k$ such that $\Pi_B(\overline{q}_k, 6) = \Pi_B(6, q_B^*(6))$, and define $\overline{q}_{k-1}$ such that $\overline{q}_{k-1} \leq q_B^*(6)$ and $\Pi_A(\overline{q}_{k-1}, 6) = \Pi_A(\overline{q}_k, 6)$. Then, since calculation shows that

$$\Pi_A(q, \phi) < \Pi_B(q_1^*, 6)(\leq F) \text{ for any } q \leq \overline{q}_{k-1},$$

A does not have an incentive to choose products smaller than $q_B^*(6)$. Hence $q_1 \geq \overline{q}_k$ if A deters entry of B. On condition that $q_1 \geq \overline{q}_k$,

$$\max \Pi_A(\{q_1, \cdots, q_{n-1}, q_n\}, \phi) - \max \Pi_A(\{q_1, \cdots, q_{n-1}\}, \phi)$$

$$< \max \Pi_A(\{q_1, \cdots, q_{n-2}, q_{n-1}\}, \phi) - \max \Pi_A(\{q_1, \cdots, q_{n-2}\}, \phi),$$

and calculation shows that

$$\max \Pi_A(\{q_1, q_2, q_3\}, \phi) = \Pi_A(\{\overline{q}_k, \sqrt{6\overline{q}_k}, 6\}, \phi)$$

$$< \Pi_A\left(\{\overline{q}_k, 6\}, \phi\right) + \Pi_B\left(q_1^*, 6\right) \leq \Pi_A\left(\{\overline{q}_k, 6\}, \phi\right) + F.$$

Namely, choosing more than two products decreases net profit. Thus A can earn $\Pi_A\left(\{\overline{q}_k, 6\}, \phi\right) - 2F$ $\left(\leq \Pi_A\left(\{\overline{q}_k, 6\}, \phi\right) - \Pi_B\left(q_1^*, 6\right) - F\right)$ at most. Since calculation shows that $\Pi_A\left(\{\overline{q}_k, 6\}, \phi\right) - \Pi_B\left(q_1^*, 6\right) < \Pi_A\left(\overline{q}_1, \phi\right)$, Π_A decreases by deviation.

Finally suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$, and that B enters the market. We will determine an upper bound of A's payoff. Since $\Pi_A\left(\hat{Q}_A, \{q_B, \cdots, q_{Bm}\}\right) \leq \Pi_A\left(\hat{Q}_A, q_B\right)$, we assume B chooses one product. If $q_k < \overline{q}_k$, B enters $q_B = 6$. Then

$$\Pi_A\left(\hat{Q}_A, 6\right) \leq \max\left\{\Pi_A\left(q_{k-1}, 6\right), \Pi_A\left(q_k, 6\right)\right\} \text{ (from Lemma 3-2)}$$
$$< \Pi_A\left(6, q_B^*(6)\right) < \Pi_A\left(\overline{q}_1, \phi\right) \text{ (from Lemma 3-1)}.$$

If $q_k \geq \overline{q}_k$, B chooses $q_B^*(q_n)$ and calculation shows that

$$2 < q_B^*\left(\overline{q}_k\right) \leq q_B^*\left(q_n\right) < 0.25\left(q_n + 3\right)$$

Then Lemma 3-3 (b) shows that A withdraws all products except q_n, and it earns $\Pi_A\left(q_n, q_B^*(q_n)\right) - nF$ $\left(< \Pi_A\left(\overline{q}_1, \phi\right) - F\right)$. Accordingly Π_A decreases in this case. Q.E.D.

Proof of Proposition 3-3.

If $Q_A = 6$, B chooses only one product because Lemma 3-2 shows that only the highest quality among $\hat{Q}_B$ has positive sales in equilibrium. Thus B chooses $q_B^*(6)$ to maximize its profit, and A earns $\Pi_A\left(6, q_B^*(6)\right) - F$. We shall prove Π_A decreases when A deviates from this.

Suppose A chooses $Q_A = q_1 < 6$. Then B enters either $q_B = 6$ or $q_B = q_B^*(q_1)$, and A earns either $\Pi_A\left(q_1, 6\right) - F$ or $\Pi_A\left(q_1, q_B^*(q_1)\right) - F$. Calculation shows that they are smaller than $\Pi_A\left(6, q_B^*(6)\right) - F$.

Next suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$. Since we consider an upper bound of A's payoff, assume that B chooses one product. If $q_k < q_1^*$, B enters $q_B = 6$. Then

$$\Pi_A\left(\hat{Q}_A, 6\right) \leq \max\left\{\Pi_A\left(q_{k-1}, 6\right), \Pi_A\left(q_k, 6\right)\right\} \text{ (from Lemma 3-2)}$$

$$< \Pi_A\left(6, q_B^*(6)\right) \text{ (from Lemma 3-1)}.$$

If $q_* \geq q_1^*$, B chooses $q_B^*(q_n)\ (\geq q_B^*(q_1^*)) > 2$) and A withdraws all products except q_n from Lemma 3-3 (b). Then it earns $\Pi_A\left(q_n, q_B^*(q_n)\right) - nF\ (< \Pi_A\left(6, q_B^*(6)\right) - F)$. Q.E.D.

Proof of Proposition 3-4.

A consumer of income y is willing to pay up to $y\left(1 - q_i^{-1}\right)$ for good q_i. When A chooses q_1 and B does not enter, A offers $P_1 = 1.5\left(1 - q_1^{-1}\right)$ in equilibrium and

$$W = 0.5\int_{1.5}^{3} y\left(1 - q_1^{-1}\right)dy - F$$

$$= \frac{27(q_1 - 1)}{16q_1} - F.$$

If $F \geq \Pi_B\left(6, q_B^*(6)\right)$, Proposition 3-1 shows that $q_1 = 6$ and $W = \frac{45}{32} - F$.

If $\Pi_B\left(q_1^*, 6\right) \leq F < \Pi_B\left(6, q_B^*(6)\right)$, A offers $\overline{q}_1$ that satisfies

$$F = \frac{(\overline{q}_1 - 1)(5r + \overline{q}_1 r - 4\overline{q}_1 - 4)}{4(\overline{q}_1 - r - 1)(\overline{q}_1 r - \overline{q}_1 - r - 1)} \text{ where } r \equiv \sqrt{\overline{q}_1 + 1}.$$

Then

$$W(\overline{q}_1) = \frac{27(\overline{q}_1 - 1)}{16\overline{q}_1} - \frac{(\overline{q}_1 - 1)(5r + \overline{q}_1 r - 4\overline{q}_1 - 4)}{4(\overline{q}_1 - r - 1)(\overline{q}_1 r - \overline{q}_1 - r - 1)}$$

and calculation shows that $\dfrac{dW}{d\overline{q}_1} > 0$ and $\dfrac{d\overline{q}_1}{dF} > 0$. Therefore $\dfrac{dW}{dF} > 0$ in this case.

If $F < \Pi_B\left(q_1^*, 6\right)$, then $(Q_A, Q_B) = \left(6, q_B^*(6)\right)$, $P_1 = \dfrac{17 - 2q_B^*}{12}$, and $P_B = \dfrac{q_B^* - 1}{q_B^*}$ in

equilibrium. Consumers whose income are less than $\dfrac{19 - 4q_B^*}{12 - 2q_B^*} \equiv b$ buy q_B, and other

consumers buy q_1. Thus

$$W = 0.5\int_1^b y\left(1 - q_B^{-1}\right)dy + 0.5\int_b^3 y\left(1 - q_1^{-1}\right)dy - 2F.$$

Calculation shows that

$$0.5\int_1^b y\left(1 - q_B^{-1}\right)dy + 0.5\int_b^3 y\left(1 - q_1^{-1}\right)dy - 2\Pi_B\left(q_1^*, 6\right) > \frac{45}{32} - \Pi_B\left(6, q_B^*(6)\right).$$

Therefore W under duopoly is always larger than W under monopoly. Q.E.D.

Proof of Proposition 3-5.

If $q_C < \min\{q_A, q_B\}$, $P_C = 0$ in equilibrium. Hence we assume $q_B < q_C < q_A$. When $q_C \leq 0.2(2q_A + 3q_B)$,

$$\Pi_C(q_A, q_B, q_C) = \frac{9(q_A - q_B)(q_A - q_C)(q_C - q_B)}{2q_C(4q_A - 3q_B - q_C)^2} \text{ and } \frac{\partial \Pi_C}{\partial q_C} > 0.$$

When $q_C \geq 0.2(2q_A + 3q_B)$,

$$\Pi_C(q_A, q_B, q_C) = \frac{q_A - q_C}{18q_C} \text{ and } \frac{\partial \Pi_C}{\partial q_C} < 0.$$

Therefore Π_C is maximized at $q_C = 0.2(2q_A + 3q_B)$ for $q_C \in [q_B, q_A]$. Entry is blockaded if $\Pi_C(6, q_B^*(6), 0.6(4 + q_B^*(6))) < F$. Otherwise B must choose closer product to A. The reason is it cannot earn positive gross profit when C enters higher quality than its product. Define

$$K \equiv \frac{\partial}{\partial q_B} \Pi_A(q_A, q_B, \phi) \Big/ \frac{\partial}{\partial q_B} \Pi_C(q_A, q_B, 0.2(2q_A + 3q_B))$$

$$- \frac{\partial}{\partial q_A} \Pi_A(q_A, q_B, \phi) \Big/ \frac{\partial}{\partial q_A} \Pi_C(q_A, q_B, 0.2(2q_A + 3q_B)).$$

A continues to choose $q_A = 6$ if $K < 0$ for $q_A = 6$. Calculation shows that

$$K = -\frac{3(3q_A - 2q_B - 1)(2q_A + 3q_B)^2}{10q_A^2 q_B(q_A - q_B)} < 0 \text{ for } q_B < 0.25(q_A + 3)$$

and $K = 0$ for $q_B \geq 0.25(q_A + 3)$.

Thus A chooses $q_A = 6$ and B chooses $\bar{q}_B$ such that $\Pi_C(6, \bar{q}_B, 0.6(4 + \bar{q}_B)) = F$ in the unique equilibrium for $\Pi_C(6, 2.25, 3.75) \leq F$.

If $F < \Pi_C(6, 2.25, 3.75)$, q_B must be larger than 2.25 to prevent entry of C. Then A chooses q_A' such that $\Pi_B(q_A', 6, \phi) \leq F$ since $K = 0$ for any q_A. B chooses q_B' such that $q_B' < q_A'$ and $\Pi_C(q_A', q_B', 0.2(2q_A' + 3q_B')) = F$. $(q_A, q_B) = (6, \bar{q}_B)$ is an example of such equilibria. Q.E.D.

Appendix C: Formal proofs of lemmata in Chapter 4.

Proof of Lemma 4-1.

There are six behavioral strategies for B in period 1; $r_1^B = u_1$, $r_1^B = l_1^B$, $r_1^B = m_1^B$, and their reverses (report d_1 instead of u_1, and vise versa). We prove that no strategy other than $r_1^B = m_1^B$ can be an equilibrium behavioral strategy for B.

First we show that $r_1^B = u_1$ is not an equilibrium. If this is an equilibrium and B reports $R_1^B = u_1$, $\hat{\theta}_1^B = \theta$ because the market cannot infer s_1^B. We show that B has an incentive to deviate regardless of the market's out-of-equilibrium belief. Suppose $\hat{\theta}_1^B\left(\left(R_1^A, d_1, o_1\right), r_1\right) = \theta_1^B\left(R_1^A, d_1, o_1\right)$. Then B reports $R_1^B = d_1$ when $R_1^A = s_1^B = d_1$, because his expected utility is

$$E\hat{\theta}_1^B = \Pr\left(u_1 \middle| d_1, d_1\right) \theta_1^B\left(d_1, d_1, u_1\right) + \Pr\left(d_1 \middle| d_1, d_1\right) \theta_1^B\left(d_1, d_1, d_1\right)$$

$$= \frac{\theta(1+\theta)}{1+\theta^2} > \theta.$$

Secondly, suppose $\hat{\theta}_1^B\left(\left(R_1^A, d_1, o_1\right), r_1\right) = \theta_1^B\left(R_1^A, u_1, o_1\right)$. Then B reports $R_1^B = d_1$ when $R_1^A = s_1^B = u_1$ for the same reason. Thirdly, suppose that $\hat{\theta}_1^B\left(\left(R_1^A, d_1, o_1\right), r_1\right) = \theta$. Then B reports $R_1^B = d_1$ when $R_1^A = s_1^B = d_1$ because $\Pr\left(d_1 \middle| d_1, d_1\right) > \Pr\left(u_1 \middle| d_1, d_1\right)$ (Remember that we assume an economist reports the forecast of the higher possibility to be correct when the expected utilities of two forecasts are the same). The same argument shows that $r_1^B = d_1$ is not an equilibrium.

Next we show that $r_1^B = l_1^B$ is not an equilibrium. If this is an equilibrium, $\hat{\theta}_1^B\left(h_1, r_1\right) = \theta_1^B\left(R_1^A, R_1^B, o_1\right)$. We show that B has an incentive to deviate when $s_1^B \neq R_1^A$. Suppose $s_1^B = d_1 \neq R_1^A = u_1$. Then $o_1 = u_1$ with probability 0.5. If B reports $R_1^B = d_1$,

$$\hat{\theta}_1^B = \theta_1^B(u_1, d_1, u_1) = \frac{2\theta(1-p)}{1+\theta}$$

when $o_1 = u_1$ (with probability 0.5) and

$$\hat{\theta}_1^B = \theta_1^B(u_1, d_1, d_1) = \frac{2\theta p}{1+\theta}$$

when $o_1 = d_1$ (with probability 0.5). If B reports $R_1^B = u_1$,

$$\hat{\theta}_1^B = \theta_1^B(u_1, u_1, u_1) = \frac{p\theta(1+\theta)}{0.5 + 2\theta(p - 0.5) + 0.5\theta^2} > \theta_1^B(u_1, d_1, d_1)$$

when $o_1 = u_1$ (with probability 0.5) and

$$\hat{\theta}_1^B = \theta_1^B(u_1, u_1, d_1) = \frac{\theta(1+\theta)(1-p)}{0.5 - 2\theta(p - 0.5) + 0.5\theta^2} > \theta_1^B(u_1, d_1, u_1)$$

when $o_1 = d_1$ (with probability 0.5). Since $\hat{\theta}_2^B$ is increasing in $\hat{\theta}_1^B$, B's expected total

utility $(\hat{\theta}_1^B + \hat{\theta}_2^B)$ increases by deviation.

Finally, reporting the reverse of s_t^B (R_t^A) cannot be an equilibrium because this is

less likely to be correct than $r_t^B = I_t^B$ ($r_t^B = m_t^B$). Q.E.D.

Proof of Proposition 4-1.

Let us assume $R_t^A = u_t$ for $t = 1,2$ without loss of generality. B chooses R_2^B that

maximizes $\hat{\theta}_2^B$. Define $E\hat{\theta}_2^B(R_2^A, s_2^B, R_2^B)$ as the expectation of $\hat{\theta}_2^B$ when A reports R_2^A,

B observes s_2^B, and he reports R_2^B. If B sticks to the equilibrium strategy and make the

same report as A,

$$E\hat{\theta}_2^B(u_2, u_2, u_2) = E\hat{\theta}_2^B(u_2, d_2, u_2) = \theta$$

because the market cannot infer s_2^B. We show that B does not have an incentive to

deviate even if the market believes that deviation implies $s_1^B = o_1$, i.e.

$$\hat{\theta}_{12}^B(h_1, R_2^A, R_2^B, r_1, r_2) = \theta_1^B(R_1^A, o_1, o_1) \text{ for } R_2^B \neq R_2^A.$$

92

Define $\overline{\theta}_1 \equiv \theta_1^i \left(R_1^A, o_1, o_1 \right)$ and $\overline{\theta} \equiv \left(\overline{\theta}_A, \overline{\theta}_B \right)$. Then

$$E\hat{\theta}_2^B \left(u_2, u_2, d_2 \right) = \theta_2^B \left(u_2, d_2, u_2 | \overline{\theta} \right) \Pr \left(u_2 | u_2, u_2 \right) + \theta_2^B \left(u_2, d_2, d_2 | \overline{\theta} \right) \Pr \left(d_2 | u_2, u_2 \right), \text{ and}$$

$$E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right) = \theta_2^B \left(u_2, d_2, u_2 | \overline{\theta} \right) \Pr \left(u_2 | u_2, d_2 \right) + \theta_2^B \left(u_2, d_2, d_2 | \overline{\theta} \right) \Pr \left(d_2 | u_2, d_2 \right)$$

Since $\theta_2^B \left(u_2, d_2, u_2 | \overline{\theta} \right) < \theta_2^B \left(u_2, d_2, d_2 | \overline{\theta} \right)$ and $\Pr \left(u_2 | u_2, u_2 \right) > \Pr \left(u_2 | u_2, d_2 \right)$,

$$E\hat{\theta}_2^B \left(u_2, u_2, d_2 \right) < E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right)$$

is always satisfied. Furthermore, $E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right)$ is increasing in $\Pr \left(d_2 | u_2, d_2 \right)$, which

is increasing in θ_1^B. Thus the necessary and sufficient condition for B not to deviate is

$E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right) < \theta$ when $s_1^B = o_1$. If $R_1^A = o_1$,

$$E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right) = \frac{2p\theta(1 + \theta)}{1 + 6p\theta - 2\theta + \theta^2 + 2p\theta^2}$$

and the condition turns out to be

$$\theta \geq \frac{1}{3} \quad \text{or} \quad p \leq \frac{(1 - \theta)^2}{2 \left(1 - 2\theta - \theta^2 \right)},$$

If $R_1^A \neq o_1$,

$$E\hat{\theta}_2^B \left(u_2, d_2, d_2 \right) = \frac{2p\theta(1 + \theta) - 4\theta^2 p(1 - p)}{(1 + \theta)^2 - 4\theta^2 p(1 - p)}$$

and the condition is

$$p \leq \frac{-1 + \theta - 2\theta^2 + \sqrt{-8\theta^3 + 9\theta^2 + 2\theta + 1}}{4\theta(1 - \theta)}. \quad \text{Q.E.D.}$$

Proof of Lemma 4-2.

The necessary and sufficient condition for B not to deviate is

$E\hat{\theta}_2^B(u_2,d_2,d_2) > E\hat{\theta}_2^B(u_2,d_2,u_2)$ for any s_1^B and o_1.

We show that the above condition is not satisfied when $s_1^B \neq o_1$. If $r_2^B = I_2^B$ is an equilibrium, we can write $\hat{\theta}_2^B = \theta_2^B(R_2^A, R_2^B, o_2)$ for given (h_1, r_1, r_2). Then, since

$$E\hat{\theta}_2^B(u_2,d_2,d_2) = \theta_2^B(u_2,d_2,u_2)\Pr(u_2|u_2,d_2) + \theta_2^B(u_2,d_2,d_2)\Pr(d_2|u_2,d_2) \text{ and}$$

$$E\hat{\theta}_2^B(u_2,d_2,u_2) = \theta_2^B(u_2,u_2,u_2)\Pr(u_2|u_2,d_2) + \theta_2^B(u_2,u_2,d_2)\Pr(d_2|u_2,d_2).$$

$$E\hat{\theta}_2^B(u_2,d_2,d_2) - E\hat{\theta}_2^B(u_2,d_2,u_2)$$

$$= \{\theta_2^B(u_2,d_2,u_2) - \theta_2^B(u_2,u_2,d_2)\}\Pr(d_2|u_2,d_2)$$

$$+ \{\theta_2^B(u_2,d_2,d_2) - \theta_2^B(u_2,u_2,u_2)\}\Pr(d_2|u_2,d_2)$$

$$+ \{\theta_2^B(u_2,d_2,u_2) - \theta_2^B(u_2,u_2,u_2)\}\{\Pr(u_2|u_2,d_2) - \Pr(d_2|u_2,d_2)\}$$

This equation is always negative for $s_1^B \neq o_1$, because

$$\Pr(u_2|u_2,d_2) \geq \Pr(d_2|u_2,d_2) \text{ (from } \theta_A^1 \geq \theta_B^1),$$

$$\theta_2^B(u_2,d_2,u_2) < \theta_2^B(u_2,u_2,d_2) < \theta_2^B(u_2,u_2,u_2),$$

and $\theta_2^B(u_2,d_2,d_2) < \theta_2^B(u_2,u_2,u_2)$. Q.E.D.

Proof of Proposition 4-2.

We show B has no incentive to deviate from the equilibrium. We assume $R_t^A = u_t$ for $t = 1, 2$ without loss of generality.

Case (a): $R_1^A = o_1 = u_1$.

$$E\hat{\theta}_2^B(u_2,u_2,u_2) = E\hat{\theta}_2^B(u_2,d_2,u_2)$$

94

$$= \frac{\Pr(u_1, d_1 | u_1) \theta_1^B(u_1, d_1, u_1) + \Pr(u_1, u_1 | u_1) \Pr(u_2, u_2) \theta_1^B(u_1, u_1, u_1)}{\Pr(u_1, d_1 | u_1) + \Pr(u_1, u_1 | u_1) \Pr(u_2, u_2)}$$

$$= \frac{4\theta(1-\theta)(1-p)\alpha^2 + 2p\theta(1+\theta)\alpha^2 + 8p^3\theta^3(1+\theta)^3}{2(1-\theta)(1+\theta)\alpha^2 + \alpha^3 + 4p^2\theta^2(1+\theta)^2\alpha},$$

where $\alpha \equiv 1 + 2\theta(2p-1) + \theta^2$.

As we show in the proof of Proposition 1, $E\hat{\theta}_2^B(u_2, d_2, d_2) > E\hat{\theta}_2^B(u_2, u_2, d_2)$.

$$E\hat{\theta}_2^B(u_2, d_2, d_2) = \theta_2^B(u_2, d_2, u_2 | \overline{\theta}) \Pr(u_2 | u_2, d_2) + \theta_2^B(u_2, d_2, d_2 | \overline{\theta}) \Pr(d_2 | u_2, d_2)$$

$$= \frac{\hat{\theta}_1^B(1 - \hat{\theta}_1^A)(1-p)}{1 + (\hat{\theta}_1^A - \hat{\theta}_1^B)(2p-1) - \hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1 + (\theta_A - \theta_B)(2p-1) - \theta_A\theta_B}{1 - \theta_A\theta_B}$$

$$+ \frac{p\hat{\theta}_1^B(1 - \hat{\theta}_1^A)}{1 - (\hat{\theta}_1^B - \hat{\theta}_1^B)(2p-1) - \hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1 - (\theta_A - \theta_B)(2p-1) - \theta_A\theta_B}{1 - \theta_A\theta_B},$$

where $\hat{\theta}_1^A = \hat{\theta}_1^B = \theta_1^A(u_1, u_1, u_1) = \dfrac{2p\theta(1+\theta)}{1 + 2\theta(2p-1) + \theta^2}$.

Case (a1): $R_1^A = o_1 = u_1 \neq s_1^B$ and $s_2^B = d_2$.

Since $\theta_A = \theta_1^A(u_1, d_1, u_1) = \dfrac{2\theta p}{1+\theta}$ and $\theta_B = \theta_1^B(u_1, d_1, u_1) = \dfrac{2\theta(1-p)}{1+\theta}$,

$$E\hat{\theta}_2^B(u_2, d_2, d_2) = \frac{2p\theta(1+\theta)}{\alpha + 2p\theta(1+\theta)} \cdot \frac{(1+\theta)^2 - 4\theta^2 p(1-p) - 2\theta(1+\theta)(2p-1)^3}{(1+\theta)^2 - 4\theta^2 p(1-p)}.$$

B chooses the equilibrium strategy if

$$E\hat{\theta}_2^B(u_2, d_2, d_2) < E\hat{\theta}_2^B(u_2, d_2, u_2). \qquad \text{(a1)}$$

Case (a2): $R_1^A = o_1 = u_1 = s_1^B$.

Since $\theta_A = \theta_B = \theta_1^1(u_1, u_1, u_1) = \dfrac{2p\theta(1+\theta)}{1 + 2\theta(2p-1) + \theta^2}$,

$$E\hat{\theta}_2^B(u_2, u_2, d_2) = \frac{2p\theta(1+\theta)}{\alpha + 2p\theta(1+\theta)} \cdot \frac{\alpha^2 + 4\theta^2 p^2(1+\theta)^2 - 4p\theta(1+\theta)(2p-1)^2\alpha}{\alpha^2 + 4\theta^2 p^2(1+\theta)^2}$$

and $E\hat{\theta}_2^B(u_2,d_2,d_2) = \dfrac{2p\theta(1+\theta)}{\alpha+2p\theta(1+\theta)}$.

B chooses the equilibrium strategy if

$$E\hat{\theta}_2^B(u_2,u_2,d_2) < E\hat{\theta}_2^B(u_2,u_2,u_2) \qquad (a2)$$

and $E\hat{\theta}_2^B(u_2,d_2,d_2) > E\hat{\theta}_2^B(u_2,d_2,u_2)$. $\qquad$ (a3)

Case (b): $R_1^A \neq o_1 = d_1$.

$$E\hat{\theta}_2^B(u_2,u_2,u_2) = E\hat{\theta}_2^B(u_2,d_2,u_2)$$

$$= \frac{\Pr(u_1,u_1|d_1)\theta_1^B(u_1,u_1,d_1) + \Pr(u_1,d_1|d_1)\Pr(u_2,u_2)\theta_1^B(u_1,d_1,d_1)}{\Pr(u_1,u_1|d_1) + \Pr(u_1,d_1|d_1)\Pr(u_2,u_2)}$$

$$= \frac{4\theta(1+\theta)^3(1-p) + 2p\theta(1-\theta)(1+\theta)^2 + 8p^2\theta^3(1-p)(1-\theta)}{2(1+\theta)^2\beta + (1-\theta)(1+\theta)^3 + 4p\theta^2(1-p)(1-\theta)(1+\theta)}$$

where $\beta \equiv 1 - 2\theta(2p-1) + \theta^2$,

$$E\hat{\theta}_2^B(u_2,d_2,d_2) = \frac{\hat{\theta}_1^B(1-\hat{\theta}_1^A)(1-p)}{1+(\hat{\theta}_1^A-\hat{\theta}_1^B)(2p-1)-\hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1+(\theta_A-\theta_B)(2p-1)-\theta_A\theta_B}{1-\theta_A\theta_B}$$

$$+ \frac{p\hat{\theta}_1^B(1-\hat{\theta}_1^A)}{1-(\hat{\theta}_1^A-\hat{\theta}_1^B)(2p-1)-\hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1-(\theta_A-\theta_B)(2p-1)-\theta_A\theta_B}{1-\theta_A\theta_B}$$

where $\hat{\theta}_1^A = \theta_1^A(u_1,d_1,d_1) = \dfrac{2\theta(1-p)}{1+\theta}$, and

$$\hat{\theta}_1^B = \theta_1^B(u_1,d_1,d_1) = \frac{2p\theta}{1+\theta}.$$

Case (b1): $R_1^A = s_1^B \neq o_1 = d_1$ and $s_2^B = d_2$.

Since $\theta_A = \theta_B = \theta_1^i(u_1,u_1,d_1) = \dfrac{2\theta(1+\theta)(1-p)}{1-2\theta(2p-1)+\theta^2}$,

$$E\hat{\theta}_2^B(u_2,d_2,d_2) = \frac{2p\theta(1-\theta+2p\theta)[(1+\theta)^2 - 2\theta(1+\theta)(2p-1)^3 - 4\theta^2p(1-p)]}{(1+\theta)^4 - 8\theta^2p(1+\theta)^2(1-p) + 16\theta^4p^2(1-p)^2 - 4\theta^2(1+\theta)^2(2p-1)^4}$$

B chooses the equilibrium strategy if

$$E\hat{\theta}_2^B(u_2,d_2,d_2) < E\hat{\theta}_2^B(u_2,d_2,u_2) \qquad \text{(b1)}$$

Case (b2): $R_1^A \neq o_1 = d_1 = s_1^B$.

Since $\theta_A = \theta_1^A(u_1,d_1,d_1) = \dfrac{2\theta(1-p)}{1+\theta}$ and $\theta_B = \theta_1^B(u_1,d_1,d_1) = \dfrac{2p\theta}{1+\theta}$,

$$E\hat{\theta}_2^B(u_2,u_2,d_2) = \frac{2p\theta(1-\theta+2p\theta)}{(1+\theta)^2 + 4\theta^2 p(1-p)}$$

$$\times \frac{(1+\theta)^4 - 4\theta(1+\theta)^2(2p-1)^2(p+\theta-p\theta) + 16\theta^3 p(1-p)^2\left[(1+\theta)(2p-1)^2 - p\theta\right]}{(1+\theta)^4 - 8\theta^2 p(1-p)(1+\theta)^2 + 16\theta^4 p^2(1-p)^2 - 4\theta^2(1+\theta)^2(2p-1)^4}$$

and $E\hat{\theta}_2^B(u_2,d_2,d_2) = \dfrac{2p\theta(1+\theta) - 4\theta^2 p(1-p)}{(1+\theta)^2 - 4\theta^2 p(1-p)}$.

B chooses the equilibrium strategy if

$$E\hat{\theta}_2^B(u_2,u_2,d_2) < E\hat{\theta}_2^B(u_2,u_2,u_2) \qquad \text{(b2)}$$

and $\quad E\hat{\theta}_2^B(u_2,d_2,d_2) > E\hat{\theta}_2^B(u_2,d_2,u_2). \qquad \text{(b3)}$

B follows the equilibrium strategy if (p,θ) satisfies all of (a1), (a2), (a3), (b1), (b2), and (b3). $(p,\theta) = (0.75, 0.36)$ is one example. Q.E.D.

Proof of Lemma 4-3.

We assume $R_t^A = u_t$ for $t = 1,2$ without loss of generality. Since

$$E\hat{\theta}_2^B(u_2,u_2,u_2) = E\hat{\theta}_2^B(u_2,d_2,u_2) = \hat{\theta}_1^B,$$

$$E\hat{\theta}_2^B(u_2,d_2,d_2) = \frac{\hat{\theta}_1^B(1-\hat{\theta}_1^A)(1-p)}{1+(\hat{\theta}_1^A - \hat{\theta}_1^B)(2p-1) - \hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1+(\theta_A - \theta_B)(2p-1) - \theta_A\theta_B}{1-\theta_A\theta_B}$$

$$+ \frac{p\hat{\theta}_1^B(1-\hat{\theta}_1^A)}{1-(\hat{\theta}_1^A - \hat{\theta}_1^B)(2p-1) - \hat{\theta}_1^A\hat{\theta}_1^B} \cdot \frac{1-(\theta_A - \theta_B)(2p-1) - \theta_A\theta_B}{1-\theta_A\theta_B},$$

and $\quad E\hat{\theta}_2^B(u_2,u_2,d_2) < E\hat{\theta}_2^B(u_2,d_2,d_2),$

97

all we have to show is

$$\hat{\theta}_1^B > E\hat{\theta}_2^B(u_2,d_2,d_2) \quad \forall(s_1^B,o_1), \quad \forall p \in (0.5,1), \text{ and } \quad \forall \theta \in (0,1).$$

Case 1: $s_1^B = u_1 = o_1$.

Since $\theta_A = \theta_B = \theta_1^i(u_1,u_1,u_1) = \dfrac{2p\theta(1+\theta)}{1+2\theta(2p-1)+\theta^2}$,

$$\hat{\theta}_1^A = \theta_1^A(u_1,d_1,u_1) = \frac{2p\theta}{1+\theta},$$

and $\hat{\theta}_1^B = \hat{\theta}_1^B(u_1,d_1,u_1) = \dfrac{2\theta(1-p)}{1+\theta}$

in this case, the condition turns out to be

$$Z \equiv (1-\theta)(1+\theta)^2 + 2p(1+\theta)^2(-2+3\theta) + 8p^3\theta(2+4\theta+\theta^2) + 4p^2(2-7\theta^2-3\theta^3) > 0.$$

This is always satisfied because $\dfrac{\partial^2 Z}{\partial p^2} > 0$, $\dfrac{\partial Z}{\partial p} > 0$ at $p = 0.5$, and $Z > 0$ at $p = 0.5$.

Case 2: $s_1^B = u_1 \neq o_1$.

Since $\theta_A = \theta_B = \theta_1^i(u_1,u_1,d_1) = \dfrac{2\theta(1+\theta)(1-p)}{1-2\theta(2p-1)+\theta^2}$,

$$\hat{\theta}_1^A = \theta_1^A(u_1,d_1,d_1) = \frac{2\theta(1-p)}{1+\theta},$$

and $\hat{\theta}_1^B = \hat{\theta}_1^B(u_1,d_1,d_1) = \dfrac{2p\theta}{1+\theta}$

in this case, the condition turns out to be

$$Y \equiv 5 - 12p + 8p^2 + (15 - 46p + 48p^2 - 16p^3)\theta + (11 - 48p + 68p^2 - 32p^3)\theta^2$$
$$+ (1 - 6p + 12p^2 - 8p^3)\theta^3 > 0.$$

This is always satisfied because $\dfrac{\partial^2 Y}{\partial \theta^2} < 0$ and $\dfrac{\partial Y}{\partial \theta} > 0$ at $\theta \in \{0,1\}$.

Case 3: $s_1^B = d_1$.

Since $\theta_A = \hat{\theta}_1^i$, $E\hat{\theta}_2^B(u_2,d_2,d_2) = \dfrac{\hat{\theta}_1^B(1-\hat{\theta}_1^A)}{1-\hat{\theta}_1^A\hat{\theta}_1^B} < \hat{\theta}_1^B$.

The above argument shows that B always mimics A in period 2. Q.E.D.

Proof of Proposition 4-3.

We assume $s_t^A = u_t$ for $t = 1, 2$ without loss of generality. Define $E\Pi_i \equiv E\hat{\theta}_i^1 + E\hat{\theta}_i^2$ as the sum of expected utility economist i earns in two periods.

We show first that B has no incentive to deviate when A follows the equilibrium strategy. Since Proposition 4-1 shows that B never deviates in period 2 when (1) and (2) are satisfied, we have only to prove that B never deviates in period 1. If B follows the equilibrium strategy, $E\Pi_B = 2\theta$. If B deviates in period 1, Lemma 4-3 shows that he always mimics A in period 2 and thus $\hat{\theta}_2^B = \hat{\theta}_1^B$. Therefore, if he deviates in period 1,

$$E\Pi_B = 2\theta_1^B(u_1, d_1, u_1)\Pr(u_1|u_1, d_1) + 2\theta_1^B(u_1, d_1, d_1)\Pr(d_1|u_1, d_1)$$

$$= \frac{2\theta}{1+\theta} < 2\theta$$

when $s_1^B = d_1$, and

$$E\Pi_B = 2\theta_1^B(u_1, d_1, u_1)\Pr(u_1|u_1, u_1) + 2\theta_1^B(u_1, d_1, d_1)\Pr(d_1|u_1, u_1)$$

$$< 2\theta_1^B(u_1, d_1, u_1)\Pr(u_1|u_1, d_1) + 2\theta_1^B(u_1, d_1, d_1)\Pr(d_1|u_1, d_1)$$

when $s_1^B = u_1$. Consequently B has no incentive to deviate.

Next we show that A has no incentive to deviate when B follows the equilibrium strategy. Since the conditional probability that u_t occurs when $s_t^A = u_t$ is

$$\Pr(u_t|u_t) = 0.5 + \theta_A(p - 0.5) > 0.5,$$

the market evaluation of A when B always mimics A, $\hat{\theta}_1^A(R_1^A, o_1)$, is

$$\hat{\theta}_1^A(u_1, u_1) = \hat{\theta}_1^A(d_1, d_1) = \frac{2p\theta}{1+\theta(2p-1)}$$ and

$$\hat{\theta}_1^A(u_1, d_1) = \hat{\theta}_1^A(d_1, u_1) = \frac{2\theta(1-p)}{1+\theta(2p-1)}.$$

Similarly we obtain $\hat{\theta}_2^A(R_2^A, o_2)$ by substituting $\hat{\theta}_1^A$ for θ in the above equations.

Suppose $s_2^A = u_2$. If A reports $R_2^A = u_2$, the expectation of $\hat{\theta}_2^A$ is

$$\hat{\theta}_2^A(u_2, u_2)\Pr(u_2|u_2) + \hat{\theta}_2^A(u_2, d_2)\Pr(d_2|u_2)$$

If A reports $R_2^A = \acute{d}_2$, the expectation of $\hat{\theta}_2^A$ decreases to

$$\hat{\theta}_2^A(d_2, u_2)\Pr(u_2|u_2) + \hat{\theta}_2^A(d_2, d_2)\Pr(d_2|u_2).$$

Therefore A does not deviate from the equilibrium in period 2. Since the same argument applies to period 1 and $\hat{\theta}_2^A$ is increasing in $\hat{\theta}_1^A$, A does not deviate in period 1. Q.E.D.

Proof of Proposition 4-4.

Assume $s_t^A = u_t$ for $t = 1, 2$. We show first that B has no incentive to deviate. Proposition 4-2 shows that B never deviates in period 2 if (p, θ) lies in the shaded area in Figure 4-2. If B deviates in period 1, the argument in the proof of Proposition 4-3 shows that $E\Pi_B = \frac{2\theta}{1+\theta}$ at most. If B follows the equilibrium strategy, $\hat{\theta}_1^B = \theta$ and the expectation of $\hat{\theta}_2^B$ is as follows (See the proof of Proposition 4-2).

$$E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_1^A = o_1\right) = \frac{4\theta(1-\theta)(1-p)\alpha^2 + 2p\theta(1+\theta)\alpha^2 + 8p^3\theta^3(1+\theta)^3}{2(1-\theta)(1+\theta)\alpha^2 + \alpha^3 + 4p^2\theta^2(1+\theta)^2\alpha}$$

where $\alpha \equiv 1 + 2\theta(2p-1) + \theta^2$,

$$E\hat{\theta}_2^B\left(R_2^B \neq R_2^A \middle| R_1^A = o_1\right) = \frac{2p\theta(1+\theta)}{\alpha + 2p\theta(1+\theta)},$$

$$E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_1^A \neq o_1\right) = \frac{4\theta(1+\theta)^3(1-p) + 2p\theta(1-\theta)(1+\theta)^2 + 8p^2\theta^3(1-p)(1-\theta)}{2(1+\theta)^2\beta + (1-\theta)(1+\theta)^3 + 4p\theta^2(1-p)(1-\theta)(1+\theta)}$$

where $\beta \equiv 1 - 2\theta(2p-1) + \theta^2$, and

$$E\hat{\theta}_2^B\left(R_2^B \neq R_2^A \middle| R_1^A \neq o_1\right) = \frac{2p\theta(1+\theta) - 4\theta^2 p(1-p)}{(1+\theta)^2 - 4\theta^2 p(1-p)}.$$

When $s_1^B = u_1 \left(= s_1^A\right)$,

$$E\Pi_B = \theta + \Pr(d_1|u_1,u_1)\, E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_1^A \neq o_1\right)$$

$$+ \Pr(u_1|u_1,u_1)\Pr(u_2,u_2) E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_1^A = o_1\right)$$

$$+ \Pr(u_1|u_1,u_1)\Pr(u_2,d_2) E\hat{\theta}_2^B\left(R_2^B \neq R_2^A \middle| R_1^A = o_1\right)$$

where $\Pr(u_2,u_2) = \dfrac{1}{2} + \dfrac{2p^2\theta^2(1+\theta)^2}{\alpha^2}$.

When $s_1^B = d_1 \left(\neq s_1^A\right)$,

$$E\Pi_B = \theta + \Pr(u_1|u_1,d_1)\, E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_1^A = o_1\right)$$

$$+ \Pr(d_1|u_1,d_1)\Pr(u_2,u_2) E\hat{\theta}_2^B\left(R_2^B = R_2^A \middle| R_3^A \neq o_1\right)$$

$$+ \Pr(d_1|u_1,d_1)\Pr(u_2,d_2) E\hat{\theta}_2^B\left(R_2^B \neq R_2^A \middle| R_1^A \neq o_1\right)$$

where $\Pr(u_2,u_2) = \dfrac{1}{2} + \dfrac{2p\theta^2(1-p)}{(1+\theta)^2}.$

Since calculation shows that both are larger than $\dfrac{2\theta}{1+\theta}$, B has no incentive to deviate in period 1.

Next we show that A never deviates from the equilibrium. Since B always mimics A in period 1, the market evaluation of A in period 1, $\hat{\theta}_1^A\left(R_1^A, o_1\right)$, is

$$\hat{\theta}_1^A(u_1,u_1) = \hat{\theta}_1^A(d_1,d_1) = \frac{2p\theta}{1+\theta(2p-1)}.$$

$$\hat{\theta}_1^A(u_1, d_1) = \hat{\theta}_1^A(d_1, u_1) = \frac{2\theta(1-p)}{1+\theta(2p-1)}.$$

The market evaluation in period 2 when $R_2^A = R_2^B$ is as follows.

$$\hat{\theta}_2^A(R_1^A = o_1, R_2^A = o_2) = \frac{p_1\theta_1 + p_2\theta_2 + p_3\theta_3}{p_1 + p_2 + p_3}$$

where $p_1 \equiv \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)$

$$= \frac{(1+\theta^2)}{8}\left[1 + \frac{4p\theta(1+\theta)(2p-1)}{\alpha} + \frac{4p^2\theta^2(1+\theta)^2}{\alpha^2}\right],$$

$$p_2 \equiv \Pr(u_1, d_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)$$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 + \frac{2\theta(2p-1)}{1+\theta} + \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_3 \equiv \Pr(u_1, d_1)\Pr(u_2, d_2)\Pr(u_2|u_2, d_2)$$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 + \frac{2\theta(2p-1)^2}{1+\theta} - \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_1\theta_1 = 0.5\alpha^{-2}p^2\theta(1+\theta)(1+\theta^2)[\alpha + 2p\theta(1+\theta)],$$

$$p_2\theta_2 = 0.5(1+\theta)^{-1}p^2\theta(1-\theta)(1+3\theta-2p\theta), \text{ and}$$

$$p_3\theta_3 = 0.5(1+\theta)^{-1}p^2\theta(1-\theta)(1-\theta+2p\theta).$$

$$\hat{\theta}_2^A(R_1^A = o_1, R_2^A \neq o_2) = \frac{p_4\theta_4 + p_5\theta_5 + p_6\theta_6}{p_4 + p_5 + p_6}$$

where $p_4 \equiv \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(d_2|u_2, u_2)$

$$= \frac{(1+\theta^2)}{8}\left[1 - \frac{4p\theta(1+\theta)(2p-1)}{\alpha} + \frac{4p^2\theta^2(1+\theta)^2}{\alpha^2}\right],$$

$$p_5 \equiv \Pr(u_1, d_1)\Pr(u_2, u_2)\Pr(d_2|u_2, u_2)$$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 - \frac{2\theta(2p-1)}{1+\theta} + \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_6 \equiv \Pr(u_1, d_1)\Pr(u_2, d_2)\Pr(d_2|u_2, d_2)$$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 - \frac{2\theta(2p-1)^2}{1+\theta} - \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_4\theta_4 = 0.5\alpha^{-2} p\theta(1-p)(1+\theta)(1+\theta^2)[\alpha + 2p\theta(1+\theta)],$$

$$p_5\theta_5 = 0.5(1+\theta)^{-1} p\theta(1-p)(1-\theta)(1+3\theta-2p\theta), \text{ and}$$

$$p_6\theta_6 = 0.5(1+\theta)^{-1} p\theta(1-p)(1-\theta)(1-\theta+2p\theta).$$

$$\hat{\theta}_2^d\left(R_1^d \ne o_1, R_2^d = o_2\right) = \frac{p_7\theta_7 + p_8\theta_8 + p_9\theta_9}{p_7 + p_8 + p_9}$$

where $p_7 \equiv \Pr(u_1, d_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 + \frac{2\theta(2p-1)}{1+\theta} + \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_8 \equiv \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)$$

$$= \frac{(1+\theta^2)}{8}\left[1 + \frac{4\theta(1-p)(1+\theta)(2p-1)}{\beta} + \frac{4\theta^2(1-p)^2(1+\theta)^2}{\beta^2}\right],$$

$$p_9 \equiv \Pr(u_1, u_1)\Pr(u_2, d_2)\Pr(u_2|u_2, d_2)$$

$$= \frac{(1+\theta^2)}{8}\left[1 - \frac{4\theta^2(1+\theta)^2(1-p)^2}{\beta^2}\right],$$

$$p_7\theta_7 = 0.5(1+\theta)^{-1} p\theta(1-\theta)(1-p)(1+\theta+2p\theta),$$

$$p_8\theta_8 = 0.5\beta^{-2} p\theta(1+\theta)(1+\theta^2)(1-p)[\alpha + 2\theta(1+\theta)(1-p)], \text{ and}$$

$$p_9\theta_9 = 0.5\beta^{-2} p\theta(1+\theta)(1+\theta^2)(1-p)[\alpha - 2\theta(1+\theta)(1-p)].$$

$$\hat{\theta}_2^A\left(R_1^A \neq o_1, R_2^A \neq o_2\right) = \frac{p_{10}\theta_{10} + p_{11}\theta_{11} + p_{12}\theta_{12}}{p_{10} + p_{11} + p_{12}}$$

where $p_{10} \equiv \Pr(u_1, d_1)\Pr(u_2, u_2)\Pr(d_2|u_2, u_2)$

$$= \frac{(1-\theta)(1+\theta)}{8}\left[1 - \frac{2\theta(2p-1)}{1+\theta} + \frac{4\theta^2 p(1-p)}{(1+\theta)^2}\right],$$

$$p_{11} \equiv \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(d_2|u_2, u_2)$$

$$= \frac{(1+\theta^2)}{8}\left[1 - \frac{4\theta(1-p)(1+\theta)(2p-1)}{\beta} + \frac{4\theta^2(1-p)^2(1+\theta)^2}{\beta^2}\right],$$

$$p_{12} \equiv \Pr(u_1, u_1)\Pr(u_2, d_2)\Pr(d_2|u_2, d_2)$$

$$= \frac{(1+\theta^2)}{8}\left[1 - \frac{4\theta^2(1+\theta)^2(1-p)^2}{\beta^2}\right],$$

$$p_{10}\theta_{10} = 0.5(1+\theta)^{-1}\theta(1-\theta)(1-p)^2(1+\theta+2p\theta),$$

$$p_{11}\theta_{11} = 0.5\beta^{-2}\theta(1+\theta)(1+\theta^2)(1-p)^2[\alpha + 2\theta(1+\theta)(1-p)], \text{ and}$$

$$p_{12}\theta_{12} = 0.5\beta^{-2}\theta(1+\theta)(1+\theta^2)(1-p)^2[\alpha - 2\theta(1+\theta)(1-p)].$$

Define $E\hat{\theta}_2^A\left(R_1^A, o_1, R_2^A\right)$ as the expectation of $\hat{\theta}_2^A$. Since

$$E\hat{\theta}_2^A(u_1, u_1, u_2) = \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)\,\theta_A^2\left(R_1^A = o_1, R_2^A = o_2\right)$$

$$+ \Pr(u_1, u_1)\Pr(u_2, u_2)\Pr(d_2|u_2, u_2)\,\hat{\theta}_A^2\left(R_1^A = o_1, R_2^A \neq o_2\right)$$

$$+ \Pr(u_1, u_1)\Pr(u_2, d_2)\Pr(u_2|u_2, d_2)\,\theta_A^2(u_1, u_1, u_1, u_2, d_2, u_2)$$

$$+ \Pr(u_1, u_1)\Pr(u_2, d_2)\Pr(d_2|u_2, d_2)\,\theta_A^2(u_1, u_1, u_1, u_2, d_2, d_2)$$

$$+ \Pr(u_1, d_1)\Pr(u_2, u_2)\Pr(u_2|u_2, u_2)\,\theta_A^2\left(R_1^A = o_1, R_2^A = o_2\right)$$

$$+ \Pr(u_1, d_1) \Pr(u_2, u_2) \Pr(d_2|u_2, u_2) \, \hat{\theta}_A^2 \left(R_1^A = o_1, R_2^A \neq o_2\right)$$

$$+ \Pr(u_1, d_1) \Pr(u_2, d_2) \Pr(u_2|u_2, d_2) \, \theta_A^2 \left(R_1^A = o_1, R_2^A = o_2\right)$$

$$+ \Pr(u_1, d_1) \Pr(u_2, d_2) \Pr(d_2|u_2, d_2) \, \theta_A^2 \left(R_1^A = o_1, R_2^A \neq o_2\right)$$

$$= p\theta\alpha^{-1}(1 + \theta)(1 + \theta^2) + p\theta(1 - \theta) > E\hat{\theta}_2^A(u_1, u_1, d_2)$$

and similarly

$$E\hat{\theta}_2^A(u_1, d_1, u_2) > E\hat{\theta}_2^A(u_1, d_1, d_2),$$

$$E\hat{\theta}_2^A(d_1, u_1, u_2) > E\hat{\theta}_2^A(d_1, u_1, d_2), \text{ and}$$

$$E\hat{\theta}_2^A(d_1, d_1, u_2) > E\hat{\theta}_2^A(d_1, d_1, d_2),$$

A never deviates in period 2.

Finally we consider period 1. Suppose $s_1^A = u_1$. If $R_1^A = u_1$, the sum of his expected utility in period 1 and 2 is

$$\Pi_A(R_1^A = s_1^A) = \left[\theta_1^A(u_1, u_1) + E\hat{\theta}_2^A(u_1, u_1, u_2)\right]\Pr(u_1|u_1)$$

$$+ \left[\theta_1^A(u_1, d_1) + E\hat{\theta}_2^A(u_1, d_1, u_2)\right]\Pr(d_1|u_1).$$

If he deviates and chooses $R_1^A = d_1$, the sum is

$$\Pi_A(R_1^A \neq s_1^A) = \left[\theta_1^A(u_1, u_1) + E\hat{\theta}_2^A(d_1, d_1, u_2)\right]\Pr(d_1|u_1)$$

$$+ \left[\theta_1^A(u_1, d_1) + E\hat{\theta}_2^A(d_1, u_1, u_2)\right]\Pr(u_1|u_1).$$

Since calculation shows that

$$\Pi_A(R_1^A = s_1^A) > \Pi_A(R_1^A \neq s_1^A),$$

A never deviates from the equilibrium in period 1. Q.E.D.

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