The Seventh Asian-Pacific Symposium on Structural Reliability and Its Applications (APSSRA2020)
October 4–7 2020, Tokyo, Japan
T. Takada, I. Yoshida & T. Itoi (editors)

Consistent Geotechnical Reliability Analysis with a Simple Deterministic Model

Te Xiao[1], Li-Min Zhang[2] and Dian-Qing Li[3]

[1]*Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong, China.*
Email: xiaote@ust.hk
[2]*Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong, China.*
Email: cezhangl@ust.hk
[3]*State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, China.*
Email: dianqing@whu.edu.cn

Abstract: Real-world geotechnical reliability analysis is limited in practice partially because of computationally time-consuming complex deterministic models involved. Auxiliary random finite element method (ARFEM) is a representative reliability method that fully utilizes the correlation between simple and complex models to achieve efficient and consistent reliability analysis. To get rid of the complex model and further improve the computational efficiency of reliability analysis, this study proposes a corrected method using a simple deterministic model only. Impacts of calculation model and geomaterial on geotechnical response are deliberately decoupled so that the response in the complex model can be inferred indirectly from that in the simple model. Through two geotechnical reliability problems in 3-D spatially varying soils, it is found that the corrected analysis, purely based on the simple-model-based preliminary analysis of ARFEM, is able to gain approximately consistent reliability results as the complex-model-based target analysis of ARFEM does but without any additional computational effort.
Keywords: Geotechnical reliability analysis; spatial variability; simple model; auxiliary random finite element method.

1. Introduction

Although reliability analysis has gained increasing interest in the geotechnical community (Phoon, 2020), its application for real-world geotechnical problems is still limited. This is partially because reliability analysis often requires repeated call of deterministic models and the computational burden is too heavy to afford particularly when a time-consuming model such as 3-D finite element model is adopted in a simulation-based reliability framework. Compared with simplified reliability methods (e.g., first-order reliability method) and surrogate-model-based methods (e.g., response surface method), simulation-based methods (e.g., Monte Carlo simulation) are often criticized for their inefficiency, but they do have many intrinsic advantages in characterizing complex uncertain systems, such as high-dimensional uncertainty modeling. In the geotechnical context, the random field modeling for spatially varying geomaterials is a typical example that advocates simulation-based reliability methods (Fenton and Griffiths, 2008).

On the other hand, it is not uncommon that various deterministic models/methods are available for the same geotechnical problem. Taking slope stability analysis as an example, there are 2-D and 3-D limit equilibrium (with circular/noncircular slip surface) and finite element (with coarse/fine mesh) models, etc. These models have positive correlation to some extent. Some models (e.g., 2-D limit equilibrium models) are simple and efficient, while others (e.g., 3-D finite element models) are complex and accurate. Correspondingly, the choice of reliability analysis using a simple or complex model is also a trade-off between accuracy and efficiency. The response conditioning method (Au, 2007) opens up the possibility to fully utilize the correlation between simple and complex models to achieve efficient and consistent reliability analysis. A complex-model-based target reliability analysis is strategically carried out with the aid of a simple-model-based preliminary reliability analysis. However, previous studies showed that the computational effort for target analysis still takes a vast majority of total effort, say more than 80% or even 98% (Li et al., 2016; Xiao et al., 2016). How to obtain a consistent reliability estimate without a complex model is an open question.

This study aims to investigate the feasibility of consistent geotechnical reliability analysis using a simple deterministic model only. The consistency herein means the reliability estimate agree with that using a complex deterministic model. The impacts of calculation model and geomaterial on system response are deliberately decoupled. Finally, two geotechnical examples are conducted to validate the feasibility.

2. Auxiliary random finite element method

To incorporate the spatial variability of geomaterials into geotechnical reliability analysis, the widely-used random finite element method (RFEM) (Fenton and Griffiths, 2008) adopts random field theory to model the spatial variability, Monte Carlo simulation for uncertainty propagation, and finite element method to assess the geotechnical problem. The auxiliary random finite element method (ARFEM) (Xiao et al., 2016) is an updated version that significantly improves computational efficiency within the framework of response conditioning method. It consists of a simple-model-based preliminary analysis and a complex-model (i.e., finite element model) -based target analysis. Since its accuracy has been validated against the traditional RFEM (Xiao et al., 2016), ARFEM will be used in this study as a benchmark of reliability analysis.

The high correlation between simple and complex models is crucial to the success of ARFEM, which can be easily satisfied in the geotechnical context as mentioned earlier. At least two universal choices can be considered in practice: (a) coarsely and finely meshed finite element models, and (b) 2-D and 3-D models, as the simple and complex models, respectively. The first choice has been applied by Xiao et al. (2016) and the second will be demonstrated later in this study.

3. Corrected reliability analysis

For a given geotechnical problem, the response g (e.g., safety factor of slope stability and foundation settlement) is the outcome of both calculation model and material. No matter which calculation model is used, the adopted material always coincides. This is why the material is taken as a bridge to link simple and complex models in the response conditioning method.

As the selection of calculation model has no relation to material, it is intuitive to assume that impacts of model and material on the system response can be decoupled into two independent components as:

$$g = f(\text{model}, \text{material}) = f_1(\text{model})\, f_2(\text{material}) \qquad (1)$$

where f, f_1 and f_2 are implicit functions. For the sake of brevity, let $g_{s,d}$ be the response obtained from model s and material d. Consider two models s and c and two materials d and r. The impact of different models on the same material can be derived as:

$$\frac{g_{s,d}}{g_{c,d}} = \frac{g_{s,r}}{g_{c,r}} = \frac{f_1(s)}{f_1(c)} \qquad (2)$$

which is independent of the material, as we expect. If models s and c are referred to simple and complex models, respectively, and materials d and r are referred to the deterministic material used in deterministic analysis (i.e., mean value) and the random material randomly generated in reliability analysis (i.e., one random field realization), respectively, we can directly correct the system response of one random realization from the simple model to the complex model by rewriting Eq. (2) as:

$$g_{c,r} = \frac{g_{c,d}}{g_{s,d}} g_{s,r} = k \times g_{s,r} \qquad (3)$$

where k is a correction constant that needs to be determined from deterministic analysis. As a matter of fact, Eq. (3) holds approximately because some simple models cannot fully capture the impact of materials, such as taking a 2-D model as the simple model of a 3-D model. The accuracy of Eq. (3) improves as the correlation between simple and complex models increases, similar to ARFEM.

By applying Eq. (3) to all samples generated in the preliminary reliability analysis of ARFEM, an approximately consistent reliability estimate using only simple model, referred to as corrected reliability analysis in this study, can be obtained easily without any extra computational effort. In theory, Eq. (3) can be applied to correct any simple-model-based reliability analysis. The reason for using ARFEM is because it also provides a target reliability analysis for further validation.

4. Illustrative examples

This section applies the corrected reliability analysis to evaluate the reliability of two geotechnical problems, including the settlement of a strip footing and the stability of a long embankment slope, in 3-D spatially varying soils. A squared exponential autocorrelation function with anisotropy in vertical and horizontal directions is used to describe the 3-D spatial variability. More details on the characterization and simulation of 3-D spatial variability can be referred to Xiao et al. (2018) and Li et al. (2019), respectively. Both problems are analyzed using 3-D finite element models developed in Abaqus, in which the soil is modeled by an elastic-perfectly plastic constitutive model with the Mohr-Coulomb failure criterion. The soil parameters in the two examples are given in Table 1.

Table 1. Soil properties in two examples.

Soil property	Footing	Slope
Cohesion, c (kPa)	20	30
Friction angle, φ (°)	30	0
Dilation angle, ψ (°)	20	0
Unit weight, γ (kN/m^3)	18	20
Young's modulus, E (MPa)	60	100
Poisson's ratio, ν	0.3	0.3

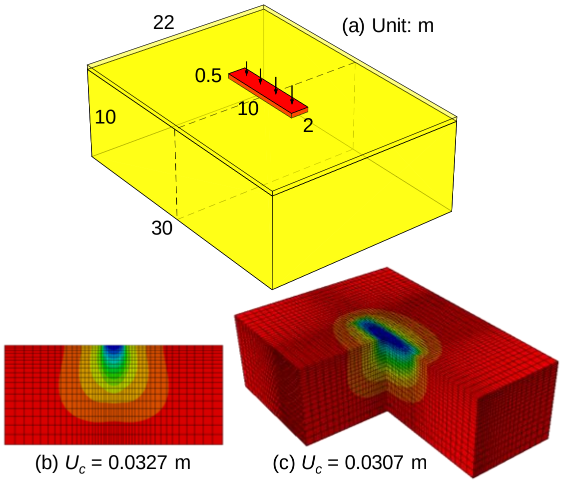

Figure 1. Example of footing: (a) basic setting; (b) simple model: 2-D model; (c) complex model: 3-D model.

4.1 Strip footing

The first example is to evaluate the reliability of a strip footing at serviceability limit state, adapted from Ahmed and Soubra (2012). The strip footing is 10 m long, 2 m wide, 0.5 m deep, and subjected to a vertical applied pressure of 500 kPa, as shown in Fig. 1(a). The tolerable central settlement is set as 2% of the width, i.e., 0.04 m. To apply ARFEM, a 2-D finite element model (Fig. 1(b)) is developed as the simple model of the 3-D finite element model (Fig. 1(c)). The central settlements (U_c) are 0.0327 m and 0.0307 m, respectively, in the 2-D and 3-D models through deterministic analysis, leading to a correction factor of 0.939. The efficiency ratio of 2-D model to 3-D model is about 120, i.e., 12 s vs. 24 min.

As a benchmark case, the Young's modulus of soil is assumed to be lognormally distributed with a coefficient of variation (COV) of 0.15 and horizontal and vertical autocorrelation distances of 10 m and 1 m, respectively. Besides, three additional soil variability cases are also considered, as shown in Table 2, to explore the impact of

Table 2. Soil variability cases.

Case	Variable	Distribution	COV	Autocorrelation distance [l_h, l_v] (m)
Case 1	E	Lognormal	0.15	[10, 1]
Case 2	E		0.3	[10, 1]
Case 3	E		0.15	[20, 2]
Case 4	E, ν		0.15	[10, 1]

Table 3. Calculation of corrected reliability analysis for Case 1.

U_c (preliminary)	U_c (corrected)	Cumulative probability
0.0274	0.0257	1.000
0.0279	0.0262	0.998
…	…	…
0.0399	0.0375	3.18×10^{-3}
<u>0.0400</u>	0.0375	<u>3.16×10^{-3}</u>
…	…	…
0.0426	0.0399	1.52×10^{-4}
0.0427	<u>0.0400</u>	<u>1.50×10^{-4}</u>
…	…	…

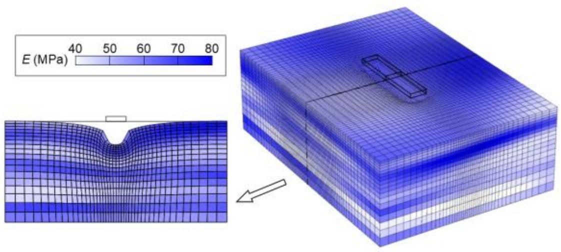

Figure 2. A typical 3-D random field realization of Case 1.

soil variability on the performance of corrected reliability analysis.

For comparison, all cases are analyzed using ARFEM with the same parameters (m = 4, N = 500, p_0 = 0.1 and N_s = 10; more details can be referred to Xiao et al. (2016)). A total of 1850 simple models and 50 complex models are generated in the preliminary and target analyses, respectively, which takes about 7 h and 14 h, respectively, using parallel computing. A typical 3-D random field realization of Case 1 is shown in Fig. 2. The 2-D random field used in the simple model is extracted from the central section of 3-D random field so that the 2-D and 3-D models can be connected through material.

The failure probabilities of Case 1 are estimated as 3.16×10^{-3} and 1.90×10^{-4} in the preliminary and target analyses, respectively, of ARFEM. Then, Eq. (3) with a correction factor of 0.939 is used to correct the failure probability from 3.16×10^{-3} to 1.50×10^{-4}, which is consistent with the complex-model-based target failure probability (i.e., 1.90×10^{-4}). The calculation procedure is provided in Table 3, in which the cumulative probability is estimated from preliminary analysis and it remains unchanged since the rank of each sample does not change. It is worth noting that no extra computational effort is needed for the corrected reliability analysis. In spite of the high efficiency of ARFEM, compared with conventional Monte Carlo simulation-based RFEM (about 50,000 3-D finite element analyses are required and need 1.6 years by estimation), the corrected reliability analysis further cuts the computational time by two thirds.

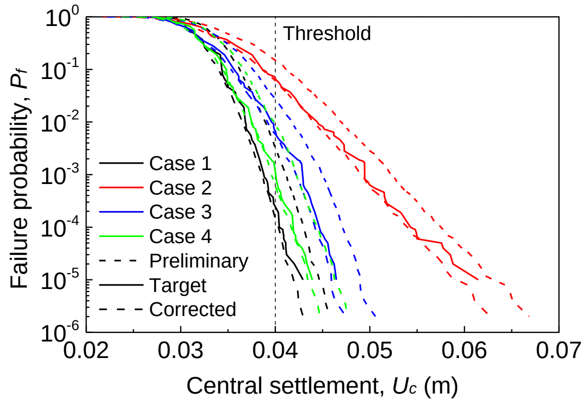

Figure 3. Cumulative distribution function of central settlement.

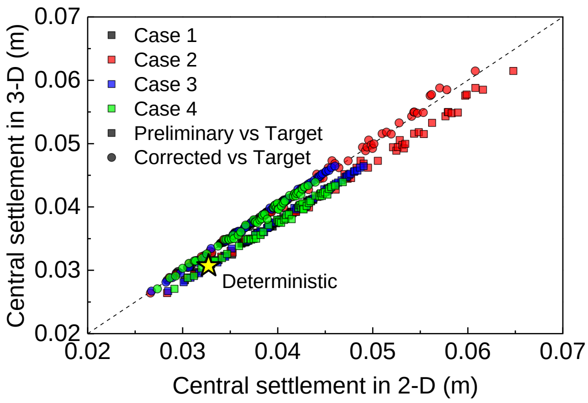

Figure 4. Central settlement calculated by different models.

It benefits from the fact that only a simple deterministic model is involved in the reliability analysis.

Similar results can also be obtained in the other three cases. Fig. 3 presents the cumulative distribution function of the central settlement in all cases. Since a smaller central settlement is more preferable, the cumulative distribution function represents the failure probability of the central settlement exceeding any specific threshold. As shown in Fig. 3, although the preliminary analysis overestimates all curves, the corrected analysis successfully updates them to agree well with the target analysis at all probability levels. Note that a uniform correction factor is adopted in all cases. This implies that the performance of the corrected reliability analysis will not be significantly affected by the soil variability.

Fig. 4 compares the central settlements calculated by both 2-D and 3-D models for those samples used in target analysis. The correlation coefficient between 2-D and 3-D models is as high as 0.99, no matter which case is considered, indicating that the 2-D model is a very good simplified model for the footing problem. Through the correction of Eq. (3), the settlements in 2-D and 3-D are well distributed along the 1:1 line, which validates the accuracy of Eq. (3) and shows that such a correction is insensitive to soil variability again.

4.2 Long embankment slope

The second example on a long embankment slope has been studied by Xiao et al. (2016) using ARFEM. Its

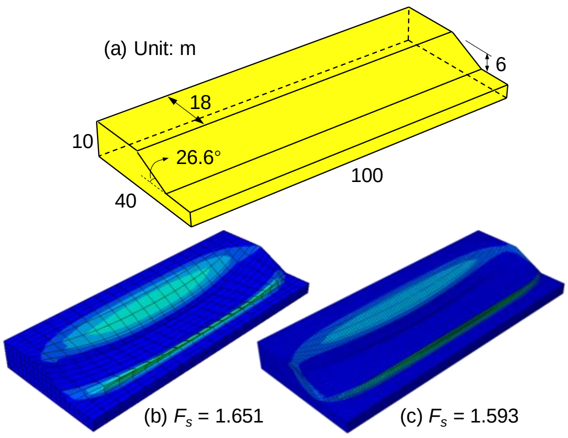

Figure 5. Example of slope: (a) basic setting; (b) simple model: coarse mesh; (c) complex model: fine mesh.

geometry is shown in Fig. 5(a) and the soil property is given by Table 1. Since the 3-D spatial variability results in irregular slip surface in both shape and location, it is not easy to choose a reasonable 2-D profile for simplification. Alternatively, a coarsely meshed finite element model (Fig. 5(b)) is taken as the simple model of a finely meshed model (Fig. 5(c)). This strategy is available for any finite element model. According to deterministic analysis, the factors of safety (F_s) of slope stability are estimated as 1.651 and 1.593, respectively, in the coarse and fine models using shear strength reduction technique. As a result, the correction factor is determined as 0.965. The efficiency ratio of coarse model to fine model is about 40, i.e., 48s vs. 35 min.

The soil cohesion is considered as a random field and modeled by a lognormal distribution with a COV of 0.3 and horizontal and vertical autocorrelation distances of 20 m and 2 m, respectively. One ARFEM run is carried out with parameters m = 4, N = 500, p_0 = 0.1 and N_s = 25. The preliminary and target analyses spend 7 h and 37 h, respectively, for analyzing 1850 coarse models and 125 fine models. The corrected analysis does not need additional effort compared to the target analysis.

Fig. 6 provides the cumulative distribution function and scatters of safety factor in both coarse and fine models. It can be seen that the preliminary analysis underestimates the failure probability at every safety level, while the corrected and target analyses nearly overlap with very high agreement. Regarding the safety margin of 1.0, the failure probabilities in the preliminary, target and corrected analyses are 8.84×10^{-4}, 2.80×10^{-3} and 2.06×10^{-3}, respectively, as shown in Table 4. Besides, the corrected safety factor using coarse model is almost identical with that using fine model (see Fig. 6), with a correlation coefficient higher than 0.99, similar to the observation in footing example.

5. Conclusions

This study proposes a corrected method for consistent geotechnical reliability analysis with a simple deterministic model and on the basis of auxiliary random finite element method (ARFEM). It assumes that impacts of calculation model and geomaterial on geotechnical

Table 4. Reliability analysis results in two examples.

Analysis	Model	Failure probability, P_f Footing (Case 1)	Slope
Preliminary	Simple	3.16×10^{-3}	8.84×10^{-4}
Target	Complex	1.90×10^{-4}	2.80×10^{-3}
Corrected	Simple	1.50×10^{-4}	2.06×10^{-3}

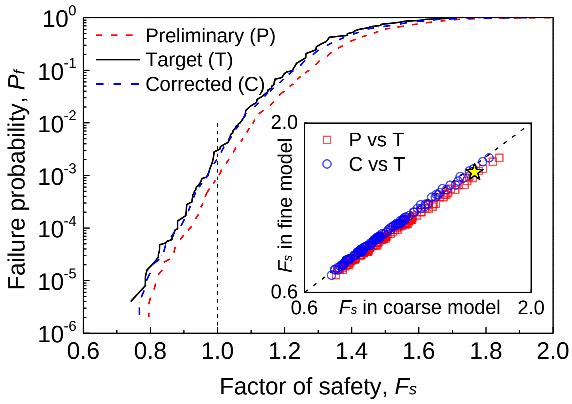

Figure 6. Results of slope reliability analysis.

response can be decoupled into two independent components. By this means, the response in the complex model can be inferred indirectly from that in the simple model. The corrected analysis is purely based on the simple-model-based preliminary analysis of ARFEM and is able to gain approximately consistent reliability results as the complex-model-based target analysis of ARFEM does, but without any additional computational effort.

Two geotechnical reliability problems on the settlement of a strip footing and the stability of a long embankment slope in 3-D spatially varying soils are investigated to validate the feasibility of corrected reliability analysis. It is found that the corrected response and cumulative distribution function using simple model are almost identical with those using complex model. Besides, the performance of the corrected reliability analysis is insensitive to material variability as long as a proper simple model is selected.

The illustrative examples also validate the feasibility of two universal simple models in ARFEM, say a coarsely meshed model and a 2-D model as the simplification of a finely meshed model and a 3-D model, respectively. Although the target analysis is necessary sometimes, such as risk assessment in which both failure probability and failure mechanism are needed, the corrected analysis is still a powerful tool to provide efficient and consistent reliability estimation and facilitate the implementation of target analysis.

Acknowledgments

This work was supported by the Research Grants Council of the Hong Kong SAR (Project Nos. AoE/E-603/18, 16206217 and 16205719), the National Natural Science Foundation of China (Project No. 51779189), and the Open Fund of Key Laboratory of Rock Mechanics in Hydraulic Structural Engineering, Ministry of Education of China (Project No. RMHSE1903).

References

Ahmed, A. and Soubra, A.H. 2012. Probabilistic analysis of strip footings resting on a spatially random soil using subset simulation approach. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 6(3), 188–201.

Au, S.K. 2007. Augmenting approximate solutions for consistent reliability analysis. *Probabilistic Engineering Mechanics*, 22(1), 77–87.

Fenton, G.A. and Griffiths, D.V. 2008. *Risk Assessment in Geotechnical Engineering*. John Wiley & Sons, Hoboken, New Jersey.

Li, D.Q., Xiao, T., Cao, Z.J., Phoon, K.K. and Zhou, C.B. 2016. Efficient and consistent reliability analysis of soil slope stability using both limit equilibrium analysis and finite element analysis. *Applied Mathematical Modelling*, 40(9–10), 5216–5229.

Li, D.Q., Xiao, T., Zhang, L.M. and Cao, Z.J. 2019. Stepwise covariance matrix decomposition for efficient simulation of multivariate large-scale three-dimensional random fields. *Applied Mathematical Modelling*, 68, 169–181.

Phoon, K.K. 2020. The story of statistics in geotechnical engineering. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 14(1), 3–25.

Xiao, T., Li, D.Q., Cao, Z.J., Au, S.K. and Phoon, K.K. 2016. Three-dimensional slope reliability and risk assessment using auxiliary random finite element method. *Computers and Geotechnics*, 79, 146–158.

Xiao, T., Li, D.Q., Cao, Z.J. and Zhang, L.M. 2018. CPT-based probabilistic characterization of three-dimensional spatial variability using MLE. *Journal of Geotechnical and Geoenvironmental Engineering*, 144(5), 04018023.