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Practical Estimation Method for Acceleration Fourier Amplitude Spectrum at an Arbitrary Point by Using Advanced Kriging Method

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Abstract: This paper reports the results of estimating an acceleration Fourier amplitude spectrum (AFS) on a ground surface at an arbitrary point. The authors have proposed a method to estimate an AFS on ground surface at an arbitrary point using a transfer function at an arbitrary point of surface strata and a limited number of observed earthquake ground motion records. The transfer functions at arbitrary points were estimated by using the advanced Kriging method, which used the transfer functions at 748 boring points both inside and within the vicinity of Owari-asahi City, Aichi prefecture, Japan. The observed records on the ground surface were obtained by 12 seismometer points located within the city. The AFS on the ground surface at one of the seismometer points was estimated by using the proposed method and the other observed records. The estimated AFS was compared with the AFS of the observed record at each of the 12 seismometer points. When the 11 observed records were used, the estimated AFS was nearly identical to the AFS of the corresponding observed record. On the contrary, when only the 7 observed records were used, the estimated AFS was less similar to the AFS of the corresponding observed record. It was found that an AFS on the ground surface at an arbitrary point inside the city could be estimated accurately if a reasonable number of seismometers were installed on the ground surface both inside and within the vicinity of the city, considering the autocorrelation distance of earthquake ground motion intensities on the engineering bedrock.

Keywords: Fourier Amplitude Spectrum, Kriging Analysis, Transfer Function, Accuracy, Objectivity

1. General

For a seismic risk assessment, earthquake ground motion intensities (GMIs), such as peak ground velocity (PGV), spectrum intensity (SI), or the Japanese seismic scale, based on scenario earthquakes have been estimated for each mesh with side length of such as 50 m, 250 m or 500 m using hypothetic ground models created on the basis of boring investigation data. Ideally, the GMI should be estimated in greater detail, such as at each construction site. Using the GMI at construction site, the seismic risk can be assessed accurately to mitigate seismic damage effectively. To accomplish this, Sugai et al. (2015) proposed an advanced Kriging method that can take into account the non-negligible errors in the estimations of the spatial distributions of the GMI at a number of boring points. The method has already been applied to the estimation of seismic hazards and risks using scenario earthquakes in Owari-asahi City, Aichi prefecture, Japan.

To improve the accuracy of the above estimation at an arbitrary point, it should be calibrated using observed earthquake ground motion records. However, it is practically impossible to estimate the spatial distribution of the GMI on ground surface directly using the observed records on the ground surface. For example, in Owari-asahi City, within which the auto-correlation distances of the peak ground acceleration (PGA), PGV, and Japanese seismic scale on the ground surface are approximately 150 to 700 m (Sugai et al., 2016), it is necessary to locate one seismometer for every 100 m^2 to estimate the GMI at an arbitrary point on the ground surface. In contrast, the auto-correlation distances of the GMI on engineering bedrock are much longer; approximately 1.5 to 2.0 km in the city (Mizutani et al., 2017). The GMI at an arbitrary point on the ground surface can be estimated using an amplification factor of surface strata and the spatial distribution of the GMI on the engineering bedrock at the same point. This is estimated as the GMI on the engineering bedrock based on an amplification factor and the observed records on the ground surface, requiring approximately one seismometer for every 1 km^2. However, the amplification factor of the GMI depends not only on the characteristics of the surface strata but also on the phase characteristics of earthquake ground motion.

Considering that a transfer function of surface strata is reported as a stable characteristic, the authors have proposed a method for estimating an acceleration Fourier amplitude spectrum (AFS) on the ground surface at an arbitrary point, using the transfer function of the surface strata and ground motion records. The transfer function is estimated by using the advanced Kriging method and the transfer functions of a number of boring points (Mizutani et al., 2019). Using the AFS, it is possible to calculate a velocity response spectrum (AIJ, 2015) at each construction site to estimate seismic risk.

In this paper, the authors investigate the proposed method using newly observed earthquake ground motion records on the ground surface at 12 seismometer points in Owari-asahi City. The transfer functions at the seismometer points are estimated by using the advanced Kriging method and the functions estimated at 748 boring points, inside and the vicinity of the city. The AFS on the ground surface at each of the seismometer point estimated by using the proposed method is compared with that of the corresponding observed record.

2. Method to Estimate an AFS on Ground Surface at an Arbitrary Point

2.1. Transfer function of surface strata at an arbitrary point

A transfer function of surface strata at an arbitrary point can be estimated per frequency by using the advanced Kriging method based on the transfer functions of a number of boring points (Mizutani et al., 2019). An overview of the advanced Kriging method (Sugai et al., 2015) is provided below.

To estimate the spatial distribution of a parameter by using Kriging method based on a set of field data, it is necessary to identify a covariance matrix and a trend function. A covariance matrix represents the correlation between the parameters at two points, and a trend function represents the overall tendency within the field (Krige, 1951; Matheron, 1963).

The optimal random field model (i.e., the covariance matrix and trend function) for the spatial distribution of the transfer function is identified as a model with a minimum Akaike information criterion (AIC) (Akaike, 1973) based on the general maximum likelihood method as

$$\min_{\boldsymbol{\mu},\boldsymbol{\theta},m} \mathbf{AIC} = -2 \times \mathrm{Max}\{\ln p(\mathbf{z}|\boldsymbol{\mu},\boldsymbol{\theta})\} + 2 \times (m) \quad (1)$$

in which $\mathbf{z}$ is the vector of the parameter, m denotes the number of random field explanatory variables, and $p(\mathbf{z}|\cdot)$ is a multivariate probability density function of $\mathbf{z}$ with parameters. $p(\mathbf{z}|\cdot)$ is often modeled by a joint lognormal probability density function as

$$p(\mathbf{z}|\boldsymbol{\mu},\boldsymbol{\theta}) = \left(\frac{1}{\sqrt{2\pi}}\right)^{\frac{n}{2}} \frac{1}{|\mathbf{z}|} |\mathbf{C}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\ln(\mathbf{z}) - \boldsymbol{\mu})^T \mathbf{C}^{-1} (\ln(\mathbf{z}) - \boldsymbol{\mu})\right\} \quad (2)$$

in which $\boldsymbol{\theta}$ denotes the explanatory variables of $\mathbf{C}$, which is the covariance matrix of $\mathbf{z}$, and n is the number of acquired data. $\boldsymbol{\mu}$ is the average value vector determined by $\mathbf{z}$ (Wackernagel, 2011) and can be expressed as a function with the maximum order, n_t, of the coordinate $\boldsymbol{\mu}=\mu(\mathbf{u}_i)=\mu(\mathrm{x}_i, \mathrm{y}_i)$ (Honda, 2000) as

$$\begin{aligned}\mu(\mathbf{u}_i) = \mathbf{b}\cdot\mathbf{f}(\mathbf{u}_i) &= \sum_{k=0}^{m_t-1} b_k f_k(\mathbf{u}_i) = \sum_{k=0}^{m_t-1} b_k f_k(x_i, y_i) \\ &= b_0 + b_1 x_i + b_2 y_i + b_3 x_i^2 + b_4 x_i y_i \\ &+ b_5 y_i^2 + \cdots + b_{m_t-2} x_i y_i^{n_t-1} \\ &+ b_{m_t-1} y_i^{n_t}\end{aligned} \quad (3)$$

in which $\mathbf{f}(\mathbf{u}_i)$ is the vector of the location of the acquired data, $\mathbf{b}$ is the coefficient vector, and m_t denotes the number of elements in $\mathbf{b}$. m_t can be expressed as a function of n_t as

$$m_t = \frac{(n_t+1)(n_t+2)}{2} \quad (4)$$

The *ij* elements of covariance matrix $\mathbf{C}$ in Eq. (2), $C(\mathbf{u}_i, \mathbf{u}_j)$, denote the covariance of the parameter between points *i* and *j*, which have the coordinates $\mathbf{u}_i$ and $\mathbf{u}_j$, respectively. $C(\mathbf{u}_i, \mathbf{u}_j)$ depends only on the distance between these two points, h_{ij}, and can be expressed as a general exponential model as

$$C(\mathbf{u}_i, \mathbf{u}_j) = C(h) = \sigma^2 \exp\left(-\frac{h_{ij}}{\ell}\right) \quad (5)$$

in which σ^2 and ℓ are the sill and range, respectively. All elements in $\mathbf{C}$ can be determined by σ^2, ℓ, and h_{ij}. It should be noted that h_{ij} is a known variable. The number of explanatory variables, m, in Eq. (1) is expressed as the sum of m_t, determined by Eq. (4), and m_c, the number of unknown parameters for determining $\mathbf{C}$, m_c, i.e., $m = m_t + m_c$.

In the case of a general Kriging method, the elements of $\mathbf{z}$ are considered as realization values in a random field. When estimating a transfer function from various specimens, such as boring data, non-negligible errors in the estimates at the boring points must be considered. To this end, some authors have proposed an advanced Kriging method (Sugai et al., 2015). In this method, $p(\mathbf{z}|\cdot)$ in Eq. (2) is estimated using the covariance matrix $\mathbf{C}'$, which is expressed as

$$\mathbf{C}' = \begin{bmatrix} C(\mathbf{u}_1,\mathbf{u}_1)+\sigma_1^2 & C(\mathbf{u}_1,\mathbf{u}_2) & \cdots & C(\mathbf{u}_1,\mathbf{u}_n) \\ C(\mathbf{u}_2,\mathbf{u}_1) & C(\mathbf{u}_2,\mathbf{u}_2)+\sigma_2^2 & \cdots & C(\mathbf{u}_2,\mathbf{u}_n) \\ \vdots & \vdots & \ddots & \vdots \\ C(\mathbf{u}_n,\mathbf{u}_1) & C(\mathbf{u}_n,\mathbf{u}_2) & \cdots & C(\mathbf{u}_n,\mathbf{u}_n)+\sigma_n^2 \end{bmatrix} \quad (6)$$

in which σ_i^2 is the variance in the estimation error ("observation errors" (Wackernagel, 2011)) of the estimates at point *i*. In this study, it is assumed that $\sigma_i^2 = \sigma_c^2$ for all $i = 1, ..., n$. Then, the number of explanatory variables in $\mathbf{C}'$, m_c, is 3, which are the explanatory variables σ^2 and ℓ of $\mathbf{C}$ and the additional explanatory variable σ_c^2.

2.2 An AFS on the ground surface at an arbitrary point

The AFS on the ground surface at an arbitrary point can be estimated by taking the following steps (Fig. 1).

1) The AFS on the engineering bedrock at all observation points is calculated using the inverse of the transfer function and the AFS of the observed record on the ground surface.
2) On the engineering bedrock, the coefficient vector ($\mathbf{b}$ in Eq. (3)) is calculated per frequency by least square method, based on the AFS on the engineering bedrock at all observation points.
3) The AFS on the ground surface at an arbitrary point can be estimated using the transfer function and the AFS on the engineering bedrock at the same point, which is obtained by substituting vector $\mathbf{b}$ from Step 2) and the latitude and longitude at the point into Eq. (3) of x and y, respectively.

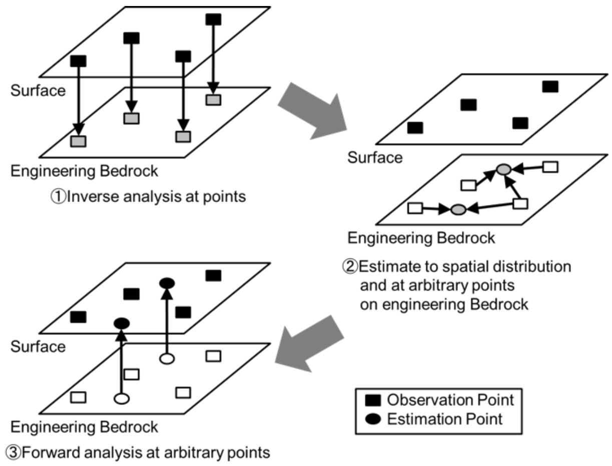


Figure 1. Schematic of the interpolation of an AFS on ground surface and engineering bedrock

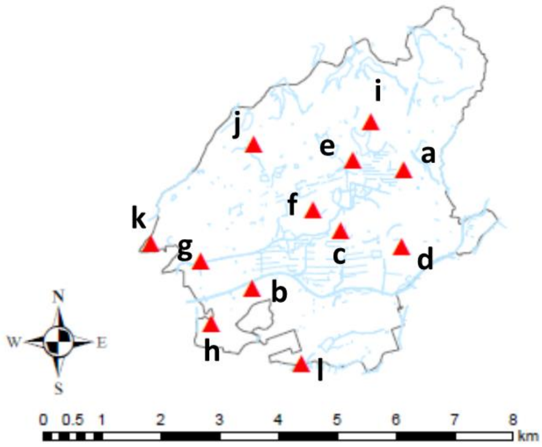


Figure 2. Installation of seismometer points in Owari-asahi City

3. Estimation of an AFS on Ground Surface at an Arbitrary Point Using Observed Earthquake Ground Motion Records on Ground Surface

3.1 Analysis of Target Area and Data

Ground motion record was observed at 12 seismometer points in Owari-asahi City, which has an area of 21.03 km^2, during the event of the M4.2 earthquake on December 3rd, 2014, shown in Fig. 2 by red triangles (▲). It was named as Earthquake in the western part of Aichi Prefecture in 2014. The observed records in the east-west direction were used in the following. The authors utilized transfer functions of surface strata at arbitrary points, including the seismometer points estimated by the advanced Kriging method based on the transfer functions at 748 boring points, inside and the vicinity of the city (Mizutani et al., 2019).

3.2 Estimation of AFS based on 11 observed earthquake ground motion records

The AFS on the ground surface at one of the seismometer points was estimated by the method described in Section 2.2, using the transfer functions and the observed records at the other 11 seismometer points. The AFS of the observed records was smoothed using a Parzen window with a bandwidth of 0.4 Hz. Vector **b** in Eq. (3) was calculated considering only the case where the trend function had a maximum order of 2, as the number of elements of vector **b** should be less than that of the seismometers.

Fig. 3 shows the AFS on the ground surface at each of the seismometer points. These were estimated by the method in Section 2.2, based on the observed records of the other 11 seismometers, hereafter referred to as “estimated values” in Fig. 3 by red line. Fig. 3 also shows the AFS of the observed records at the respective point (the observed value, in Fig. 3 by blue line) for the Earthquake in the western part of Aichi Prefecture in 2014. As shown in the figures, the estimated values such as points, a, b, c, and e are almost identical to those of the observed values. Fig. 4 shows the degree of fitting of the estimated values to those of the observed values at three levels (○: matched, △: roughly matched, ×: discrepant) in Fig.3. As shown in the figures, the degree of fitting is either ○ or △ at those points where seismometers were installed in the vicinity, such as points a, b, and c. On the other hand, the degree of fitting becomes × at points where seismometers were not installed in the vicinity, such as points j, k, and l.

In the case of Owari-Asahi City, the autocorrelation distance of GMI’s on the engineering bedrock was 1.5 to 2.0 km (Mizutani et al., 2017). Therefore, to estimate the AFS on the ground surface at a point accurately, a reasonable number of seismometers should be installed on the ground surface within this distance from the estimated point, considering the autocorrelation distance on the engineering bedrock.

3.3 Estimation of AFS on ground surface based on 6 earthquake ground motion records

To investigate the effect of the number of seismometers on the accuracy of estimation, the authors estimated the AFS on the ground surface at each of the 7 seismometer points by the same method as in Section 3.2. However, only the observed records at the other 6 seismometer points were used. Two cases of “7 seismometer points” were considered here; 7 seismometer points located in the northeastern part of the city and 7 located in the southwestern part were selected for a case.

Figs. 5 and 6 show the estimated values of the AFS on the ground surface at each of the seismometer points, along with the observed values of the AFS at the two case of northeastern part and southwestern part, respectively. As shown in the figures, the estimated values of the seismometer points and the corresponding observed values are not quite identical.

Figs. 7 and 8 show the degree of fitting of the estimated values compared to those of the observed values at three levels, as shown in Fig. 4, in Figs. 5 and 6, respectively. As shown in the figures, it can be seen that the degree of fitting becomes × at almost all seismometer points in both the northeastern and southwestern parts of the city.

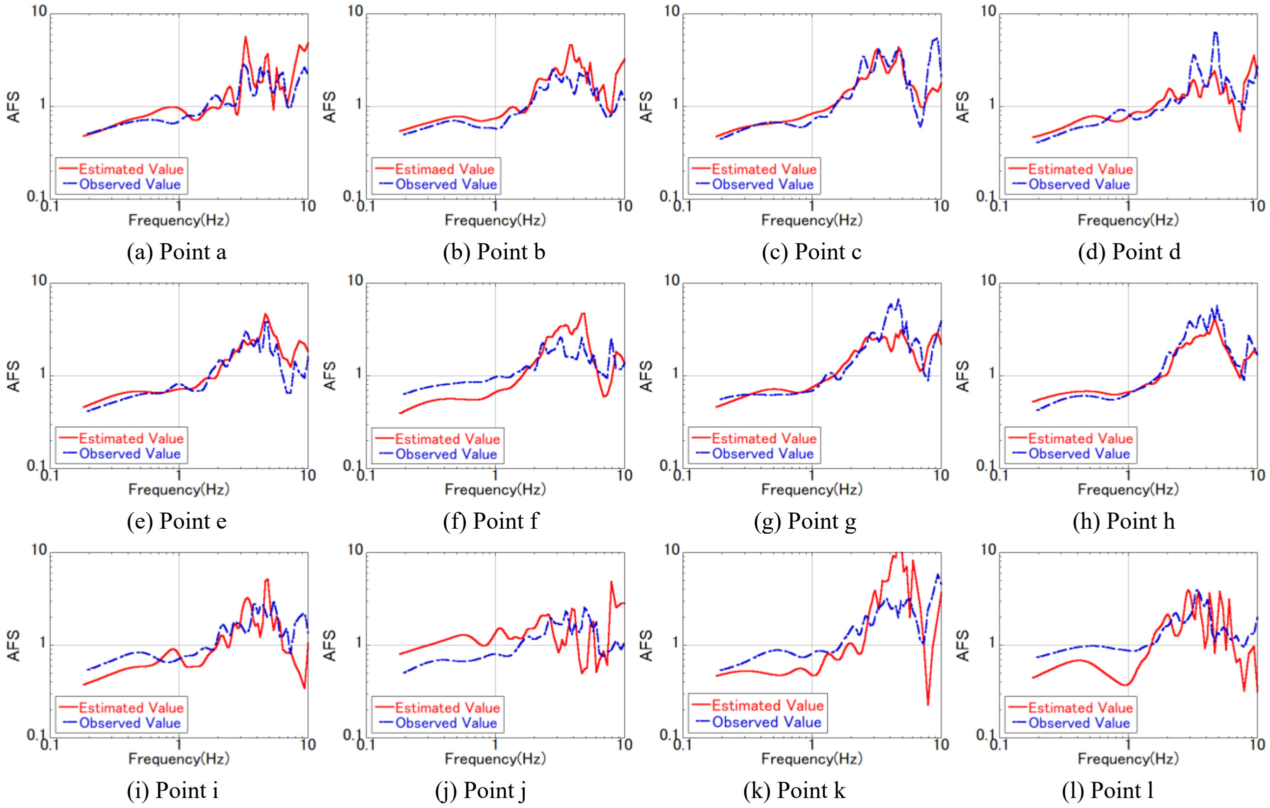


Figure 3. Comparison of estimated values using the observed records of 11 seismometer points of Owari-asahi City with observed value (Earthquake in the western part of Aichi Prefecture in 2014, east-west direction)

From these results, it was found that an AFS on the ground surface at an arbitrary point inside Owari-ashi City could be estimated accurately if a reasonable number of seismometers were installed on the ground surface both inside and within the vicinity of the city, considering the autocorrelation distance of earthquake ground motion intensities on the engineering bedrock.

4. Conclusions

This paper reports the results of estimating an AFS on a ground surface at arbitrary points using the transfer functions of surface strata and newly observed earthquake ground motion records on the ground surface at 12 seismometer points in Owari-asahi City.

The major findings of this analysis are outlined below.

1) The AFS on the ground surface at one of 12 seismometer points in Owari-asahi City was estimated using the transfer functions and the observed records at the other 11 seismometer points for the Earthquake in the western part of Aichi Prefecture in 2014 (east-west direction). The estimated AFS and observed AFS of the 11 seismometers were almost identical.
2) To investigate the estimation accuracy of the effect of the number of seismometers, the AFS on the ground surface at one of 7 seismometer points was estimated using the transfer functions and the observed records at the other 6 seismometer points, the northeastern part and the southwestern part in the city, respectively. The estimated AFS and observed AFS of the 7 seismometers were not quite identical in both the northeastern and southwestern parts of the city.

In addition, to improve the accuracy of the method to estimate an AFS proposed by the authors, it should be verified using other observed earthquake ground motion records.

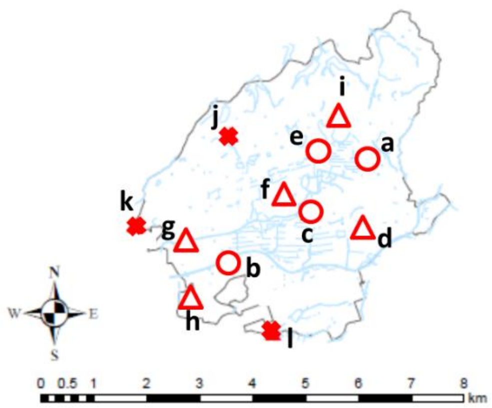
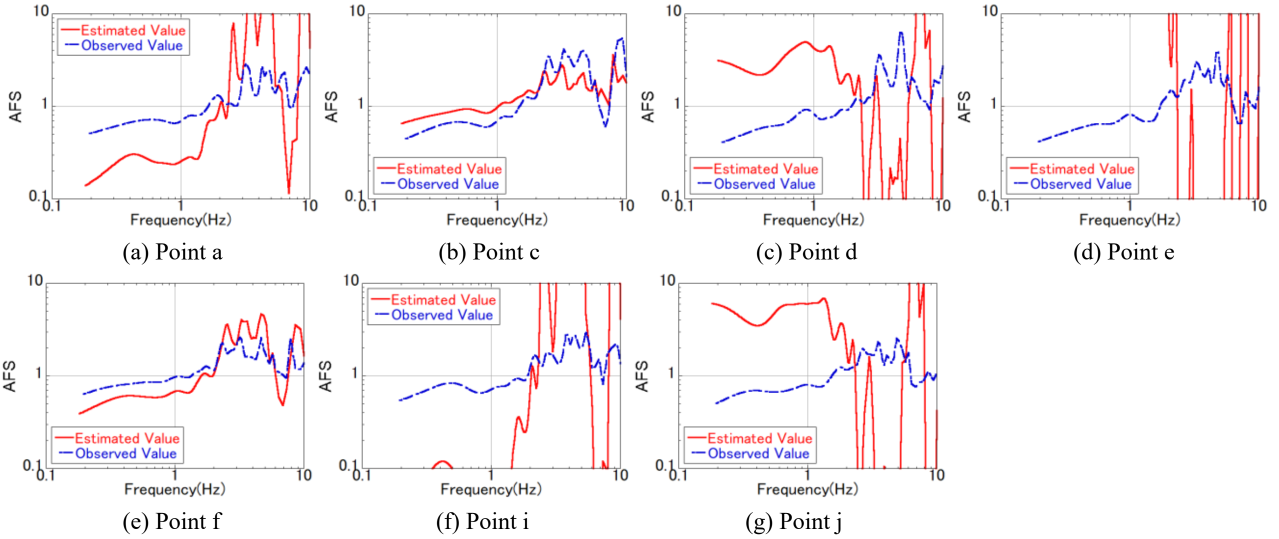
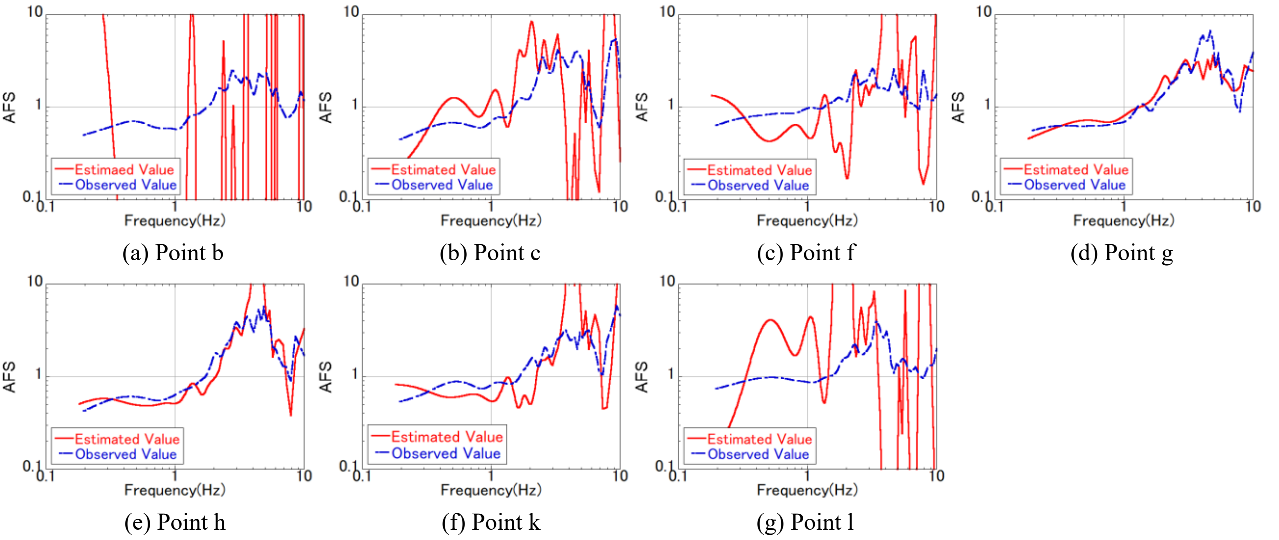


Figure 4. The degree of fitting of the estimated value to those of the observed value at the seismometer points in Fig. 3



(a) Point a (b) Point c (c) Point d (d) Point e
(e) Point f (f) Point i (g) Point j

Figure 5. Comparison of estimated values using the observed records of 6 seismometer points in the northern part of Owari-asahi City with observed value (Earthquake in the western part of Aichi Prefecture in 2014, east-west direction)



(a) Point b (b) Point c (c) Point f (d) Point g
(e) Point h (f) Point k (g) Point l

Figure 6. Comparison of estimated values using the observed records of 6 seismometer points in the southern part of Owari-asahi City with observed value (Earthquake in the western part of Aichi Prefecture in 2014, east-west direction)

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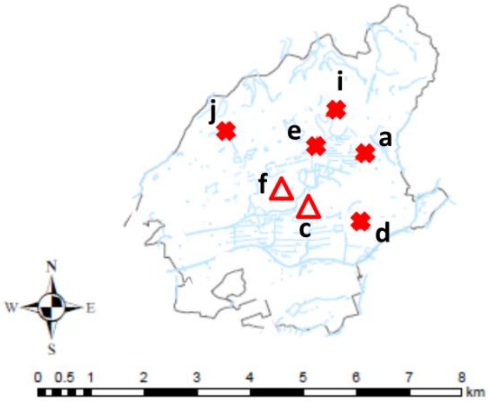

Figure 7. The degree of fitting of the estimated value to those of the observed value at the seismometer points in Fig. 5

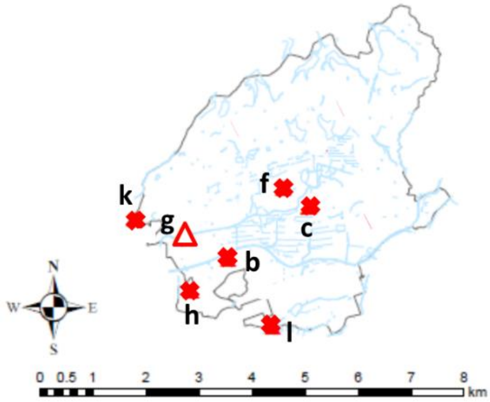

Figure 8. The degree of fitting of the estimated value to those of the observed value at the seismometer points in Fig. 6

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