論文の内容の要旨

論文題目 Three Point Functions in AdS_5/CFT_4 Correspondence and Integrability
(AdS_5/CFT_4 対応における三点関数と可積分性)

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String theory, first conceived in the late 1960s as a theory of hadrons, was once abandoned after the advent of non-Abelian gauge theories and quantum chromodynamics (QCD), because it predicted the existence of unrealistic massless particles and required the spacetime dimension to be 10. However, it was subsequently realized that these "unrealistic" massless particles could describe gravitons and the theory resurrected as a leading candidate for the unified theory of all interactions including gravity.

String theory, comprised not of zero-dimensional particles but of one-dimensional strings, has two main features which make it distinct from conventional quantum field theories. The first feature, perhaps the most well-known one, is that a single string is capable of describing infinitely many kinds of particles including gravitons as vibration modes. This is often regarded as the most important and the most fundamental property which makes string theory a candidate for a theory of everything. However, this feature captures only a partial aspect of the theory as it tells just about the spectrum of the particles and nothing about their interactions. What is truly important for string theory to be a consistent theory of all interactions is the following second feature: Infinitely many types of interactions for infinitely many kinds of particles are described in a unified way by the geometric structure of the string worldsheet, which glues or splits when the interaction occurs. This is quite a remarkable property since it is inconceivable anyhow in conventional field theories that the interactions admit such sort of geometric interpretation.

Vigorous studies in the past have revealed another interesting but at the same time slightly bizarre facet of string theory, called *duality*. Duality is a phenomenon that two seemingly different theories secretly describe the same physical situation. A distinctive feature of string theory is that it predicts various new dualities and make the connection between theories with completely different properties. Today, duality has become one of the essential ingredients in string theory, which helps to bring together diverse theoretical ideas.

Among numerous dualities found in string theory, the one which has been intensively studied in the past fifteen years is the *AdS/CFT correspondence*. The AdS/CFT correspondence is a conjectural duality between a d-dimensional conformally invariant field theory and a $d+1$-dimensional quantum gravity on the anti-de Sitter space. It is a concrete realization of the old idea, called *holography*, that the dynamics of quantum gravity is describable in terms of the degrees of freedom living on the boundary of the spacetime. As such, the AdS/CFT correspondence has had an immense impact on diverse areas of theoretical physics: On the one hand, it is considered to provide a non-perturbative formulation of quantum gravity in terms of conventional quantum field theories. On the other hand, it has enabled us to explore the strongly-coupled regime of interacting gauge theories via gravity. Looking back on the history of string theory, the discovery of the AdS/CFT correspondence marks a major milestone. It fulfills the dearest dream in the early days of string theory in rather an unexpected way: String theory indeed describes the dynamics of non-Abelian gauge theories, but only indirectly through the holography.

Despite its theoretical importance, fundamental understanding as to why and how the AdS/CFT correspondence holds is still missing even after fifteen years. Then, the next best thing we could do is to compute the physical observables on both sides of the duality and compare them. As the dual gauge theory is conformally invariant, there is no clear notion of particles or asymptotic states, with which we compute S-matrices. Instead, the most natural and the most fundamental objects in the theory are *local operators* and the counterparts of the S-matrices are provided by their *correlation functions*. Of particular interest among these are two- and three-point functions, which together constitute the building blocks of the dual gauge theory. Indeed, one can express any correlation functions in the theory in terms of two- and three-point functions by performing the operator product expansion. In the AdS/CFT correspondence, the two-point function is considered to describe a free propagation of a vibrating string in the AdS spacetime and the three-point function is believed to describe their interaction. Thus, these two quantities are of crucial importance also on the string-theory side and we would hopefully gain new insights into the fundamental mechanism of the duality by closely examining two- and three-point functions.

This thesis is devoted to the study of three-point functions in the most typical example of the AdS/CFT correspondence, called the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence, which is a conjectural duality between the $\mathcal{N}=4$ super Yang-Mills theory in four-dimension and the type IIB superstring theory on a certain ten-dimensional curved space, called $AdS_5 \times S^5$ background. The main objective of the thesis is to explain how the integrability-based approaches help to simplify the computation of three-point functions and enable us to extract the structures common to both sides of the duality.

In Part I, we first give a general introduction of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence. Then, we review how the integrability-based techniques enabled us to compute two-point functions on each side of the duality. In Part II, we review the perturbative computation of three-point functions in the gauge theory and explain that the computation can be reformulated as a problem of evaluating the scalar products between two different states of a certain integrable spin-chain. Next we summarize several existing expressions for such scalar products and discuss their behavior in the *semi-classical* limit, in which the length of the chain and the number of excitations are both large. Then we derive a new concise formula for such scalar products, which is expressed as a multiple integral akin to the eigenvalue integrals in the matrix models, based on the author's original work. We also discuss that the integral expression we derived is potentially useful to get a more physical picture of the semi-classical limit.

In Part III, which is the main part of this thesis, we explain the computation of three-point functions on the string theory side. After recalling the well-known fact that the strong coupling limit of the gauge theory can be described alternatively by the classical string theory on $AdS_5 \times S^5$, we explain that the three-point function in such a limit can be computed by evaluating the classical action of a three-pronged worldsheet plus the boundary terms, which come from the semi-classical wave function of each string. Then we describe the integrability-based method to compute these two contributions: First, we show that both of the contributions are expressible in terms of the important quantities, called *Wronskians*, which fit naturally into the framework of integrability. Next, we examine the analytic properties of the Wronskians and set up the Riemann-Hilbert problem. Then, we determine each Wronskian by solving the Riemann-Hilbert problem in terms of certain convolution integrals, and present a fairly general formula for three-point functions at strong couping. In spite of the complexity of the contributions from various parts in the intermediate stages, the final answer for the three-point function takes a remarkably simple form, exhibiting the structure reminiscent of the one obtained in the gauge theory. We then perform a detailed comparison with the results in the gauge theory and discuss the implications.

Finally, in Part IV, we summarize the results explained in this thesis and conclude by mentioning future directions.