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Effect of Multiple Thrust Patterns in a Long-Term Spiral Trajectory Design

–長期間多周回遷移軌道の設計における複数推力パターンの効果–

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1 Introduction

Spiral trajectories are efficient transfer trajectories used by spacecrafts with low-thrust propulsion. Spiral trajectories will enable maximization of payloads and will also expand possibilities of small class deep space probes. Though, design of spiral trajectories is a challenging problem due to its exceedingly large number of design variables resulting from its long transfer time. Despite its difficulty in design, many studies have been done, but under conditions of many constraints on the thrust, such as constraint on the magnitude, direction, and location of the thrust. These constraints were problematic because it limits the solution that can be obtained. To solve this problem, Hudson[1] proposed an method that enables calculation of change in orbit from an thrust with infinite-degree of freedom, with only just 14 coefficients, called TFCs. Although many studies using TFCs to design spiral trajectories were performed, design of long-term spiral trajectories still remains with a room for research.

2 Objective

Because optimal TFCs depend on the shape of the orbit, in long-term spiral trajectories, the change in shape of the orbit cannot be disregarded, therefore using a single set of TFCs as in conventional studies will not be sufficient enough to obtain sufficient trajectories. In this study, we will perform multi-objective genetic algorithm approach to optimize TFCs for multiple thrust patterns in a long-term trajectory. The objectives are as follows.

- To design multiple thrust profiles for a long-term spiral trajectory using TFCs to obtain better results against conventional single thrust pattern design.
- To clarify how many thrust patterns are sufficient enough for design of long-term spiral trajectories.

3 Methods

3.1 Thrust Fourier Coefficients[1]

The thrust Fourier coefficients, or TFCs is a method that enables to chase secular change in the orbit caused by a thrust pattern with infinite-degree of freedom, expressed by only 14 coefficients of the Fourier's theorem. In our study, only planar thrusts will be considered meaning that we will have only 9 of the TFCs to optimize for each of the thrust patterns.

3.2 Genetic Algorithm: NSGA-II[2]

Genetic algorithm is a type of optimization method that imitates the process of natural selection. the optimizer will tweak variables (TFCs in this study), by performing genetic processes such as selection, mutation, and crossover. From multi-objective optimization, a set of solutions called pareto front can be obtained, which is a set of the most optimal individuals that appeared throughout the optimization. The pareto front is a very powerful tool in mission design, for its visualization of trade-off between multiple objectives.

4 Results and Discussion

4.1 Preparation and Settings

In this research, mass of the spacecraft was set to 480[kg], with maximum thrust of 40[mN], which are referenced from a small class space probe, DESTINY+. Below is the initial state for the spacecraft.

Table 4-1 Initial State

Semi-major Axis	a [km]	42240
Eccentricity	e [-]	0.1
Inclination	i [deg]	10
Right Ascension of Ascending Node	Ω [deg]	20
Argument of Perigee	ω [deg]	90
Mean Anomaly	M [deg]	0

The terminal condition is apogee altitude of $r_{apogee,f}$ =300,000[km]. From the equation of $r_{apogee} = a(1+e)$, the initial apogee altitude is $r_{apogee,i}$ =46464[km].NS-GA-II was performed with a population of 100. The objectives of the optimization are minimization of ToF (time of flight) and ΔV.

4.2 Comparison of 1, 2, and 4 Sections of Thrust Patterns

Figure 4.1 below is the comparison of pareto fronts for 1,2 and 4 thrust patterns. From this result, because pareto fronts that are closer to the point of origin are better in minimization problems, we can see that improvement in the pareto front were seen with multiple thrust patterns.

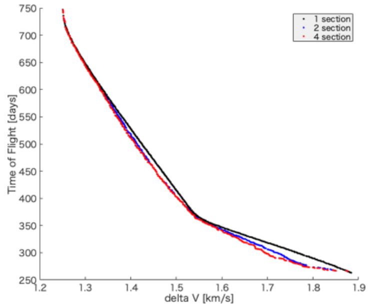

Figure 4.1 Pareto Front for 1,2, and 4 Sections

To see why multiple thrust patterns perform better, we compared the thrust patterns and the change of orbital elements for each of the cases that had a ToF value of 289 [days]. Figure 4.2 shows the thrust patterns for each of the cases, in which the first section of each case is plotted with a solid line, and the latter sections are the dashed lines. The latter sections are not distinguished, due to their similarity. Figure 4.3 below shows the change of parameters of cases with 1,2, and 4 sections. The circle markers represent the position of change in sections.

Observing the thrust patterns in figure 4.2, we can notice that for the multiple section cases, the thrust patterns are arranged in a way where in the first section, high thrust is applied, and in the latter sections, a rather efficient thrust are applied. From the observation of individuals in the pareto front, we have found that applying larger thrust prioritizes the rise in semi-major axis. If we look at figure 4.3, we can see that the high thrust in the first section enables cases with multiple sections to raise the semi-major axis quickly. To clarify the reason for the priority in semi-major axis we observed the equation for average change of a, and e, which directly affect the value of apogee altitude, where μ is the gravitational constant, and α, β are the TFCs.

$$\bar{\dot{a}} = 2\sqrt{\frac{a^3}{\mu}}\left(\frac{1}{2}e\beta_1^R + \sqrt{1-e^2}\alpha_0^S\right) \quad (4.1)$$

$$\bar{\dot{e}} = \sqrt{\frac{a}{\mu}}\left(\frac{1}{2}\sqrt{1-e^2}\beta_1^R + \alpha_1^S - \frac{3}{2}e\alpha_0^S - \frac{1}{4}e\alpha_2^S\right) \quad (4.2)$$

Here, we can see that for both of the equations, the value of semi-major axis is the overall coefficient. This means that by raising the semi-major axis, the sensitivity of $\dot{a}$, and $\dot{e}$ rises. Therefore, efficient trajectories are realized with multiple thrust patterns by performing a maneuver that raises the overall sensitivity of the orbital elements, and under the high sensitivity, efficient thrust patterns are performed. Further observing, if we look at the thrust patterns for a 4-section case,

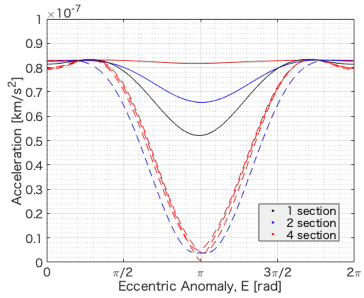

Figure 4.2 Superposition of Thrust Patterns

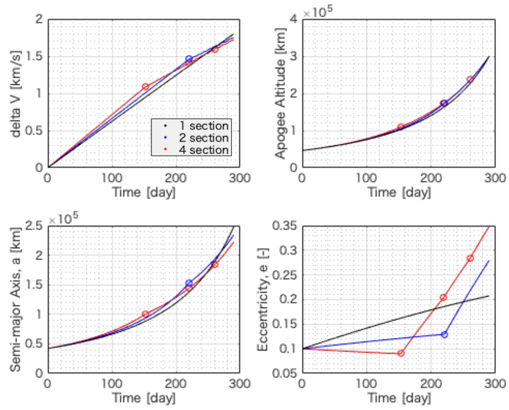

Figure 4.3 Change of Parameters

we can see that the thrust patterns in the 2~4 sections are almost identical. This was observed with other 4-section cases also. From this we can consider the 4-section case as a 2-section case. Now, the difference between these cases is the timing of the section change. Because the sections were divided in a way in which each of the sections of a n-section transfer need to achieve $(r_{apogee,f} - r_{apogee,i})/n$, this means that the more number of sections, shorter time is spent with each of the thrust profiles. With the 4-section case, the shorter time spent with the first high-thrusting section allowed for a larger change in a during a shorter time, which enabled the 4-section case to outperform the 2-section case.

5 Conclusion

- Multiple thrust patterns in a long-term spiral trajectory allowed efficient use of thrust to occur which lead to acquirement of better trajectories against the single thrust pattern case.
- Based on the results, an optimization with 2 thrust patterns with a variable switching point in between the sections will be sufficient enough for our study case.

References

[1] J. S. Hudson and D. J. Scheeres, "Reduction of Low-Thrust Continuous Controls for Trajectory Dynamics," *J. Guid. Control. Dyn.*, vol. 32, no. 3, pp. 780–787, 2009.

[2] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.