Optimal Planning of Rockfall Protection Structures Based on Hazard Assessment for Road

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Abstract: This study proposed a methodology for an optimal selection of protection structures against rockfalls based on minimization of the total cost on the entire roads. The total cost is the sum of the construction cost of protection structures and risk of the exceedance of the limit states. Risk is the product of probability of exceedance and its consequence when the limit state is exceeded. After modeling distribution of size of rockfall, we calculated the probabilities of limit state exceedance from the collision hazard, which is relationship between probability of exceedance and energy of falling rock, which is evaluated using three dimensional rockfall simulation by lumped mass method. A three dimensional slope model is constructed based on the elevation data. The road is divided into segments to calculated rockfall hazard and evaluated optimal placement and selection of protection structures. The proposed method is capable of determining an optimal placement of protection structures that minimizes the total cost on the entire segments, while satisfying a given budget constraint. We applied the proposed method to actual road under slopes and show examples of the optimal placement of rockfall protection structures. The protection structures were arranged according to the rockfall hazard while satisfying a given budget constraint.

Keywords: optimal planning, rockfall protection structure, total cost, rockfall hazard, constraint optimization problem.

1. Introduction

Rockfall is one of major natural hazard to a road in mountainous terrain. Safety of a road against rockfall is regarded as an important item to assess. Countermeasures against rockfall are carried out for the slope near the road as road disaster prevention in Japan (Japan Road Association 2017). Influence of slope shape to road are usually assessed qualitatively for rockfall countermeasure by engineers. For further sophistication of risk management of rockfall, comprehensive evaluation based on quantitative risk evaluation is necessary. The comprehensive evaluation method considering quantitative risk due to rockfall for road is composed of three steps, 1) Rockfall hazard assessment for road, 2) quantitative risk assessment by assessing the consequence of rockfall for road, 3) decision-making of countermeasure based on risk assessment. This report discusses the step 2 and 3 because the step 1 is presented in Tsuda et al. (2019).

In order to quantitatively evaluate the hazard of rockfall for roads and structures, a method based on the velocity of rockfall is proposed (Yoshida et al. 2018). However, the energy of rockfall is not discussed. The hazard assessment with respect to energy, force or impulse of collision is necessary to evaluate the damage of protection facilities. For the countermeasure of decision-making, Kanno et al. (2019) evaluated the risk from the energy of rockfall and consequence, and an optimum placement of protection structures that minimizes the total risk on the entire road, while satisfying a given budget constraint. However, the countermeasure of injury or human life loss, and economic loss due to long-term traffic interruption is not discussed.

In this study, a methodology for optimal selection of protection structures against rockfalls is proposed based on minimization of the total cost with budget constraint. The total cost is the sum of the construction cost of protection structures and risk of the exceedance of the limit states. Risk is the product of probability of exceedance and its consequence when the limit state is exceeded. The probabilities of exceedance are calculated from the rockfall hazard which is evaluated using three dimensional rockfall simulation by lumped mass method. The rockfall hazard curve shows relationship between probability of exceedance and the collision energy when the falling rocks reach to the road. In order to calculate the collision energy, we tried to model probabilistic distribution of rock size. We apply the proposed method to actual road under slopes and show examples of the optimal placement of rockfall protection structures.

2. Assessment of Rockfall Hazard

2.1 Modeling of Slope Shape

Fig.1 indicates the slope model at a model site. Digital elevation model data in Japan can be downloaded from Geospatial Information Authority of Japan. A three dimensional slope model along a road of the site with 1500m x 1500m is constructed with using 90,000 points of the digital elevation data. Initial location of falling rocks are allocated to the slope surface depending on the slope angle calculated by 5m x 5m mesh. Rockfall hazard to assumed road segments under the slope is assessed by using the lumped mass method. The road under the slope is divided into 147 segments, s1 to s147 with length 10 m. Initial location of falling rocks should be presumed based on topography, geology and inspection data performed by local government. It, however, needs lots of effort to allocate the dangerous regions in a large area considering above all information. It is reported that slopes of which angle is 45 degree or more likely cause rockfall (Guzzetti et al. 2003). As a preliminary study, initial location of

falling rocks are simply allocated depending on the slope angle as shown in Fig.1. At first we select area which may affect the segment of road s1 to s147 under the slopes, and then allocate falling rocks depending on the slope angle.

2.2 Outline of Rockfall Simulation

It is difficult to predict the behavior of rockfall deterministically if a sophisticated simulation method such as Distinct Element Method (DEM) (Cundall et al. 1979) is used because the movement of rockfall is strongly affected by small irregularity of slope surface and shape of falling rock, and it is impossible to model every detail of them. This study uses simple lumped mass simulation with random numbers (Yoshida et al. 2018). Calculation cost of this method is much smaller than that of DEM. This method only considers bouncing and ignores the rolling and slide of falling rocks. The parameters of the simple lumped mass simulation were roughness parameter and restitution coefficients in lateral and normal direction. The restitution coefficient is a parameter that related to energy loss when a point mass hits ground surface. The roughness is a parameter that represents the irregularity of the slope surface and the shape of rock.

In the simulation, the roughness parameter is assumed to be 0.3, restitution coefficients in normal direction is 0.43. The restitution coefficient in lateral direction is assumed to be not deterministic but random with uniform distribution [0.85, 1.05]. They are determined to obtain good agreement with experiment results of falling rocks (Yoshida et al. 2018).

2.3 Modeling of Size of Rockfall

It is necessary to model the distribution of rock size in order to evaluate the hazard with respect to energy. We investigated several literatures of the distribution of rock size. We compared them by diameter because the index that show the size of rockfall differ depending on the literature. The maximum diameter in literatures was assumed to be the diameter. The diameter was calculated by assuming that the rockfall shape was a sphere when volume of rock is specified in the literature. The diameter was calculated by assuming that the rockfall shape was a sphere with the density of soil particles (2.7 g / cm³) when weight of rock is specified in the literature. Fig.2 shows an example of cumulative distribution of rock diameter, e.g., it shows that the probability of rockfall with a diameter of 2 m or less is around 80%. The rockfall size is modeled using an exponential distribution. cumulative probability distribution The of the exponential distribution is shown below:

$$p(x \mid \lambda) = \lambda \exp(-\lambda x) \tag{1}$$

where, λ is a parameters indicating inverse of mean, *x* is the diameter of rockfall. Fig.3 shows cumulative distributions indicated in each literature and exponential distribution. Large differences in the distributions are observed depending on the literature. The exponential distributions at $\lambda = 0.6$ and 3.0 show the upper and lower bounds of the data presented in the literature. Small rockfalls occur at $\lambda = 3.0$, and large rockfalls occur at $\lambda = 0.6$.



Figure 1. Model slope and segments s1 to s147for the study.



Figure 2. An example of frequency distribution and cumulative distribution.



Figure 3. Cumulative distribution of rock size from literatures and the model of rock size based on exponential distribution.

2.4 Hazard Assessment based on Rockfall Collision Energy

The hazard curve with respect to collision energy of rockfall was calculated by the following procedure.

- 1. Calculate the velocity when a rock reaches a road segment from rockfall simulation
- 2. Assume rockfall size by random numbers generated by the exponential distribution
- 3. Calculate energy from the velocity and the rockfall size
- 4. Calculate the probability of exceedance from the energy when the falling rocks reach to the segment.

The energy of rockfall is defined as follows:

$$E = mv^2/2$$
 where, $m = \rho V$, $V = 4\pi (d/2)^3/3$ (2)

where *E* is the collision energy, *m* is the mass, *v* is the velocity of rockfall, ρ is the density of soil particle, *V* is the volume of rock, *d* is diameter of rock. Cumulative distribution of the collision energy when the falling rocks reach to the road segment is calculated with 1000000(= one million) particles.

As an example, Fig.4 shows the rockfall hazard curve of s53 (segment 53 shown in Fig.1) which indicates the relationship between the probability of exceedance and the collision energy when the falling rocks reach to s53. Black lines show the rockfall hazard curve simulated 100 times by changing the seed of random numbers for rock size generation. Though the variation of the hazard curve at large energy is large, that at small energy is small. The red line shows 0.5 fractile hazard curve calculated from the all rockfall hazard curves. This is called the collision hazard in this paper. The probability of exceedance corresponding to the energy of rockfall 0, 100 and 1000kJ of road segments s1-s147 are shown in Fig.5. Fig.5(1) shows the probability of exceedance calculated based on exponential distribution with λ =0.6. Fig.5(2) shows the of exceedance calculated probability based on exponential distribution with λ =3.0. For example, there is 100 times difference between the probability of s53 of 1000kJ with $\lambda = 0.6$ and =3.0. The collision energy is greatly affected by the value of λ . The difference of probability of exceedance with respect to energy is small when λ =0.6. On the other hand, it is large when λ =3.0. The small difference of probability of exceedance means the most of rocks that reach the road have large collision energy. The large difference of probability of exceedance means the most of rocks which reach the road have small energy, i.e., small impact of collision. Three dimensional slope shape of mountain causes such difference of hazard.



Figure 4. The collision hazard curves of s53 with 100 runs for rock size distribution.



Figure 5. Estimated probability of exceedance at 0, 100, 1000kJ.

Table 1. Cost, consequence, and mint state energy of each protection structure types.					
Type of protection structure	Construction $\cos c_0$	Limit state 1 consequence c_1	Limit state energy 1 (kJ)	Limit state 2 consequence c_2	Limit state energy 2 (kJ)
Non-structure	0	0	0	3.0×10 ⁵	10
А	35	17.5	60	3.0×10 ⁵	100
В	75	37.5	150	3.0×10 ⁵	200
С	200	100	600	3.0×10 ⁵	800

Table 1. Cost, consequence, and limit state energy of each protection structure types.

3. Methodology for optimal placement of rockfall protection structures

3.1 Formulation of Total Cost

The two kinds of limit state of road facilities were assumed based on manual for anti-impact structures against falling rocks (Japan Road Association 2017). When limit state 1 is exceeded, the road has minor damage which causes short-term closure, but there is no serious impact on road, vehicles, or human life. The other is major damage which causes failure of road, injury or human life loss, and economic loss due to long-term traffic interruption. Limit states are defined by collision energy of rockfall in this study. Energies according to limit states are called as limit state energy in this paper. Total cost of protection structure at a segment can be defined as follows:

$$c_t = c_0 + (p_{t1} - p_{t2})c_1 + p_{t2}c_2 \tag{3}$$

$$p_{t1} = p_e p_{f1}, \ p_{t2} = p_e p_{f2} \tag{4}$$

where c_0 is the construction cost of protection structures; p_{t1} and p_{t2} are the probabilities that the limit state is exceeded; c_1 is the consequence when limit state 1 is exceeded (repair costs); c_2 is the consequence when limit state 2 is exceeded (costs for failure of road, injury or human life, and economic lass due to long-term traffic); p_f is conditional probabilities to exceed the limit state energy when a rockfall occurs, which is calculated from the collision hazard shown in Chapter 2. As shown in Fig.6, the probability corresponding to limit state energy 1, 2 are obtained from collision hazard. p_e is probability of rockfall occurrence. The risk is the product of probability of exceedance and its consequence when the limit state1, 2 is exceeded. The total cost is the sum of the construction cost of protection structures and the risk of the exceedance of the limit states.



Figure 6. The relation between probability of exceedance and limit state energy 1, 2.

3.2 Optimization problem for placement of protection structures

An optimal placement of protection structures can be proposed to minimize the total cost on the entire segments which is estimated with Eq.(3). However, it is not practical to determine the optimal placement without budget constraint, so that we determine an optimal placement of protection structures that minimizes the total cost on the entire segments, while satisfying a given budget constraint. It is difficult to solve optimization problem with the constraint empirically because it becomes complicated as the types of protection structures and the number of segments increased. Therefore, the optimization problem is solved based on linear programming as follows:

min
$$C_t = \sum_{j=1}^{m} \sum_{k=1}^{n} a_{jk} c_{tjk}$$

s.t $\sum_{j=1}^{m} \sum_{k=1}^{n} a_{jk} c_j \leq B$
 $a_{jk} \in \{0, 1\}, \ j = 1,...,m, \ k = 1,...,n$
(5)

where C_t is the total cost; a_{jk} is the indicator which is 1 when protection structure *j* is placed on segment *k*, and which is 0 when not; c_j is the construction cost of protection structures *j*; *B* is the budget constraint.

4. Results

In this study, four types of protection structures were assumed as shown in Table 1. Protection sturecure A is strong and expensive, C is weak and cheap, and B is between A and C. Total probability of rockfall occurrence p_e is assumed to be 0.9, the parameters for the exponential distribution with respect to the size of rockfall λ is assumed to be 0.6. Fig.7 shows total cost each protection structures in segments s1 to s147. Fig.8 shows optimal placement of protection structures based on minimization of the total cost. For example, structure C (strong) are arranged near segment s53, structure A (weak) are arranged near segment s110.

Fig.9 shows an optimal placement of protection structures when budget constraint is 25 million yen. Compared with Fig.8, more structure A (weak) are generally arranged at entire road. The protection structures are not changed from structure C (strong) irrespective of the budget constraint, near segment s53 where the rockfall hazard is high, i.e., the slope is steep. On the other hand, the protection structures near segment s110 where the hazard of rockfall is small, i.e., the slope

is gentle, changed from structure A (weak) to without protection structure. The protection structures were arranged according to the rockfall hazard while satisfying a given budget constraint.



Figure 7. Total cost for each protection structures in segments s1 to s147.



Figure 8. Optimal placement of protection structures based on the total cost without budget limitation, probability of rockfall occurrence p_e is 0.9.



Figure 9. Optimal placement of protection structures that minimizes the total cost on the entire segments, under a given budget constraint. Budget constraint is 25 million yen, probability of rockfall occurrence p_e is 0.9.

5. Conclusions

This study proposed a methodology for an optimal selection of protection structures against rockfalls based on minimization of the total cost with budget constraint. After modeling distribution of size of rockfall, a method to assess the collision hazard, which is relationship between probability of exceedance and energy of falling rock, is proposed. We applied the proposed method to actual road under slopes and show examples of the optimal placement of rockfall protection structures.

More studies are required in order to apply this proposed method for actual road management because many parameters were assumed in this study. The parameter with respect to the size of rockfall λ should be determined based on the types of rock in the region. We quantitatively evaluated the consequence of failure of road, injury or human life loss, and economic loss due to rockfalls with many assumptions. These assumptions need to be reviewed and discussed with stakeholders of the road.

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