論文の内容の要旨

論文題目　Boundaries and domain walls in two-dimensional supersymmetric theories
（二次元超対称理論における境界とドメインウォール）

氏　名　　本田　大悟

Two-dimensional $\mathcal{N} = (2, 2)$ gauged linear sigma models are simple quantum field theories that exhibit very rich structures. As such, they have a variety of applications. At low energy, these theories reduce to non-linear sigma models whose target spaces are Kähler manifolds, in particular Calabi-Yau manifolds under certain conditions. When we put these theories on a surface with boundary, the boundary conditions describe D-branes. A boundary condition in the product of two theories can be regarded as a domain wall that connects two regions where the two theories live.

In this thesis we study boundaries and domain walls in $\mathcal{N} = (2, 2)$ gauged linear sigma models using supersymmetric localization. We focus on the hemisphere geometry, which has a single boundary component. The resulting hemisphere partition function is roughly a half of the sphere partition function which is the partition function of the gauged linear sigma models on a sphere. This thesis is based on the results obtained in collaboration with Takuya Okuda, assistant professor at the University of Tokyo.

There are two broad motivations for studying the hemisphere partition function. The first is the study of D-branes in Calabi-Yau manifolds, with applications to mirror symmetry, Gromov-Witten invariants, D-brane stability, string phenomenology, etc. In such contexts the two dimensional theory describes the worldsheet of a superstring, and one is especially interested in theories that flows to a non-linear sigma model with target space a compact Calabi-Yau manifold. Generically such a theory possesses no flavor symmetries.

The hemisphere partition function depends analytically on the complexified FI parameters, which we collectively denote as t and use to parametrize the Kähler moduli space. The second motivation, the main one for us, is to study the dynamics of the two-dimensional quantum field theory in its own right. It is known that $\mathcal{N} = (2, 2)$ theories are closely related to integrable models. Such a theory also arises as the defining theory for a surface operator embedded in a four-dimensional theory. It is natural to turn on twisted masses $m = (m_a)$, or equivariant parameters for flavor symmetries, in these contexts. Boundaries are interesting ingredients in the physics of the theory, while domain walls ($\simeq$ line operators in two dimensions) provide a natural example of non-local disorder operators, and are akin to 't Hooft loops, vortex loops, surface operators, and domain walls in higher dimensions.

The type of boundary conditions $\mathcal{B}$ we study preserve B-type supersymmetries. For abelian gauge theories general B-type boundary conditions were formulated by Herbst, Hori, and Page. We extend these boundary conditions, in a straightforward way, to theories with non-abelian gauge groups and twisted masses. We will argue that the hemisphere partition function $Z_{\text{hem}}(\mathcal{B}; t; m)$ is the overlap $\langle \mathcal{B} | \mathtt{1} \rangle$ of two states, where both the boundary state $\langle \mathcal{B}|$ and the state $|\mathtt{1}\rangle$ created by a topological twist are zero-energy states in the Hilbert space for the Ramond-Ramond sector.

When the gauge theory flows to a non-linear sigma model with a smooth target space, there are coarse and refined classifications of B-branes:

$$\begin{aligned} \{\text{B-branes}\} &\simeq \text{derived category of coherent sheaves} \\ \{\text{topological charges}\} &\simeq \text{K theory} \end{aligned}$$

The latter amounts to classifying B-branes up to dynamical creation and annihilation (tachyon condensation) processes. In type II string theory compactified on a Calabi-Yau manifold, such topological charges of branes determine the central charges of the extended supersymmetry algebra in non-compact dimensions. This central charge is given precisely by the overlap $\langle \mathcal{B} | \mathtt{1} \rangle$. We will argue that the hemisphere partition function $Z_{\text{hem}}(\mathcal{B})$ indeed depends only on the K theory class of the brane. The known formula for the central charge, which is valid in the large volume limit and was obtained by an anomaly inflow argument, provides a useful check of our result and is completed by our exact formula.

More generally, our localization computation yields a pairing $\langle \mathcal{B} | \mathtt{f} \rangle$ between the boundary state $\langle \mathcal{B}|$ and an arbitrary element $\mathtt{f}$ of the quantum cohomology ring. With twisted masses for the flavor symmetry group G_{F} turned on, the sheaves, K theories, and quantum/classical cohomologies are replaced by their G_{F}-equivariant versions. It was found by Nekrasov and Shatashvili that the relations in the equivariant quantum cohomology of certain models are precisely the Bethe ansatz equations of spin chains. Our work is thus related to, and in fact most directly motivated by, the study of integrable structures in supersymmetric gauge theories. Integrability suggests the presence of infinite dimensional quantum group symmetries, whose generators are expected to be realized as domain walls. As mentioned domain walls are D-branes in product theories, and the quantum group symmetries are known to be realized geometrically as so-called convolution algebras in equivariant K theories and derived categories. In this work we take a modest step in this direction by realizing the $sl(2)$ affine Hecke algebra as the domain wall algebra.

Relatedly, the 2d $\mathcal{N} = (2, 2)$ theories can also be embedded in a 4d $\mathcal{N} = 2$ theory to define a surface operator. Domain walls in the 2d theory can then be regarded as 4d line operators bound to the surface operator, and via the AGT correspondence is related to certain defects in Toda conformal field theories. We use our results to identify the precise domain walls that correspond to the defects.

We also study Seiberg-like dualities. In some dual pairs of theories, the hemisphere partition functions are found to be identical, while in the others they turn out to differ by a simple overall factor. Such dualities also serve as nice checks of our result.