Path Tracking Control of a Manipulator Considering Torque Saturation

Hirohiko Arai, *Member, IEEE,* Kazuo Tanie, *Member, IEEE,* and Susumu Tachi, *Member, IEEE*

***Abstract*—When the minimum-time trajectory of a manipulator along a geometrically prescribed path is planned taking into consideration the manipulator's dynamics and actuator's torque limits, at least one of the joints should be at the torque limit. The execution of such a trajectory by a conventional feedback control scheme results in torque saturation. Consequently, the tracking error cannot be suppressed and the manipulator may deviate from the desired path. In this paper, we propose a feedback control method for path tracking which takes the torque saturation into account. Based on the desired path, a coordinate system called *path coordinates* is defined. The path coordinates are composed of the component along the path and the components normal to the path. The equation of motion is described in terms of the path coordinates. Control of the components normal to the path is given priority in order to keep the motion of the manipulator on the path. Simulations of a two-degree-of-freedom manipulator show the effectiveness of this method.**

I. Introduction

HIGH-SPEED manipulation is one of the most important performance requirements of a robot manipulator. It is especially necessary in industrial applications. However, joint actuators should be as small and as light as possible. In the future, robot applications will spread into the medical field, domestic life, and various other areas in which robots operate in a common space with humans. In those applications from the viewpoint of safety, it is not desirable that robots have actuators as powerful as the ones of current industrial robots. In space manipulators in order to save energy, it is required that long arms be controlled by small actuators efficiently. Thus, a control scheme which provides faster operation of a weaker manipulator by making the best of dynamic characteristics of the manipulator will be more important.

There are two types of minimum-time control problems in manipulator dynamics. One is where only the starting point and final point of the path are given, and the path and control input are determined [1]–[3]. The other is where the path geometry as well as the starting and final points are specified. The latter is more practical when it is combined with path planning algorithms for obstacle avoidance, etc. In that case, the solution of the problem is represented as an acceleration/deceleration profile along the desired path. Several off-line planning algorithms for the minimum-time trajectory with a specified path and with torque limits of the actuators have previously been proposed [4]–[6].

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H. Arai and K. Tanie are with the Robotics Department, Mechanical Engineering Laboratory, MITI, 1-2 Namiki, Tsukuba, Ibaraki 305, Japan.

S. Tachi is with RCAST, University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153, Japan.

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When the minimum-time trajectory of a manipulator along a geometrically prescribed path is planned taking into consideration the manipulator's dynamics and its actuators' torque limits, at least one of the joints should be at the torque limit. Trajectory execution is usually based on the position feedback following of a target point on the desired path. The execution of minimum-time trajectory by such a control scheme results in torque saturation. Consequently, the control has no margin to suppress the tracking error and the manipulator may deviate from the desired path.

In this paper, we propose a feedback control method for path tracking which takes the torque saturation into account. Dahl and Nielsen [7] proposed an on-line path following algorithm, in which the time scale of the desired trajectory is modified in real time according to the torque limit. Tam [8] used a perturbation scheme of modifying switching time and nonsaturated torque for the same purpose. We define a coordinate system, called the *path coordinates* [9], based on the desired path. The path coordinates are composed of the component along the path and the components normal to the path. The equation of motion is described in terms of the path coordinates. Control of the components normal to the path is given priority in order to keep the motion of the manipulator on the path. In this method, dynamics of the manipulator are described not only on the path, but also out of the path. Thus, the control is guaranteed to converge to the desired path under torque saturation and a transient response can be chosen. Furthermore, in this method, it is not required that the nominal trajectory be the minimum-time trajectory because this method does not depend on characteristics of the nominal trajectory (e.g., bang-bang characteristics). The simulations of a two-degree-of-freedom manipulator show the effectiveness of this method.

II. Equation of Motion in Terms of Path Coordinates

A. Path Coordinates

Control of an n DOF manipulator in n-dimensional operational space is considered here. A mathematical description of the desired path is considered first. The desired path is geometrically specified as a continuous curve in operational space. It is not associated with a time variable. In the minimum-time trajectory planning problem, this type of path is often parameterized by a path parameter [4]–[6]. A position of a point on the path is represented as a vector function of a scalar

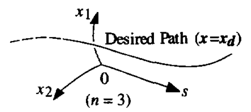


Fig. 1. Path coordinate system.

parameter. When the operational space is n-dimensional, a point $\boldsymbol{q} \in \mathbb{R}^n$ on the path is represented as

$$\boldsymbol{q} = \boldsymbol{q}(s), \qquad s_0 \le s \le s_f \tag{1}$$

where s is the path parameter. $\boldsymbol{q}(s_0)$ is the starting point of the path and $\boldsymbol{q}(s_f)$ is the final point. s can be considered the distance along the path. Since s is a scalar value, this method can represent a point only on the path itself.

The path tracking control considered here is one in which the feedback control makes the manipulator return to the path if the manipulator deviates from the path due to disturbances. Therefore, points which are not on the path as well as points on the path should be represented. Furthermore, the tracking error should be measured. We propose the concept of *path coordinates* [9], which is an extension of path parameter. A curvilinear coordinate frame is defined in the operational space. The coordinates are composed of a component s along the path and components, $x_1, \cdots, x_{n-1}$ which are normal to s. These coordinates are called path coordinates (Fig. 1). A point $\boldsymbol{p} \in \mathbb{R}^n$ represented in terms of the path coordinates is

$$\boldsymbol{p} = [x_1, \cdots, x_{n-1}, s]^T = [\boldsymbol{x}^T \quad s]^T. \tag{2}$$

The desired path is represented as

$$\boldsymbol{x} = \boldsymbol{x}_d(\text{constant}), \qquad s_0 \le s \le s_f \tag{3}$$

in terms of the path coordinates. The desired path is also represented as

$$\boldsymbol{q} = \boldsymbol{q}([\boldsymbol{x}_d^T \quad s]^T), \qquad s_0 \le s \le s_f \tag{4}$$

in terms of the operational coordinates. Equation (4) is extended to all the points in the operational space. A point $\boldsymbol{q}$ in the operational space is represented as a vector function of the path coordinates $\boldsymbol{p}$:

$$\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{p}). \tag{5}$$

Equation (5) represents the coordinate transformation from the path coordinate space to the operational space. The motion on the path is of the s component only and has one degree of freedom. It corresponds to motion on the straight line $\boldsymbol{x} = \boldsymbol{x}_d$ in the path coordinate space. It means that the $\boldsymbol{x}$ components of the path coordinates remain constant $\boldsymbol{x}_d$, irrespective of time if the manipulator is on the path. $\boldsymbol{x} - \boldsymbol{x}_d$ represents the deviation of the point $\boldsymbol{p} = [\boldsymbol{x}^T \quad s]^T$ from the desired path $\boldsymbol{x} = \boldsymbol{x}_d$.

Example: Operational coordinates: Cartesian coordinates $(n = 3)$.

Desired path: A circle of radius r_0, centered at $[x_0, y_0, z_0]$, parallel to the xy plane

$$\boldsymbol{q}(s) = [r_0 \cos(\omega s) + x_0, \ r_0 \sin(\omega s) + y_0, \ z_0].$$

Path coordinates: Cylindrical coordinates

$$\begin{aligned}\boldsymbol{q}(\boldsymbol{p}) &= \boldsymbol{q}([x_1, x_2, s]^T) \\ &= [x_1 \cos(\omega s) + x_0, \ x_1 \sin(\omega s) + y_0, \ x_2].\end{aligned}$$

The desired path is represented as

$$\boldsymbol{x} = [x_1, x_2]^T = [r_0, z_0]^T$$

in terms of the path coordinates.

Note that there exist many path coordinate systems for one desired path. In the case of this example, spherical coordinates which have an origin at the center of the desired path can also be path coordinates.

B. Equation of Motion

The equation of motion of the manipulator in joint space can be written as follows:

$$\boldsymbol{M}(\theta)\ddot{\theta} + \boldsymbol{b}(\theta, \dot{\theta}) = \tau \tag{6}$$

where

$\boldsymbol{b}(\theta, \dot{\theta}) = \boldsymbol{h}(\theta, \dot{\theta}) + \Gamma\dot{\theta} + \boldsymbol{g}(\theta)$
$\theta \in \mathbb{R}^n$: joint angle vector
$\tau \in \mathbb{R}^n$: joint torque vector
$\boldsymbol{g}(\theta) \in \mathbb{R}^n$: gravity torque vector
$\boldsymbol{h}(\theta, \dot{\theta}) \in \mathbb{R}^n$: Coriolis and centrifugal torque vector
$\boldsymbol{M}(\theta) \in \mathbb{R}^{n \times n}$: inertia matrix
$\Gamma \in \mathbb{R}^{n \times n}$: viscosity friction matrix.

The equation of motion is rewritten in terms of path coordinates $\boldsymbol{p}$ defined in Section II-A. From (5), the operational coordinates $\boldsymbol{q}$ and the path coordinates $\boldsymbol{p}$ are related as $\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{p})$. The operational coordinates $\boldsymbol{q}$ is calculated by forward kinematics, $\boldsymbol{q} = \boldsymbol{q}(\theta)$. When

$$\boldsymbol{J}_1 = \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{p}}, \quad \boldsymbol{J}_2 = \frac{\partial \boldsymbol{q}}{\partial \theta} \qquad (\boldsymbol{J}_1, \boldsymbol{J}_2 \in \mathbb{R}^{n \times n}),$$

the Jacobian matrix, $\boldsymbol{J}(\theta) \in \mathbb{R}^{n \times n}$ of path coordinate $\boldsymbol{p}$ for joint coordinates θ is represented as

$$\boldsymbol{J} = \frac{\partial \boldsymbol{p}}{\partial \theta} = \boldsymbol{J}_1^{-1}\boldsymbol{J}_2.$$

Consequently, $\boldsymbol{p}$ and θ are related as

$$\dot{\boldsymbol{p}} = \boldsymbol{J}\dot{\theta}. \tag{7}$$

When (7) is differentiated with respect to time, we obtain

$$\ddot{\boldsymbol{p}} = \boldsymbol{J}\ddot{\theta} + \dot{\boldsymbol{J}}\dot{\theta}. \tag{8}$$

If $\boldsymbol{J}$ is nonsingular,

$$\ddot{\theta} = \boldsymbol{J}^{-1}(\ddot{\boldsymbol{p}} - \dot{\boldsymbol{J}}\dot{\theta}). \tag{9}$$

Here, $\boldsymbol{p}$, $\boldsymbol{M}$, $\boldsymbol{b}$, and $\boldsymbol{H} \equiv \boldsymbol{J}^{-1}$ are partitioned as follows:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{m}_1 \\ \vdots \\ \boldsymbol{m}_n \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}$$
$$\boldsymbol{H} = [\underset{n-1}{\boldsymbol{H}_x} \quad \underset{1}{\boldsymbol{h}_s}]n. \tag{10}$$

When (9) and (10) are substituted in (6), we obtain

$$\boldsymbol{m}_i(\boldsymbol{H}_x\ddot{\boldsymbol{x}} + \boldsymbol{h}_s\ddot{s} - \boldsymbol{H}\dot{\boldsymbol{J}}\dot{\theta}) + b_i = \tau_i \qquad (i = 1, \cdots, n). \tag{11}$$

The equation of motion (11) relates accelerations of path coordinates to the torque of each joint actuator.

III. Minimum-Time Trajectory Planning

An off-line algorithm for minimum-time trajectory planning [4]–[6] is reviewed in this section. A minimum-time trajectory is planned for the case when the torque of the joint actuators is limited. The planned trajectory is used as a nominal trajectory, and the path tracking control in the following sections is applied to it. The torque limit is represented as a domain $\boldsymbol{T}$ in torque vector space $\mathbf{R}^n$:

$$\tau \in \boldsymbol{T}, \boldsymbol{T} = \{\tau \mid \tau_i^{\min} \le \tau_i \le \tau_i^{\max} \qquad (i = 1, \cdots, n)\}. \quad (12)$$

From (3), $\boldsymbol{x}$ components of the path coordinates are constant when the manipulator moves along the desired path. Thus, $\dot{\boldsymbol{x}} = \boldsymbol{0}$ and $\ddot{\boldsymbol{x}} = \boldsymbol{0}$ on the path. Since $\boldsymbol{H}\dot{\boldsymbol{J}} = -\dot{\boldsymbol{H}}\boldsymbol{J}$ and $\dot{\boldsymbol{p}} = \boldsymbol{J}\dot{\theta}$, acceleration along the path is

$$\ddot{s} = (\boldsymbol{m}_i\boldsymbol{h}_s)^{-1}(\tau_i - \boldsymbol{m}_i\dot{\boldsymbol{h}}_s\dot{s} - b_i) \qquad (i = 1, \cdots, n) \quad (13)$$

from (11). The maximum value $\ddot{s}_i^{\max}$ and the minimum value $\ddot{s}_i^{\min}$ of the path acceleration $\ddot{s}$ for the available torque of each joint are obtained when the torque in (13) is limited according to (12). The admissible region of $\ddot{s}$, $[\ddot{s}^{\min}, \ddot{s}^{\max}]$ for the whole manipulator is obtained as a product of admissible regions for each joint torque limit:

$$[\ddot{s}^{\min}, \ddot{s}^{\max}] = \bigcap_{i=1}^{n} [\ddot{s}_i^{\min}, \ddot{s}_i^{\max}]$$

$$\ddot{s}^{\max} = \operatorname*{MIN}_{i=1}^{n}(\ddot{s}_i^{\max}), \qquad \ddot{s}^{\min} = \operatorname*{MAX}_{i=1}^{n}(\ddot{s}_i^{\min}). \quad (14)$$

Note that the range of the region $[\ddot{s}^{\min}, \ddot{s}^{\max}]$ depends on the path velocity $\dot{s}$. If the velocity is too large, the admissible region of $\ddot{s}$ cannot exist and the motion along the path is impossible. It determines the admissible velocity. The minimum-time trajectory is composed by connecting maximum/minimum acceleration trajectory segments so as to obtain maximum velocity without exceeding the admissible velocity.

An example of the trajectory planning for a two DOF horizontally articulated manipulator (Fig. 2) is shown. The joint torque limit is $-2 \le \tau_1 \le 2$, $-1 \le \tau_2 \le 1$ (N · m). The desired path is a circle (Fig. 3; center: [0.4, 0], radius: 0.1 m). In this case, the path coordinate system is a polar coordinate system whose origin is at the center of the path. The s component along the path is angle α, and the x component normal to the path is radius r. In the planned trajectory, the velocity $\dot{\alpha}$ along the path (Fig. 4(a), bold line) is obtained by switching maximum acceleration/deceleration without exceeding the admissible velocity (Fig. 4(a), thin line). One of the joint torques is always saturated [Fig. 4(b), (c)]. Therefore, a conventional feedback control cannot be used.

In this section, the equation of motion is represented in terms of the path coordinates. However, (13) can also be rewritten as

$$\ddot{s} = (\boldsymbol{m}_i\boldsymbol{q}')^{-1}(\tau_i - \boldsymbol{m}_i\boldsymbol{q}''\dot{s}^2 - b_i) \qquad (i = 1, \cdots, n)$$

since $\boldsymbol{h}_s = d\boldsymbol{q}/ds = \boldsymbol{q}'$ on the desired path. Thus, this expression is equivalent to the expressions in [4]–[6] using the path parameters.

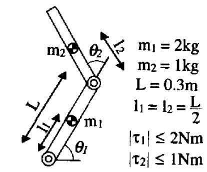


Fig. 2. Two-degree-of-freedom manipulator.

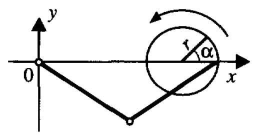


Fig. 3. Desired path.

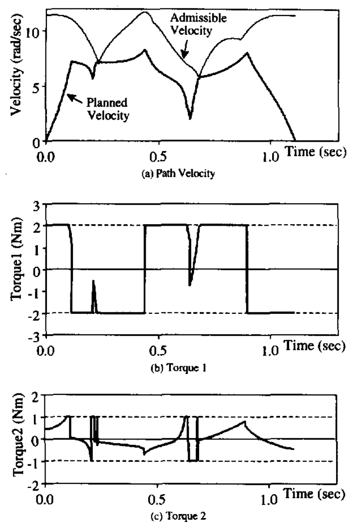


Fig. 4. Minimum-time trajectory. (a) Path velocity. (b) Torque 1. (c) Torque 2.

IV. Path Tracking Control

In the trajectory planned by the algorithm in Section III, the torque of at least one joint is saturated. Under conventional feedback control, it may be possible that the manipulator can deviate from the desired path. Path tracking control which takes into account torque saturation is proposed in this section. In this method, the tracking capability is achieved by independently controlling the $\boldsymbol{x}$ components normal to the path and the s component along the path. Since the characteristics of a minimum-time trajectory (e.g., bang-bang joint torques) are not used, this method can also be applied to the nominal trajectories which are not minimum time.

The equation of motion in terms of path coordinates is

$$\boldsymbol{M}(\boldsymbol{H}_{\boldsymbol{x}}\ddot{\boldsymbol{x}} + \boldsymbol{h}_s\ddot{s} - \boldsymbol{H}\dot{\boldsymbol{J}}\dot{\theta}) + \boldsymbol{b} = \tau. \quad (15)$$

This equation is rewritten as follows:

$$\tilde{\boldsymbol{M}}_{\boldsymbol{x}}\ddot{\boldsymbol{x}} + \tilde{\boldsymbol{m}}_s\ddot{s} = \tilde{\tau} \quad (16)$$

where

$$\tilde{M} = MH, \qquad \tilde{M}_x = \begin{bmatrix} \tilde{m}_{x1} \\ \vdots \\ \tilde{m}_{xn} \end{bmatrix} = MH_x,$$

$$\tilde{m}_s = \begin{bmatrix} \tilde{m}_{s1} \\ \vdots \\ \tilde{m}_{sn} \end{bmatrix} = Mh_s, \qquad \tilde{b} = b - \tilde{M}\dot{J}\dot{\theta}, \qquad \tilde{\tau} = \tau - \tilde{b}. \tag{17}$$

The joint torque limit is

$$\tilde{\tau} \in \tilde{T}, \qquad \tilde{T} = \{\tilde{\tau} \mid \tilde{\tau}_i^{\min} \le \tilde{\tau}_i \le \tilde{\tau}_i^{\max} \qquad (i = 1, \cdots, n)\} \tag{18}$$

where

$$\tilde{\tau}_i^{\min} = \tau_i^{\min} - \tilde{b}_i, \qquad \tilde{\tau}_i^{\max} = \tau_i^{\max} - \tilde{b}_i. \tag{19}$$

The domain T is moved parallel to $\tilde{T}$ by $\tilde{b}$ in the torque vector space.

A. Control of x Components

The control of the x components normal to the path is given priority in order to keep the motion of the manipulator on the path. The tracking error is suppressed by the following PID feedback:

$$\ddot{x}_c = -K_v\dot{x} + K_p(x_d - x) + K_i\int(x_d - x)\,dt. \tag{20}$$

x_d is the desired position and x is the measured position of the x coordinates. $\ddot{x}_c$ is the acceleration to suppress the error. If acceleration of the x coordinates is $\ddot{x}_c$,

$$(\ddot{x}_d - \ddot{x}) + K_v(\dot{x}_d - \dot{x}) + K_p(x_d - x) + K_i\int(x_d - x)\,dt = 0. \tag{21}$$

Dynamic characteristics of x are determined by the gain matrices K_v, K_p, and K_i. x converges to x_d and the manipulator tracks the path if K_v, K_p, and K_i are selected appropriately.

In fact, it may be possible that the acceleration $\ddot{x}_c$ cannot be generated due to torque limit ($\forall\ddot{s}$, $\tilde{M}_x\ddot{x}_c + \tilde{m}_s\ddot{s} \notin \tilde{T}$). The possibility to obtain $\ddot{x}_c$ is judged by the geometrical relation in the acceleration space. If $\ddot{x}_c$ cannot be obtained, the acceleration is limited to the best value that can be realized and the actual acceleration $\ddot{x}_a$ is determined.

The acceleration $\ddot{x}_c$ is substituted in (16), If $\tilde{m}_{si} \ne 0$,

$$\ddot{s} = \tilde{m}_{si}^{-1}(\tilde{\tau}_i - \tilde{m}_{xi}\ddot{x}_c). \tag{22}$$

If the torque of (22) is restricted according to (18), the maximum acceleration $\ddot{s}_i^{\max}$ and the minimum acceleration $\ddot{s}_i^{\min}$ along the path are determined for the torque limit of each joint. If there exists a common admissible region $[\ddot{s}^{\min}, \ddot{s}^{\max}]$ for all the joints, $\tilde{M}_x\ddot{x}_c + \tilde{m}_s\ddot{s} \in T$ for this region. In this case, $\ddot{x}_c$ can be obtained and $\ddot{x}_a = \ddot{x}_c$. (If $\tilde{m}_{si} = 0$, there is no limit of $\ddot{s}$ for joint i. In this case, $\ddot{x}_c$ can be realized when $\tilde{\tau}_i^{\min} \le \tilde{M}_x\ddot{x}_c \le \tilde{\tau}_i^{\max}$.)

The inertia matrix $\tilde{M}$ can be considered a linear transformation from the torque space $\mathbb{R}^n$ to the acceleration space $\mathbb{R}^n$. Torque limit $\tilde{T}$ is transformed to an admissible acceleration domain $\tilde{U}$ by the inertia matrix $\tilde{M}$. For example, $\tilde{T}$ is a rectangular prism and $\tilde{U}$ is a parallel hexahedron for $n = 3$. $\ddot{x} = \ddot{x}_c$ is a straight line parallel to the $\ddot{s}$ axis in the acceleration space. $\ddot{x}_c$ can be realized when $\ddot{x} = \ddot{x}_c$ intersects with domain $\tilde{U}$. Intersection between $\tilde{U}$ and $\ddot{x} = \ddot{x}_c$ means an admissible region of $\ddot{s}$.

On the other hand, if $\tilde{U}$ and $\ddot{x} = \ddot{x}_c$ do not intersect, the admissible region of $\ddot{s}$ does not exist and $\ddot{x}_c$ cannot be obtained. It is necessary to limit $\ddot{x}_c$ and the actual acceleration $\ddot{x}_a$ should be determined. It is desirable that $\ddot{x}_a$ is as close to $\ddot{x}_c$ as possible; thus, $\ddot{x}_a$ is determined so that $|\ddot{x}_a - \ddot{x}_c|^2$ is minimum. Projection $\tilde{U}'$ of domain $\tilde{U}$ on the plane $\ddot{s} = 0$ is considered. $\ddot{x}_a$ should belong to $\tilde{U}'$ on the plane $\ddot{s} = 0$. Therefore, $\ddot{x}_a$ is chosen as the closest point to $\ddot{x}_c$ in the region $\tilde{U}'$. $\ddot{x}_a$ is the closest acceleration to $\ddot{x}_c$ which can be achieved by the available torque.

Equation (20) is not valid since $\ddot{x} \ne \ddot{x}_c$. However, it does not directly mean that the manipulator deviates from the desired path. When one of $\ddot{x}_i$, $\dot{x}_i$, and $x_i - x_{di}$ is positive (negative) and the others are negative (positive), the tracking error of the ith component is inclined to decrease. If the domain $\tilde{U}'$ includes the origin $\ddot{x} = 0$, each component of $\ddot{x}$ can have either a positive or negative value. Then there exists $\ddot{x}$ which satisfies the condition described above. Since $\ddot{x} = 0$ on the nominal trajectory, the domain $\tilde{U}'$ includes $\ddot{x} = 0$ near the nominal trajectory, and the condition of convergence to the path can be satisfied.

B. Control of s Component

Next, the s component is controlled and the acceleration/deceleration along the path is determined. The path acceleration $\ddot{s}_c$ is calculated from the planned nominal velocity $\dot{s}_n$, nominal acceleration $\ddot{s}_n$, and measured velocity $\dot{s}$. We used the feedback of $\dot{s}^2$ proposed in [7]:

$$\ddot{s}_c = \ddot{s}_n(s) + K_s(\dot{s}_n(s)^2 - \dot{s}^2). \tag{23}$$

The differentiation of $\dot{s}^2$ with regard to s is

$$\frac{d\dot{s}^2}{ds} = 2\frac{ds}{dt}\cdot\frac{d\dot{s}}{ds} = 2\frac{d^2s}{dt^2} = 2\ddot{s}. \tag{24}$$

The feedback of (23) provides a linear system with regard to $\dot{s}^2$, and $\dot{s}^2$ converges to $\dot{s}_n^2$ if $\ddot{s} = \ddot{s}_c$.

The actual acceleration $\ddot{s}_a$ is determined by the torque limit in the same way as in the case of $\ddot{x}_a$. If $\ddot{x}_c$ can be generated, the admissible region $[\ddot{s}^{\min}, \ddot{s}^{\max}]$ of $\ddot{s}$ is already determined (see Section IV-A). If $\ddot{x}_c$ is limited, the admissible region of $\ddot{s}$ is determined by substituting $\ddot{x}_a$ to $\ddot{x}_c$ of (22). In the latter case, the admissible $\ddot{s}$ is generally a unique value. Since $\ddot{s}$ is a scalar value, the closest value to $\ddot{s}_c$ in the admissible region of $\ddot{s}$ is chosen as $\ddot{s}_a$. The limitation described above means that the acceleration of s is limited in order to realize the acceleration $\ddot{x}_a$. In particular, when the admissible region $[\ddot{s}^{\min}, \ddot{s}^{\max}]$ for the acceleration $\ddot{x}_c$ exists, $\ddot{x}_c$ itself can be realized even if $\ddot{s}_c$ is limited. It corresponds to the case when one joint torque is saturated. The tracking capability is guaranteed by (21). Note that $\ddot{s}_n$ and $\dot{s}_n$ are functions of s and are not functions of time. The time trajectory of s can

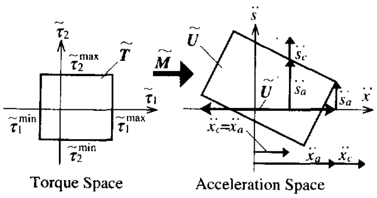


Fig. 5. Limit of accelerations.

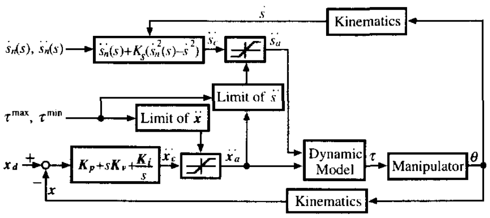


Fig. 6. Path tracking control system.

be different from that of the nominal trajectory when $\ddot{s}_c$ is limited. (The planning of $\ddot{s}_n$ and $\dot{s}_n$ is based on the shape of the desired path and the dynamics of the manipulator, which are functions of s. Consequently, $\ddot{s}_n$ and $\dot{s}_n$ can be represented as functions of s.)

Fig. 5 illustrates the limit of the accelerations when $n = 2$. The matrix $\tilde{\boldsymbol{M}}$ can be regarded as a linear transformation from the torque vector space to the acceleration vector space. The admissible torque domain $\tilde{\boldsymbol{T}}$ is transformed by $\tilde{\boldsymbol{M}}$ to the admissible acceleration domain $\tilde{\boldsymbol{U}}$. Line segment $\tilde{\boldsymbol{U}}'$, which is a projection of $\tilde{\boldsymbol{U}}$ on the $\ddot{x}$ axis, represents the admissible region of $\ddot{x}$. If $\ddot{x}_c$ obtained by feedback (20) is outside $\tilde{\boldsymbol{U}}'$, the actual acceleration is limited to $\ddot{x}_a$. In this case, $\ddot{s}_a$ is determined uniquely. If $\ddot{x}_c$ is inside $\tilde{\boldsymbol{U}}'$, $\ddot{s}_c$ is limited within the admissible region of $\ddot{s}$.

The joint torque can be calculated by substituting the accelerations $\ddot{\boldsymbol{x}}_{\boldsymbol{a}}$ and $\ddot{s}_a$ in the equation of motion (16). (Generally, the torque is already calculated by previous procedures.) Fig. 6 illustrates the control system. All the inputs of the system do not depend on time. $\boldsymbol{x_d}$ and the torque limits are constant values. $\ddot{s}_n$ and $\dot{s}_n$ are functions of s. Therefore, this control system can stretch or shrink the time axis in order to achieve path tracking.

As the time to complete the path tracking may be different from that of the nominal trajectory, the end of the tracking should be watched:

$$\lambda = \frac{s - s_0}{s_f - s_0}. \tag{25}$$

The value of λ is monitored during the path tracking. If the manipulator is on the path, $0 \leq \lambda \leq 1$. The path tracking control is terminated when $\lambda = 1$.

V. Simulations

Path tracking simulations were carried out with a two DOF horizontally articulated manipulator (Fig. 2). A minimum-time trajectory (Fig. 4) was planned as the nominal trajectory for a desired path of Fig. 3. Viscosity friction and coulomb friction [Joint 1: $0.2\,\mathrm{sgn}(\dot{\theta}) + 0.4\dot{\theta}$ (N · m), Joint 2: $0.1\,\mathrm{sgn}(\dot{\theta}) + 0.2\dot{\theta}$ (N·m)] were applied to the joints as disturbances. Friction was not considered in the dynamic model used for the trajectory planning algorithm and the controllers. Fig. 7 shows the simulation results. Although the manipulator deviates from the desired path in the case of the conventional method [(b), the computed torque method in operational space], tracking error in the case of the proposed method (a) is small. Fig. 8(a) shows the velocity along the path for tracking by the proposed method. In the result, the velocity is slowed down in comparison with the nominal trajectory (Fig. 4) and the tracking is achieved. However, the joint torques (b), (c) are almost saturated and the capacities of the actuators are fully utilized.

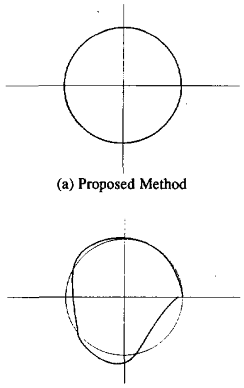


Fig. 7. Simulation results with joint friction. (a) Proposed method. (b) Conventional method.

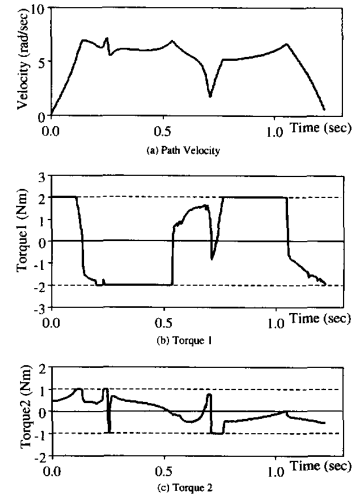
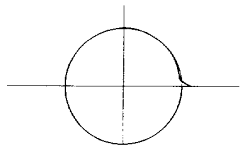
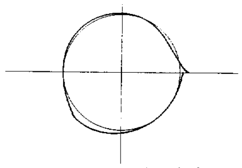


Fig. 8. Simulation results with joint friction. (a) Path velocity. (b) Torque 1. (c) Torque 2.

In the next simulations, the initial position is not on the path (Fig. 9). The initial error $(r - r_d)$ is 0.015 m. The conventional method (b) expands the error due to the torque limit. In the

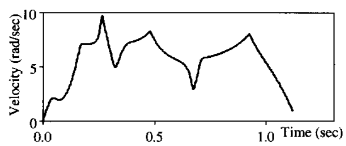


(a) Proposed Method

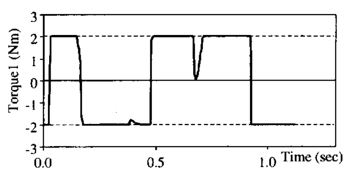


(b) Conventional Method

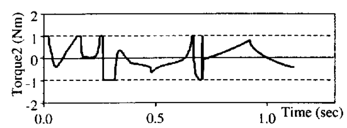
Fig. 9. Simulation results with initial error. (a) Proposed method. (b) Conventional method.



(a) Path Velocity



(b) Torque 1



(c) Torque 2

Fig. 10. Simulation results with initial error. (a) Path velocity. (b) Torque 1. (c) Torque 2.

proposed method (a), the error is suppressed quickly, and the manipulator can subsequently track the path. By shifting the time axis, the velocities and torques (Fig. 10) after the error suppression nearly coincide with the minimum-time trajectory of Fig. 4.

When the feedback gain is increased, the tracking error increases by the conventional method since the difference between the desired torque and the actual torque grows in both simulations. On the other hand, the tracking error by the proposed method decreases when the feedback gain is increased.

VI. Conclusions

A feedback control method for path tracking which takes the torque saturation into account is proposed. The equation of motion of the manipulator is described in terms of the *path coordinates*. The components normal to the path and the component along the path are independently controlled considering the torque limit. Since the inputs to the control system are not functions of time, path tracking can be achieved by modifying the time axis of the nominal trajectory. In simulations with a two-degree-of-freedom manipulator, tracking error is well suppressed even if disturbances or initial errors exist.

Presently, we plan to apply this method to a real manipulator and to confirm the tracking ability experimentally. This method depends on the dynamic model of a manipulator, and it requires real-time calculation of dynamics. We are also investigating methods to deal with model variations and to reduce calculations.

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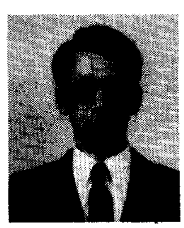


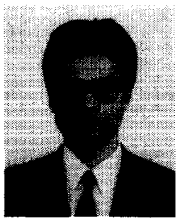
Hirohiko Arai (M'91) was born in Tokyo, Japan, on July 9, 1959. He received the B.E. and Ph.D. degrees in measurement and control engineering from the University of Tokyo in 1982 and 1993, respectively.

From 1982 to 1984 he was with Honda Engineering Company, Saitama, Japan. He joined the Mechanical Engineering Laboratory, Ministry of International Trade and Industry, Tsukuba, Ibaraki, Japan, in 1984. He is currently a Senior Researcher at the Biorobotics Division of the Robotics Department. He was a Visiting Scholar at the Mechanical Engineering Department, Stanford University, CA, from March 1993 to February 1994. His research interests include dynamic control of manipulators, and man–machine interface for teleoperation.

Kazuo Tanie (M'85) was born in Yokohama, Japan, on November 6, 1946. He received the B.S., M.S., and Dr.Eng. degrees in mechanical engineering from Waseda University, Tokyo, Japan, in 1969, 1971, and 1980, respectively.

He joined the Mechanical Engineering Laboratory, Ministry of International Trade and Industry, Tsukuba, Ibaraki, Japan, in 1971, where he is currently the Director of the Biorobotics Division of the Robotics Department. He has also been a Professor in the Graduate School of the University of Tsukuba since 1992, and was a Visiting Scholar at the Biotechnology Laboratory, University of California, Los Angeles, from August 1981 to August 1982. His research interests currently relate to tactile sensors, control for multifingered hands, force control for robotic arms, virtual reality and its application to telerobotics, and the practical application of robotics.

Dr. Tanie is a member of the Board of Directors of the Robotics Society of Japan and the Society of Instrumentation and Control Engineers. He is a member of the Advisory Board of the Society of Biomechanisms.



Susumu Tachi (M'82) was born in Tokyo, Japan, on January 1, 1946. He received the B.E., M.S., and Ph.D. degrees in mathematical engineering and information physics from the University of Tokyo in 1968, 1970, and 1973, respectively.

He joined the Faculty of Engineering of the University of Tokyo in 1973, and in 1975 joined the Mechanical Engineering Laboratory, Ministry of International Trade and Industry, Tsukuba Science City, Japan, where he served as the Director of the Biorobotics Division. In 1989 he rejoined the University of Tokyo as an Associate Professor. Since 1992 he has been a Professor in the Research Center for Advanced Science and Technology (RCAST) of the University of Tokyo. From 1979 to 1980 he was a Japanese Government Award Senior Visiting Scientist at the Massachusetts Institute of Technology, Cambridge. His scientific achievements include intelligent mobile robot systems for the blind, called Guide Dog Robot (1976–1983), and a national large-scale project on advanced robotics, especially advanced human robot systems with a real-time sensation of presence, known as Tele-Existence (1983–1990). His present research covers robotics information systems, tele-existence, and virtual reality.

Dr. Tachi is a Founding Director of the Robotics Society of Japan, and is a Fellow of the Society of Instrument and Control Engineers. Since 1988 he has been Chairman of the IMEKO (International Measurement Confederation) Technical Committee on Robotics.