

## Implementation of Adaptive Kriging Surrogate Model for Seismic Reliability Analysis of Existing Bridges

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**Abstract:** The deterioration of seismic-isolated rubber bearings affects the seismic performance of existing bridges and it is required to estimate the residual seismic performance of the existing bridges by seismic reliability analysis considering uncertainties due to the deterioration. However, with the complex performance function and time-demanding computation of nonlinear seismic responses, estimation of small failure probabilities is a challenging task and the surrogate modeling, especially for the adaptive Kriging model, has been paid attention to effectively evaluate the failure probabilities. In this study, a newly proposed method, namely AK-MCMC, is implemented to seismic reliability analysis of existing bridges with deteriorated rubber bearings. AK-MCMC approximates a set of intermediate failure surfaces by the adaptive Kriging model, which converges to the true failure surface, and is applicable for estimation of very small failure probabilities. The accuracy and efficiency of AK-MCMC are examined for two cases; the healthy and deteriorated conditions at the rubber bearings, and the results demonstrated that, as compared with Subset simulation (SS) and other adaptive Kriging models, AK-MCMC provides accurate results more efficiently regardless of the order of the failure probability.

**Keywords:** adaptive Kriging, surrogate model, Markov Chain Monte Carlo, seismic reliability analysis, existing bridge.

### 1. Introduction

Over the past decades, seismic-isolated rubber bearings have been considered as an attractive technology to mitigate the risk of seismic damages on bridges (Bhuiyan and Alam 2013). On the other hand, it is well known that the structural property of rubber bearings is varied due to the aging deterioration. Hence, it is important to estimate the residual seismic performance of existing bridges with deteriorated rubber bearings considering uncertainties due to the deterioration.

Seismic reliability analysis plays a key role in this context; however, with the complex performance function and time-demanding computation of nonlinear seismic responses, estimation of small failure probabilities is a challenging task. Surrogate modeling is an important approach to efficiently evaluate the performance function with approximate surrogate models. The commonly used surrogate models include the response surface (Sudret 2012), neural network (Hurtado and Diego 2001), and Kriging model (Kaymaz 2005).

Recently, attention has been paid to methods by combining sampling procedures with surrogate models. Echard et al. (2011) proposed AK-MCS by combining the adaptive Kriging surrogate model with Monte Carlo simulation (MCS) and demonstrated its accuracy and efficiency. Huang et al. (2016) developed AK-SS, which combines AK-MCS and Subset simulation (SS) (Au and Beck 2001). However, these methods are not applicable for estimating very small failure probabilities as AK-MCS is not effective in approximating the failure surface when it is far away from the distribution center of input variables. To solve this problem, Wei et al. (2019) proposed a new procedure, namely AK-MCMC. Based on Markov chain Monte Carlo (MCMC) samples, this method approximates a set of intermediate failure surfaces by the adaptive Kriging surrogate model, which converges to the true failure surface.

This study aims to show the applicability of AK-MCMC to seismic reliability analysis of existing bridges. The failure probability against the acceptance plastic modulus of the reinforced concrete (RC) pier is estimated for two cases; the healthy and deteriorated conditions at the rubber bearings, and compared with the results by SS and other adaptive Kriging surrogate models.

### 2. Description of AK-MCMC

AK-MCMC (Wei et al. 2019) is derived from SS (Au and Beck 2001). In the SS procedure, a sequence of intermediate failure domains is defined as  $F_i = \{G(\mathbf{x}) \leq b_i\}$ , where  $\mathbf{x} = [x_1, \dots, x_n]$  is the uncertain input variables with the joint probability density function (PDF)  $p(\mathbf{x})$ ,  $G(\mathbf{x})$  is the performance function and  $b_i$  is the corresponding failure threshold ( $b_1 > b_2 > \dots > b_m = 0$ ). The failure probability  $P_f$  can be then expressed as:

$$P_f = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1}|F_i) \quad (1)$$

The intermediate probabilities  $P(F_1)$  and  $P(F_{i+1}|F_i)$  are set to be a constant value  $p_0$  and each intermediate failure threshold  $b_i$  is calculated based on MC or MCMC samples.

Based on the SS procedure, the AK-MCMC procedure is described as follows and its flowchart is shown in Fig. 1.

- (1) Let  $i = 1$ . Generate  $N_1$  MC samples  $\mathbf{W}_1$  according to the joint PDF  $p(\mathbf{x})$ .
- (2) Randomly select  $N_0$  samples from  $\mathbf{W}_1$  and estimate the true performance function  $G(\mathbf{x})$  on these samples. Attribute these  $N_0$  samples to the training population  $\mathbf{W}_t$ .
- (3) Train or update the Kriging model  $\hat{G}_i(\mathbf{x})$  with  $\mathbf{W}_t$ .
- (4) Predict the performance function for each non-training sample in  $\mathbf{W}_i$  by the Kriging model  $\hat{G}_i(\mathbf{x})$  and obtain or update the intermediate failure threshold  $b_i$  based on the principle that  $\lfloor p_0 N_1 \rfloor$  samples in  $\mathbf{W}_i$  drop into the intermediate failure domain  $F_i$ .

- (5) Compute the learning function proposed by Echard et al. (2011) as  $U(\mathbf{x}) = |\hat{G}(\mathbf{x}) - b_i|/\sigma_G(\mathbf{x})$ , where  $\hat{G}(\mathbf{x})$  is the Kriging predictor and  $\sigma_G(\mathbf{x})$  is the Kriging standard deviation. If  $\min(U(\mathbf{x})) \geq 2$  for all the  $N_1$  samples, go to the next step. Otherwise, find the non-training sample in  $\mathbf{W}_i$  with the minimum value of the learning function, compute the corresponding true performance function, add this point to  $\mathbf{W}_t$ , and return to step (3).
- (6) If  $b_i \leq 0$ , let  $m = i$ , save the Kriging model  $\hat{G}_m(\mathbf{x})$ , and end the algorithm. Otherwise, generate  $N_1$  MCMC samples  $\mathbf{W}_i$  following the conditional PDF  $p(\mathbf{x}|F_i)$  by calling the Kriging model  $\hat{G}_i(\mathbf{x})$  based on the modified Metropolis-Hastings (M-H) algorithm (Au and Beck 2001), let  $i = i + 1$  and  $\hat{G}_i(\mathbf{x}) = \hat{G}_{i-1}(\mathbf{x})$ , and return to step (4).

With the final Kriging surrogate model  $\hat{G}_m(\mathbf{x})$ , the failure probability can be estimated by any sampling procedures, e.g. MCS and SS.

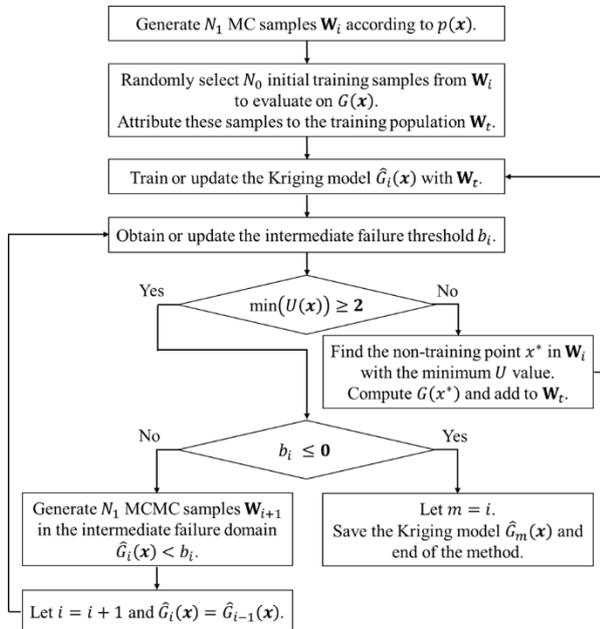


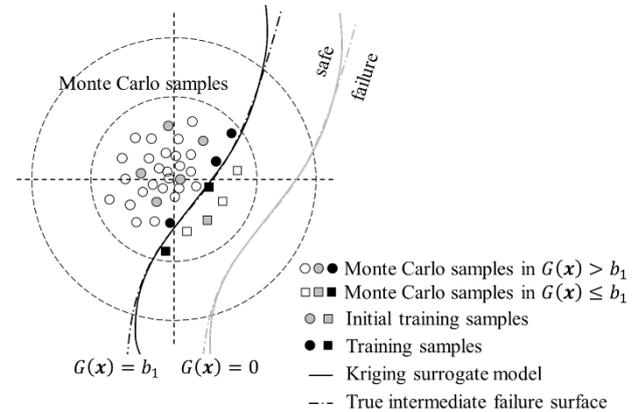
Figure 1. Flowchart of AK-MCMC algorithm.

With regards to the learning function employed in step (5), since the Kriging predictor follows the standard Gaussian distribution,  $\Phi(U(\mathbf{x}))$  is the probability of making a wrong classification on the sign of  $G(\mathbf{x}) - b_i$ , where  $\Phi$  is the standard normal cumulative distribution function. Thus, the stopping criterion,  $\min(U) \geq 2$ , corresponds to the case that the probability of making a wrong classification on the sign of  $G(\mathbf{x}) - b_i$  is less than  $\Phi(-2) = 0.023$ .

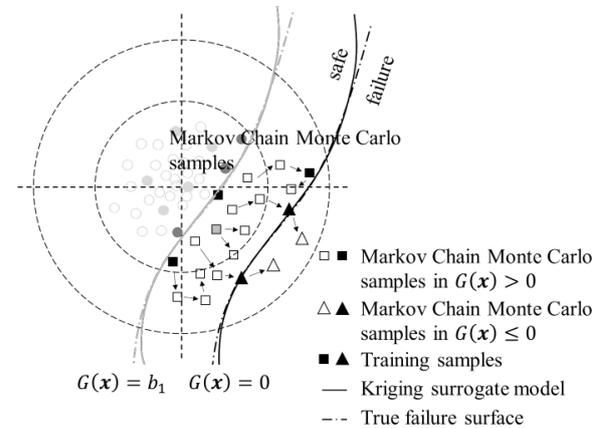
AK-MCMC adaptively generates a sequence of intermediate failure surfaces approximated by the Kriging surrogate model, which converges to the true failure surface. An illustration of AK-MCMC algorithm is provided in Fig. 2. Fig 2(a) shows an illustration of the approximation of the intermediate failure surface by MC samples. Here, the points represent the MC samples. In

particular, the circles are the samples which drop into the region where  $G(\mathbf{x}) > b_1$  and the squares are the samples which drop into the intermediate failure domain  $F_1$ . The grey points indicate the initial training samples and the black points show training samples which are adaptively selected based on the learning function. The dashed line illustrates the true intermediate failure surface and the solid line describes its Kriging surrogate model. Fig. 2(b) shows an illustration of the approximation of the failure surface by MCMC samples. Here, the points represent the MCMC samples, and, in particular, the triangles denote the samples in the failure domain. Similarly to Fig. 2(a), the black points show the training samples which are adaptively selected based on the learning function. The dashed line shows the true failure surface and the solid line indicates its Kriging surrogate model. Note that, no prior information about the failure probability is required for implementing this method. When the failure probability is larger than  $p_0$ , AK-MCMC degrades into AK-MCS.

Comparing with SS, MCMC samples is generated based on the Kriging surrogate model instead of the true performance function in AK-MCMC. The approximation of the Kriging surrogate model only requires to compute the true performance function for a small set of the training samples adaptively chosen from MC or MCMC samples; hence, AK-MCMC needs a much smaller number of calls to the true performance function than the classical SS.



(a). Approximation of the intermediate failure surface.



(b). Approximation of the failure surface.

Figure 2. Illustration of AK-MCMC algorithm.

### 3. Target bridge and analytical conditions

#### 3.1 Target bridge and its analytical model

The target bridge is a seismic-isolated bridge designed based on the design specifications of highway bridges (Japan Road Association (JRA) 2016) and manual on bearings for highway bridges (JRA 2004). The RC pier with rubber bearings is modeled as a two degree of freedom (DOF) lumped mass system, as shown in Fig. 3. Descriptions of the target bridge are also listed in Table 1. Force-displacement relationships of the rubber bearing and RC pier are described by hysteresis loops using the bi-linear and stiffness degradation bi-linear modes (so-called Takeda model) (Takeda et al. 1970), respectively. The ultimate strength of the pier is idealized as same as its yield strength; thus, the pier has no post-yield stiffness. Rayleigh damping is assumed in which damping ratios of the bearing and pier are given as 0% and 2%, respectively.

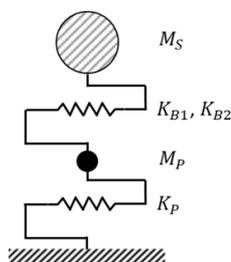
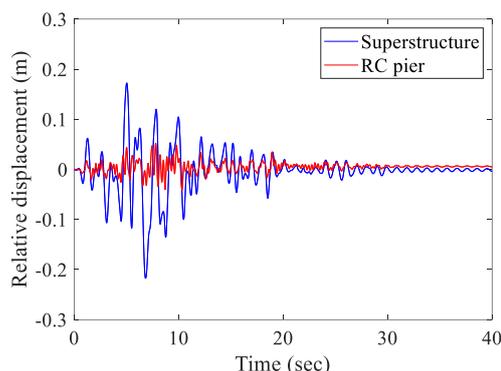


Figure 3. 2-DOF lumped mass model for the target bridge.

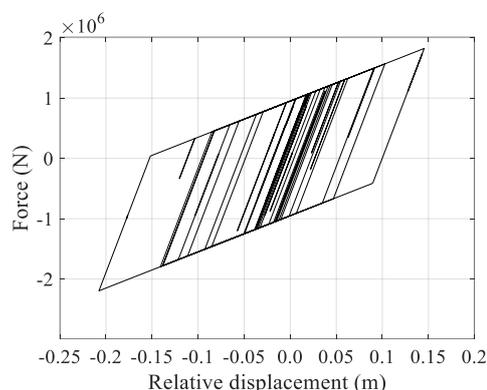
Table 1. Descriptions of the target bridge.

Model parameter		Nominal value
Superstructure	Mass $M_s$ (ton)	604.0
Rubber bearing	Yield strength (kN)	1118
	Yield stiffness $K_{B1}$ (kN/m)	40000
	Post-yield stiffness $K_{B2}$ (kN/m)	6000
RC pier	Mass $M_p$ (ton)	346.2
	Yield strength (kN)	3374
	Yield displacement (m)	0.0306
	Ultimate displacement (m)	0.251
	Yield stiffness $K_p$ (kN/m)	110100

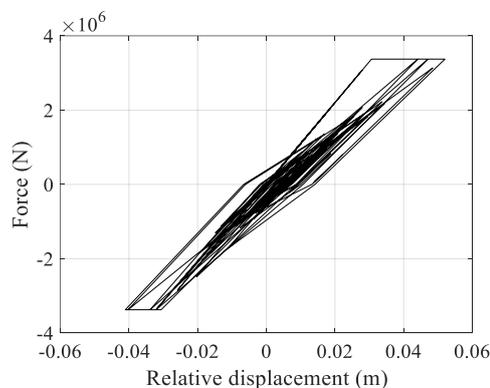
A dynamic response analysis is conducted by Newmark  $\beta$  method ( $\gamma=1/2$  and  $\beta=1/4$ ) with a time step  $\Delta t = 0.001$ sec. The level-2 type-II-I-1 earthquake, defined in JRA (2016), is used as an input ground motion. This earthquake is a ground acceleration corresponding to an inland direct strike type earthquake with the low probability of occurrence, strong acceleration, and short duration, such as Kobe earthquake. Fig. 4(a) shows time-histories of the relative displacement response at the superstructure and RC pier. It can be seen that the time-histories of the relative displacement response at the pier is much smaller than those at the superstructure because the rubber bearings work properly as the isolator. Fig. 4(b) and (c) also show the response of the rubber bearing and RC pier, respectively. The plasticity of the RC pier is limited compared with that of the rubber bearing.



(a). Time-histories of the relative displacement response.



(b). Rubber bearing.



(c). RC pier.

Figure 4. Responses of the target bridge.

#### 3.2 Model parameter uncertainties

The uncertain model parameters affecting the target bridge considered in this study are listed in Table 2. The uncertainties are considered as multiplicative coefficients applied to the nominal parameter values of Table 1. All uncertain parameters follow a Gaussian distribution.

Both healthy and deteriorated conditions at the rubber bearings are considered in this study. Statistical values for the healthy condition, as shown in Table 2(a), are based on Adachi (2002). On the other hand, statistical values for the deteriorated condition, as shown in Table 2(b), are set such that the yield and post-yield stiffnesses of the rubber bearings are 1.2 times those of the healthy condition and the yield strength of the rubber bearings is 0.5 times that of

the healthy condition based on Matsuzaki et al. (2017). The coefficient of variations (COVs) for the deteriorated condition are assumed identical to those for the healthy condition due to the lack of statistical information about deteriorated rubber bearings.

Table 2. Model parameter uncertainties (Gaussian distribution).

(a). Healthy condition.			
Model parameter		Coefficient	
		Mean	COV
Superstructure	Mass	1.05	0.05
Rubber bearing	Yield strength	1.13	0.18
	Yield stiffness	1.00	0.07
RC pier	Post-yield stiffness	1.00	0.07
	Mass	1.05	0.05
	Ultimate displacement	1.062	0.181
	Yield stiffness	1.00	0.07

(b). Deteriorated condition.			
Model parameter		Coefficient	
		Mean	COV
Superstructure	Mass	1.05	0.05
Rubber bearing	Yield strength	0.5×1.13	0.18
	Yield stiffness	1.2×1.00	0.07
RC pier	Post-yield stiffness	1.2×1.00	0.07
	Mass	1.05	0.05
	Ultimate displacement	1.062	0.181
	Yield stiffness	1.00	0.07

### 3.3 Seismic reliability analysis

Seismic reliability analysis is conducted by AK-MCMC taking into account the uncertainties listed in Table 2. The performance function associated with the maximum relative displacement of the RC pier is considered in this study. A threshold value is given as the following equation based on JRA (2016):

$$D_0(\mathbf{x}) = u_{py} + (u_{pu}(\mathbf{x}) - u_{py})/2.4 \quad (2)$$

where,  $u_{py}$  and  $u_{pu}(\mathbf{x})$  are the yield and ultimate displacements of the pier. The ultimate displacement of the pier is also taken into account as the random variable and its distribution information is listed in Table 2. Hence, the performance function is defined as:

$$G(\mathbf{x}) = D_0(\mathbf{x}) - D(\mathbf{x}) \quad (3)$$

where,  $D(\mathbf{x})$  is the maximum displacement of the pier.

The parameters of AK-MCMC are set as follows. The number of initial training samples is set to be  $N_0 = 12$ , as suggested in literatures (Echard et al. 2011, Huang et al. 2016, and Wei et al. 2019), the number of initial MC samples is set to be  $N_1 = 1000$ , and the intermediate failure probability is set to be  $p_0 = 0.01$ . Note that, these samples are employed to select the training samples based on the Kriging model, hence the total number of calls to the true performance function in AK-MCMC is different from this value. With the final Kriging surrogate model, the failure probability is estimated by SS and the initial number of MC samples in SS is chosen as 20,000. Reference values are defined as the failure probabilities

assessed by MCS with a number of 500,000 samples. Furthermore, AK-MCMC is compared with the results by SS, AK-MCS, and AK-SS.

## 4. Analytical results

### 4.1 Healthy condition

The results of seismic reliability analysis for the healthy condition at the rubber bearings are summarized in Table 3. Table 3(a) shows the accuracy of the results. Here,  $\delta_{P_f}$  and  $\varepsilon_{P_f}$  are the COV of the failure probability and percentage error of the failure probability in comparison with the reference value obtained by MCS, respectively. Table 3(b) details the efficiency of the results. Here,  $N_{initial}$  and  $N_{call}$  are the initial number of MC samples and number of calls to the true performance function, respectively. The total computation time consumed in the five methods, all performed with Intel core 1.9 GHz – 4 cores, is also listed in the last column of Table 3(b).

The accuracy and efficiency of AK-MCMC are compared with those of SS, AK-MCS, and AK-SS in terms of the COV of the failure probability, relative error compared with MCS, number of calls to the true performance function, and total computation time. The MCS result with the number of 500,000 samples is defined as the reference value. The reference failure probability is assessed as  $8.0 \times 10^{-5}$  and the corresponding COV of the failure probability is 15.8%. For the healthy condition at the rubber bearings, the failure probability is quite small because the rubber bearings work property as the isolator and the relative displacement response at the RC pier is suppressed. In addition, the total computation time is  $1.7 \times 10^6$ s and it is completely beyond the acceptable level for the practical engineering even if the local parallelization is employed.

SS is performed with a number of 1,000 initial samples and provides a result of  $P_f = 8.2 \times 10^{-5}$  with 3,400 calls to the true performance function. SS gives a very accurate result (the relative error is  $\varepsilon_{P_f} = 2.5\%$ ), while the COV of the failure probability is 44.9% which is too large to be accepted. The total computation time is  $1.2 \times 10^4$ s and is still beyond an acceptable level for practical engineering.

Methods applying the adaptive Kriging surrogate model, AK-MCS, AK-SS, and AK-MCMC, require considerably fewer calls to the true performance function and total computation time; hence, they are significantly more efficient than MCS and SS. AK-MCMC provides a result of  $P_f = 7.6 \times 10^{-5}$  with 91 calls to the true performance function. AK-MCMC gives a less accurate result than SS, but since its relative error is  $\varepsilon_{P_f} = 5.0\%$ , it is still acceptable. Moreover, the COV of the failure probability and total computation cost are 9.6% and 133s, respectively, thus giving a definite improvement over the results of the classic SS. AK-MCMC adaptively produces three intermediate failure surfaces and the last Kriging surrogate model is an accurate approximation of the true failure surface.

On the other hand, AK-MCS gives a result of  $P_f = 9.4 \times 10^{-5}$  with 249 calls to the true performance function, which is both less accurate and less efficient than AK-MCMC. The relative error of the failure probability is

17.5% and it is beyond an acceptable level. AK-MCS requires iterative procedures with enlarging MC samples until the acceptance level for the COV of the failure probability is achieved. The acceptance level is set as 15% due to the quite small failure probability. Furthermore, the initial number of MC samples is set as 50,000 in order to ensure that there are enough samples in the failure domain. It causes a significant decrease in the efficiency of selecting optimal training samples from the MC samples. AK-MCS requires 10 times iterations and thus the selection of optimal training samples is performed on 1,000,000 MC samples at the last iteration. As a result, it leads a large total computational cost (the total computational time is 923s). On the other hand, AK-MCMC avoids this problem by adaptively approximating a set of intermediate failure surfaces and this procedure makes it possible to set the MC samples to be much smaller.

Finally, AK-SS gives a result of  $P_f = 1.1 \times 10^{-4}$  with 30 calls to the true performance function. AK-SS provides the minimum number of calls to the true performance function and total computational time (52s). However, the relative error of the failure probability is 37.5% and is much larger than that of AK-MCS and AK-MCMC. AK-SS also requires iterative procedures with increasing MC samples until the acceptance level for the COV of the failure probability is achieved and the acceptance level is set as 10% to get an almost same COV as AK-MCMC. Furthermore, the initial number of MC samples is set as 10,000. AK-SS employs the SS procedure to estimate the failure probability; hence, the number of MC samples can be smaller than that of AK-MCS, however, it is still larger than that of AK-MCMC. In addition, AK-SS only requires two times iterations.

Table 3. Results of seismic reliability analysis for the healthy condition.

(a). Accuracy of the results			
Method	$P_f$	$\delta_{P_f}(\%)$	$\varepsilon_{P_f}(\%)$
MCS	$8.0 \times 10^{-5}$	15.8	-
SS	$8.2 \times 10^{-5}$	44.9	2.5
AK-MCS	$9.4 \times 10^{-5}$	14.6	17.5
AK-SS	$1.1 \times 10^{-4}$	9.5	37.5
AK-MCMC	$7.6 \times 10^{-5}$	9.6	5.0

(b). Efficiency of the results			
Method	$N_{initial}$	$N_{call}$	Time (s)
MCS	500,000	500,000	$1.7 \times 10^6$
SS	1,000	3,400	$1.2 \times 10^4$
AK-MCS	50,000	249	923
AK-SS	10,000	30	52
AK-MCMC	1,000	91	133

As a consequence, adaptive Kriging surrogate models are significantly efficient to assess the quite small failure probability compared with MCS and SS. AK-MCMC is superior to AK-MCS and AK-SS considering its accuracy and efficiency thanks to its advantage that AK-MCMC

approximates a sequence of intermediate failure surfaces by the Kriging surrogate model, which converges to the true failure surface, instead of directly approximates the true failure surface like AK-MCS and AK-SS.

#### 4.2 Deteriorated condition

The results of seismic reliability analysis for the deteriorated condition at the rubber bearings are summarized in Table 4. Table 4(a) shows the accuracy of the results and Table 4(b) lists the efficiency of the results. The total computation time consumed in the five methods, all performed with Intel core 1.9 GHz – 4 cores, is also listed in the last column of Table 4(b).

The accuracy and efficiency of AK-MCMC are compared with those of SS, AK-MCS, and AK-SS in terms of the COV of the failure probability, relative error compared with MCS, number of calls to the true performance function, and total computation time. The MCS result with the number of 500,000 samples is defined as the reference value. The reference failure probability is obtained as  $2.4 \times 10^{-3}$  and the corresponding COV of the failure probability is 2.9%. For the deteriorated condition at the rubber bearings, the failure probability is larger than that for the healthy condition because the deterioration of the rubber bearings leads to the large relative displacement response at the RC pier.

SS is performed with a number of 1,000 initial samples and provides the same result as the probability obtained by MCS, with 2,249 calls to the true performance function. However, the COV of the failure probability is 27.9% and is too large to be accepted. The total computation time is  $7.5 \times 10^3$ s and it is still beyond an acceptable level for practical engineering.

Same as the healthy condition, methods applying the adaptive Kriging surrogate model, AK-MCS, AK-SS, and AK-MCMC, require considerably fewer calls to the true performance function and total computation time; hence, they are significantly more efficient than MCS and SS. AK-MCMC provides a result of  $P_f = 2.2 \times 10^{-3}$  with 63 calls to the true performance function. AK-MCMC gives a less accurate result than SS, while its relative error is  $\varepsilon_{P_f} = 8.3\%$  and is still acceptable. Moreover, the COV of the failure probability and total computation cost are 6.5% and 91s, respectively, and they are quite less than those of SS. AK-MCMC adaptively produces two intermediate failure surfaces and the last Kriging surrogate model is an accurate approximation of the true failure surface. Compared with the healthy condition, AK-MCMC gives the result with the almost same accuracy, regardless of the order of the failure probability. However, the number of calls to the true performance function and total computational time are reduced because the less intermediate failure surfaces are approximated for estimation of the relatively larger failure probability.

On the other hand, AK-MCS gives a result of  $P_f = 2.0 \times 10^{-3}$  with 206 calls to the true performance function, which is less accurate and efficient than AK-MCMC. The relative error of the failure probability is 16.7% and it is beyond acceptable level. Furthermore, the acceptance level for the iterative procedures is set as 15% to get an almost same COV as the healthy condition. The

initial number of MC samples is set as 5,000 in order to ensure that there are enough samples in the failure domain and AK-MCS requires five times iterations. The total computational time is 313s and is quite less than that for the healthy condition thanks to much less initial MC samples and iterations.

Finally, AK-SS gives a result of  $P_f = 2.8 \times 10^{-3}$  with 95 calls to the true performance function. AK-SS provides an efficient result with small number of calls to the true performance function and short total computational time (145s). However, the relative error of the failure probability is same as AK-MCS and is too large to be accepted. Moreover, the acceptance level for the iterative procedures is set as 10% to get an almost same COV as AK-MCMC. The initial number of MC samples is set as 1,000 and AK-SS requires eight times iterations. Comparing with the healthy condition, more iterative procedures are employed and thus the number of calls to the true performance function and total computational time are much larger, while the initial number of MC samples is much smaller than that for the healthy condition.

Table 4. Results of seismic reliability analysis for the deteriorated condition.

(a). Accuracy of the results

Method	$P_f$	$\delta_{P_f}(\%)$	$\varepsilon_{P_f}(\%)$
MCS	$2.4 \times 10^{-3}$	2.9	-
SS	$2.4 \times 10^{-3}$	27.1	0
AK-MCS	$2.0 \times 10^{-3}$	14.3	16.7
AK-SS	$2.8 \times 10^{-3}$	8.6	16.7
AK-MCMC	$2.2 \times 10^{-3}$	6.5	8.3

(b). Efficiency of the results

Method	$N_{initial}$	$N_{call}$	Time (s)
MCS	500,000	500,000	$1.7 \times 10^6$
SS	1,000	2,249	$7.5 \times 10^3$
AK-MCS	5,000	206	313
AK-SS	1,000	95	145
AK-MCMC	1,000	63	91

As a consequence, adaptive Kriging surrogate models are still significantly efficient to estimate the relatively large failure probability. AK-MCMC is still superior to AK-MCS and AK-SS considering its accuracy and efficiency, while the differences in these methods decrease compared with the healthy condition due to the relatively large failure probability.

## 5. Conclusions

In this study, a newly proposed adaptive Kriging surrogate model, namely AK-MCMC, is implemented to seismic reliability analysis of existing bridges with seismic-isolated rubber bearings for two cases; the healthy and deteriorated conditions at the rubber bearings, and compared with the results by MCS, SS, and other adaptive Kriging models, AK-MCS and AK-SS.

The results demonstrated that AK-MCMC provides accurate results more efficiency compared with SS and

other adaptive Kriging surrogate models, regardless of the order of the failure probability, by approximating a sequence of intermediate failure surfaces, which converges to the failure surface, instead of directly approximating the failure surface. Hence, AK-MCMC can be a promising approach for seismic reliability analysis involving small failure probabilities and time-consuming simulation codes in the practical engineering for estimation of the residual seismic performance of existing bridges.

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