The Seventh Asian-Pacific Symposium on Structural Reliability and Its Applications (APSSRA2020)
October 4–7 2020, Tokyo, Japan
T. Takada, I. Yoshida & T. Itoi (editors)

Space Product Reliability Evaluation Based on Data from Two-stage Development

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Abstract: Turbopumps of liquid rocket engines require high reliability because unreliable turbopumps will certainly lead to catastrophic failure of a rocket launch. Aerospace products usually undergo multi-stage development and will be tested after each stage. Due to budget and time pressures, such tests typically have small sample sizes and short durations. For highly reliable products, few failures or zero failure may be observed in such tests. At the end of a new phase, relying solely on the data from the corresponding tests, the reliability assessment is not satisfactory. So, the data from the previous stage is better included in the reliability assessment of the current stage, which helps to develop the experimental design, improve the accuracy of the reliability assessment and make it more practical. In this study, we assume that the lifetime of turbopump components follow Weibull distribution, and there is only one failure occurred in the 1st stage and the 2nd stage has zero failure. Then a scaling factor method is presented to deal with the conversion problem of 1st stage data, to the 2nd stage. And the 2nd stage Bayes estimations of the Weibull distribution parameters are obtained by the Markov Chain Monte Carlo (MCMC) sampling method. Finally, the effect of the failure time in the 1st stage on the evaluation results of the 2nd stage is studied in this paper.

Keywords: MCMC, Multistage test, Reliability Evaluation, Scaling Factor.

1. Introduction

Liquid rocket engines require high reliability, as the lack of reliability of such products will lead to catastrophic failure of rocket launches. Reliability tests usually proceed to assess whether the product has reached the required reliability. Due to the pressure of budget and time to market, the reliability tests of aerospace products are performed with small sample size and short duration. In such tests, there are often very few or even no failures, which makes it difficult to assess reliability through traditional methods (Jia 2015). Meanwhile, products will experience multi-stage development with respective tests after each stage, by which, weaknesses are identified and corresponding modifications will be made in the next stage. This is an important process of product development and reliability growth (Tian 1992 and Wayne 2014). To improve the accuracy of reliability assessment, it is necessary to make full use of the available information, especially, the data from previous stages. Based on the motivation, it is reasonable to design a more appropriate evaluation method.

A Bayesian evaluation method based on information conversion has been proposed. This method not only solves the problems mentioned above, but also provides an interval estimate of reliability, which is of great importance to engineering practice. Firstly, a new scaling factor method is proposed under Weibull lifetime distribution. Then, the prior distributions of the Weibull parameters are obtained by expertise, and finally, the Bayesian method is used to obtain estimates by applying of converted data.

After verifying the effectiveness of this method, we hope to obtain some regular conclusions through further exploration. In this paper, Part 2 describes the experimental background and the data; the specific implementation process of this method is reflected in Part 3; Part 4 is the main point of this article, which is to explore the influence of the failure time on the evaluates. The paper is summarized in Part 5.

2. Background

As a certain type of turbopump, the rated mission working time τ is 500s and two-unknown parameter Weibull life distribution is chosen to describe the lifetime. Two stages of reliability tests are arranged and the test design arrangement is listed in Table 1.

Table 1 Test design of two stages

Stage	Censored time	Sample size
1	$t_a = 500s$	$n_1 = 10$
2	$t_b = 500s$	$n_2 = 10$

There is one failure that occurred in the 1st stage, denoted by t_1=316s and the others are censored, which are denoted by t_2=...=t_{10}=t_a=500s. According to experts, the shape parameter is m_1=1.5. Referring to the MLE (Maximum Likelihood Estimation) method (Song 2014), the characteristic life η_1 (also called scale parameter) can be calculated by

$$\eta_1 = \left(\sum_{i=1}^{n_1} t_i^{m_1} \right)^{1/m_1} = 2243.1 \qquad (1)$$

The product design would be revised in the 2nd stage. So, the distribution parameters, m_2 and η_2, differ from those in the 1st stage. Besides, due to the reliability growth along with the design revisions, there is no failure occurred in the 2nd stage, which makes it difficult to evaluate reliability by the MLE method. Thus, Bayesian method, by fusing multi-source information, is desirable to obtain more accurate results (Jiang 2010).

3. Bayesian evaluation

3.1 Data from the first stage

According to Zhang (2004), the converted data of the 1st stage can be used in the 2nd stage by scaling factor which is defined as

$$C = \frac{p_i}{p_{i+1}} \tag{2}$$

where p_i is the failure probability of the ith stage.

As the life distribution of turbopump is assumed to follow Weibull distribution, it is improper to define the scaling factor by Eq.2. Moreover, it only converts the sample size between two stages, while ignoring the important time information. Therefore, inspired by this method, a modified scaling factor method is introduced here in the two-stage test. The failure probability of the 1st stage is

$$p_{1j} = P(T < t_j) = 1 - \exp(-(\frac{t_j}{\eta_1})^{m_1}),\ j = 1,...,n_1 \tag{3}$$

where p_{1j} is the failure probability when $t = t_j$ in the 1st stage. The failure probability of the 2nd stage p_2 is calculated by an empirical formula which was proposed by Bailey (1997).

$$p_2 = 1 - 0.5^{\frac{1}{n_2+1}} \tag{4}$$

Correspondingly, the scaling factor is changed to

$$C_j = \frac{p_{1j}}{p_2} \tag{5}$$

Next, the converted data will be obtained by Eq.6 and the results are listed in Table 2. In this case, p_2 is only related to the sample size n_2. Therefore, p_2 is a constant in Table 2. p_{1j} is treated as time-dependent because the data in the 1st stage is sufficient and the Weibull distribution is assumed. The parameter η_1 has been obtained by the MLE method in Eq.1. Next, the failure probabilities at different times are known by Eq.3. So, there are two sets in p_{1j}, as well as C_j and t_{1j} in Table 2. The second row corresponds with the converted results of the failure time (t_1=316*s*), and the third row corresponds with converted results of the right-censored data (t_2=…=t_{10}=t_a=500*s*).

$$t_j \to t_{1j} = t_j \times C_j \tag{6}$$

Table 2 The scaling factor between the two-stage test

t_j	p_{1j}	p_2	C_j	t_{1j}
316	0.0515	0.0611	0.8431	266.39
500	0.0999	0.0611	1.6357	817.84

3.2 Determination of Bayesian prior distribution

The prior distributions should be determined primarily. Obviously the relationship between η_2 and the mission reliability R_τ is

$$R_\tau = \exp\left(-\left(\frac{\tau}{\eta_2}\right)^{m_2}\right) \tag{7}$$

Due to the reliability growth, the mission reliability of the 2nd stage is greater than that of the 1st stage and it cannot exceed 1 (Liu 2006 and Qiu 2018). Simply, we can use a uniform distribution to describe the prior distribution of R_τ.

$$\pi(R_\tau) = \frac{1}{1-R_L} \tag{8}$$

where R_L is the mission reliability of the 1st stage, determined by Eq.9.

$$R_L = \exp\left(-\left(\frac{\tau}{\eta_1}\right)^{m_1}\right) = 0.9 \tag{9}$$

The shape parameter m_2 is a material-related parameter. In engineering practice, an interval that contains the true value can be provided by experience. Due to this reason, uniform distribution in Eq.10 is adopted.

$$\pi(m_2) = \frac{1}{m_b - m_a}, m_a \le m_2 \le m_b \tag{10}$$

where the lower and upper limits m_a and m_b are determined by experts.

3.3 Bayesian assessment

In this case, the likelihood function is

$$L(D \mid m_2, \eta_2) = \frac{(n_1+n_2)!}{(n_1+n_2-1)!} f(t_1) R(t_{1a})^{n_1-1} R(t_b)^{n_2} \tag{11}$$

Substituting Eq.7-10 into Eq.11, the joint posterior distribution of R_τ and m_2 is

$$\pi(m_2, R_\tau \mid D) = \frac{\pi(m_2)\pi(R_\tau)L(D \mid m_2, R_\tau)}{\iint \pi(m_2)\pi(R_\tau)L(D \mid m_2, R_\tau)dm_2 dR_\tau} \tag{12}$$

The kernel of the posterior distribution is also obtained.

$$\pi(m_2, R_\tau \mid D) \propto \frac{-m_2 \ln R_\tau}{\tau^{m_2}} t_{11}^{\ m_2-1} R_\tau^{\frac{\sum_{i=1}^{n_1} t_{1i}^{\ m_2} + n_2 * t_b^{\ m_2}}{\tau^{m_2}}} \tag{13}$$

Since the kernel function has the same dimension as the target distribution and there are two unknown parameters in Eq.13, the MCMC method is proposed here for complex distribution without an analytical solution (Aslett 2017). Metropolis-Hastings (M-H) sampling and Gibbs sampling are two widely used sampling plans in MCMC. What's more, the Markov process constructed by the M-H sampling algorithm satisfies the meticulous and stationary conditions.

Estimated values of (m_2, R_τ) are (1.99, 0.96). By substituting to Eq.7, the value of η_2 is solved and η_2=2942.0. Then a comparison of the CDFs (Cumulative Distribution Functions) between two stages is shown in Fig.1.

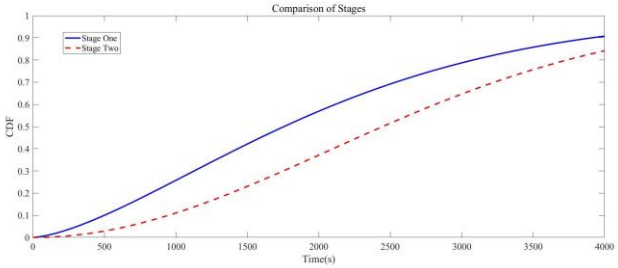

Figure 1. The comparison between two stages.

Furthermore, an interval estimate is more practical in engineering projects. By solving Eq.14 and Eq.15, the confidence lower limit R_{LL}, at a given confidence level γ, will be obtained. And the estimates under different γ are listed in Table 3.

$$\pi(R_\tau \mid D) = \int_{ma}^{mb} \pi(m_2, R_\tau \mid D) dm_2 \tag{14}$$

$$\int_{R_{LL}}^{1} \pi(R_\tau \mid D) dR_\tau = \gamma \tag{15}$$

Table 3 The estimates under different γ

γ	0.95	0.90	0.75
R_{LL}	0.9108	0.9190	0.9380

4. The effect of failure time on evaluates

The entire process of reliability assessment has been completed in Part 3. This part will explore the relationship between the failure time t_1 and the estimate of η_2.

Let the failure time t_1 in [0,500] (second) change in 5*s* steps, the corresponding estimates of η_2 can be calculated and plotted in Fig.2.

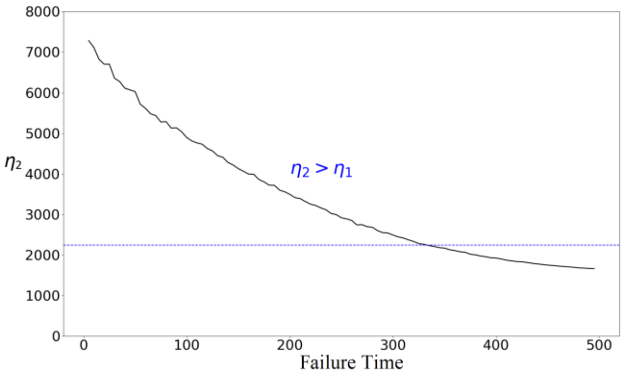

Figure 2. Different results over different failure time. (m, η)

The reason why this tendency appears will be explored here. Considering the effect of the scaling method on the results, we replace the scaling factor method in Section 3.1 with the following Binomial Scaling method (Zhang 2004).

$$p_1 = \frac{r}{n} \tag{16}$$

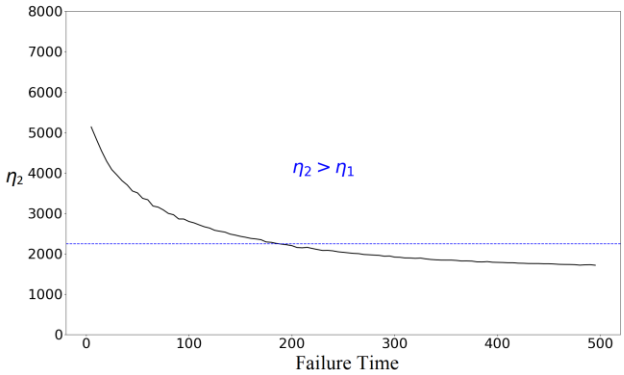

Figure 3. Different results over different failure times. (0-1)

Comparing Fig.2 with Fig.3, the scaling method under Weibull distribution is less affected by the failure time distinctly. As we have assumed that there is a reliability growth between two stages, the characteristic life must meet $\eta_2 > \eta_1$. Take this as the basis for comparison, the range of failure time in Fig.2 (0,330) is greater than the Binomial scaling factor method in Fig.3 (0,195). However, the trend of η_2 under the two different methods is uniform. As the failure time approaches the censoring time, the characteristic life of the 2nd stage decreases. This indicates that the choice of scaling methods is not the main reason for the decreasing trend, other reasons need to be considered to continue exploring.

Then we hope to explain this curve by studying the trend of the parameters. There are many variables during the reliability assessment in Part 3. The diversifications of involved variables are summarized in Table 4. Variables before M-H in Table 4 are used in M-H arithmetic while Variables in M-H are the results of MCMC sampling. In particular, the trends of m_2 and R_τ over the increasing failure time are plotted in Fig.4 and 5, respectively.

Table 4 Variables during reliability assessment

Indepen-dent variable	Interme-diate variable	Variables before M-H	Variables in M-H	Final Evaluate
$t_1 \uparrow$	$\eta_1 \uparrow$ $C_1 \uparrow$	$R_L \uparrow$ $t_{11} \uparrow$	$m_2 \uparrow$ $R_\tau \uparrow$	$\eta_2 \downarrow$

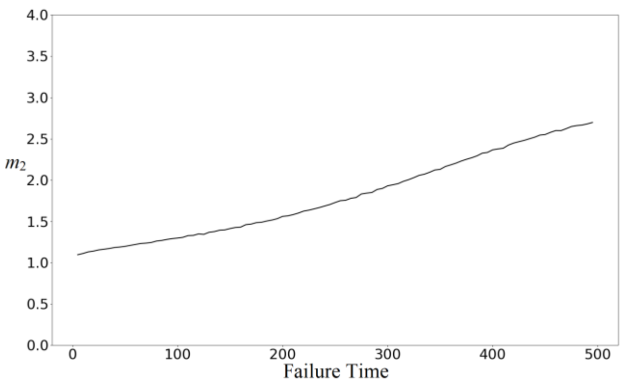

Figure 4. The trend of m_2 over increasing failure time.

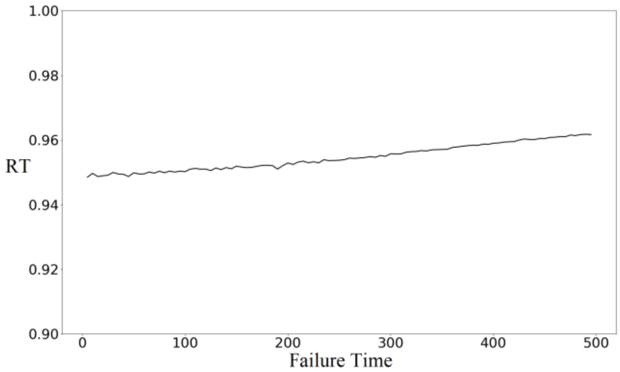

Figure 5. The trend of R_τ over increasing failure time.

After obtaining the estimates of m_2 and R_τ, we can calculate η_2 by Eq.7. The following conclusions are drawn after exploring the interactions between variables in Eq.7:

C(1) When η_2 is constant, R_τ is positively correlated with m_2.

C(2) When m_2 is constant, R_τ is positively correlated with η_2.

C(3) When R_τ is constant, m_2 is negatively correlated with η_2.

Focusing on C(1), Table 4 is consistent with it, and both parameters R_τ and m_2 are increasing. From C(2) and C(3), when R_τ and m_2 are both increasing, they have contrary impacts on η_2. Note that in Table 4, η_2 decreases over failure time t_1, while R_τ and m_2 are both increasing over t_1. This indicates that C(3) is consistent with our proposed method in Table 4. Namely, m_2 has a greater influence on η_2 than R_τ. To improve the quality of estimated value of m_2 in the algorithm, the upper and lower limits given by experts should be taken into account. Therefore, the fusing of multi-source information to make more accurate judgments and narrow the interval of shape parameter can effectively improve the accuracy of reliability assessment.

5. Conclusion

For a multi-stage test with few or no failures data, a feasible solution to evaluate products reliability was proposed. The new scaling factor method under Weibull distribution is developed from the Binomial Scaling method, and it enables wider applications.

Based on this method, the relationship between the failure time and the evaluation results is discussed in this paper. After rigorous research, it has been revealed that the impact of shape parameter of 2nd stage on the reliability assessment is greater than mission reliability. To improve the accuracy of reliability assessment, multiple sources of information are highly recommended and the interval of (m_a, m_b) determined by experts, should be better shortened.

Appreciation

This work was supported by the National Science Foundation of China under Grant number 71871218.

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