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On the reliability assessment approaches of a short column subjected to vertical loading

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Abstract: The point strength of short columns (i.e. cement-treated columns) often exhibits significant spatial variability. Because of the high heterogeneity in the unconfined compressive strength of columns, the first-order second-moment method, second-order second-moment method and variance reduction approach for reliability assessment are generally adopted in practice. In this study, one of the main sources for the heterogeneity in the unconfined compressive strength of columns is considered, namely, the intra-column non-uniformity of point strength. This study presents a model for random strength distribution in columns, based on the assumption that spatial variation in strength arises from spatial variation in material strength. The point strength is simulated as a three-dimensional random field, which serves as the input parameter in finite element analysis. Based on such kind of finite-element analysis, the result of the probability of failure and reliability index is assessed. For comparisons, the reliability index is also calculated by the first-order second-moment method, second-order second-moment method and Monte-Carlo simulation with random variables. Comparison results indicate that the reliability assessed by finite element method with random field is generally greater than it is assessed by other methods. The results of this study are likely to offer guidelines for comparable projects in practice.

Keywords: column, unconfined compressive strength, random finite-element method

1. Introduction

Cement-treated column is the method used to strengthen soft saturation soft clay foundation. It uses cement as the curing agent, through a special mixing machine, the soft soil and the curing agent are forced to mix in the depth of the foundation, and a series of physical and chemical reactions between the curing agent and the soft soil are used to make the soft soil harden into a high-quality foundation with integrity, water stability and certain strength. At present, it is widely used in soft soil foundation treatment (Lee et al. 2013).

For the cement-treated columns formed by forced mixing, it can be seen from the results of sampling that the strength of cement soil presents high heterogeneity (Liu et al. 2016). Lee et al. (2005) give the empirical relation, which is an extension of the relationship given by Gallavresi (1992), is adapted. In this relationship, the unconfined compressive strength, q_u, is given by

$$q_u = q_0 e^{0.62x} y^{-3} \tag{1}$$

where x and y are soil-cement ratio and water-cement ratio in the soil-cement admixture, respectively. Hence, the heterogeneity of cement-treated column strength mainly comes from soil-cement ratio and water-cement ratio. The first-order second-moment method is widely accepted in solving the reliability assessment of cement-treated columns. However, the research and analysis show that when the nonlinear degree of the limit state equation is high, the convergence cannot be guaranteed (Gong et al. 2003). For this reason, the second-order second-moment method (Breitung 1984), Monte-Carlo simulation with random variables (Rubinstein and Kroese 1981) and random finite-element method (Namikawa and Koseki 2013) can be used in this situation. In this research, the cement-treated column is simulated as random field according to the heterogeneity of cement-treated column, and the unconfined compressive strength of the cement-treated column is analyzed by random finite-element method. The result of random finite-element method is compared with the results of first-order second-moment method, second-order second-moment method and Monte-Carlo simulation with random variables, which can provide reference for the design of piles.

2. Methodology

In structural reliability analysis, the limit state of a structure is generally described by a performance function. According to Eq. 2, the load effect is set to 1500 kPa, then the performance function, z is given by

$$z = 150 e^{0.62x} y^{-3} - 1500 \tag{2}$$

The value of the performance function strictly distinguishes the working state of the cement-treated column into three different states: reliable state, limit state and failure state.

If $z > 0$, the cement-treated column is in the reliable state;

If $z = 0$, the cement-treated column is in the limit state;

If $z < 0$, the cement-treated column is in the failure state.

The probability that the performance function appears less than zero is called the failure probability of the cement-treated column, P_f. The performance function approximately obeys the normal distribution, so the probability density function of Z is given by

$$f_z(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp[-\frac{1}{2}(\frac{z-\mu_z}{\sigma_z})^2] \tag{3}$$

In Eq. 3 μ_z represents the mean value of the random variable z. Integrate the part less than zero in Eq. 3, and the failure probability is given by

$$\begin{aligned} P_f &= P(z<0) = \int_{-\infty}^{0} f_z(z)\mathrm{d}z \\ &= \int_{-\infty}^{0} \exp[-\frac{1}{2}(\frac{z-\mu_z}{\sigma_z})^2]\mathrm{d}z \end{aligned} \tag{4}$$

Table 1 Summary of some existing investigations on reliability assessment.

Reference	Method	Investigation approach	Research object
Paloheimo& Hannus, 1974	FOSM	Analytical	Composite members
Rackwitz & Fiessler, 1978	FOSM	Analytical	A section of a wall
Hohenbichler & Rackwitz, 1981	FOSM	Analytical	Composite members
Augusti et al., 1984	MCS	Numerical	Structural engineering
Faravelli, 1989	RS	Analytical	Composite members
Schuëller & Bucher, 1989	RS	Analytical	Frame structure
Bucher, 1990	RS	Analytical	Frame structure
Kim & Na, 1997	RS	Analytical	Structural and non-structural problems
Das & Zheng, 2000	RS	Analytical	Stiffened plated structure
Zheng & Das, 2000	RS	Analytical	Stiffened plated structure
Namikawa & Koseki, 2013	RFEM	Numerical	Cement-treated column
Liu et al., 2018	RFEM	Numerical	Deep cement-mixed clay

Note: FOSM: First-order second-moment; SOSM: Second-order second-moment; MCS: Monte-Carlo simulation; RS: response surface; RFEM: Random finite element method.

The area shaded in Fig. 1 (a) is the failure probability, P_f. By transforming $Y = \frac{(Z-\mu_Z)}{\sigma_Z}$, convert Z from the normal distribution $Z \sim N(\mu_z, \sigma_z)$ to the standard normal distribution $Y \sim N(0,1)$. The failure probability is given by

$$P_f = \int_{-\infty}^{-\frac{\mu_z}{\sigma_z}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{y^2}{2}) dy \tag{5}$$

It can be seen from Fig. 1 (b) that the distance from the origin O to the average Y can be measured by the standard deviation z

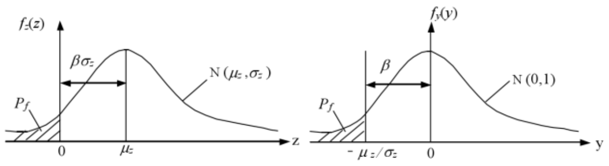
$$\beta = \frac{\mu_z}{\sigma_z} \tag{6}$$

where β is a dimensionless number, termed the reliability index. Therefore, P_f can be expressed as

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{7}$$

2.1 First-order second-moment method

The factors that affect the reliability of the structure are complex, but the mean and variance of random variables are easier to obtain. The first-order second-moment is a method that uses a mathematical model with only mean and standard deviation to solve structural reliability. The center-point method of the first-order second-moment method expands the performance function at the mean point with a Taylor series, and linearize it to calculate the reliability of the structure.



(a) (b)

Figure 1. Relationship between failure probability and reliability index.

Eq. 1 is the performance function, and the soil-cement ratio x follows normal distribution with mean μ_x = 0.2 and standard deviation σ_x = 0.051962, and water-cement ratio y also follows normal distribution with mean μ_y = 0.4 and standard deviation σ_y = 0.070711. Performing first-order Taylor expansion for x = 0.2 yields

$$z = -\frac{1500y^3 - 150e^{\frac{31}{250}}}{y^3} + \frac{93e^{\frac{31}{250}}(x-0.2)}{y^3} \tag{8}$$

Performing first-order Taylor expansion for y = 0.4 yields

$$z = \frac{9375e^{0.62x} - 6000}{4} - \frac{140625e^{0.62x}(y-0.4)}{8} \tag{9}$$

The mean value of z is given by

$$\mu_z = E(z) = g(\mu_x, \mu_y) \tag{10}$$

The variance of z is given by ($x_1 = x, x_2 = y$)

$$\sigma_z = \sqrt{\sum_{i=1}^{n} [\frac{-\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2} \tag{11}$$

The reliability index of the structure β_c in the center-point method can be calculated as

$$\beta_c = \frac{\mu_z}{\sigma_z} = \frac{g(\mu_{x_1}, \mu_{x_2})}{\sqrt{\sum_{i=1}^{n} [\frac{-\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2}} = 0.8180 \tag{12}$$

The failure probability P_f is given by

$$P_f = \Phi(-\beta_c) = 0.2061 \tag{13}$$

2.2 Second-order second-moment method

The first-order second-moment method does not consider the local nature of the performance function in the vicinity of the design checking point. When the nonlinearity of the performance function is high, a large error may occur. The second-order second-moment method uses the gradient of the nonlinear performance function and calculates the nonlinear properties such as the curvature of its second derivative near the checkpoint, thus improving the reliability of the analysis accuracy.

Performing second-order Taylor expansion for x = 0.2 yields

$$z = -\frac{1500y^3 - 150\mathrm{e}^{\frac{31}{250}}}{y^3} + \frac{93\mathrm{e}^{\frac{31}{250}}(x-0.2)}{y^3} + \frac{2883\mathrm{e}^{\frac{31}{250}}(x-0.2)^2}{100y^3} \tag{14}$$

Performing second-order Taylor expansion for y = 0.4 yields

$$z = \frac{9375\mathrm{e}^{0.62x} - 6000}{4} - \frac{140625\mathrm{e}^{0.62x}(y-0.4)}{8} + \frac{703125\mathrm{e}^{0.62x}(y-0.4)^2}{8} \tag{15}$$

The reliability index of the structure in the center-point method can be estimated as

$$\beta_c = \frac{\mu_z}{\sigma_z} = \frac{z(x^*) + \sum_{i=1}^{n}\frac{\partial g_x(x^*)}{\partial x_i}(\mu_{x_i} - x_i^*)}{\sqrt{\sum_{i=1}^{n}[\frac{\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2}} + \frac{\sum_{i=1}^{n}\frac{\partial g_x{}^2(x^*)}{\partial x_i{}^2}(\mu_{x_i} - x_i^*)^2}{\sqrt{\sum_{i=1}^{n}[\frac{\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2}} \tag{16}$$

Iterations are conducted based on the following equations.

$$x_i^* = \mu_{x_i} + \beta_c \cos\theta x_i \tag{17}$$

$$\cos\theta x_i = -\frac{\frac{\partial g_x(x^*)}{\partial x_i}\sigma_{X_i}}{\sqrt{\sum_{i=1}^{n}[\frac{\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2}} \tag{18}$$

If the difference of $|x_i^*|$ between the two sequential iterations is less than 0.1, the iteration process is considered to be completed.

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{z(\mu_X)}{\sqrt{\sum_{i=1}^{n}[\frac{\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2}} = 1.1812 \tag{19}$$

where β is the reliability index of the structure in second-order second-moment method.

The failure probability, P_f is given by

$$P_\mathrm{f} = \Phi(-\beta_c) = 0.119 \tag{20}$$

2.3 Monte-Carlo simulation

Monte-Carlo simulation is to randomly sample random variables that affect its reliability, and then substitute these sampled values into the performance function group by group to determine whether the structure fails. After carrying out a large number of tests, the failure probability of a structure is the frequency at which the number of structural failures accounts for the total number of samplings. When the number of samples is large enough, the frequency is approximately equal to the probability.

Sampling N times, if the number of times $z \leq 0$ is M, then the failure probability of the problem, $P_\mathrm{f} = N / M$. The random variables X, Y which follow the normal distribution are sampled 100,000 times, and these samples are brought into the performance function z for calculation.

The frequency distribution histogram and cumulative distribution curve of q_u are shown in Fig. 2 and Fig. 3 when q_u < 1500, z < 0, respectively.

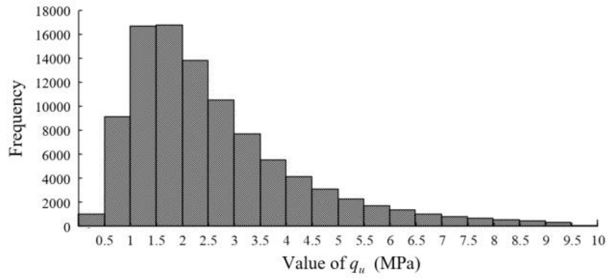


Figure 2. Frequency distribution histogram of q_u.

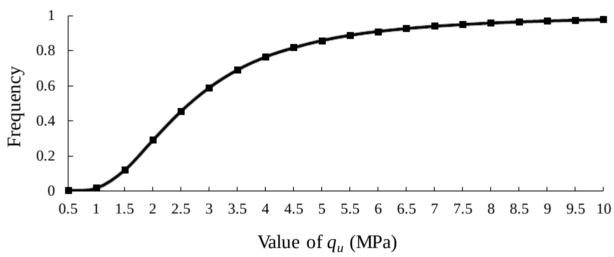


Figure 3. Cumulative distribution curve of q_u.

$$P_\mathrm{f} = \frac{N}{M} = 0.11844 \tag{21}$$

When it is necessary to compare the degree of dispersion of multiple sets of data, if the data scale or data unit of the two sets of data is different, it is not appropriate to use the standard deviation directly, and the coefficient of variations (*COV*) can eliminate the influence of the measurement scale and data dimension.

It can be seen in Fig. 4 that the *COV* of the *y* has a great influence on the *COV* of the q_u, while the *COV* of the *x* has little influence on the *COV* of the q_u.

$$\sigma_{q_u} = \sqrt{\sum_{i=1}^{n}[\frac{\partial g_x(\mu_x)}{\partial X_i}]^2 \sigma_{X_i}{}^2} = 1095.94\sqrt{\sigma_x{}^2 + 111.74\sigma_y{}^2} \tag{22}$$

$$\mu_{q_u} = 2609.0625 \tag{23}$$

$$COV_{q_u} = \frac{\sigma_{q_u}}{\mu_{q_u}} = 0.42\sqrt{\sigma_x{}^2 + 111.74\sigma_y{}^2} \tag{24}$$

Since μ_x = 0.2 and μ_y = 0.4 and the *COV* ranges of variables *x* and *y* are both [0,1], the ranges of σ_x and σ_y are [0,0.2] and [0,0.4], respectively. According to Eq. 23, the coefficient before σ_y^2 is much greater than the coefficient before σ_x^2. Therefore, the main influence factor of COV_{q_u} is the *COV* of *y*.

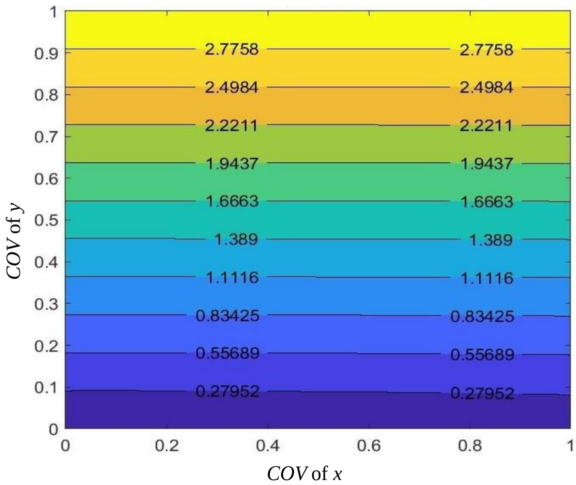


Figure 4. Relationship among coefficients of variation of *x*, *y* and q_u.

2.4 Random finite-element method

This research mainly considers the reliability analysis of the pile under vertical load. As shown in Fig. 5, the cement-treated column model with a diameter of 3 m and a height of 5 m is established. A displacement of 0.5 m is applied on the upper surface of the pile to calculate the unconfined compressive strength of the pile surface. The unconfined compressive strength of the cement-treated column follows the normal distribution, and the specific parameters are shown in Table 2.

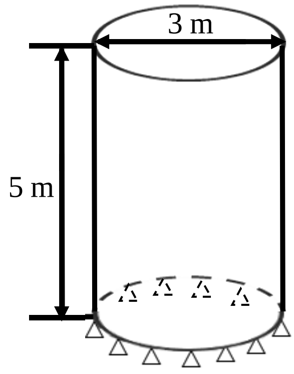


Figure 5. Cement-treated column model.

Table 2 Unconfined compressive strength of cement-treated column

Average value μ_{q_u} (MPa)	Standard deviation σ_{q_u} (MPa)	Auto-correlation lengths along *X* and *Y* direction (m)	Auto-correlation lengths along *Z* direction (m)
2609.06	1355.75	0.50	2.00

According to the above parameters, 100 random finite-element simulations are carried out. The schematic diagram of the random field generation and calculation results of unconfined compressive strength of the cement-treated column is shown in Fig. 6.

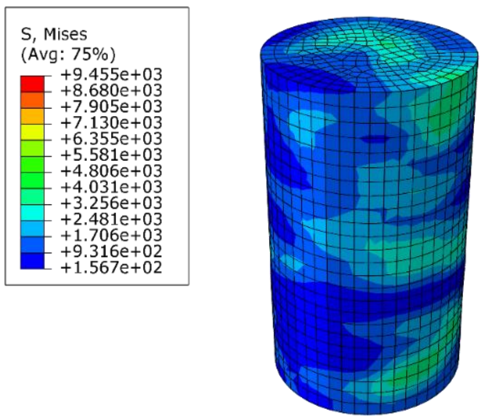


Figure 6. Finite element analysis results of cement-treated column.

The results of 100 random finite element simulations of the unconfined compressive strength of the cement-treated column in this example is counted. The histogram of frequency distribution of unconfined compressive strength of the cement-treated column is shown in Fig. 7.

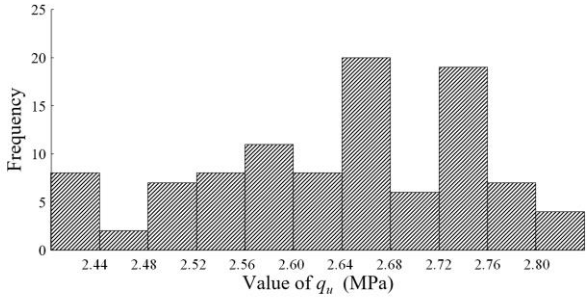


Figure 7. Histogram of frequency distribution of unconfined compressive strength.

According to the above calculation results, the cumulative distribution function graph of the simulation results of 100 random finite elements can be drawn as shown in Fig. 8.

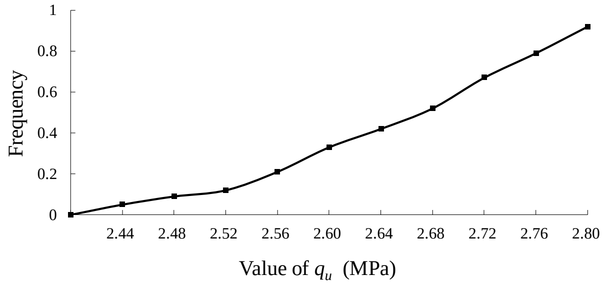


Figure 8. Cumulative distribution function graph of unconfined compressive strength.

3. Results and Discussions

The reliability index calculated by the first-order second-moment, second-order second-moment, Monte-Carlo simulation and random finite-element method is shown in Table 3.

Table 3 Unconfined compressive strength of cement-treated column

Failure probability	FOSM	SOSM	MCS	RFEM
P_f	0.2061	0.1190	0.1184	0

It can be seen from the above table that the calculation result of the second-order second-moment method is closer to the calculation result of Monte-Carlo simulation. This is because the first-order second-moment method does not take into account the local nature of the performance function in the vicinity of the checkpoint, and at the same time, the nonlinearity of the performance function in this research is high, resulting in a large error.

In the process of random finite-element simulation, although there are some elements with relatively high unconfined compressive strength in the pile, the unconfined compressive strength of each element in the same simulation process generally follows a normal distribution. At the same time, the numerical values of the unconfined compressive strength of the elements in the range of relevant length are similar. The higher strength elements support the cement-treated columns, resulting in the consequence that there is no damage to the cement-treated columns.

4. Conclusions

According to the above calculation process, it can be seen that the simulation of the first-order second-moment method, second-order second-moment method and Monte-Carlo simulation can only simulate the situation when the unconfined compressive strength of a single cement-treated column is a fixed value.

The non-uniformity of the unconfined compressive strength of the cement-treated column was characterized by the random field method. The random finite-element simulations that the uneven spraying of the cement pastes makes the unconfined compressive strength has a strong randomness. This random finite-element method can calculate the failure situation of the cement-treated column which is closer to the actual engineering situation.

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