論文の内容の要旨

論文題目　Effect of Particle Statistics and Frustration on Ground-State Energy

（粒子統計とフラストレーションの基底状態エネルギーに対する影響）

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We study the connections among particle statistics, frustration, and ground-state energy, in many-particle systems. Statistics of identical particles is one of the most fundamental concepts, which pervades all of quantum mechanics. A consequence of the particle statistics appears in the ground-state energy. We compare the ground-state energies of bosons and fermions with the same Hamiltonian. In non-interacting systems, the influence of particle statistics on the ground-state energy is trivial: the ground-state energy of non-interacting bosons is lower than that of free fermions because of Bose-Einstein condensation (BEC) and Pauli exclusion principle. The relation that bosons have a lower ground-state energy than fermions is described as natural inequality in this thesis. However, the comparison of the ground-state energies of bosons and fermions is not trivial in presence of interaction, because the simple argument based on the perfect BEC breaks down. In a system of interacting bosons, it is in fact already a nontrivial question whether the BEC actually takes place. In strongly correlated systems, the influence of particle statistics on the ground-state energy is still a relatively unexplored area.

We have found a sufficient condition for the natural inequality to hold for spinless and spinful case respectively, without relying on the occurrence of BEC. That is, if all the hopping amplitudes are non-negative, the ground-state energy of hard-core bosons is still lower than that of fermions. The same argument implies that, once we relax the condition of non-negative hopping amplitudes, it is possible to reverse the inequality so that the ground-state energy of bosons can be higher than that of fermions. By relaxing the condition, we indeed have found several concrete examples in which the ground-state energy of hard-core bosons is proved to be higher than that of fermions. Many of the examples are even proved rigorously in the thermodynamic limit.

Our study leads to a novel physical understanding of the effects of particle statistics, in terms of frustration in quantal phase. This is more general than the picture based on the perfect BEC, and is indeed applicable to systems with interaction. We can map a quantum many-particle problem to a single-particle problem on a fictitious lattice in higher dimensions. When all the hopping amplitudes are non-negative and the particles are bosons, the corresponding single-particle problem also has only non-negative hopping amplitudes. In such a case, there is no frustration in the quantal phase of the wavefunction. On the other hand, Fermi statistics of the original particles gives an effective magnetic flux in the corresponding single-particle problem. This implies a frustration in the phase of the wavefunction, which can be regarded as ``statistical frustration'': it is induced by the Fermi statistics and it leads to a destructive interference among propagation along different paths.

In presence of a non-vanishing flux in the original many-body problem, we observe that there is also a magnetic flux in the corresponding single-particle problem, inducing a frustration among quantal phases, which we name hopping frustration. In the original many-particle problem, the statistical frustration appears rather different from the hopping frustration. However, upon mapping to the single-particle problem on the fictitious lattice, both hopping frustration and statistical frustration are represented by non-vanishing flux in the fictitious lattice. This provides a unified understanding of frustration. Based on the unified understanding, we find the mechanism why the ground-state energy of hard-core bosons can be higher than that of fermions. For many-body bosonic system, in which there is no statistical frustration, introduction of hopping frustration will not decrease the ground-state energy. This is known as Simon's universal diamagnetism of bosons. On the other hand, in many-body fermionic system, where statistical frustration already exists, hopping frustration if introduced, is expected to compete with statistical frustration and sometimes partially cancel with each other, resulting in the reversed inequality between the ground-state energies of the hard-core bosons and fermions.