研 究 速 報

On the Characteristics of Eddies in DNS, GS, and SGS Velocity Fields in Homogeneous Isotropic Turbulence

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1. INTRODUCTION

It is known that turbulent flows contain various types of vortical structures and there is a large range of scales. Theorist believed that tube-like structure is a type of eddy, which is the candidate of fine scale structure, particularly, in the small-scale motions in turbulence[1,2,3]. In recent studies by direct numerical simulations[4,5,6,7], fine scale tube-like eddies in homogeneous turbulence are observed, and the visualization of this small-scale eddies becomes possible. By direct use of local flow pattern in different flow fields in turbulence, Tanahashi *et al.*[7,8,9,10] have shown that the existence of 'coherent fine scale eddies' in turbulence is universal.

Since turbulent flow contains three-dimensional and unsteady structures, more strictly tube-like coherent structures or eddies but development of SGS (Subgrid scale) model based on coherent structures is relatively scarce. Concerning the structure base SGS model, it seems quite important to know what happened in the filtered field for LES (Large Eddy Simulation) including appearance or disappearance of the tube-like eddies as well as the characteristics of these eddies in resolved or unresolved field.

In the previous study[11], we have identified the GS (Grid scale) and SGS (Subgrid scale) eddies in which GS and SGS velocity fields were obtained by filtering the DNS (Direct Numerical Simulations) velocity field in homogeneous isotropic turbulence for performing *a priori test*. We have also studied the characteristics of these GS and SGS eddies for a low Re_λ (Taylor microscale Reynolds number)case[12]. The objective of this study is to investigate the characteristics of the eddies in GS and SGS velocity fields comparing with that of in the DNS velocity fields using different Re_λ cases in homogeneous isotropic turbulence.

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2. GENERATION OF GS AND SGS VELOCITY FIELDS

2.1 DNS Database

In this study, two DNS data sets of decaying homogeneous isotropic turbulence have been used, which are conducted by Tanahashi *et al.*[7] using 128^3 and 216^3 grid points. Reynolds numbers based on u_{rms} and Taylor microscale, λ of the DNS data are $Re_\lambda = 64.9$ and 87.9, respectively.

2.2 GS and SGS Velocity Fields

For generating GS and SGS velocity fields, we have directly filtered above DNS velocity fields using Gaussian and Sharp cutoff filters for LES. In LES, a velocity component u can be decomposed into $\overline{u}$ (GS) and u' (SGS) components so that:

$$u = \overline{u} + u' \qquad (1)$$

We filter DNS velocity fields in the Fourier space rather than physical space. The filtered operation in the Fourier space is given by the relation:

$$\hat{\overline{u}}(\mathbf{k}) = \hat{G}(\mathbf{k})\hat{u}(\mathbf{k}), \mathbf{k} = 0, \pm1, \pm2, \ldots \qquad (2)$$

where $\hat{G}$ is the transfer function associated with the filter kernel G.

For a filter width Δ_i in i-direction, the filter functions in Fourier space are written as follows:

(I) Gaussian filter:

$$\hat{G}(k_i) = \exp\left\{-\frac{(\Delta_i k_i)^2}{24}\right\} \qquad (3)$$

(II) Sharp cutoff filter:

$$\hat{G}(k_i) = \begin{cases} 1, \left(|k_i| \le \dfrac{\pi}{\Delta_i}\right) \\ 0, \left(|k_i| > \dfrac{\pi}{\Delta_i}\right) \end{cases} \qquad (4)$$

Using these two filters for LES in the Fourier space, above two sets of DNS data are filtered and the exact GS velocity fields $\overline{u}$ are obtained. After generating, $\overline{u}$ the SGS velocity field can be obtained by performing the operation:

$$u' = u - \overline{u} \quad \cdots\cdots (5)$$

The characteristic filter width Δ_i is commonly used by most of the researchers as the length, approximately proportional to the grid interval Δ[13,14]. The structures represented by the GS and SGS velocities consequently depend both on the grid interval and on the type of filter employed. In the previous studies[7,8], it is shown that the mean diameter of the coherent fine scale eddy is about 10 times of Kolmogorov microscale (η) in turbulent flows. Therefore, in this study, Δ_i is considered as the length of η in the DNS field with a constant multiplication. Since we are dealing with homogeneous isotropic turbulence, Δ_i is same in each direction and hereafter it is denoted by $\overline{\Delta}$. We considered $\overline{\Delta} = 10\eta$ for $Re_\lambda = 64.9$ and $\overline{\Delta} = 20\eta$ for $Re_\lambda = 87.9$.

Although filter width, $\overline{\Delta} = 10\eta$ or 20η is a small value, but we have seen that the generation of GS velocity field as well as SGS velocity filed from these Re_λ cases using the above filter widths was well possible for both filter functions[11]. Accuracy of this filtering process was also confirmed by comparing the three-dimensional energy spectra for GS as well as SGS velocity fields with DNS velocity field. Detailed of this procedure is reported in our previous studies[11,12,16].

3. DNS, GS, AND SGS EDDIES

Fig.1 shows the contour surfaces of normalized second invariant of the velocity gradient tensor Q in DNS as well as GS and SGS velocity fields in which GS and SGS fields are obtained by using Gaussian filter for Re_λ = 87.9. The second invariant is defined as follows:

$$Q = -\frac{1}{2}\left(S_{ij}S_{ij} - W_{ij}W_{ij}\right), \quad \cdots\cdots (6)$$

where S_{ij} and W_{ij} are the symmetric and antisymmetric part of the velocity gradient tensor A_{ij}. In Fig. 1, the visualized region is 1/8 of the whole calculation domain and the viewpoint for all cases is same. The level of the isosurface is selected to be $Q^* = 0.03$ for DNS field, $Q^* = 0.005$ for GS field and $Q^* = 0.025$ for SGS field. Hereafter, * denotes the normalization by Kolmogorov microscale η and root mean square of velocity fluctuations, u_{rms} obtained from DNS field. Fig. 1 clearly indicates that GS and SGS velocity fields contain lots of distinct tube-like vortical structures somewhat similar to the structures in the DNS velocity fields but different in sizes. Of course, however, the visualization of these structures significantly depends on the threshold values of Q. We define these structures as coherent eddies in DNS, GS and SGS velocity fields. It is also clear that the size or length of the eddies in the GS fields in all cases seems to be larger than that of in the SGS fields. It is known from the classical idea of fluid dynamics that several small-scale structure together form a large-scale structure, i.e., several small (SGS) eddies entirely lie inside a large (GS) eddy in the order of its size. But our DNS database study suggests that the appearance of coherent fine scale eddies in turbulence are quite distinct and unique. The present study also

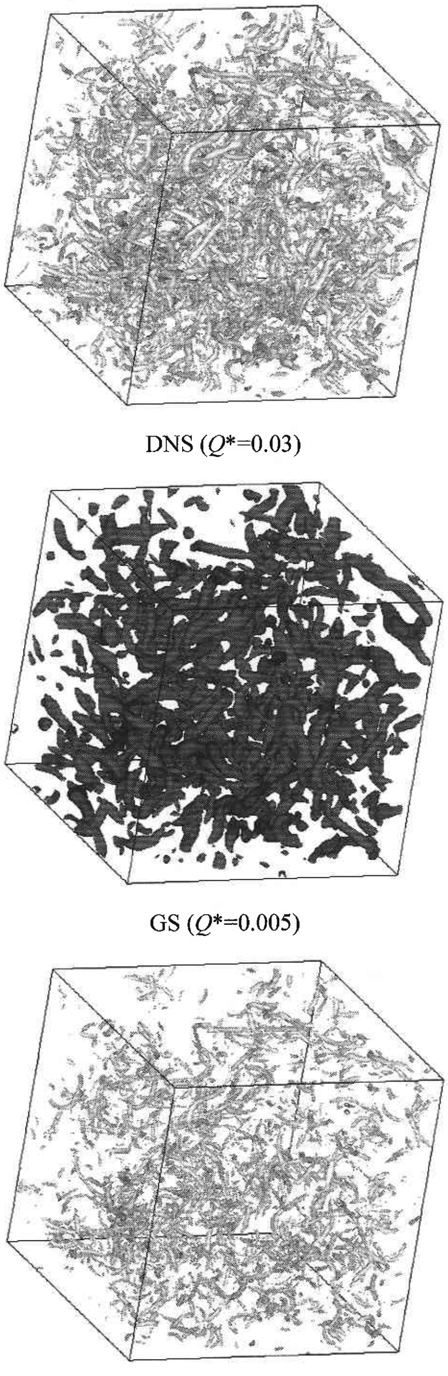

Fig. 1 Contour surfaces of the second invariant of the velocity gradient tensor Q in DNS, GS and SGS velocity fields for the case of Re_λ =87.9. GS and SGS fields are obtained by filtering the DNS field using Gaussian filter. Visualized region is 1/8 of the whole calculation domain. Second invariant is normalized by η and u_{rms} obtained from the DNS field.

reveals the uniqueness of small-scale and large-scale eddies in turbulent flows. The detail definition of the DNS, GS and SGS eddies are given in our previous studies (please see[11,12,16]).

4. CHARACTERISTICS OF DNS, GS, AND SGS EDDIES

4.1 Identification Procedure

For visualization of flows in Fig. 1, positive second invariant Q is used and the existence of many tube-like eddies in DNS, GS and SGS velocity fields have shown in homogeneous isotropic turbulence. Obviously these tube-like eddies contain at least one local maximum of Q on its' axis. In this study, using an auto-tracing algorithm the points with local Q maximal on the cross sections of the axis of this eddy are identified and then the characteristics are investigated. Detailed of this identification procedure are described in our previous studies[7,8,15].

4.2 Characteristics of the Eddies

Figs. 2–3 show the comparisons of normalized mean azimuthal velocity (v_θ*) profiles of the eddies in DNS, GS and SGS velocity fields for both Gaussian and Sharp cutoff filters for Re_λ = 64.9 and 87.9, respectively. In these figures, r represents the radius of the eddy, which is determined by the distance between the center and the location where the mean azimuthal velocity reaches the maximum value. It is revealed that v_θ* profiles of the eddies in GS velocity fields for both filter functions collapse with DNS profile for lower Re_λ case at $r^* = 20\eta$, while for higher Re_λ case it collapse at $r^* = 40\eta$. On the other hand, v_θ* of the eddies in SGS velocity fields for both filter functions collapse each other at $r^* = 10\eta$ for Re_λ = 64.9, and for Re_λ = 87.9 it is at $r^* = 20\eta$, but not with DNS profile in both cases. The maximum of mean azimuthal velocity ($v_{\theta,\max}$*) and mean diameter (D*) of the eddies in DNS velocity fields for both Re_λ cases are about $0.6u_{rms}$ and 10η, respectively, which are same as reported in the previous researches[7,8]. In all cases, the $v_{\theta,\max}$* of the eddies in GS fields is higher than that of in DNS fields, and in SGS fields is lower than that of in DNS fields. Although the $v_{\theta,\max}$* of the eddies in GS fields for Gaussian filter is close to DNS field for both cases, but it is larger than DNS field for Sharp cutoff filter. Similarly, D* of the eddies in GS and SGS fields, respectively, larger and smaller than that of in the DNS fields. Moreover, D* of the eddies for Gaussian filter shows lower values than that of for Sharp cutoff filters in all cases.

The probability density functions (pdf) of mean diameter (D*) of the eddies in DNS, GS and SGS velocity fields, which are normalized by η are shown in Figs. 4–5 for both filter functions and for tow Re_λ cases, respectively. It is clear that the peak of pdf of the eddies in GS velocity fields for all cases do not coincide with each other or with DNS profile. Moreover, the peak of pdf of the eddies in GS fields for Sharp cutoff filters shows higher value than that of

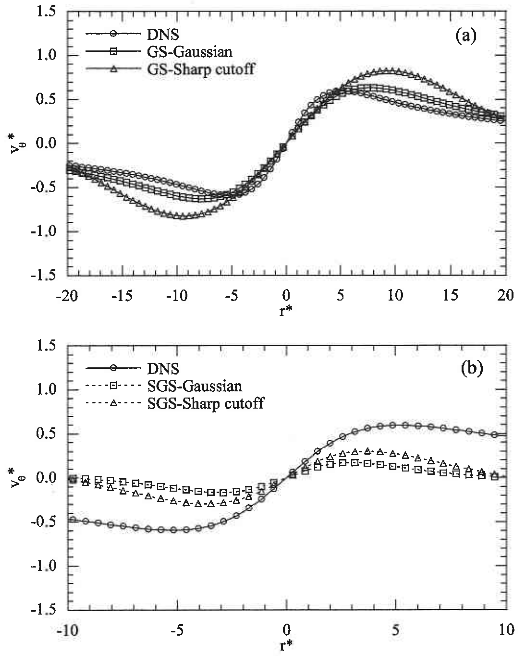

Fig. 2 Comparison of normalized mean azimuthal velocity profile (v_θ*) of the eddies in DNS, GS and SGS velocity fields for the case of Re_λ = 64.9. (a) DNS and GS fields, (b) DNS and SGS fields. In all cases, mean azimuthal velocity profile is normalized by η and u_{rms} obtained from the DNS field.

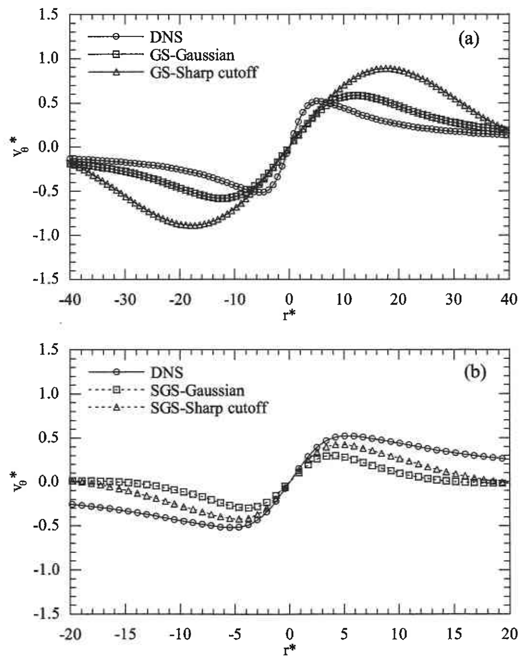

Fig.3 Same as in Fig.2, but Re_λ = 87.9.

for Gaussian filter for both Re_λ cases. However, the collapse of pdf of diameter in the SGS fields is good, and the peak of pdf for SGS profile increases from the DNS profile for both filter functions.

Figs. 6–7 shows the pdf of $v_{\theta,\max}$* of the eddies in the same

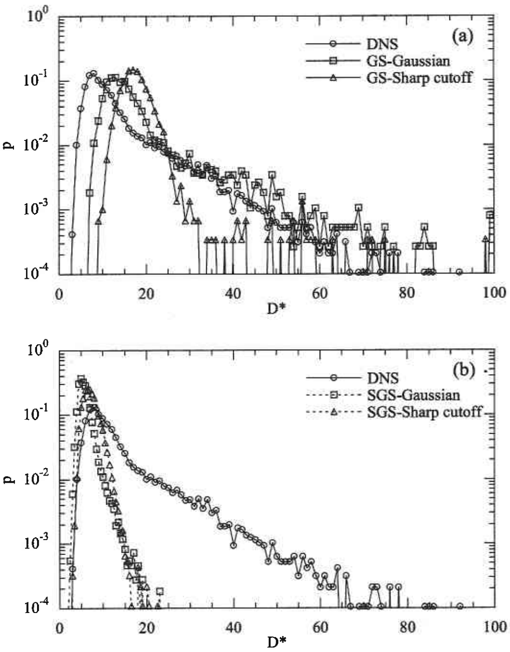

Fig. 4 Probability density function of diameter of the eddies in DNS, GS and SGS velocity fields for the case of Re_λ = 64.9, which is normalized by η. (a) DNS and GS fields, (b) DNS and SGS fields.

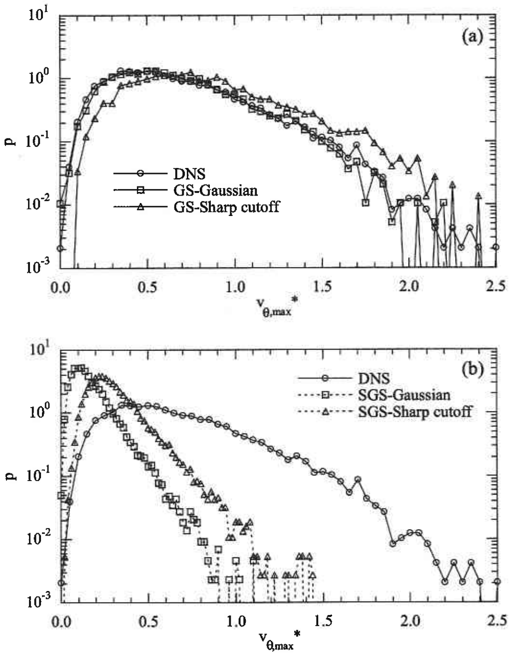

Fig. 6 Probability density function of the maximum of mean azimuthal velocity of the eddies in DNS, GS and SGS velocity fields for the case of Re_λ = 64.9, which is normalized by u_{rms}. (a) DNS and GS fields, (b) DNS and SGS fields.

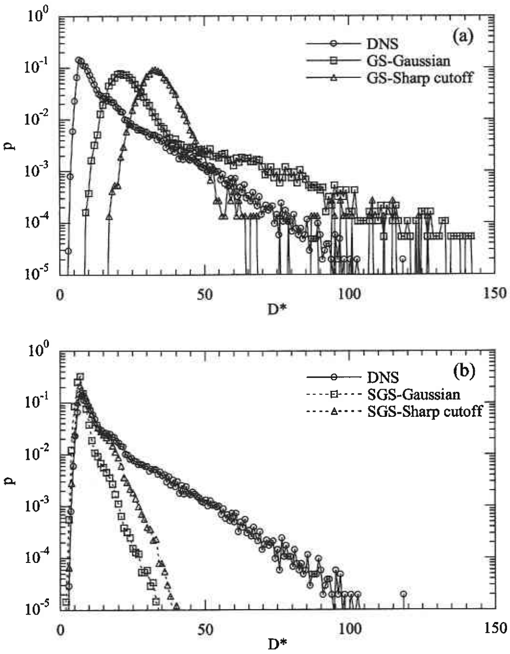

Fig. 5 Same as in Fig. 4, but Re_λ = 87.9.

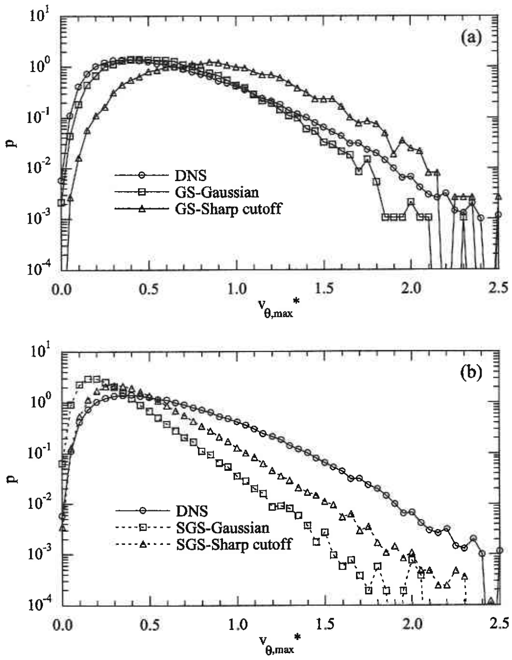

Fig. 7 Same as in Fig. 6, but Re_λ =87.9.

DNS, GS and SGS velocity fields using both filter functions and the two Re_λ cases, respectively, which are normalized by u_{rms}. It is revealed that, the profile of $v_{\theta,max}$* for GS-Gaussian fields collapse with DNS profile well in the whole range for both Re_λ cases. The $v_{\theta,max}$* profile in the GS-sharp cutoff fields is smaller than that of DNS profile at the range of small azimuthal velocity and larger than that of in the DNS fields at the range of large azimuthal velocity of the tube-like eddies. The maximum value of mean azimuthal velocity in GS and DNS fields seems to be same. On the other hand, the profiles of mean azimuthal velocity in the SGS fields for both filter functions do not coincides each other or with DNS profile for both Re_λ cases. The maximum value of $v_{\theta,max}*$ in SGS fields for both filter functions is lower than $1.5u_{rms}$, i.e., about half of DNS value for lower Re_λ case while it increases for higher Re_λ cases and shows very close to DNS value. However, using large and different filter width, we have also seen that the pdf of $v_{\theta,max}*$ of the eddies in the SGS fields becomes very close with that of in the DNS fields and the maximum value in the SGS field never exceed the maximum value in the DNS field. That is, the characteristics of the eddies in GS and SGS fields significantly depend on the filter widths. Using DNS database study[7,8,9], it was shown that the coherent fine scale eddies can be scaled by η and u_{rms}. The above results in the present study also suggest that the scaling of the eddies in GS and SGS velocity fields may possible by η and u_{rms} as well as in DNS.

5. CONCLUSIONS

In this study, characteristics of eddies are investigated by using DNS, GS and SGS velocity fields in homogeneous isotropic turbulence up to $Re_\lambda = 87.9$. It is shown that the DNS velocity fields can be separated well in GS and SGS velocity fields by using classical filter functions for LES. Visualizations of flows suggests that the GS and SGS fields itself contain distinct tube-like eddies, of course different in sizes but somewhat similar to the eddies in the DNS fields. It is also shown that the characteristics of the eddies in GS and SGS fields are likewise similar to the eddies in the DNS fields, which suggests the DNS field may contain small-scale and relatively large-scale structures together in turbulent flows. The eddies in GS and SGS velocity fields can be scaled by Kolmogorov microscale (η) and *r.m.s* of velocity fluctuations (u_{rms}) as well as in DNS.

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