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Data Assimilation of Seepage Analysis Model including Boundary Condition based on Field Measurement Data

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Abstract: Data assimilation methods are effective tools to estimate analytical models based on measurement data. Therefore, this study first evaluates seepage analysis models based on the field measurement data of volumetric water content using data assimilation methods. In existing studies, seepage behaviors were simulated by setting a free drainage boundary condition. However, during heavy rain, such simulations could not reproduce the field measurement data. Therefore, this paper proposes a method for estimating the in-situ seepage analysis model, including the boundary condition, and discusses the validity of the proposed method. The simulation results obtained using the proposed method were significantly more accurate than the results of the previous studies that set the free drainage boundary condition, thus validating the proposed method to estimate the in-situ seepage analysis model.

Keywords: data assimilation, seepage analysis model, boundary condition, field measurement data.

1. Introduction

With the progression of global warming, the intensity of rainfall experienced worldwide has increased, inducing multiple landslide disasters. Soil moisture conditions such as volumetric water content, pressure head, and groundwater level affect the occurrence of landslide disasters. Therefore, it is important to evaluate seepage analysis models to simulate soil moisture conditions. It includes unsaturated soil hydraulic properties, such as the soil-water characteristic curve, unsaturated hydraulic conductivity function, as well as initial and boundary conditions. Generally, laboratory tests help to determine the unsaturated soil hydraulic properties. However, there are various difficulties encountered while estimating in-situ seepage analysis model.

Recently, field monitoring systems, in which the soil moisture conditions can be observed in real-time, have been developed to assess the risk of landslide disasters (Koizumi et al. 2012, Sakuradani et al. 2018). The systems can not only measure present soil moisture conditions but also accumulate measurement data automatically. Predicting future soil moisture conditions utilizing these measurement data is valuable for the landslide disaster mitigation. Estimating the in-situ seepage analysis model, which can reproduce the field measurement data with high accuracy, is a necessity to achieve this.

Data assimilation methods are inverse analysis methods that originate from modifying the numerical simulation model based on field measurement data. They have been developed in the field of meteorology and oceanography. Several data assimilation methods, such as 4D-VAR (Talagrand and Courtier 1987), ensemble Kalman filter method (Evensen 1994), particle filter (PF) method (Gordon et al. 1993), and merging particle filter (MPF) method (Nakano et al. 2007) have been proposed. In geotechnical engineering, the PF method is applied to identify the mechanical parameters of an elasto-plastic constitutive model based on the measurement data of consolidation settlements (Shuku et al. 2012). The authors applied the PF method to estimate the in-situ unsaturated soil hydraulic properties, based on field measurement data, at three slopes with different types of soils, and to validate the PF method (Ito et al. 2019).

Figure 1 shows the simulated results in an existing study by Nishimura et al. (2019). The simulated results could not reproduce the field measurement data, due to boundary conditions. In this study, one-dimensional seepage analysis models were assumed and unsaturated seepage behaviors were simulated by setting the free drainage boundary condition at the bottom of one-dimensional models. However, during heavy rain, because of the rising groundwater level at the field slope, the simulation set with the free drainage boundary condition could not reproduce the field measurement data.

In this paper, a method estimating the in-situ seepage analysis model has been proposed, including the boundary condition at the bottom of a one-dimensional model. In the proposed method, a seepage coefficient β is introduced as the parameter at the bottom of the one-dimensional model, and the MPF is adopted as the data assimilation method. The study discusses the validity of the proposed method through the data assimilation of the in-situ seepage analysis model.

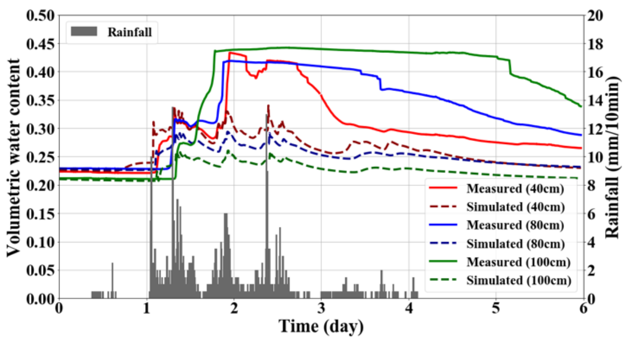


Figure 1. Simulated results in the previous study by Nishimura et al. (2019).

2. Analytical methods

2.1 Unsaturated-saturated seepage analysis

In this study, an unsaturated-saturated seepage finite element analysis is used to reproduce the soil moisture conditions. The following equation, Richards equation (Richards 1931), is applied in numerical analysis.

$$C\cdot\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial x_3}\left\{k(\psi)\left(\frac{\partial\psi}{\partial x_3}+1\right)\right\} \tag{1}$$

Here, C ($=\partial\theta/\partial\psi$) is the hydraulic capacity function, θ is the volumetric water content, ψ is the pressure head, and $k(\psi)$ is the unsaturated hydraulic conductivity. The following van Genuchten model (van Genuchten 1980) is adopted to express the soil-water characteristic curve, and the Mualem model (Mualem 1976) is utilized to estimate the unsaturated hydraulic conductivity.

$$S_e=\frac{\theta-\theta_r}{\theta_s-\theta_r}=\left\{\frac{1}{1+(-\alpha\cdot\psi)^n}\right\}^{1-\frac{1}{n}} \tag{2}$$

$$k(\psi)=k_s\cdot S_e^{0.5}\left\{1-\left(1-S_e^{\frac{n}{n-1}}\right)^{1-\frac{1}{n}}\right\}^2 \tag{3}$$

Here, S_e is effective soil water saturation, θ_r is residual volumetric water content, θ_s is saturated volumetric water content, α and n are material parameters, and k_s is saturated hydraulic conductivity. In this study, θ_s, θ_r, α, n, and k_s are unknown parameters corresponding to unsaturated soil hydraulic properties.

2.2 Boundary condition

In the previous study by Nishimura et al. (2019), one-dimensional seepage analysis models were assumed, and unsaturated seepage behaviors were simulated by setting the free drainage boundary condition at the bottom of the one-dimensional models. The free drainage boundary condition assumed that the diffusion flux was negligible. As Chen et al. (2018) highlighted, the free drainage boundary condition is suitable when there is slight variation in the moisture near the bottom of the soil, such as when the groundwater level is deeper than that in the one-dimensional model. The following equation presents the free drainage boundary condition.

$$\left.\frac{\partial\psi}{\partial x_3}\right|_{x_3=\Gamma}=0 \tag{4}$$

Here, Γ is the boundary at the bottom of the one-dimensional model. The drainage flux, v_{out}, equals the unsaturated hydraulic conductivity at the boundary Γ as given by the following equation.

$$v_{out}=k(\psi) \tag{5}$$

On setting the free drainage boundary condition, soil moisture condition in the seepage analysis model maintained an unsaturated condition as it drained a large amounts of pore water. To express raising the groundwater level, a new boundary condition, that reduced the amount of water being drained, was necessary.

This study focuses on the tank model (Ishihara and Kobatake 1979) to propose a new boundary condition. Figure 2 shows a conceptual diagram of the tank model. The following equation expresses the amount of drainage Z.

$$Z=\beta\times S \tag{6}$$

Here, β is the seepage coefficient and S is storage discharge. The seepage coefficient, β, controls the amount of drainage, Z, in the tank model. This study proposes a drainage boundary condition using the seepage coefficient β. The following equation gives the boundary condition.

$$v_{out}=\beta\times k(\psi) \tag{7}$$

Here, the seepage coefficient, β, is $0\le\beta\le 1$. β equals to 1 indicates the free drainage boundary condition at the bottom of the one-dimensional model, and β equals to 0 indicates an undrained boundary condition. Introducing the seepage coefficient, β, can control the amount of drainage from the bottom of the one-dimensional model. The proposed method regards not only unsaturated soil hydraulic properties (θ_s, θ_r, α, n, and k_s) but also the seepage coefficient β as an unknown parameter. These unknown parameters are estimated by the MPF based on field measurement data.

2.3 Merging particle filter (MPF)

The MPF (Nakano et al. 2007) is a sequential data assimilation method in which the probability distribution of a physical quantity is approximated with its realizations. Each realization is called a particle, and each set is called an ensemble. The MPF evaluates the particles at a discrete time, using Bayes' theorem. Figure 3 schematically shows the computational procedure of the MPF. First, several numerical simulations, in which different sets of unknown parameters are applied for each particle, are conducted ((a) Prediction in Figure 3). When the number of particles is N, N Monte Carlo simulations are conducted. Second, the likelihood is evaluated for each particle by comparing measurement data and simulated results ((b) Filtering in Figure 3). The particles are then resampled into samples ((c) Resampling in Figure 3). When the number of particles is N, $l\times N$ samples are drawn based on the likelihood. The particles with high likelihood are resampled more than l, and the samples of particles with low likelihood are decreased. Finally, the samples are merged into new particles ((d) Merging in Figure 3). Each particle is generated as a weighted sum of l samples from the $l\times N$ sample set as,

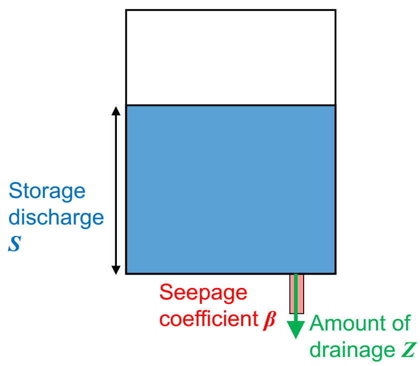


Figure 2. Conceptual diagram of the tank model.

$$x_{k|k}^{(i)} = \sum_{j=1}^{l} \alpha_j \hat{x}_{k|k}^{(j,i)} \tag{8}$$

where α_j is the merging weight. The merging weights α_j satisfy,

$$\sum_{j=1}^{l} \alpha_j = 1 \tag{9-a}$$

$$\sum_{j=1}^{l} \alpha_j^2 = 1 \tag{9-b}$$

where each α_j is a real number. As demonstrated by Nakano (2007), the number of merged particles was set to l=3, and the weights α_j were set as follows.

$$\alpha_1 = \frac{3}{4} \tag{10-a}$$

$$\alpha_2 = \frac{\sqrt{13}+1}{8} \tag{10-b}$$

$$\alpha_3 = -\frac{\sqrt{13}-1}{8} \tag{10-c}$$

Through iteration of these four steps (Prediction, Filtering, Resampling, Merging) each discrete time, the MPF modified the seepage analysis model based on field measurement data.

3. Field measurement data

Photo 1 shows the field monitoring system at the target slope. The slope has an angle of ~35 degrees, and the surface layer is composed of weathered granite. Table 1 shows the physical properties of the sample at the slope. Figure 4 shows the result of a portable cone penetration test at the target point of this study. The portable cone penetration test investigates hardness at each depth. The N_d value indicates the number of falling of portable cone. The layer from the ground surface up to a depth of 140 cm has a small N_d value, while the layer below it is a hard layer. It can be presumed that the difference of the hardness near the depth of 140 cm may generate groundwater.

A rain gage was used to measure the amount of rainfall and soil moisture sensors were used to measure the volumetric water content at each depth. Photo 2 shows the method for installing the soil moisture sensors. Vertical trenches were dug, and the sensors were installed at right angles to the trenches to measure the in-situ unsaturated infiltration behaviors. The sensors were installed at depths of 40 cm, 80 cm, and 100 cm. The monitoring system measured the amount of rainfall and volumetric water content at ten-minute intervals since November 2017. Figure 5 shows the field measurement data used. The heavy rain from July 5-7 raised groundwater level over 40 cm depth temporarily, and the soil moisture condition, at 100 cm depth, remained saturated until July 9. It was reported that the heavy rain had caused several surface failures near the target slope.

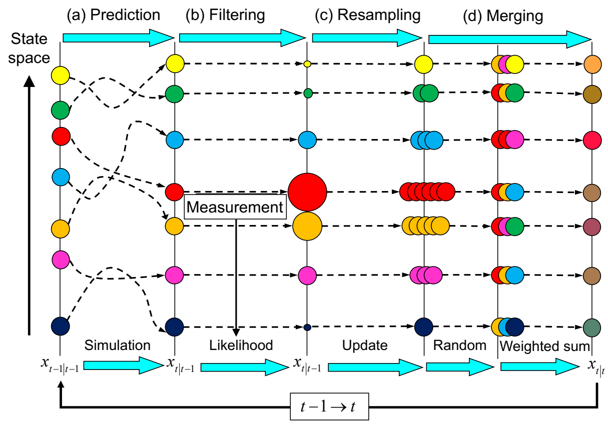


Figure 3. Computational procedure of the MPF.

Photo 1. Field monitoring system at the target slope.

Table 1. Physical properties of the sample.

Soil particle density ρ_s (g/cm^3)	2.648
Maximum dry density $\rho_{d\,max}$ (g/cm^3)	1.858
Optimum water content w_{opt} (g/cm^3)	13.5

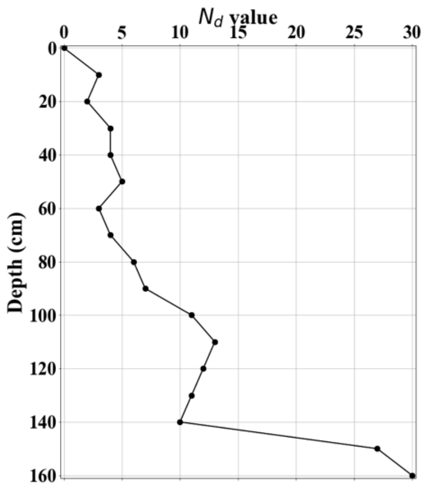


Figure 4. Result of a portable cone penetration test.

4. Analytical results

4.1 Analytical conditions

Figure 6 shows the one-dimensional analytical model. The model was divided into three layers (upper, middle, and bottom layer), corresponding to the depth of the installed soil moisture sensors. The top of the model inputted rainfall boundary conditions, and the bottom of the model inputted drainage boundary conditions, introducing the seepage coefficient β. As the initial condition, the upper layer had the initial value of volumetric water content of $\theta_{t=0}^{40cm}$, measured at 40 cm depth, the middle layer set the value of $\theta_{t=0}^{80cm}$, measured at 80 cm depth, and the bottom layer set the value of $\theta_{t=0}^{100cm}$, measured at 100 cm depth.

Photo 2. Installation of the soil moisture sensors.

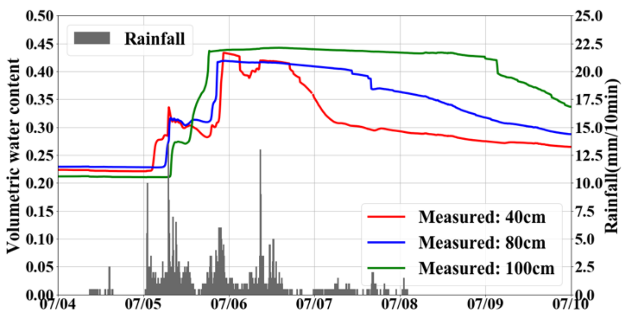


Figure 5. Field measurement data – daily variation of volumetric water content in the soil and the rainfall received.

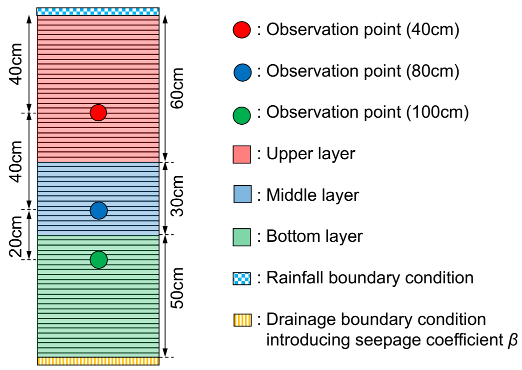


Figure 6. One-dimensional analytical model.

This study used 500 particles. In the MPF, the set of particles approximate probability distribution of unknown parameters. The upper and the middle layers had information of only five parameters (θ_s, θ_r, α, n, and k_s), while the bottom layer included information of all six parameters (θ_s, θ_r, α, n, k_s, and β). Table 2 shows the minimum and maximum values of each parameter. All particles were generated randomly within a range. The observation noise followed a three-dimensional normal distribution. Assuming that all observation errors were independent of each other, covariance matrix R_t was set as follows.

$$R_t = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} 0.0636 & 0.0 & 0.0 \\ 0.0 & 0.0573 & 0.0 \\ 0.0 & 0.0 & 0.0696 \end{pmatrix} \quad (11)$$

Here, the non-diagonal covariance terms were assumed to be zero.

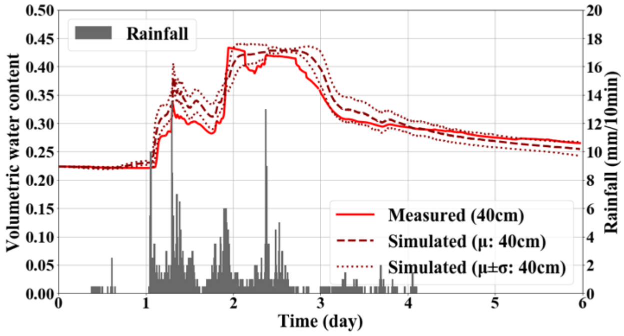
4.2 Data assimilation results

Figure 7 shows the comparison of field measurement data with data assimilation results. The data assimilation results indicate the variation of the posterior distribution of volumetric water content. As the number of Monte Carlo simulations corresponded to the number of particles, the mean values and standard deviations of the simulated volumetric water content could be calculated. From the results of Figure 7, the proposed method could express raising the groundwater level after the heavy rain and could reproduce falling groundwater level and decreasing volumetric water content with enough accuracy. Moreover, the data assimilation results of Figure 7 were significantly more accurate, than in the previous study (Figure 1), setting the free drainage boundary condition at the bottom of the one-dimensional models. This result revealed that introducing the seepage coefficient β was a valid method to estimate the in-situ seepage analysis model.

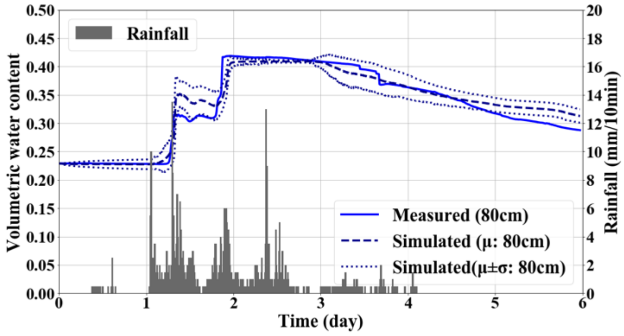
Figure 8 shows the histograms of the distribution of parameters. All particles had a seepage coefficient β less than 0.1, and many were in the order of one in one-thousandth. Thus, some pore water was drained from the drainage boundary condition after introducing the seepage coefficient β. As shown in Figure 8, the MPF estimated the posterior distribution of unknown parameters, to evaluate the soil moisture condition probabilistically.

Table 2. Minimum and maximum values of each parameter.

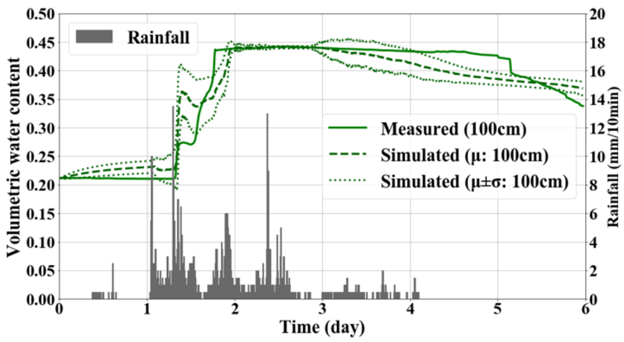
		θ_s	θ_r	α (1/cm)	n	k_s (m/s)	β
Upper	min	0.42	0.12	0.02	1.20	1.0×10^{-5}	
	max	0.44	0.20	0.10	1.80	1.0×10^{-3}	
Middle	min	0.40	0.12	0.02	1.20	1.0×10^{-5}	
	max	0.42	0.20	0.10	1.80	1.0×10^{-3}	
Bottom	min	0.43	0.12	0.02	1.20	1.0×10^{-5}	1.0×10^{-3}
	max	0.45	0.18	0.10	1.80	1.0×10^{-3}	1.0×10^{0}



(a) 40cm

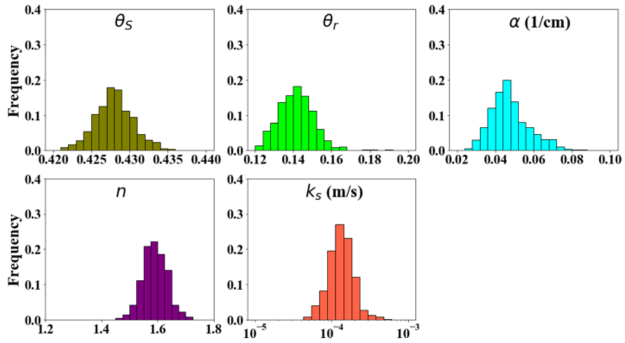


(b) 80cm

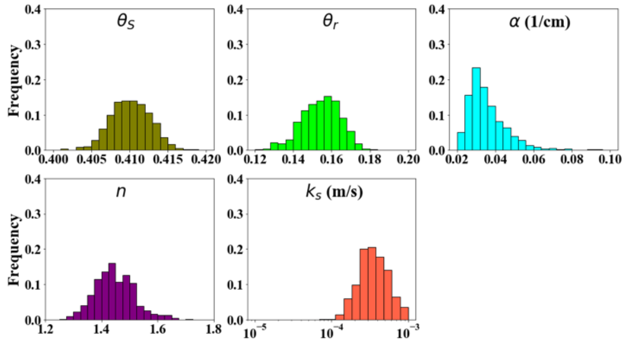


(c) 100cm

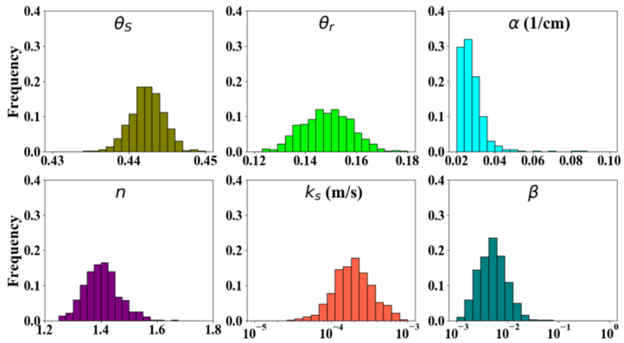
Figure 7. Comparison of field measurement data with data assimilation results at the selected depths from ground surface.



(a) 40cm



(b) 80cm



(c) 100cm

Figure 8. histograms of the distribution of parameters.

5. Conclusions

This paper proposed a method for estimating the seepage analysis model, including the seepage coefficient β. The validity of the proposed method is then discussed through the data assimilation of the in-situ seepage analysis model. The main conclusions of this study are summarized as follows:

1. The proposed method could express raising the groundwater level after the heavy rain.
2. The proposed method could also reproduce falling groundwater levels and decreasing volumetric water content with sufficient accuracy.
3. The simulated results obtained using the proposed method were significantly more accurate than the results of existing studies that set the free drainage boundary condition at the bottom of one-dimensional models.
4. The proposed method, introducing the seepage coefficient β, was a valid method to estimate the in-situ seepage analysis model.

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