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Novel Techniques for Efficient Slope Reliability Analyses Combining Active-Learning SVM Classifier and Judgement-based Strength Reduction Method

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Abstract: To enhance computational efficiency in slope reliability analyses, this paper develops a novel binary classification method (BCM) that takes advantages of support vector machine (SVM) classifier and judgement-based strength reduction method (SRM). Given that it is able to judge the stability state (e.g., stable or unstable) of a slope using SRM without calculating its exact factor of safety, the SVM classifier is naturally selected to establish a response surface to approximate the limit state function. Moreover, the active-learning technique is applied to iteratively search training samples near the boundary of safe and failure domains to update the SVM classifier, which can further improve the computational efficiency and meanwhile ensure computational accuracy. Latin hypercube sampling (LHS) is then employed to compute the system probability of failure of a slope based on the obtained SVM classifier. The proposed method is applied to a two-layer soil slope example taken from literature, which shows great effectiveness and efficiency. The combination of active-learning SVM classifier and judgement-based SRM reduces the computational time of a common slope to several minutes with good accuracy, thus providing a powerful and promising vehicle for efficient system reliability analyses of soil slopes.

Keywords: Soil slope; System reliability; SVM; Judgement-based strength reduction method; Active-learning strategy

1. Introduction

Slope stability evaluation is a complex geotechnical engineering problem because of the uncertainties in the material properties (Kang et al. 2016). Evaluating the factor of safety (*FS*) of a slope is one of the most fundamental aspects of reliability analysis. Conventionally, the *FS* is often determined using the limit equilibrium method (LEM) or strength reduction method (SRM) incorporated into the finite-element/difference method (FEM/FDM). The LEM has been widely adopted to calculate the *FS* with a critical slip surface for reliability analyses owing to its simplicity (see e.g., Ji and Low 2012; Zeng et al. 2015; Zhang et al. 2013). However, in these studies, the slip surfaces were often assumed to be circular, which may not be suitable for complex slope systems, particularly when a weak layer exists (Ching et al. 2010).

The SRM embedded in the FEM/FDM can outperform the LEM for *FS* determination in many aspects. As reported by Ching et al. (2010) and Ma et al. (2017), the main advantage of the SRM is that it can provide a credible *FS* estimation considering the slope as a whole system and automatically identify critical slip surfaces with arbitrary shapes. Thus, it is potentially an ideal tool for system reliability analyses of soil slopes (see e.g., Griffiths and Fenton 2004; Huang et al. 2017). However, as reported by Li et al. (2016), there have been few applications of the SRM in slope reliability analysis, despite the aforementioned advantages. One reason for this could be the time-consuming numerical analysis process of the SRM.

This study proposes a binary classification method (BCM) to further improve the computational efficiency of system reliability analyses of layered soil slopes and to satisfy the computational-cost requirement of engineering applications, while maintaining its accuracy. Rather than using the conventional time-consuming SRM to evaluate the exact *FS* of a slope, this method directly determines the stability state (stable or unstable) by setting the *FS* as 1. A support vector machine (SVM) binary classifier with a modified initial sampling rule and an active-learning strategy is employed together with the judgment method to construct a binary classifier that approximates the true limit state function (LSF) in the variable space. Finally, Latin hypercube sampling (LHS) is applied to estimate the $P_{f,s}$ of the slopes based on the binary classifier. A two-layer soil slope was employed to evaluate the performance of the proposed method. The advantages of the proposed method are illustrated by comparing the computed results with those of the existing methods.

2. Binary classification-based reliability analysis framework

For improving the computational efficiency of the system reliability analyses of layered soil slopes, a binary classification method (BCM) was developed to provide a new perspective for system probability of failure estimation. The binary classification framework contains two main parts: (*i*) a judgment-based strategy, i.e., a binary judgment of the stability state of a slope (instead of computing its exact *FS*) and (*ii*) a surrogate-based strategy, i.e., a binary surrogate classifier instead of the true performance function for separating the LHS samples into two categories.

Given the nature of MCS-based slope reliability analysis process, instead of computing the exact value of $G(\mathbf{u})$, we focus on the state of the given slope (stable or unstable), which can be expressed as

$$Y(\mathbf{u}) = \text{sign}[G(\mathbf{u})] \tag{1}$$

where $Y(\mathbf{u}) = +1$ indicates that the slope is stable with the current soil parameters (i.e., $G(\mathbf{u}) > 0$), and $Y(\mathbf{u}) = -1$ corresponds to the unstable state (i.e., $G(\mathbf{u}) \leqslant 0$). The SRM embedded in FLAC allows such a judgment-based strategy to be used to identify the stability of a slope without calculating the exact *FS*, making it ideal for directly judging the sign of $G(\mathbf{u})$. With the aid of the judgment-based strategy, once a limited number of training samples have been generated, their corresponding stability state can be determined. Then, a classification model, such as an SVM, an ANN, a naïve Bayes classifier, or a random forests (Caruana and Niculescu-Mizil 2006), can be trained to construct a binary classifier to approximate the LSF, where $G(\mathbf{u}) = 0$. Finally, a large number of LHS samples can be generated together with the binary classifier to efficiently estimate the system probability of failure of the slope considered. The schematic in Fig. 1 shows the basic idea of the proposed binary classification framework. Details of the implementation procedure are presented in the following sections.

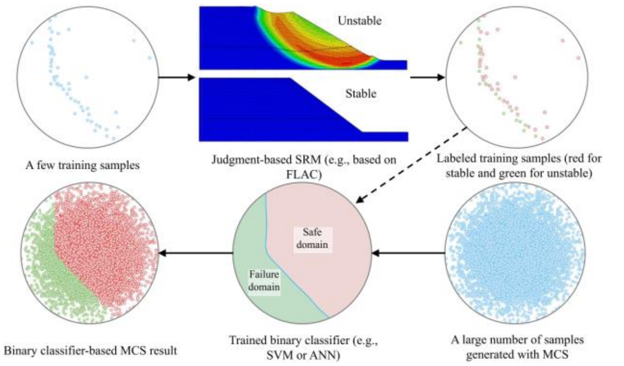

Figure 1. Schematic of binary classification-based reliability analyses of slopes

3. Judgment-based SRM

Rather than computing the exact value of the *FS*, we only need to know the stability state of a slope with given input parameters. Therefore, the SRM embedded in the FLAC was used as an example to implement the proposed judging technique. The basic idea of this technique is to replace the iterative process of searching for the exact *FS* with directly setting it to 1, whereby the SRM can be simplified. If the slope stability is judged as stable with the current shear-strength parameters, the exact *FS* is >1, and an unstable state corresponds to *FS* < 1. Once a slope numerical model is constructed within the FLAC, the inner command "*solve fos bracket 1,1*" can be used to determine the stability of the slope, and the output message "*the FS is larger than 1*" or "*the FS is smaller than 1*" can be found in the current log file.

4. Active-learning SVM

In this study, an active-learning algorithm is introduced to update the SVM classifier dynamically to reduce the total number of training samples required without losing fitting accuracy. As the primary objective of an response surface method (RSM) is to find an explicit function to separate the safe and failure domains, the key to the active-learning technique is selecting the training samples near the border of the safe and failure domains (i.e., LSF, where $G(\mathbf{u}) = 0$), which contributes significantly to the probability of failure and discarding samples far from this border. Thus, a credible SVM classifier can be established with a few training samples and high computation accuracy.

4.1 Initial samples

The *FS* of a soil slope is a monotonically non-decreasing function of the soil strength parameters (Ma et al. 2017), which reveals a useful property of the overall trend of the partition within the sampling space for the slope reliability problem. Moreover, we often regard these strength parameters as random variables when performing a reliability analysis. Herein, a modified three-sigma rule is proposed according to this observation. We first generate a set of initial samples via the traditional three-sigma rule in the U space. Then, we remove each sample, $\mathbf{u}$, from this set unless n-1 elements of $\mathbf{u}$ are equal to –3 (i.e., $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n = -3$ and $u_i = 0$ or 3, where $i = 1, 2, \ldots, n$) or $u_1 = u_2 = \ldots = u_n = k$, where k = -3, 0 or 3. Finally, $2n+3$ initial samples are selected. For slope reliability analysis, the proposed strategy can significantly reduce the number of initial samples and inherit the advantage of the traditional three-sigma rule in the case of many variables.

4.2 Active-learning function

The active-learning algorithm is combined with a pool-based method (Pan and Dias 2017) in this study, which aims to identify the most informative samples in the pool (using LHS), $\mathbf{T}$, to enrich the training sample set, $\mathbf{S}$, iteratively. A candidate sample should simultaneously satisfy two conditions: (*i*) it is located close to the LSF, and (*ii*) it avoids redundant information (i.e., it is not too close to existing training samples). The active-learning function is crucial for performing such an iterative process, and a rational choice of this function can increase the convergence speed, reducing the computational cost of the reliability analysis.

Recently, Li et al. (2018) proposed a comprehensive function with a weighting factor and a distance penalty term that has been proven to provide both concision and precision for reliability problems. The function needs to be established with an exact fitting model in the U space. In the present study, this learning function is extended to be applicable to the binary SVM classifier, which is given as follows:

$$\mathbf{u}_c = \arg\min_{\mathbf{u_T}} \frac{(1+\|\mathbf{u_T}\|)|F(\mathbf{u_T})|}{\left[\dfrac{d(\mathbf{u_T},\mathbf{S})}{1+\exp(-20(d(\mathbf{u_T},\mathbf{S})-d(\mathbf{S})))}\right]} \quad (2)$$

where $\mathbf{u}_c$ represents the selected optimal sample, $\mathbf{u_T}$ represents a sample within the LHS pool, **T**, $d(\mathbf{u_T},\mathbf{S})$ represents the minimum distance between $\mathbf{u_T}$ and the existing training samples, and $d(\mathbf{S})$ represents a reasonable value of the target minimum distance.

$$d(\mathbf{S}) = \lambda \max_{\mathbf{u}_\mathbf{S}^{(i)}} \left(\min_{\mathbf{u}_\mathbf{S}^{(j)}} \left\| \mathbf{u}_\mathbf{S}^{(i)} - \mathbf{u}_\mathbf{S}^{(j)} \right\| \right) : \mathbf{u}_\mathbf{S}^{(i)}, \mathbf{u}_\mathbf{S}^{(j)} \in \mathbf{S} \quad (3)$$

where, λ is a scale factor in the range of 0.1-0.5 (0.2 is used in this study). After the candidate sample is selected from the LHS pool, **T**, it is added to the training sample set, **S**, to update the SVM classifier. With the initial sampling strategy and the active-learning function, a sequential training process of the active-learning SVM (ASVM) can then be started.

4.3 Stopping criterion

A reasonable convergence criterion should stop the training process in a timely manner, reducing the required number of training samples, when the current surrogate model becomes stable. In this study, we apply the criterion where the coefficient of variation (COV) of the $P_{f,s}$ for the slope system that is computed for the last five iterations is equal to or less than a threshold value:

$$\mathrm{COV}_{P_{f,s}^{\mathrm{iter}(5)}} \le \eta \quad (4)$$

where η represents the threshold value (η = 0.001 is used in this study, according to the authors' experience).

5. Case study

A two-layer soil slope taken from the literature was employed to demonstrate the applicability of the proposed BCM, which combined the ASVM classifier and judgment-based SRM. This example is taken from Cho (2009). A cross section with meshes is shown in Fig. 2. The soil strength parameters affecting the stability of the slope, including the friction angle and cohesion, are all considered as random variables. The unit weights of both layers are assumed to be deterministic variables with 19 kN/m^3. Because the shear modulus and bulk modulus of soils have little effect on the *FS* of a slope (Griffiths and Lane 1999), their values were assumed to be 30 and 100 MPa, respectively. Table 1 presents the statistical properties of the soil parameters for the slope. The *FS* calculated with FLAC according to the mean values of the soil properties is 1.59, which is close to the value of 1.60 reported by Cho (2009).

For comparison, the widely used active-learning Kriging (AK) method with the learning function *U* (Zhang et al. 2019) was employed with a conventional FLAC SRM analysis to compare computational efficiency and accuracy. To measure the computational accuracy, the LHS was directly applied to the judgment-based SRM, and the $P_{f,s}$ provided by the LHS was considered as the 'reference' or 'exact' solution.

Fig. 3 shows the training process of BCM and AK. The fluctuation of the predicted $P_{f,s}$ based on the BCM is larger than that for AK, which is mainly attributed to two factors: (*i*) only the label information is used to construct the BCM model, rather than the exact *FS* value, and (*ii*) the fitting ability of the SVM is inferior to that of Kriging. In contrast, comparing the CPU times required by these two methods reveals that surprisingly, the proposed BCM needs only 273 s to achieve a stable binary classifier, whereas the AK needs 1256 s. Thus, the proposed method reduces the amount of computational effort needed for estimating the $P_{f,s}$. This implies that the judgment-based SRM significantly reduces the computational cost of $P_{f,s}$ estimation; even if twice as many training samples are used in the BCM, the CPU time can be reduced to approximately 22% of that required by AK. Regarding the computational accuracy, as shown in Table 2, both the BCM and AK provide good estimates of $P_{f,s}$, with absolute relative errors of <1.5%, compared with the LHS result (1.62%). However, other reliability methods reported in the literature (ANN, linked list-based sifting method, and polynomial RSM considered herein) can hardly provide an accurate $P_{f,s}$; their absolute relative errors vary from 4.93% to 14.8%.

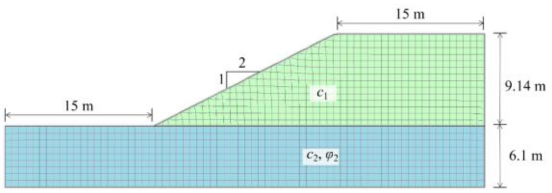

Figure 2. Geometry and mesh of the slope

Table 1 Statistical information of the soil parameters

Slope layer	Parameters	Mean	COV	Distribution type
Top layer	c_1 [kPa]	38.31	0.2	Normal
	γ_1 [kN/m^3]	19	0	Constant
Bottom layer	c_2 [kPa]	23.94	0.2	Normal
	φ_2 [°]	12	0.1	Normal
	γ_2 [kN/m^3]	19	0	Constant

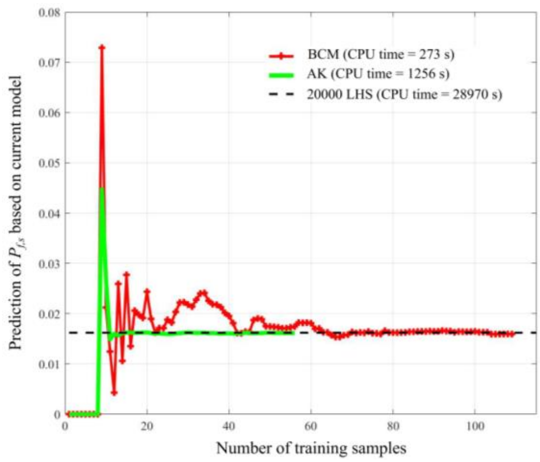

Figure 3. $P_{f,s}$ prediction during the training process of active-learning models

Table 2 Reliability results obtained via different methods

Method	NS[a]	$P_{f,s}$ [%]	Δ [%][b]	Time [s]	Reference
BCM	109	1.59	–1.23	273	This study
AK	56	1.61	–0.62	1256	This study
ANN	-	1.38	–14.8	-	Cho (2009)
Linked list-based sifting method	-	1.70	4.93	-	Luo et al. (2012)
Polynomial RSM	-	1.46	–9.88	-	Xu and Low (2006)
Direct LHS (20000)	-	1.62	-	28970	This study

[a] NS = Number of training samples
[b] Deviations with respect to the mean of LHS

6. Summary and conclusions

In this study, we proposed a BCM that combines a judgment-based SRM embedded in FLAC and an ASVM to efficiently and accurately estimate the system probability of failure of layered soil slopes. Based on the computed results, the following conclusions are drawn.

(1) The proposed approach provided a good estimation of the probability of failure for the two-layer slope considered in this study. The estimation was validated against the results from the literature and direct LHS in this study. Compared with the direct LHS results, the probability of failure prediction obtained via the proposed approach was within an absolute relative error of 1.5%. This accuracy level is acceptable in practical applications.

(2) The proposed method can efficiently estimate system reliability of a layered soil slope within several minutes. The CPU times required by the proposed method was only 22% of those required by the widely used AK method.

(3) With the combination of the ASVM and judgment-based SRM, the proposed BCM can facilitate reliability-based slope design in geotechnical engineering practice, because the high computational cost is currently one of the most significant problems hindering the application of reliability analysis in routine engineering design.

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