The Seventh Asian-Pacific Symposium on Structural Reliability and Its Applications (APSSRA2020)
October 4–7 2020, Tokyo, Japan
T. Takada, I. Yoshida & T. Itoi (editors)

Conditional slope stability analysis considering spatial variability of soils and site investigation information

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Abstract: In slope risk analysis, random field theory is often utilized to characterize the inherent spatial variability of soil properties. Most of the researches neglect the actual site investigation data at certain locations. In order to incorporate those available data from site investigation into simulation, a conditional random field is employed in this study. The influence range of available data is related to the scale of fluctuation. By virtue of Monte-Carlo simulations, the factors of safety and the probability of failure of a slope modelled by unconditional and conditional random field are analyzed in a statistical manner. The results suggest that, compared with the unconditional random field simulations, a conditional random field yields reduction in the spatial variability. This finding is of practical importance if site investigation is conducted on a specific project.

Keywords: slope stability, risk, conditional random field, material properties, spatial variability, Monte-Carlo simulation

1. Introduction

The spatial variability in material properties is often described by a second-order random field. Various applications considering the spatial variability have been reported, such as slope stability analyses (Griffiths and Fenton 2004, Vanmarcke 1977b, Li and Lumb 1987, El-Ramly et al. 2002, Li et al. 2015), geotechnical site characterizations (Vanmarcke 1977a, Fenton 1999), aseismic-response analyses (Yeh and Rahman 1998), foundation–settlement assessments (Fenton and Griffiths 2002), liquefaction risk analyses (Fenton and Vanmarcke 1998), and groundwater levels analyses (Delhomme 1979). A second-order random field can be characterized by the mean, standard deviation and correlation function of material properties. In some cases, the material properties at some portions are available by virtue of site investigations or sampling. When ignoring the available information, an unconditioned random field is likely to overestimate the variability of the field (Tsutomu 2016). As such, a conditional random field shall be more applicable for these cases.

Various investigations can be found in literature on adopting the concept of conditional random field. Vanmarcke and Fenton (1991) extended the methodology of Kriging to simulation of a local field of earthquake ground motions. Baker et al. (2008) evaluated the potential spatial extent of liquefaction by using sample data. Wang et al. (2010) described a Bayesian approach for probabilistic characterization of soil properties from a limited number of tests. In addition, Cao et al. (2013) applied this method to provide information on the number or thicknesses/boundaries of the statistically homogenous layers of soil. Li et al. (2016) combined 3-D Kriging with an existing random field generator to identify optimum sampling locations and the cost-effective design of a slope. Liu et al. (2017) discovered that the ratio of the sample distance to the autocorrelation length is an important factor to ensure the uncertainty reduction by the conditional simulation. Gong et al. (2018) adopted the Hoffman method to generate conditional random field of soil properties for probabilistic analysis of tunnel longitudinal performance. Johari and Gholampour (2019) presented an approach by conditional random finite element method for reliability analysis of slopes in unsaturated soils. Huang et al. (2019) investigated the effect of rotated anisotropy on slope reliability evaluation that considers conditional random field. Gholampour and Johari (2019) extended the application of conditional random fields into the reliability analysis of braced excavation in unsaturated soils. These researches drew a common conclusion that conditional random field simulations yield reduction in the spatial variability, and subsequently offer more reasonable parameters of the material in modelling.

It is found that the accurate simulation of the conditional random field depends highly on the ratio of the sample distance and the autocorrelation distance (Liu et al. 2017). While, in some situations, there are usually few values based on site investigations, and the statistical data used to generate unconditional random field are often obtained through empirical estimation. In the process of generating conditional random field, the overall parameters, such as the mean value, coefficient of variation (COV) fluctuate with few measurement point values, which are inconsistent with the previous estimated one, thus, it may erroneously estimate the spatial variation of soil properties. Therefore, the effect of the true values should be limited to a smaller range in such cases.

This study adopts a patching algorithm for conditional random field in modelling material properties. Available data from site investigation are incorporated into simulation. By virtue of Monte-Carlo simulations, the factors of safety and the probability of failure of a slope modelled by unconditional and conditional random field are calculated.

2. Methodology

2.1 Simulation of unconditional random field

Regarding the simulation of a stationary random field, several methods are available in the literature, such as

Karhunen–Loève expansion, spectral representation method (SRM), local average subdivision method, and Cholesky decomposition technique. The SRM is adopted in this study due to its theoretical elegance and relative simplicity in implications (Shinozuka and Deodatis 1996).

A second-order normal random field with zero mean value and unit variance can be simulated by SRM as

$$f(x_1,x_2)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} [\sqrt{G(\omega_{1n},\omega_{2n})\Delta\omega_1\Delta\omega_2}\times\cos(\omega_{1n}\times x_1+\omega_{2n}\times x_2+\Phi^{(1)}{}_{n_1n_2})+ +\sqrt{G(\omega_{1n},-\omega_{2n})\Delta\omega_1\Delta\omega_2}\times\cos(\omega_{1n}\times x_1-\omega_{2n}\times x_2+\Phi^{(2)}{}_{n_1n_2})] \quad (1)$$

$$\omega_{1n}=n_1\Delta\omega_1,\omega_{2n}=n_2\Delta\omega_2 \quad (2)$$

$$\Delta\omega_1=\frac{\omega_{1u}}{N_1},\Delta\omega_2=\frac{\omega_{2u}}{N_2} \quad (3)$$

where x_1 and x_2 are position vectors in the horizontal and vertical directions, respectively; ω_{1n} and ω_{2n} are the corresponding wave numbers in the horizontal and vertical directions, respectively; $G(\omega_{1n}, \omega_{2n})$ and $G(\omega_{1n}, -\omega_{2n})$ are one-side power spectral density functions (PSDF); ω_{1u} and ω_{2u} are the upper cut-off wave numbers in the horizontal and vertical directions, respectively; $\Phi^{(1)}{}_{n_1n_2}$ and $\Phi^{(2)}{}_{n_1n_2}$ are independent random phase angles uniformly distributed in the range $[0,2\pi]$.

A second-order Gaussian correlation function is utilized in this study and defined as

$$\rho(\tau_1,\tau_2)=\exp[-\pi(\frac{\tau_1}{\theta_1})^2-\pi(\frac{\tau_2}{\theta_2})^2] \quad (4)$$

where $\tau_1=|x_{1m}-x_{1n}|$ and $\tau_2=|x_{2m}-x_{2n}|$ are the absolute distances between two position vectors; θ_1 and θ_2 are the autocorrelation distances in the horizontal and vertical directions, respectively.

The corresponding PSDF (Wiener-Khintchine transform of correlation function) is

$$G(\omega_1,\omega_2)=\frac{\theta_1\theta_2}{\pi^2}\times\exp(-\frac{\theta_1^2\omega_1^2}{4\pi}-\frac{\theta_2^2\omega_2^2}{4\pi}) \quad (5)$$

The simulated random field is shown in Fig. 1. This method can be easily applied to multidimensional stochastic field with arbitrary distribution and multiple variables.

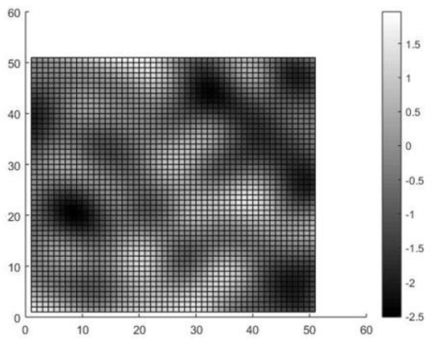

Figure 1. Simulation of unconditional random field.

2.2 Simulation of conditional random field

A patching algorithm (Ouyang et al. 2020) for conditional random field is utilized in this study. There are n_e values at n_e points to be determined, in which p values have been measured from practice. A second-order conditional random field $f^C(x_1, x_2)$ is generated to simulate the soil properties with the following steps:

Step 1: Generate an unconditional random field $f(x_1,x_2)$ using the SRM described in the proceeding section.

Step 2: Assume that the coordinate of the first known point (x_{1k1}, x_{2k1}) $(k1 = 1)$ is (20, 20), at which replace the value with real value, as shown in Fig. 3.

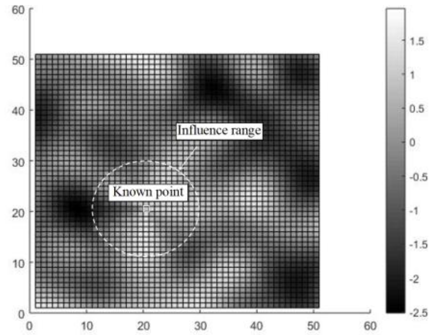

Figure 3. Simulation of the conditional random field

Step 3: Assume that the autocorrelation distance is equal to $10\sqrt{\pi}$, and the influence range of known point is illustrated in Fig. 2. The points within the range are unknown points to be interpolated (x_{1k3}, x_{2k3}) $(k3 = 1 \sim n_e\text{-}p)$.

Step 4: On the edge of the influence range, find the interpolation point (x_{1k2}, x_{2k2}) $(k2 = 1 \sim n_e\text{-}p)$ corresponding to each unknown point, which meets both of the following conditions:

(1) The interpolation point and the unknown point are located on the same side of the known point.

(2) The interpolation point, the unknown point and the known point are located on the same line. The schematic diagram of linear interpolation is shown in Fig. 2.

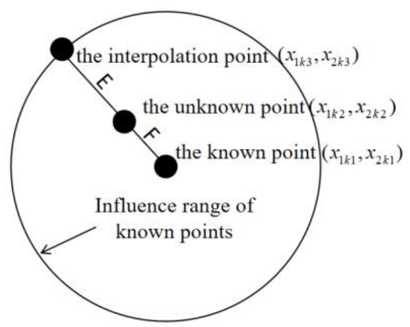

Figure 2. Schematic diagram of linear interpolation. Radius of the influence range is $10\sqrt{\pi}/\sqrt{\pi} = 10$ (see Ouyang et al. 2020).

Step 5: The value at the unknown point is calculated through a linear interpolation method:

$$f(x_{1k2},x_{2k2})=\sqrt{F/(F+E)}\times f(x_{1k3},x_{2k3}) +\sqrt{E/(F+E)}\times f(x_{1k1},x_{2k1}) \quad (6)$$

where E and F are the distances between the interpolation point and the unknown point, and between the known point and the unknown point, respectively.

Step 6: For each other known point (x_{1k1}, x_{2k1}) $(k1=2\sim p)$, repeat step 2 to step 5, and finally the conditional random field $f^C(x_1, x_2)$ is generated, which is shown in Fig. 3.

Note that, the conditional random field exactly matches the known data. As $(\sqrt{F/(F+E)})^2+(\sqrt{E/(F+E)})^2=1$, after linear interpolation, the values of all points follow the normal distribution with mean value equal to 0 and variance value equal to 1.

3. Illustrative example

3.1 Basic model

For illustration, a hypothetical cohesion-frictional slope model is used as the basic model. The slope has also been successively studied in the literature (Cho 2010, Li et al. 2015, Liu et al. 2017). The geometric dimensions of the slope are shown in Fig. 4. Following previous researches, related parameters of soil properties are summarized in Table 1.

Table 1. Statistics of soil properties.

Parameters	Mean	COV	Distribution	Cross-correlation
c (kPa)	10	0.3	Lognormal	$\rho_{c,\varphi}$=-0.5
φ (°)	30	0.2	Lognormal	$\rho_{c,\varphi}$=-0.5
γ (kN/m^3)	20	-	-	-

The cohesion c and friction angle φ are modeled as lognormal random fields with a cross-correlation coefficient $\rho_{c,\varphi}$=-0.5, and the horizontal and vertical autocorrelation distances are chosen as l_h=20 m and l_v=2 m, respectively. The finite element model is discretized into 1765 elements with 1859 nodes, consisting of 4-noded quadrilateral elements. Based on the mean values of c and φ, the deterministic slope stability model provides a similar FS value (1.186) to the value (1.210) calculated by Bishop's simplified method. The plastic strain magnitude (PEMAG) at the failure of slope is also schematically shown in Fig. 4.

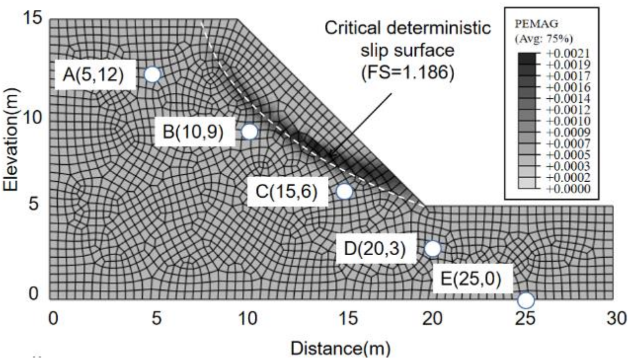

Figure 4. The geometry of the slope, layout of known points and calculation results of the deterministic model

For conditional random field simulations, virtual samples are used as replacements of known data of soil properties at particular locations. Referring to Liu et al. (2017), Fig. 4 shows the layout of the five virtual samples, which are marked consecutively as A, B, C, D and E. Shear strength values are assigned to each of the samples, which are determined based on the statistical properties of soil properties, as illustrated in Table 2. With these known data, conditional random field can be simulated using the method suggested in this study. The influence range of each known value is an ellipse with long axis equal to $20/\sqrt{\pi}$ m and short axis equal to $2/\sqrt{\pi}$ m (see Ouyang et al. 2020). A typical realization of friction angle φ based on unconditional and conditional random field is shown in Figs. 5 and 6, respectively. Besides, A typical realization of cohesion c based on unconditional and conditional random field is revealed in Figs. 7 and 8, respectively.

Table 2. The known data of the five virtual samples

	A	B	C	D	E
c (kPa)	9.07	9.80	9.38	7.82	9.16
φ (°)	32.30	33.60	29.96	42.20	28.15

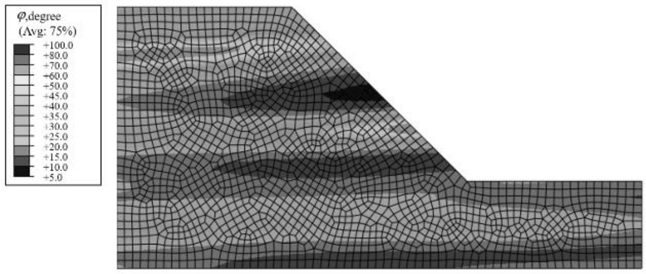

Figure 5. A typical realization of friction angle φ based on unconditional random field.

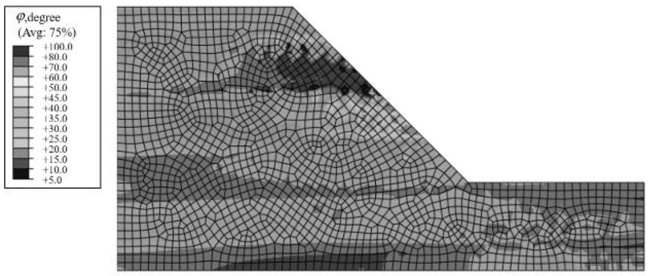

Figure 6. Conditional random field of Fig. 5 with five known data.

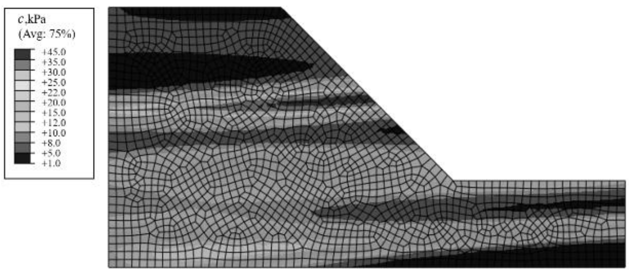

Figure 7. A typical realization of cohesion c based on unconditional random field.

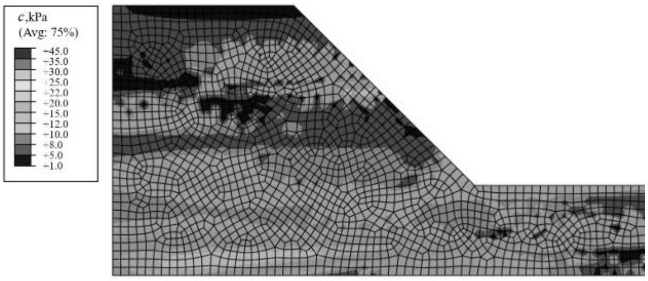

Figure 8. Conditional random field of Fig. 7 with five known data.

3.2 Results and discussions

In this study, the number of known data points (N_d) in each case is set to 0, 2, 3 and 5, as listed in Table 3. Monte Carlo simulations (MCS) are conducted for numerical analysis. It is essential to choose a proper MCS realization times N_{sim}. As the statistics of the FS present little difference when N_{sim} is chosen as 200 and 500, N_{sim} is selected as 200. For these cases, the factor of safety (FS) and the probability of failure (P_f) of slopes are obtained, which are summarized in Table 3, and shown in Figs. 9 and 10, respectively.

Table 3. Statistical results of random finite element analysis.

No. of case	N_d	Known point	Factor of Safety		P_f
			Mean	Coefficient of variation (COV)	
Case1	0	/	1.185	0.183	0.14
Case2	2	A, E	1.202	0.161	0.12
Case3	3	A, C, E	1.195	0.127	0.07
Case4	5	A, B, C, D, E	1.214	0.123	0.05

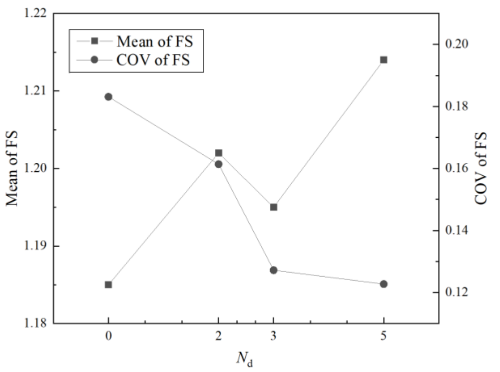

Figure 9. Mean and COV of FS calculated using unconditional and conditional random field for various cases.

It is observed that the mean value of FS increases as the number of known point considered increases of the whole. When N_d=3, the small value may be due to the point C is close to the slope slip surface and the known value is relatively small. Meanwhile, the COV of FS decreases indicating that the simulation variance of the conditional random field can be efficiently reduced by the known data, and more known data result in the spatial variation of soil properties being better represented.

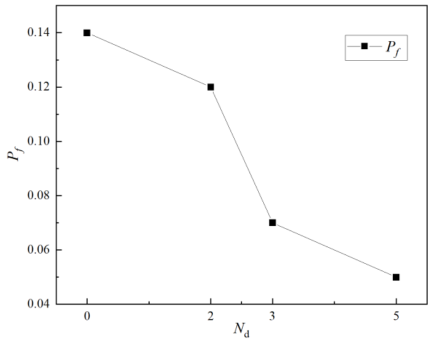

Figure 10. Comparison of P_f calculated using unconditional and conditional random field for various cases.

Moreover, the P_f of slope decreases by the known value increases, which reflects that the P_f may be overestimated by the traditional unconditional random field simulations, so that the conditional random field simulation can effectively reduce the uncertainties and provide more reasonable results.

Some studies have found that the relationship between the sample interval and the autocorrelation distance is critical to the establishment of the conditional random field. When the sample interval is greater than the autocorrelation distance or the autocorrelation distance is too small, the advantage of some conditional random field simulations is not evident, for example, the Kriging random field commonly used (Wu et al. 2009, Li et al. 2016, Liu et al. 2017). In general, the mean value, COV et al. of known data are not consistent with the overall statistical parameters. For these simulation methods, if there is no sufficient known information provided, the volatility cannot be offset; thus, it may erroneously estimate the spatial variation of soil properties. While for the patching algorithm method proposed in this paper, after considering known points, all points of the random field follow the original distribution, so that it will not be limited by the number and spacing of known points.

4. Conclusions

In this paper, a patching algorithm is proposed to combine MCS for assessing the reliability of a slope in spatially variable soils where some known data at particular locations are available. Several cases have been performed to investigate the effects of different layouts of known samples on the FS and P_f, for comparison, the results obtained by the unconditional random field simulations are also provided herein. Based on the present study, several conclusions can be drawn:

(1) The unconditional random field underlying the soil properties can be effectively simulated by the SRM. Through the patching algorithm method, the conditional random field simulated exactly matches the known data and follows the original normal distribution.

(2) The conditional random field can effectively reduce the simulation variance of the random field and the P_f of slopes, indicating that after taking the known data into consideration, the spatial variation of soil properties can be better represented.

(3) Compared with other simulation methods of conditional random field, the method proposed in this paper does not need to be limited by the autocorrelation distances and the number and spacing of known points. Thus, it is more efficient and more suitable for slopes with relatively fewer known data.

Acknowledgements

This research is supported by the National Natural Science foundation of China (Grant No. 51879203) and the NRF-NSFC 3rd Joint Research Grant (Earth Science) (Grant No. 41861144022).

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