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First Excursion Probability Bounds under Imprecise Stochastic Loading

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Abstract: This paper presents a highly efficient and accurate approach to determine the bounds on the first excursion probability of a linear oscillator that is subjected to an imprecise stochastic load. Traditionally, determining these bounds involves solving a double loop problem, where the aleatory uncertainty has to be fully propagated for each realization of the epistemic uncertainty or vice versa. When considering realistic structures such as building models, often containing thousands of degrees of freedom, such approach becomes quickly computationally intractable. In this paper, we introduce an approach to decouple this propagation by applying operator norm theory. In practice, the method determines those epistemic parameter values that yield the bounds on the probability of failure, given the epistemic uncertainty. The probability of failure, conditional on those epistemic parameters, is then computed using the recently introduced framework of Directional Importance Sampling. A case study involving a modulated Clough-Penzien spectrum is included to illustrate the efficiency and exactness of the proposed approach.

Keywords: Stochastic loading, First excursion probability, Linear structure, imprecise probabilities, interval analysis.

1. Introduction

Most dynamically loaded systems exhibit some sort of randomness, for instance caused by several uncontrollable sources of variability of the loading conditions. However, in many practical engineering applications such as assessing the effect of wind loads on buildings, the statistical properties of a large number of samples of the phenomenon are often found to be constant (Li & Chen, 2009). This motivates the application of probabilistic methods to assess the reliability (i.e., the probability that the structure does not fail given a probabilistic description of the load) of a structure. Dynamic loads that act on structures and components are in this context often modeled via a stochastic process representation, where the time-domain behavior of the loading phenomenon is modeled by an auto-correlation function or a power spectrum. In the case of realistic natural structural loading conditions such as earthquakes or wind loads, these auto-correlation functions or power spectra are intricate and require the definition of a set of governing parameters. For example, a stochastic representation of an earthquake spectrum can be modeled by means of a Clough-Penzien power spectrum, potentially combined with the Shinozuka - Sato modulating function (Deodatis, 1996). However, the accuracy of the representation of this stochastic process is highly dependent on the accuracy of the estimation of the governing site-specific soil parameters. As such, it is reasonable to question to which extent a computed crisp value for the reliability of the structure given such model representation is realistic when only a very limited data set is available to estimate the governing parameters from.

As an alternative approach, the framework of imprecise probabilistic analysis (Beer, Ferson, & Kreinovich, 2013) offers a variety of tools to relax the need for a crisp probabilistic description of the variable dynamic loading, and hence, allows for explicitly taking the (epistemic) uncertainty an analyst has concerning the parameters of the stochastic process model into account in the reliability estimation. Instead of a crisp value, in this case, bounds are obtained between which the *true* crisp probability of failure is believed to lie, are obtained. Concerning the estimation of the bounds on the probability of failure, many efficient approaches have been introduced for scalar imprecise probability problems, such as e.g., those based on Chebyshev polynomial schemes (Wu, Luo, Zhang, & Zhang, 2015) or variants of the Sobol-Hoeffding decomposition (Wei et al., 2019). However, it is as yet unclear how these bounds can be efficiently obtained when the structure is subjected to a load modelled as a stochastic process with an imprecise autocorrelation structure without resorting to computationally demanding double-loop approaches. A theoretical study on such imprecise stochastic processes is presented in (Faes & Moens, 2019a) or (Dannert et al., 2018). This paper goes beyond these works and presents an efficient and highly accurate approach for the computation of the bounds on the failure probability of linear structures subjected to loads that are modelled as imprecise stochastic processes. By applying the operator norm theorem and fully exploiting the linearity of the problem, the proposed method is capable to fully decouple the epistemic uncertainty in the parameters of the stochastic model from the stochastic process itself, and as such allows determining those values for the epistemically uncertain parameters that yield the bounds on the probability of failure a priori. This drastically reduces the computational cost of the computation, as no double loop approach is required. The paper is structured as follows; Section 2 provides the theoretical background on stochastic dynamic motion simulation; Section 3 discusses the proposed approach for imprecise stochastic load propagation; Section 4 provides a case study on a single degree-of-freedom oscillator and Section 5 lists the conclusions of the work.

2. Stochastic dynamic motion simulation

Consider a structural system modeled as linear, elastic and with classical damping. The model possesses n_D degrees-of-freedom, its structural matrices are deterministic, and it is subjected to a stochastic ground motion loading, which is modelled as a base excitation. The equation of motion for the degrees of freedom of this system, represented as a finite element model, is:

$$\boldsymbol{M}\ddot{x}(t,\boldsymbol{z}) + \boldsymbol{C}\dot{x}(t,\boldsymbol{z}) + \boldsymbol{K}x(t,\boldsymbol{z}) = \boldsymbol{\rho}p(t,\boldsymbol{z}) \quad (1)$$

where $\ddot{x},\dot{x},x \in \mathbb{R}^{n_D}$ are vectors respectively representing the acceleration, velocity and displacement of the degrees of freedom. The matrices $\boldsymbol{M},\boldsymbol{C},\boldsymbol{K} \in \mathbb{R}^{n_D\times n_D}$ are respectively the mass, damping and stiffness matrices of the system. The vector $\boldsymbol{\rho}$ couples the stochastic ground acceleration loading $p(t,\boldsymbol{z})$ to the corresponding degrees of freedom of the system. Since the system is linear, propagation of $p(t,\boldsymbol{z})$ towards predefined degrees of freedom is performed via convolution. Specifically, consider n_η responses of interest, denoted $\eta_i(t,\boldsymbol{z})$, which are computed as follows:

$$\eta_i(t,\boldsymbol{z}) = \int_0^t h_i(t-\tau)\,p(t,\boldsymbol{z})d\tau \quad (2)$$

for $i = 1,\dots,n_\eta$ and with $h_i(t)$ the corresponding unit response functions, which are computed as:

$$h_i(t) = \sum_{v=1}^{n_D} \frac{\boldsymbol{\gamma}_i^T\boldsymbol{\phi}_v\boldsymbol{\phi}_v^T\boldsymbol{\rho}}{\boldsymbol{\phi}_v^T\boldsymbol{M}\boldsymbol{\phi}_v}\cdot\frac{1}{\omega_{d,v}}\cdot e^{-\zeta_v\omega_v t}\sin(\omega_{d,v}t) \quad (3)$$

where $\boldsymbol{\phi}_v, v = 1,\dots,n_D$ are the eigenvectors associated with the eigenproblem of the undamped equation of motion; $\omega_v, v = 1,\dots,n_D$ are the natural frequencies of the system; $\zeta_v, v = 1,\dots,n_D$ are the corresponding damping ratios; $\omega_{d,v} = \omega_v\sqrt{1-\zeta_v^2}, v = 1,\dots,n_D$ are the damped frequencies; and $\boldsymbol{\gamma}_i$ is a constant vector such that $\eta_i = \boldsymbol{\gamma}_i^T\boldsymbol{x}$.

In the context of propagating stochastic ground acceleration models, the loading $p(t,\boldsymbol{z})$ is represented as a zero-mean non-stationary stochastic process, where the inherent uncertainty associated with the stochastic ground acceleration is considered to follow a Gaussian distribution. To simulate sample paths from $p(t,\boldsymbol{z})$, a modulated Clough-Penzien (CP) autocorrelation model is applied (see e.g., (Li & Chen, 2009)). Specifically, the autocorrelation function R^m of the stochastic process $p(t,\boldsymbol{z})$ is represented as:

$$R^m = m(t_1)m(t_2)R^{CP}(t_2-t_1) \quad (4)$$

with R^{CP} the Clough-Penzien autocorrelation model and $m(t)$ a deterministic modulation function. In this paper, we consider the Shinozuka-Sato modulation function (Shinozuka & Sato, 1967) which is given by:

$$m(t) = \left(\left(\frac{c_1}{c_2-c_1}\right)e^{\frac{c_2}{c_2-c_1}\ln\left(\frac{c_2}{c_1}\right)}\right)^{-1}\left(e^{-c_1 t} - e^{-c_2 t}\right) \quad (5)$$

with c_1 and c_2 parameters of the model that have to be set by the analyst. The time-behavior of a stochastic process governed by a Clough-Penzien autocorrelation model is governed by 7 parameters, gathered in a vector $\boldsymbol{\theta}$:

$$\boldsymbol{\theta} = \left[\omega_g,\ \omega_f, \zeta_g, \zeta_f, S_0, c_1, c_2\right] \quad (6)$$

with $\omega_g,\ \omega_f, \zeta_g, \zeta_f$ filter parameters associated to the CP spectrum, which are soil specific, S_0 the spectral intensity associated with the bedrock excitation and c_1, c_2 the parameters of the Shinozuka-Sato modulation function.

Samples of the stochastic process are then generated by applying the Karhunen-Loève expansion (Vanmarcke, 1983). Hereto, we assume that the loading time of the ground acceleration is T, and that the time is discretized such that $t_k = (k-1)\Delta t, k = 1,\dots,n_T$, where Δt is the time step and n_T the number of time steps. This allows discretizing R^m into a discrete autocovariance matrix $C \in \mathbb{R}^{n_T\times n_T}$, with $C_{ij} = m(t_i)m(t_j)R^{CP}(t_i - t_j)$. Finally, samples of the loading $\boldsymbol{p}(t,\boldsymbol{z})$ can be generated according to:

$$\boldsymbol{p}(t,\boldsymbol{z}) = \boldsymbol{\Psi}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{z} \quad (7)$$

with $\boldsymbol{z} \in \mathbb{R}^{\boldsymbol{n_{KL}}}$ a realization of an n_{KL}-dimensional standard uncorrelated Gaussian distribution; $\boldsymbol{\Psi} \in \mathbb{R}^{n_{KL}\times n_{KL}}$ a matrix whose columns contain the eigenvectors associated with the largest n_{KL} eigenvalues of the discrete covariance matrix C and $\boldsymbol{\Lambda} \in \mathbb{R}^{n_{KL}\times n_{KL}}$ a diagonal matrix containing the ordered n_{KL} largest eigenvalues of C.

Taking this excitation model into account, the dynamic response of interest, evaluated at a time t_k is computed as:

$$\eta_i(t_k,\boldsymbol{z}) = \sum_{l_1=1}^{k}\Delta t\epsilon_{l1}h_i(t_k - t_{l_1})\left(\sum_{l_2=1}^{n_{KL}}\psi_{l_1,l_2}\sqrt{\lambda_{l2}}z_{l2}\right) \quad (8)$$

with ϵ_{l1} a coefficient depending on the applied integration scheme (e.g., Gauss or Trapezoidal) to solve the convolution integral. This can be translated to a matrix-vector equation as:

$$\eta_i(t_k,\boldsymbol{z}) = \boldsymbol{a}_{i,k}^T\boldsymbol{z} \quad (9)$$

with $i = 1,\dots,n_\eta, k = 1,\dots,n_T$ and where $\boldsymbol{a}_{i,k} \in \mathbb{R}^{n_{KL}}$ is a vector such that:

$$\boldsymbol{a}_{i,k}^T = \begin{bmatrix} \sum_{l=1}^{k}\Delta t\epsilon_l h_i(t_k - t_{l_1})\psi_{l_1,1}\sqrt{\lambda_1} \\ \sum_{l=1}^{k}\Delta t\epsilon_l h_i(t_k - t_{l_1})\psi_{l_1,2}\sqrt{\lambda_2} \\ \vdots \\ \sum_{l=1}^{k}\Delta t\epsilon_l h_i(t_k - t_{l_1})\psi_{l_1,n_{KL}}\sqrt{\lambda_{n_{KL}}} \end{bmatrix} \quad (10)$$

In the context of assessing the reliability of a structure subjected to a stochastic ground excitation load such as an earthquake, especially the first excursion probability is of interest, which measures the probability that any of the considered responses $\eta_i(t_k), i = 1,\dots,n_\eta$ exceeds a predefined threshold level $b_i, i = 1,\dots,n_\eta$ within the duration T of the stochastic loading. Specifically, this probability P_F is computed as:

$$P_F = \int_{z \in \mathbb{R}^{n_{KL}}} I_F(\boldsymbol{z}) f_Z(\boldsymbol{z}) d\boldsymbol{z} \tag{11}$$

with $f_Z(\boldsymbol{z})$ an n_{KL} dimensional standard Gaussian distribution and $I_F(\boldsymbol{z})$ an indicator function which is equal to one in case of failure and zero otherwise. Failure in terms of first excursion exceedance occurs whenever the performance function $g(\boldsymbol{z})$ assumes a value equal or smaller than zero; $g(\boldsymbol{z})$ is computed as:

$$g(\boldsymbol{z}) = 1 - \max_{i=1,\ldots,n_\eta} \left(\max_{k=1,\ldots,n_T} \left(\frac{|\eta_i(t_k, \boldsymbol{z})|}{b_i} \right) \right) \tag{12}$$

where $|\cdot|$ denotes the absolute value. Since for a general stochastic ground acceleration process, the number of terms n_{KL} in the KL expansion can easily become high, the application of quadrature schemes to solve Eq.(11) is computationally intractable, we apply Directional Importance Sampling, as introduced by (Misraji, Valdebenito, Jensen, & Mayorga, 2020), to compute the first excursion probability of the system.

3. Imprecise stochastic failure probability calculation

As implied in section 2, the time-behavior of a stochastic process governed by a Clough-Penzien autocorrelation model is determined by a set of parameters $\boldsymbol{\theta}$**.** However, the selection of appropriate parameters for the modulated Clough-Penzien autocorrelation to represent the stochastic ground acceleration realistically is highly case- and site dependent. Therefore, rather than assuming a crisp autocorrelation model, we propose to define bounds between which the autocorrelation model parameters lie. These bounds can, for instance, be based on a set of measurement of similar sites, expert opinion, indirect measurements (Broggi et al., 2018; Faes et al., 2019) or past measurement campaigns (Imholz, Faes, Vandepitte, & Moens, 2020). By applying intervals to these parameters, the stochastic ground acceleration model becomes an imprecise stochastic process, as e.g. discussed in (Faes & Moens, 2019a), governed by an interval vector $\boldsymbol{\theta}^I \in \mathbb{IR}^7$, with $\mathbb{IR}$ the space of real-valued intervals. For more information concerning interval methods in engineering, the reader is referred to (Faes & Moens, 2019c) for a recent overview paper.

This section focuses on the calculation of the failure probability for the case where the effects of imprecision are included in the description of the stochastic loading process by the interval vector $\boldsymbol{\theta}^I$. This implies that both the lower bound and upper bound of the intervals associated the failure probability P_F^I must be determined, which leads to the following two optimization problems:

$$\underline{P}_F = \min_{\boldsymbol{\theta} \in \boldsymbol{\theta}^I} \left(P_F(\boldsymbol{\theta}) \right) \tag{13}$$

$$\overline{P}_F = \max_{\boldsymbol{\theta} \in \boldsymbol{\theta}^I} \left(P_F(\boldsymbol{\theta}) \right) \tag{14}$$

with $P_F(\boldsymbol{\theta})$ determined according to Eq.(11).

The calculation of the bounds associated with the failure probability can be extremely demanding from a numerical viewpoint. On one hand, the calculation of the failure probability for a fixed value of the parameters associated with the stochastic process is quite costly, especially when a full finite element model of a building has to be considered; even when highly efficient methods such as Directional Importance Sampling are applied. On the other hand, solving the associated optimization problems to take the epistemic uncertainty into account is far from trivial, as it constitutes a double loop problem, where the inner loop comprises probability calculation, while the outer loop explores the possible values that the parameters $\boldsymbol{\theta}$ may assume. Hence, apart from considering near-trivial simulation models, such computation is intractable without resorting to some sort of surrogate modelling strategies.

However, by exploiting the linearity in the structural system under consideration, a large gain in computational cost can be obtained. Indeed, this allows to decouple the double loop problem described by Eq.(13) and (14) by determining those parameter values in $\boldsymbol{\theta}^I$ that yield the bounds in the failure probability a priori. Such decoupling is obtained by applying the operator norm theorem.

Consider a continuous map $A: \mathbb{R}^{n_{KL}} \mapsto \mathbb{R}^{n_T}$, a real number $c \in \mathbb{R}$ and an arbitrary vector $v \in \mathbb{R}^{n_{KL}}$. It can be shown that in this case following inequality always holds:

$$\|Av\|_{p_2} \leq |c| \cdot \|v\|_{p_1} \tag{15}$$

where $\|\cdot\|_p$ denotes a norm on the vector spaces $\mathbb{R}^{n_{KL}}$ and $\mathbb{R}^{n_T}$ and p $\geqslant$ 1 constructs a particular $\mathcal{L}_p$ norm. When we consider A_i to be defined as $A_i(\boldsymbol{\theta}) = [\boldsymbol{a}_{i,1}^T(\boldsymbol{\theta}), \boldsymbol{a}_{i,2}^T(\boldsymbol{\theta}), \ldots, \boldsymbol{a}_{i,n_T}^T(\boldsymbol{\theta})]$, and integrating Eq.(10), then Eq.(15) becomes:

$$\|\eta_i(t, \boldsymbol{\theta}, \boldsymbol{z})\|_{p_2} \leq |c_i(\boldsymbol{\theta})| \cdot \|\boldsymbol{z}\|_{p_1} \tag{16}$$

where $\eta_i(t, \boldsymbol{\theta}, \boldsymbol{z})$ denotes the i^{th} dynamic response as a function of t, and $\boldsymbol{z}$ are the i.i.d. Gaussian variables stemming from the KL expansion in Eq.(7). The computation of the maximum possible amplification of $\boldsymbol{z}$ is represented by the operator norm $\|\boldsymbol{A}\|_{p_1,p_2}$, which in its turn is related to the selection of the type of $\mathcal{L}_p$ norm that is chosen on both sides of the equation. In this particular case, i.e., bounding the first excursion probability of a linear dynamical system under imprecise stochastic loading, we consider following problem:

$$\|\eta_i(t, \boldsymbol{\theta}, \boldsymbol{z})\|_\infty \leq |c_i(\boldsymbol{\theta})| \cdot \|\boldsymbol{z}\|_2 \tag{17}$$

The choice for an ∞ −norm is motivated by the notion that those values in $\boldsymbol{\theta}$ that yield the extreme structural responses in $\eta_i(t, \boldsymbol{\theta}, \boldsymbol{z})$ are of highest interest, as it are these extremes in the responses that drive the probability of failure. The 2-norm on the right-hand side can loosely be interpreted as a measure for the energy in the stochastic signal. In this case, it can be shown that $\|\boldsymbol{A}\|_{p_1,p_2}$, can be computed as:

$$\|\boldsymbol{A}\|_{p_1,p_2'} = \max_{l} \left\| A_{i,l:}(\boldsymbol{\theta}) \right\|_2 \quad (18)$$

where the subscript l: denotes taking the l[th] row of the matrix $A_i(\boldsymbol{\theta})$. As such, $c_i(\boldsymbol{\theta})$ is computed as the maximum 2-norm of a row of $A_i(\boldsymbol{\theta})$. Finally, to determine which values in $\boldsymbol{\theta}^I$ yield the bounds on p_F, two optimization problems have to solved:

$$\underline{\theta^*} = \mathrm{argmin}_{\theta\in\theta^I} \max_{l} \left\| A_{i,l:}(\boldsymbol{\theta}) \right\|_2 \quad (19)$$

$$\overline{\theta^*} = \mathrm{argmax}_{\theta\in\theta^I} \max_{l} \left\| A_{i,l:}(\boldsymbol{\theta}) \right\|_2 \quad (20)$$

These analyses show that the parameters of the stochastic ground acceleration model that yield the bounds on the first excursion probability of structure can be determined in two optimization calls. Furthermore, this only requires a single deterministic call to the FE solver, namely, to determine the impulse response functions $h_i(t)$ that are required to assemble $\boldsymbol{a}_{i,k}^T$, as shown in Eq.(10). Therefore, since the interval problem corresponding to Eq.(13) and Eq.(14) can be solved completely a priori, only 2 computations of the probability of failure are required:

$$\underline{p}_F = \int_{z\in\mathbb{R}^{n_{KL}}} I_F(\boldsymbol{z}, \underline{\boldsymbol{\theta}^*}) f_Z(z)dz \quad (21)$$

$$\overline{p}_F = \int_{z\in\mathbb{R}^{n_{KL}}} I_F\left(\boldsymbol{z}, \overline{\boldsymbol{\theta}^*}\right) f_Z(z)dz \quad (22)$$

to obtain the upper bound, strongly reducing the computational cost of the determination of the bounds on the first excursion probability of the structure subjected to an imprecise stochastic ground motion acceleration.

4. Case study: a linear oscillator

This example comprises a single-degree-of-freedom oscillator with mass m = 1 [kg], stiffness k = 225 [N/m] and classical damping d = 5% subject to a stochastic ground acceleration $p_z(t)$. The ground acceleration follows a modulated Clough-Penzien model Nominal parameters for the modulated Clough-Penzien model are set equal to $[\omega_g, \omega_f, \zeta_g, \zeta_f, S_0, c_1, c_2] = [6\pi,\ 0.6\pi,\ 0.6,\ 0.6, 0.04, 0.14, 0.16]$. The total duration of the acceleration is 20 [s]. The time discretization is taken to be Δt = 0.01 [s]. The prescribed threshold level is b = 0.1 [m]. The oscillator is at rest at the beginning of the stochastic. The K-L expansion of the stochastic process is truncated at 99% of the total variance, yielding approximately 1300 terms in the expansion. Directional importance sampling with a sample size of 500 deterministic model evaluations is used to compute the crisp probability of failure. Using this set of parameters, the probability of failure of the mass-spring system is 0.0053 with a coefficient of variation of 0.0359.

To illustrate the performance of the developed approach, a study is performed with wide interval widths on the parameters in $\boldsymbol{\theta}^I$, as illustrated in Table 2. These bounds are derived from the data in (Deodatis, 1996) and expert knowledge estimates and correspond to a case of nearly non-informative estimates on the parameters. For the soil conditions, parameters spanning the full range between Soft and Firm soil are considered. Three different approaches are applied to compute the bounds: (1) a vertex analysis (see e.g., (Hanss, 2005) for the theoretical basis or (Faes & Moens, 2019b) for an extension towards multivariate interval uncertainty propagation), where all combinations of the bounds of the parameters in $\boldsymbol{\theta}^I$ are explored, leading to 2^7 = 128 computations of the probability of failure and hence, 64000 deterministic model evaluations, (2) Quasi Monte Carlo simulation under the assumption of a uniform distribution between the bounds in $\boldsymbol{\theta}^I$ comprising of a Sobol sequence with 500 points, leading to 500 computations of the probability of failure and hence, 250000 deterministic model evaluations and (3) the optimization approach explained in Section 3, leading to 2 computations of the probability of failure and hence, 1000 deterministic model evaluations.

Table 1. Tested values for $\boldsymbol{\theta}^I$.

	ω_g^I	ω_f^I	ζ_g^I	ζ_f^I	S_0	c_1^I	c_2^I
Lower bound	2.4π	0.24π	0.6	0.6	0.03	0.12	0.14
Upper bound	8π	0.8π	0.85	0.85	0.05	0.16	0.18

The results of the three propagation schemes are shown in Table 2. As may be noted, the bounds obtained by the proposed decoupling approach based on the operator norm (denoted optimization in the table) are the widest among the three tested methods. Concerning the lower bound on the first excursion probability, the Vertex approach and the proposed method predict the same lower bound for the probability, whereas the sampling approach provides a lower bound that is higher with almost an order of magnitude. Concerning the upper bound of the first excursion probability, it is clear that the Vertex method underestimates the probability as compared to the upper bound computed by both Sobol sampling and the proposed approach. This indicates that the bounds of the first excursion probability do not vary monotonically with respect to the parameters of the Clough-Penzien autocorrelation model of the stochastic process. The origin of this non-monotonicity lies in the interplay between the frequency content of the non-stationary stochastic base excitation with resonances inside the structure. Since both Sobol sampling and the proposed optimization approach do not assume any monotonicity, they are not affected by this effect. Comparing these two methods, it is furthermore clear that the upper bound, predicted by the optimization method, is higher than the one computed by Sobol sampling, indicating that the optimization procedure is in this case indeed capable of identifying the upper bound correctly. Furthermore, these bounds are computed at a far smaller computational cost as compared to both Sobol sampling as the Vertex approach; respectively a factor 64 and 250 fewer deterministic model evaluations are required for the proposed approach.

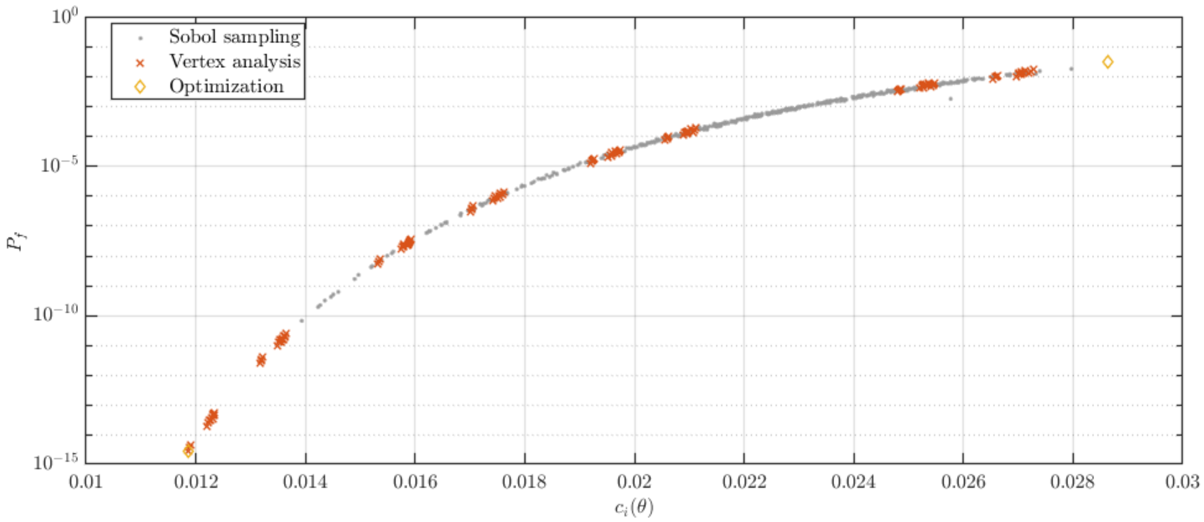

Figure 1. Probability of failure compared to $\|\boldsymbol{A}\|_{p_1,p_2}$, , as computed in Eq.(18), together with the data obtained via Sobol sampling (grey dots) and the Vertex analysis (orange crosses)

Table 2. Computed bounds on the failure probability.

	Vertex analysis	Sobol sampling	Optimization
Low. bound	2.7e-15	4.7e-14	2.78e-15
Up. bound	0.018	0.0192	0.0328

Finally, Figure 1 shows the computed values of the first excursion probability for a realization of $\boldsymbol{\theta}^I$ compared to the corresponding value of $c_i(\boldsymbol{\theta})$. It can be noted that a reasonably smooth and perfectly monotonic relation between these two quantities exists in this case study, and that furthermore the bounds on the former correspond to the bounds on the latter and vice-versa. As such, it can be concluded that computing those realizations in $\boldsymbol{\theta}^I$ that bound $c_i(\boldsymbol{\theta})$ also provides the bounds on P_F, be it at a far reduced computational cost since no costly propagation of the stochastic process is required.

5. Conclusions

This paper discusses the application of imprecise probabilistic methods to account for epistemic uncertainty in commonly applied autocorrelation models for stochastic process in the context of computing the reliability of a linear structure in terms of its first excursion probability. In case insufficient data are available to determine a crisp autocorrelation model to represent a stochastic load such as e.g., an Earthquake, we propose to model the parameters of the autocorrelation function using intervals. To allow for propagating these imprecise stochastic processes within a reasonable computational budget, we introduce an efficient approach based on operator norm theory. The main idea is to decouple the propagation of the epistemic uncertainty from the stochastic variation by determining which values in the autocorrelation parameter intervals provide a bound on the probability of failure following an optimization approach. As such, no double loop propagation is required.

The results of the case study considering a single degree-of-freedom oscillator illustrate that the method is indeed capable of bounding the first excursion probability of this system, subjected to an imprecise stochastic loading, accurately and at a greatly reduced computational cost. In fact, we show that this approach is more accurate than a Quasi Monte Carlo or Vertex approach for propagating the epistemic uncertainty, at a computational cost that is smaller with several orders of magnitude.

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References

Beer, M., Ferson, S., & Kreinovich, V. (2013). Imprecise probabilities in engineering analyses. *Mechanical Systems and Signal Processing*, *37*(1), 4–29. https://doi.org/http://dx.doi.org/10.1016/j.ymssp.2013.01.024

Broggi, M., Faes, M., Patelli, E., Govers, Y., Moens, D., & Beer, M. (2018). Comparison of Bayesian and interval uncertainty quantification: Application to the AIRMOD test structure. In *2017 IEEE Symposium Series on Computational Intelligence, SSCI 2017 - Proceedings* (Vol. 2018-Janua, pp. 1–8). https://doi.org/10.1109/SSCI.2017.8280882

Dannert, M. M., Fau, A., Fleury, R. M. N., Broggi, M., Nackenhorst, U., & Beer, M. (2018). A probability-box approach on uncertain correlation lengths by stochastic finite element method. *PAMM (Proceedings in Applied Mathematics and Mechanics)*, *18*(1), e201800114. https://doi.org/10.1002/pamm.201800114

Deodatis, G. (1996). Non-stationary stochastic vector processes: seismic ground motion applications. *Probabilistic Engineering Mechanics*, *11*(3), 149–167. https://doi.org/https://doi.org/10.1016/0266-8920(96)00007-0

Faes, M., Broggi, M., Patelli, E., Govers, Y., Mottershead, J., Beer, M., & Moens, D. (2019). A multivariate interval approach for inverse uncertainty quantification with limited experimental data. *Mechanical Systems and Signal Processing*, *118*, 534–548. https://doi.org/10.1016/j.ymssp.2018.08.050

Faes, M., & Moens, D. (2019a). Imprecise random field analysis with parametrized kernel functions. *Mechanical Systems and Signal Processing*, *134*, 106334. https://doi.org/10.1016/j.ymssp.2019.106334

Faes, M., & Moens, D. (2019b). Multivariate dependent interval finite element analysis via convex hull pair constructions and the Extended Transformation Method. *Computer Methods in Applied Mechanics and Engineering*, *347*, 85–102. https://doi.org/10.1016/j.cma.2018.12.021

Faes, M., & Moens, D. (2019c). Recent Trends in the Modeling and Quantification of Non-probabilistic Uncertainty. *Archives of Computational Methods in Engineering*. https://doi.org/10.1007/s11831-019-09327-x

Hanss, M. (2005). *Applied Fuzzy Arithmetic: an Introduction with Engineering Applications*. *Control*. Berlin: Springer.

Imholz, M., Faes, M., Vandepitte, D., & Moens, D. (2020). Robust uncertainty quantification in structural dynamics under scarse experimental modal data: A Bayesian-interval approach. *Journal of Sound and Vibration*, *467*, 114983. https://doi.org/10.1016/j.jsv.2019.114983

Li, J., & Chen, J. (2009). *Stochastic Dynamics of Structures*. Singapore: John Wiley & Sons. https://doi.org/10.1002/9780470824269

Misraji, M. A., Valdebenito, M. A., Jensen, H. A., & Mayorga, C. F. (2020). Application of directional importance sampling for estimation of first excursion probabilities of linear structural systems subject to stochastic Gaussian loading. *Mechanical Systems and Signal Processing*, *139*, 106621. https://doi.org/10.1016/j.ymssp.2020.106621

Shinozuka, M., & Sato, Y. (1967). Simulation of nonstationary random process. *Journal of the Engineering Mechanics Division*, *93*(1), 11–40.

Vanmarcke, E. (1983). *Random Fields: Analysis and Synthesis*. *MIT Press, Cambridge, MA*. Cambridge: MIT Press.

Wei, P., Song, J., Bi, S., Broggi, M., Beer, M., Lu, Z., & Yue, Z. (2019). Non-intrusive stochastic analysis with parameterized imprecise probability models: I. Performance estimation. *Mechanical Systems and Signal Processing*, *124*, 349–368.

Wu, J., Luo, Z., Zhang, N., & Zhang, Y. (2015). A new uncertain analysis method and its application in vehicle dynamics. *Mechanical Systems and Signal Processing*, *50–51*, 659–675. https://doi.org/10.1016/j.ymssp.2014.05.036