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Rapid Prediction of Wildfire Spread by Ensemble Kalman Filter Employing Polyline Simplification Algorithm

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Abstract: Wildfires tend to spread rapidly using various natural materials including trees and plants as fuels. This naturally leads to widespread damage to the natural areas and nearby cities. Besides, prompt responses to wildfires during the firefighting operation are quite challenging due to topographical and environmental conditions. A possible reduction of the time required for wildfire spread prediction would provide decision-makers with additional time to prepare for the pattern of the upcoming wildfire spread. Therefore, to facilitate the successful suppression of wildfire, a rapid prediction of wildfire spread is just as essential as an accurate prediction. This study proposes a new method developed for quick prediction of wildfire spread by predicting the present conditions by a computational simulation model and assimilating them with the actual observation data. To this end, the FARSITE software is used to predict the wildfire spread at the next time step, and the ensemble Kalman Filter (EnKF) is used to assimilate between model-based predictions and observation data. In order to maintain accuracy and expedite the prediction and assimilation, the perimeter of the wildfire area is simplified using a 2-D polyline simplification algorithm. The proposed method can predict wildfire spread significantly faster using fewer points than existing methods that do not employ the line simplification method while suppressing the loss of accuracy as much as possible. It is expected that the developed method will serve as a core algorithm for near-real-time prediction and data-driven updating of wildfires in natural and urban areas.

Keywords: Wildfire, FARSITE, Ensemble Kalman Filter, Data assimilation, Polyline simplification

1. Introduction

Wildfires tend to spread rapidly using natural fuels including various plants in the mountains. This may result in not only the destruction of the wildland vegetation, but also serious problems such as the transition to quasi-wasteland in the burned areas, the occurrence of landslides, and the destruction of ecosystems growing in the wildland. Furthermore, nearby urban areas can suffer from various issues and socioeconomic impacts. It is thus essential to predict wildfire spread in order to reduce damage as much as possible. However, it is challenging to incorporate all of the various factors that affect wildfire spread to prediction process, entailing errors that increase as the spread continues. In order to solve this issue, Mandel et al. (2004) first proposed a method to improve the accuracy of wildfire spread prediction by applying the actual wildfire spread observation dataset. To incorporate actual wildfire spread observation data, data assimilation through the ensemble Kalman Filter (EnKF) was applied (Mandel et al. 2008). This method applied the data assimilation to newly developed hydrodynamic models associated with wildfire spread. As a result of the application, the prediction accuracy of wildfire spread and related parameters were greatly improved.

Among various Bayesian filters, EnKF is considered an efficient option for predicting wildfire because EnKF can effectively handle problems with a large number of variables and nonlinear system. However, because wildfire prediction model is extremely nonlinear, implementation of the standard EnKF may cause large errors. To address this issue, various modified EnKFs have been developed and applied. Johns and Mandel (2008) developed a two-stage EnKF to apply ensembles with large gradients to the nonlinear convection-diffusion-reaction partial differential equation. In addition, Beezely and Mandel (2008) developed a morphing EnKF suitable for consistent nonlinear problems, using intermediate states obtained by morphing technique instead of the linear combination of ensembles. Rochoux et al. (2014) developed a polynomial chaos EnKF that reduces computational cost using the surrogate model based on polynomial chaos expansion.

On the other hand, various wildfire spread simulation programs have been developed to facilitate providing wildfire spread predictions. Examples of computational fluid dynamics-based models include FIRETEC (Linn and Harlow 1997) and WFDS (Mell et al. 2007) while examples of regional-scale fire spread models are FARSITE (Finney 1998) and PROMETHEUS (Tymstra et al. 2010). Although these simulation programs have enabled us to predict the wildfire spread without going through complicated calculations, they do not support data assimilation using actual wildfire spread observation dataset. Recently, research efforts are being made to improve prediction accuracy by applying data assimilation to simulation programs. For instance, Srivas et al. (2016) proposed an algorithm that applies data assimilation using EnKF to wildfire spread prediction using FARSITE. In addition, Zhou et al. (2019) applied the ensemble transform Kalman Filter, which was developed to ignore perturbed observations required for the time-updating in EnKF, to predict wildfire spreads using FARSITE.

This paper proposes a new algorithm to reduce the time required for EnKF data assimilation using FARSITE. Because wildfire spread prediction using FARSITE does not have unified dimensions, EnKF should be applied after dimensional unification using re-interpolation. The proposed new algorithm improves performance by applying two new techniques: (1) the dimension of ensembles representing prediction result is reduced by applying 2-D polyline simplification to accelerate the data assimilation; and (2) Minimized the accuracy degradation caused by re-interpolation considering the complexity of each part of the perimeter. Using the proposed algorithm, it is possible to rapidly predict wildfire spread while suppressing the accuracy decrease as much as possible.

2. Theoretical Backgrounds

This section introduces FARSITE, a 2-D wildfire spread simulator used in the methodology proposed in this paper. The section also briefly explains the 2-D polyline simplification algorithm, which is proposed such that the topological relations between the polylines and polygons are not changed for a rapid prediction by the proposed data assimilation methodology.

2.1 FARSITE: Wildfire spread simulator

FARSITE (Finney 1998) is a 2-D wildfire growth simulator that is widely used by the U.S. Forest Service and National Park Service. It computes wildfire growth and various behavior for specified time periods under the heterogeneous conditions of fuel moistures, weather streams, ignitions, and terrains. FARSITE incorporates existing fire behavior models for crown fires (Wagner 1977, Rothermel 1991), spotting fires (Albini 1979), and dead fuel moistures (Nelson 2000) into a Rothermel's 2-D surface fire growth model (Rothermel 1972). Using incorporated fire behavior models, FARSITE can provide various outputs related to wildfire, e.g. fire perimeters, arrival time, flame length, rate of spread, and spread direction.

FARSITE requires the input of a set of parameters related to various environmental factors affecting the wildfire spread. Parameters required for FARSITE simulation can be divided into two types: time-invariant and time-variant parameters. First, time-invariant parameters consist of topography-related and vegetation-related values, which vary over spaces. FARSITE landscape file contains time-invariant parameters, which are supported by LANDFIRE project (Rollins 2009). This file include raster maps that combine various static values that describe terrains. It contains elevation, slope, aspect, fuel model, and canopy cover. Each fuel model additionally requires 1-hour fuel moisture content, 10-hour fuel moisture content, 100-hour fuel moisture content, live herbaceous moisture content, and live woody moisture content. A total of 13 fuel models are required when Anderson's model (Anderson 1981) is used, and 40 fuel models are required when Scott and Burgan's model (Scott 2005) is used.

Second, time-variant parameters generally require the input of common value in all spaces. This requires temperature, relative humidity, hourly precipitation amount, wind speed, wind direction, and cloud cover percentage on an hourly basis. In addition, because the regional wind speed and direction vary greatly depending on the mountainous terrain, an external program such as WindNinja by Missoula Fire Sciences Laboratory is needed to correct wind speed and direction for each grid based on the mean value. It is also possible to apply the customize gridded winds dataset directly.

2.2 2-D polyline simplification algorithm without changing the topological relations

Polyline simplification, or polygon simplification, is an algorithm that simplifies polylines and polygons with excessive complexity within a given cost limit. Higher complexity increases computational cost, while lower complexity eliminates important details of the polyline. A number of algorithms have been developed to minimize the difference from the original figure while satisfying the given complexity limits. Examples include Douglas-Peucker algorithm (Douglas and Peucker 1973) and Visvalingam-Whyatt algorithm (Visvalingam and Whyatt 1993).

Douglas-Peucker algorithm is widely used with a predetermined limit of complexity. The algorithm recursively divides the line between the original polyline's first and end point. At each step, this algorithm finds the farthest point from the line segment and keeps the point if it is located farther than a predetermined complexity limit. If the point is closer than the limit, then all points between the line segment are discarded. On the contrary, Visvalingam-Whyatt algorithm is widely used with a predetermined limit on the number of points. Points are sequentially removed in the order of the smallest area of the triangle formed by three consecutive points.

However, there is one caution in applying a polyline simplification algorithm to multiple polylines or polygons. Fig. 1 shows the result of an undesirable removal of the point. When polyline simplification is applied to two consecutive line segments $[a, b]$ and $[b, c]$ with the single line segment $[a, c]$, the line segments of other adjacent polygons $[p, q]$ and $[q, r]$ create new intersections (unfilled circles), and thus, change the topological relationship between polygons. When the time interval for wildfire spread estimation is short, or the wildfire spread is slow, a change of topological relationship may occur between wildfire perimeters if the general polyline simplification algorithm is applied. Therefore, it is necessary to apply a polyline simplification algorithm that preserves the topological relationship.

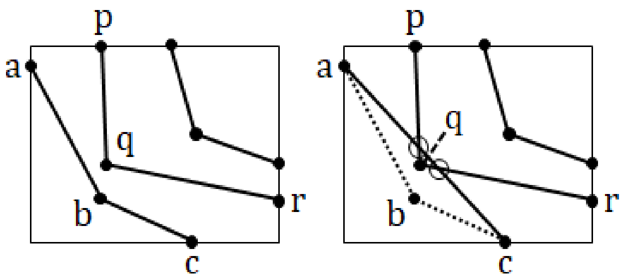

Figure 1. Change of topological relationship caused by polyline simplification.

In this paper, the polyline simplification algorithm proposed by Dyken et al. (2009) is introduced to preserve the topological relationships between polylines and polygons. To this end, this algorithm only removes points that satisfy some requirements: (1) existing intersections are maintained, (2) new intersections are not generated, (3) polylines are not degenerate into a single point, and (4) polygons are not degenerate into a single line segment. To satisfy these requirements, the triangulation technique is used in this algorithm. An example of polyline simplification using triangulation is shown in Fig. 2. Before applying a polyline simplification algorithm, we construct triangulation including all points and line segments of polylines and polygons, and determine the removal point b and consecutive points a and c. Next, we create a new polygon B that is the union of all triangles which has b as a corner. The grey part of Fig. 2 shows the new polygon B. Finally, b is removed only when the line segment [a, c] is totally inside of B, as shown in the left figure. If the line segment [a, c] intersects the boundary of B, as shown in the right figure, b cannot be removed without changing the topological relationship.

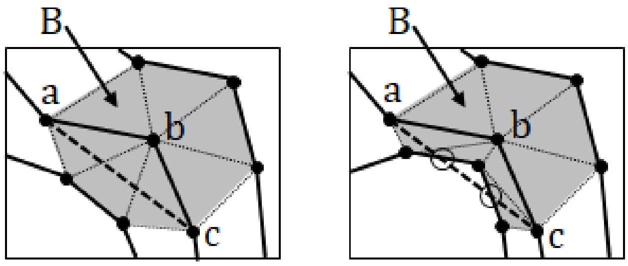

Figure 2. Polyline simplification using triangulation. Left: b should be removed, right: b should not be removed.

To perform polyline simplification in the proposed framework, this paper uses the polyline simplification algorithm implemented in *The Computational Geometry Algorithms Library* (*CGAL*) in C++.

3. Rapid Prediction of Wildfire Spread by Ensemble Kalman Filter

This section describes an improved EnKF-based algorithm that facilitates the rapid prediction of wildfire spread. In contrast to the standard Kalman Filter which updates entire state distribution, EnKF updates the ensemble of vectors that approximate the state distribution, instead of updating state distribution directly. This can also be described as an update of the state distribution using Monte Carlo simulation. It is widely known that the EnKF can provide successful results in data assimilation problems of nonlinear climate prediction (Hargreaves 2004). The improved algorithm proposed in this paper is based on the method presented by Gillijins et al. (2006) and Srivas et al. (2016).

3.1 Forecast step: forward simulations with FARSITE

In the forecast step of the EnKF, various forward models are applied to each ensemble to estimate the state and uncertainty of the ensemble at the next time step. In this methodology, FARSITE simulation was used as a forward model applied to wildfire. The forecast steps using FARSITE can be summarized as follows:

1. Estimate initial wildfire perimeter $\bar{x}_{0|0}$, and assume covariance matrix $P_{0|0}^{xx}$. Next, generate set of initial sample ensembles $x_{0|0}^1, x_{0|0}^2, \dots, x_{0|0}^N$ using initialized distribution:

$$x_{0|0}^i \sim \left(\bar{x}_{0|0}, P_{0|0}^{xx}\right), x_{0|0}^i \in \mathbb{R}^{n_{0|0}^i}, i = \{1, 2, \dots, N\} \qquad (1)$$

where N represents the number of ensembles, and $n_{a|b}^i$ denotes the dimension of ith ensemble applied at a forecast steps and b update steps. To represent x as a vector, which is a set of coordinates representing the wildfire perimeter, ensembles are expressed in the form

$$x_{0|0}^i = (u_1 \quad v_1 \quad u_2 \quad v_2 \quad \cdots \quad u_{m_{0|0}^i} \quad v_{m_{0|0}^i})^T \qquad (2)$$

in which $m_{0|0}^i$ is the number of coordinates consisting $x_{0|0}^i$, therefore $m_{0|0}^i = n_{0|0}^i/2$.

The above step is applied only when obtaining initial sample ensembles. After obtaining initial sample ensembles, this process is omitted. That is, updated ensembles $x_{k|k}^1, x_{k|k}^2, \dots, x_{k|k}^N$ are used directly in the subsequent steps. $x_{a|b}^i$ means the ith ensemble applied a forecast steps and b update steps. The second step of the algorithm is described as below.

2. Generate forward ensembles after time update by applying FARSITE to the ensembles $\{x_{k|k}^1, x_{k|k}^2, \dots, x_{k|k}^N\}$:

$$x_{k+1|k}^i = FARSITE\left(x_{k|k}^i, h_k\right), i = \{1, 2, \dots, N\} \qquad (3)$$

where h_k is the set of parameters required for FARSITE simulation as described in part 2.1 of this paper.

In order to use the forward ensembles obtained through FARSITE simulation in the update step, the coordinate values constituting the perimeter are required. However, FARSITE provides a perimeter dataset as ESRI shapefiles, which requires additional process to extract coordinate values. QGIS and ArcGIS software are generally used to extract the coordinate values from ESRI shapefiles. In this paper, the coordinate values are extracted through QGIS.

3.2 Update step 1: re-interpolate wildfire perimeter to apply data assimilation

In the update step of the Ensemble Kalman filter, the data assimilation is applied between the forward ensemble and the observed data with uncertainty. In this paper, the update step is roughly divided into two steps. In the first step of the update step, forward ensembles are adjusted in order that EnKF can be applied to the forward ensemble. Additionally, reducing the time required for the update step is also the purpose of this step.

3. Unify the dimension of each ensemble to $n_{k+1|k}$ by performing the re-interpolation and polyline simplification algorithm that makes topological relationship remain constant.

Since the number of points for expressing the forward ensembles by FARSITE are entirely different, the dimension of each ensemble is also different. However, in order to apply the EnKF to the forward ensembles, the dimension of each forward ensemble must be equivalent. In addition, the corresponding condition between points of similar positions constituting each forward ensemble is also required.

The algorithm proposed in this paper estimates wildfire spread with similar accuracy to the previous researches but using a relatively small number of points. The perimeter of wildfire has a relatively simple and complex parts depending on the complexity, and the number of points required to represent the perimeter also differ considerably. Therefore, re-interpolating the entire perimeter with the same length is inefficient in the simple part, and has low accuracy in the complex part. To address this issue, the re-interpolation is performed by using a small number of points in the simple part, and a large number of points in the complex part. This enables us to maintain the accuracy with a relatively small number of points when representing wildfire perimeter and applying EnKF. This process is achieved through the following steps 3-1 to 3-4.

3-1. Generate simplified forward ensembles $x^{1,sim}_{k+1|k}, x^{2,sim}_{k+1|k}, \dots, x^{N,sim}_{k+1|k}$ by applying polyline simplification to the forward ensembles $x^1_{k+1|k}, x^2_{k+1|k}, \dots, x^N_{k+1|k}$. It is not necessary to unify the number of points in every simplified forward ensemble. In this paper, polyline simplification is applied to all points at which the squared distance between the simplified perimeter and the existing point is less than a certain value.

3-2. Assume the number of wildfire perimeter observation points r, considered the interval at which the actual wildfire perimeter should be observed. Afterwards divide each simplified forward ensemble $x^{i,sim}_{k+1|k}, i = 1, \dots, N$ into r pieces $x^{i,sim,1}_{k+1|k}, \dots, x^{i,sim,r}_{k+1|k}$.

3-3. Re-interpolate pieces $x^{1,sim,j}_{k+1|k}, x^{2,sim,j}_{k+1|k}, \dots, x^{N,sim,j}_{k+1|k}$ with the same order $j = 1, \dots, r$. The number of points at the jth piece, $m^{j,piece}_{k+1|k}$ is

$$m^{j,piece}_{k+1|k} = \frac{1}{N}\sum_{i=1}^{N} m^{i,sim,j}_{k+1|k} \tag{4}$$

3-4. Obtain the re-interpolated forward ensembles $x^{1,re}_{k+1|k}, x^{2,re}_{k+1|k}, \dots, x^{N,re}_{k+1|k}$ that should apply the EnKF by combining re-interpolated pieces. The dimension of each re-interpolated forward ensemble is determined as

$$n^{re}_{k+1|k} = \sum_{j=1}^{r} n^{j,piece}_{k+1|k} \tag{5}$$

The re-interpolated forward ensembles obtained from this part have the same dimension, and since the corresponding condition exists also between the points of similar positions, the EnKF can be applied to the subsequent steps. In addition, the accuracy of the perimeter should be maintained while reducing the number of points using polyline simplification.

3.3 Update step 2: adjustment of wildfire perimeter using data assimilation with observations

In the second step of the update step, EnKF is applied to re-interpolated forward ensembles $x^{1,re}_{k+1|k}, x^{2,re}_{k+1|k}, \dots, x^{N,re}_{k+1|k}$ generated in the first step. Raw data of forward ensembles are re-interpolated at the first step to apply EnKF. The subsequent process is described as follows.

4. Obtain the set of estimated observation points of wildfire perimeter $\bar{y}_{k+1|k}$ at time $k+1$, and assume covariance estimate $V_{k+1|k}$. Next, generate a set of observed ensembles $y^1_{k+1|k}, y^2_{k+1|k}, \dots, y^N_{k+1|k}$ using $\bar{y}_{k+1|k}$ and $V_{k+1|k}$.

$$y^i_{k+1|k} \sim (\bar{y}_{k+1|k}, V_{k+1|k}), \bar{y}_{k+1|k} \in \mathbb{R}^{2r}, V_{k+1|k} \in \mathbb{R}^{2r\times 2r} \tag{6}$$

Form of $y^i_{k+1|k}$ are equivalent to $x^i_{0|0}$ in Eq. (2).

5. Define the re-interpolated ensemble error matrix $E^{x,re}_{k+1|k}$ and approximated sample covariance of the state $P^{xx}_{k+1|k}$ by

$$E^{x,re}_{k+1|k} = \left(x^{1,re}_{k+1|k} - \bar{x}^{re}_{k+1|k}, \dots, x^{N,re}_{k+1|k} - \bar{x}^{re}_{k+1|k}\right) \tag{7}$$

$$P^{xx}_{k+1|k} = \frac{1}{N-1} E^{x,re}_{k+1|k}\left(E^{x,re}_{k+1|k}\right)^T \tag{8}$$

6. Generate the Kalman gain

$$K_{k+1} = P^{xx}_{k+1|k} C^T_{k+1|k}\left(C_{k+1|k} P^{xx}_{k+1|k} C^T_{k+1|k} + V_{k+1|k}\right)^{-1} \tag{9}$$

where $C_{k+1|k}$ is a spatial down-sampling matrix that defines the correspondence between $y^i_{k+1|k}$ and $x^{i,re}_{k+1|k}$. Assume that $x^{re}_{k+1|k}$ is the re-interpolated true state data of the wildfire perimeter, and $y_{k+1|k}$ is the true state date of the observed point. Therefore, the following relation is satisfied:

$$y_{k+1|k} = C_{k+1|k} x_{k+1|k} \tag{9}$$

in which $C_{k+1|k}$ that satisfies the above relationship is a binary matrix defined as

$$\begin{cases} C_{k+1|k}\left(2q-1, \sum_{p=1}^{q-1} n^{p,piece}_{k+1|k} + 1\right) = 1, q = 1,2,\dots,r \\ C_{k+1|k}\left(2q, \sum_{p=1}^{q-1} n^{p,piece}_{k+1|k} + 2\right) = 1, q = 1,2,\dots,r \\ C_{k+1|k}(i.j) = 0 \ \text{otherwise} \end{cases} \tag{10}$$

7. Generate assimilated forward ensembles $x^1_{k+1|k+1}, x^2_{k+1|k+1}, \dots, x^N_{k+1|k+1}$ which is next sample ensembles.

$$x^i_{k+1|k+1} = x^{i,re}_{k+1|k} + K_{k+1}\left(y^i_{k+1|k} - C_{k+1|k} x^{i,re}_{k+1|k}\right) \tag{11}$$

Afterward, repeat from Step 2 in Section 3.1 to Step 7 above.

Using the assimilated forward ensembles $x^1_{k+1|k+1}, x^2_{k+1|k+1}, \dots, x^N_{k+1|k+1}$ to which EnKF is applied during $k+1$ time steps, it is possible to estimate the wildfire perimeter after $k+1$ time steps as

$$x^{est}_{k+1} = \sum_{i=1}^{N} x^i_{k+1|k+1} \tag{12}$$

Fig. 3 visualizes the updating process by the proposed EnKF. A total of 32 ensembles and 10 observation points are used, and the first 16 ensembles are visualized. Fig. 3(a) shows initial wildfire perimeter ensembles at $t = 0$ while (b) shows the forward wildfire perimeter ensembles at $t = 1$ after the forecast step using FARSITE. Fig. 3(c) shows assimilated forward wildfire perimeter ensembles at $t = 1$ after the update step using polyline simplification and data assimilation. The grey areas in Fig. 3(b) and (c) represent the actual area of the wildfire spread. These figures confirm that the ensembles are concentrated around the actual wildfire perimeter after the data assimilation. Fig. 3(d) shows how the ensemble changes as going through the forecast and update steps. The observation points (including errors) used in the update step are also displayed. By comparing the perimeters before and after the update step, it is shown that the update step can move the ensemble closer to the actual perimeter.

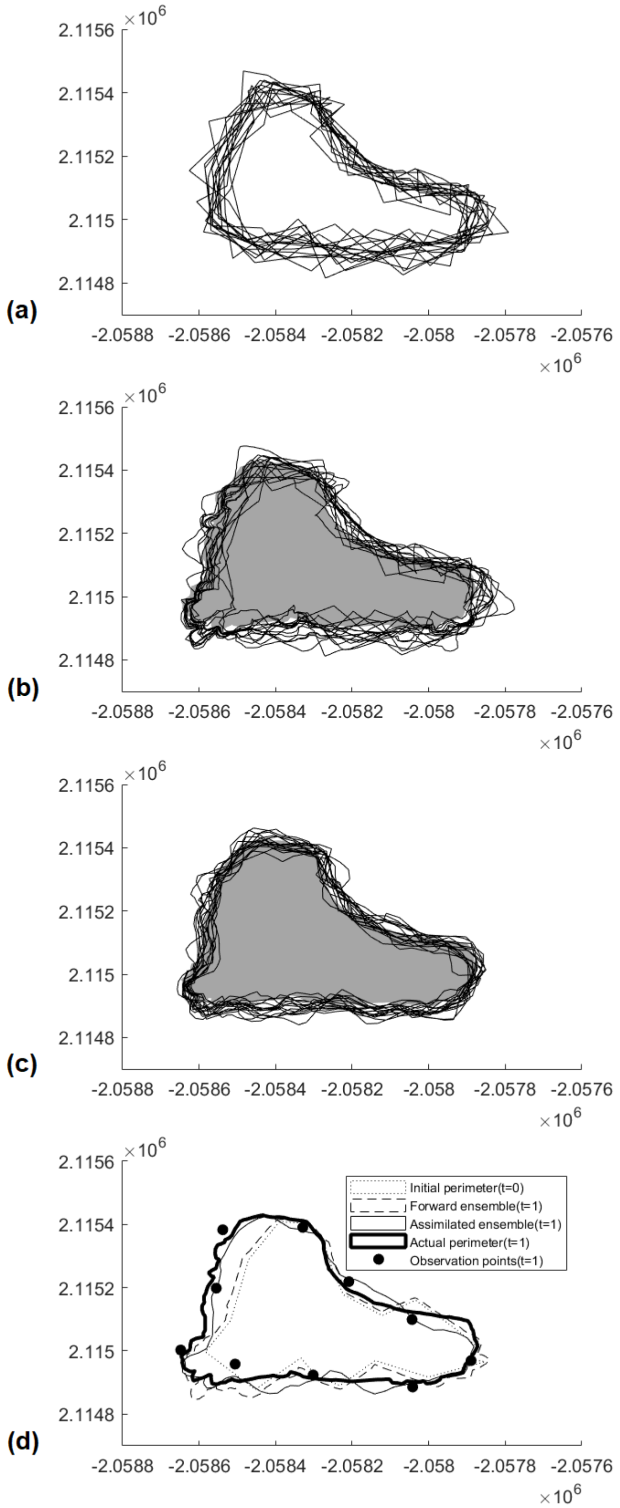

Figure 3. Updating results by the EnKF: (a) initial ensembles, (b) forward ensembles, (c) assimilated forward ensembles, and (d) ensemble updates.

4. Conclusion

In this paper, a new algorithm was proposed for improving the speed of high-accuracy wildfire spread prediction using real-time observation data. To facilitate rapid prediction, a 2-D polyline simplification that can preserve the topological relationship was applied to the FARSITE forward simulation results. The number of points to represent wildfire perimeters is decreased adaptively when the polyline simplification is applied. Therefore, it is possible to improve data assimilation speed using the EnKF. Instead of re-interpolating the whole perimeter to have the same density, different densities were achieved depending on the complexity of each part of the perimeter. The proposed method minimized the increase of the perimeter estimation error due to the application of polyline simplification and re-interpolation introduced for the application of the EnKF.

This study identified possible future research topics: (1) It is required to check the efficiency of the proposed algorithm when applied to actual wildfires. Actual wildfires feature much more complicated climate factors whose applications to computational simulations are limited. In addition, wildfire suppression is also difficult to incorporate into simulations, which may hamper accurate data assimilation. For this reason, additional modifications of the proposed algorithm needed to be explored; and (2) It is noted that the fire spread prediction after wildfire spread affect the cities is also important. The algorithm proposed in this paper is applicable if wildfire does not reach the artificially developed area, e.g. cities, roads. It is desirable to conduct further research to investigate a potential impact of a decrease in the rate of fire spread inside the city on the rate of fire spread outside.

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References

Albini, F. A. 1979. *Spot Fire Distance from Burning Trees: A Predictive Model* (Vol. 56). Intermountain Forest and Range Experiment Station, Forest Service, US Department of Agriculture.

Anderson, H. E. 1981. *Aids to Determining Fuel Models for Estimating Fire Behavior* (Vol. 122). US Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station.

Beezley, J. D., and Mandel, J. 2008. Morphing Ensemble Kalman Filters. *Tellus A: Dynamic Meteorology and Oceanography*, 60(1), 131-140.

Douglas, D. H., and Peucker, T. K. 1973. Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or its Caricature. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 10(2), 112-122.

Dyken, C., Dæhlen, M., and Sevaldrud, T. 2009. Simultaneous Curve Simplification. *Journal of Geographical Systems*, 11(3), 273-289.

Finney, M. A. 1998. *FARSITE, Fire Area Simulator - Model Development and Evaluation* (No. 4). US Department of Agriculture, Forest Service, Rocky Mountain Research Station.

Gillijns, S., Mendoza, O. B., Chandrasekar, J., De Moor, B. L. R., Bernstein, D. S., and Ridley, A. 2006. What is the Ensemble Kalman Filter and How Well Does It Work?. In *2006 American Control Conference* (pp. 4448-4453.)

Hargreaves, J. C., Annan, J. D., Edwards, N. R., and Marsh, R. 2004. An Efficient Climate Forecasting Method Using an Intermediate Complexity Earth System Model and the Ensemble Kalman Filter. *Climate Dynamics*, 23(7-8), 745-760.

Johns, C. J., and Mandel, J. 2008. A Two-stage Ensemble Kalman Filter for Smooth Data Assimilation. *Environmental and Ecological Statistics*, 15(1), 101-110.

Linn, R. R., and Harlow, F. H. 1997. *FIRETEC: A Transport Description of Wildfire Behavior* (No. LA-UR-97-3920; CONF-980121-). Los Alamos National Lab., NM (United States).

Mandel, J., Chen, M., Franca, L. P., Johns, C., Puhalskii, A., Coen, J. L., ... and Zhao, W. 2004. A Note on Dynamic Data Driven Wildfire Modeling. In International Conference on Computational Science (pp. 725-731). Springer, Berlin, Heidelberg.

Mandel, J., Bennethum, L. S., Beezley, J. D., Coen, J. L., Douglas, C. C., Kim, M., and Vodacek, A. 2008. A Wildland Fire Model with Data Assimilation. Mathematics and Computers in Simulation, 79(3), 584-606.

Mell, W., Jenkins, M. A., Gould, J., and Cheney, P. 2007. A Physics-based Approach to Modelling Grassland Fires. *International Journal of Wildland Fire*, 16(1), 1-22.

Nelson Jr, R. M. 2000. Prediction of Diurnal Change in 10-h Fuel Stick Moisture Content. *Canadian Journal of Forest Research*, 30(7), 1071-1087.

Rollins, M. G. 2009. LANDFIRE: A Nationally Consistent Vegetation, Wildland Fire, and Fuel Assessment. *International Journal of Wildland Fire*, 18(3), 235-249.

Rochoux, M. C., Ricci, S., Lucor, D., Cuenot, B., and Trouvé, A. 2014. Towards Predictive Data-driven Simulations of Wildfire Spread–Part I: Reduced-cost Ensemble Kalman Filter Based on a Polynomial Chaos Surrogate Model for Parameter Estimation. *Natural Hazards and Earth System Sciences*, 14, 2951-2973

Rothermel, R. C. 1972. *A Mathematical Model for Predicting Fire Spread in Wildland Fuels* (Vol. 115). Intermountain Forest and Range Experiment Station, Forest Service, United States Department of Agriculture.

Rothermel, R. C. 1991. *Predicting Behavior and Size of Crown Fires in the Northern Rocky Mountains* (Vol. 438). US Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station.

Scott, J. H. 2005. *Standard Fire Behavior Fuel Models: A Comprehensive Set for Use with Rothermel's Surface Fire Spread Model*. US Department of Agriculture, Forest Service, Rocky Mountain Research Station.

Srivas, T., Artés, T., De Callafon, R. A., and Altintas, I. 2016. Wildfire Spread Prediction and Assimilation for FARSITE Using Ensemble Kalman Filtering. *Procedia Computer Science*, 80, 897-908.

Tymstra, C., Bryce, R. W., Wotton, B. M., Taylor, S. W., and Armitage, O. B. 2010. Development and Structure of Prometheus: The Canadian Wildland Fire Growth simulation Model. *Natural Resources Canada, Canadian Forest Service, Northern Forestry Centre, Information Report NOR-X-417.(Edmonton, AB)*.

Visvalingam, M., and Whyatt, J. D. 1993. Line Generalisation by Repeated Elimination of Points. *The Cartographic Journal*, 30(1), 46-51.

Wagner, C. V. 1977. Conditions for the Start and Spread of Crown Rire. *Canadian Journal of Forest Research*, 7(1), 23-34.

Zhou, T., Ding, L., Ji, J., Li, L., and Huang, W. 2019. Ensemble Transform Kalman Filter (ETKF) for Large-scale Wildland Fire Spread Simulation Using FARSITE Tool and State Estimation Method. *Fire Safety Journal*, 105, 95-106.